

Colloquium: Strongly dipolar molecular Bose-Einstein condensates: From few- to many-body physics

Andreas Schindewolf 

Vienna Center for Quantum Science and Technology, Atominstitut, [TU Wien](#), Stadionallee 2, A-1020 Vienna, Austria

Jens Hertkorn 

*MIT-Harvard Center for Ultracold Atoms, Research Laboratory of Electronics, and Department of Physics, [Massachusetts Institute of Technology](#), Cambridge, Massachusetts 02139, USA
and 5. Physikalisches Institut and Center for Integrated Quantum Science and Technology, [Universität Stuttgart](#), Pfaffenwaldring 57, 70569 Stuttgart, Germany*

Ian Stevenson 

Department of Physics, [Columbia University](#), New York, New York 10027, USA

Matteo Ciardi 

Institute for Theoretical Physics, [TU Wien](#), Wiedner Hauptstraße 8-10/136, 1040 Vienna, Austria

Phillip Groß 

Vienna Center for Quantum Science and Technology, Atominstitut, [TU Wien](#), Stadionallee 2, A-1020 Vienna, Austria

Dajun Wang 

Department of Physics, [The Chinese University of Hong Kong](#), Shatin, Hong Kong Special Administrative Region, China

Tijs Karman 

Institute for Molecules and Materials, [Radboud University](#), Nijmegen, The Netherlands

Goulven Quémener 

[Université Paris-Saclay](#), CNRS, [Laboratoire Aimé Cotton](#), 91405 Orsay, France

Sebastian Will 

Department of Physics, [Columbia University](#), New York, New York 10027, USA

Thomas Pohl 

Institute for Theoretical Physics, [TU Wien](#), Wiedner Hauptstraße 8-10/136, 1040 Vienna, Austria

Tim Langen *

*Vienna Center for Quantum Science and Technology, Atominstitut, [TU Wien](#), Stadionallee 2, A-1020 Vienna, Austria
and 5. Physikalisches Institut and Center for Integrated Quantum Science and Technology, [Universität Stuttgart](#), Pfaffenwaldring 57, 70569 Stuttgart, Germany*



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*Contact author: tim.langen@tuwien.ac.at

Recent advances in molecular cooling have enabled the realization of strongly dipolar Bose-Einstein condensates (BECs) of molecules, and BECs of many different molecular species may become experimentally accessible in the near future. Here the unique properties of such BECs and the new insights they may offer into dipolar quantum fluids and many-body physics are explored. The Colloquium examines which parameter regimes can realistically be achieved using currently available experimental techniques, discusses how to implement these techniques, and outlines which molecular species are particularly well suited to exploring exotic new states of matter. The Colloquium further determines how state-of-the-art beyond-mean-field theories, originally developed for weakly dipolar magnetic gases, can be pushed to their limits and beyond, and what other long-standing questions in the field of dipolar physics may realistically come into reach using molecular systems.

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I. INTRODUCTION

Systems of dipolar particles play a distinctive role in quantum many-body physics owing to the long-range and anisotropic character of their interactions. The anisotropy of the dipole-dipole interaction—whose sign and magnitude depend on the relative orientation of the dipoles—combined with the tunability afforded by external fields and geometric confinement enables a degree of experimental control over the interaction potential that is rarely available. These properties can profoundly alter the spreading of entanglement and the scrambling of quantum information (Hauke and Tagliacozzo, 2013; Foss-Feig *et al.*, 2015; Chen and Lucas, 2019; Tran

et al., 2021), affect transport and localization phenomena (Yao *et al.*, 2014), and enable the exploration of exotic quantum magnetism and phases of matter in and out of equilibrium (Koffel, Lewenstein, and Tagliacozzo, 2012; Yao *et al.*, 2018; Carroll *et al.*, 2025). In ultracold quantum gases, the realization of dipolar interactions has yielded numerous discoveries that have deepened our understanding of correlations and quantum fluctuations in bosonic systems and have enabled explorations of long-elusive quantum many-body states (Lahaye *et al.*, 2009; Baranov *et al.*, 2012; Bohn, Rey, and Ye, 2017; Böttcher *et al.*, 2021; Chomaz *et al.*, 2023). Great advances in the study of atomic dipolar gases have been achieved using Bose-Einstein condensates (BECs) of different species of magnetic atoms (Griesmaier *et al.*, 2005; Lu *et al.*, 2011; Aikawa *et al.*, 2012; Lu, Burdick, and Lev, 2012; Miyazawa *et al.*, 2022). Most notably, such systems have enabled the observation of dipolar quantum droplets and supersolids (Chomaz *et al.*, 2019; Tanzi, Lucioni *et al.*, 2019; Böttcher *et al.*, 2021), and they are predicted to show intriguing pattern-formation phenomena yet to be explored (Hertkorn, Schmidt *et al.*, 2021; Zhang, Pohl, and Maucher, 2021). These possibilities provide a unique laboratory platform for broad studies of exotic phases of matter. Quantum droplets are a rare example of self-bound quantum matter that remains stable in free space without external confinement (Baillie *et al.*, 2016; Chomaz *et al.*, 2016; Schmitt *et al.*, 2016). They emerge from the attractive part of dipole-dipole interactions and are stabilized by quantum fluctuations, somewhat akin to the stabilization of nuclei or neutron stars (Poli *et al.*, 2023). Supersolids, by contrast, simultaneously exhibit the crystalline density order of a solid and the frictionless transport of a superfluid (Boninsegni and Prokof'ev, 2012). The possibility of such a peculiar state was first suggested in the context of low-temperature helium more than 50 years ago (Gross, 1958; Andreev and Lifshits, 1969; Chester, 1970; Leggett, 1970), but this state evaded experimental verification there (Balibar, 2010). Its realization in dipolar BECs has enabled thorough experimental investigations of supersolidity in recent years (Guo *et al.*, 2019; Natale *et al.*, 2019; Tanzi, Rocuzzo *et al.*, 2019).

Dipolar interactions between highly magnetic atoms have also been used to study thermalization (Tang, Kao, Li, Seo *et al.*, 2018), to explore the physics of extended Bose-Hubbard models (Baier *et al.*, 2016; Fraxanet *et al.*, 2022; Su *et al.*, 2023), to investigate spin dynamics in synthetic quantum magnets (Lepoutre *et al.*, 2019), and to demonstrate protocols

for quantum-enhanced sensing of magnetic fields (Chalopin *et al.*, 2018).

In parallel to these developments, important progress has been made in the cooling of even more strongly dipolar molecules (Bohn, Rey, and Ye, 2017; Langen *et al.*, 2024). Decades of effort have resulted in the creation of the first degenerate Fermi gases of ground-state molecules (De Marco *et al.*, 2019; Valtolina *et al.*, 2020; Schindewolf *et al.*, 2022; Cao *et al.*, 2023; Duda *et al.*, 2023), directly laser-cooled molecules with significant phase-space density (Anderegg *et al.*, 2018; Jurgilas *et al.*, 2021; Wu *et al.*, 2021; Burau *et al.*, 2023; Jorapur *et al.*, 2024), near-degenerate assembled bosonic gases (Son *et al.*, 2020; Bigagli *et al.*, 2023; Lin *et al.*, 2023), and, most recently, the first dipolar BECs and droplets of molecules (Bigagli *et al.*, 2024; Shi *et al.*, 2025; Zhang *et al.*, 2026).

Taken together, these recent landmark advances raise an important question: How will dipolar physics affect a BEC of polar molecules? Conversely, what novel insights might BECs of different molecular species, with different individual properties, offer to the study of dipolar physics in the near future?

Although conceptually similar to magnetic atoms and historically treated on the same theoretical footing (Lahaye *et al.*, 2009), molecules, with their substantially larger electric dipole moments, present unique challenges and opportunities when it comes to dipolar many-body physics. Molecules are often assumed to provide a straightforward route to strongly interacting dipolar systems. However, realizing this potential in practice entails subtle challenges and limitations, which we examine in detail in the Colloquium. On the few-body side, significant experimental effort is required to control long-range interactions and short-range losses (de Miranda *et al.*, 2011; Matsuda *et al.*, 2020; Valtolina *et al.*, 2020; Anderegg *et al.*, 2021; Li *et al.*, 2021; Schindewolf *et al.*, 2022; Bause *et al.*, 2023; Bigagli *et al.*, 2023; Chen *et al.*, 2023; Karam *et al.*, 2023; Lin *et al.*, 2023), which influence achievable parameter regimes. On the many-body side, the important role that quantum fluctuations play in dipolar systems (Böttcher *et al.*, 2021) and their connection to the limits of mean-field theory for molecular systems have only rarely been considered to date (Schmidt *et al.*, 2022; Polteraueur and Zillich, 2025; Cardinale, Bland, and Reimann, 2026).

Here we address these open questions and explore them from both theoretical and experimental perspectives. Building on the established and well-tested theoretical framework to describe atomic dipolar quantum gases, we discuss differences and extensions for molecules, outline the principles that govern the characteristics of different molecular species, and explore the various existing collisional shielding mechanisms that are essential to the formation of stable ultracold molecular gases. We also discuss the experimental challenges in achieving ultracold molecular samples. We then explore known limitations in the theoretical framework of dipolar physics to highlight areas where molecules can provide a new perspective, go beyond existing mean-field approaches, benchmark more advanced theoretical models, and realize hitherto unexplored states of matter.

II. INTERACTIONS AND FEW-BODY PHYSICS

We begin by examining the nature of interactions in ultracold molecular gases. We trace their evolution from

fundamental few-body scattering processes to emergent many-body collective behavior.

A. Long-range interactions

In the literature many researchers have discussed the treatment of two-body interactions and collisions between ultracold dipolar magnetic atoms and electrically polar molecules (Napolitano, Weiner, and Julienne, 1997; Avdeenkov and Bohn, 2002; Krems and Dalgarno, 2004; Micheli *et al.*, 2007; Lahaye *et al.*, 2009; Tscherbul *et al.*, 2009; Baranov *et al.*, 2012; Lepers *et al.*, 2013; Wang and Quémener, 2015; Lepers and Dulieu, 2017; Quémener, 2017; Karman, Yan, and Zwierlein, 2022; Chomaz *et al.*, 2023), as well as their connection to ultracold chemistry (Liu and Ni, 2022; Karman, Tomza, and Pérez-Ríos, 2024).

The interaction potential between two molecules can be generically written as the superposition of different power law contributions of the form

$$V_n(\mathbf{r}) = \frac{C_n}{r^n} f(\theta, \varphi), \quad (1)$$

where C_n parametrizes the strength of the respective contributions and the dimensionless function $f_n(\theta, \varphi)$ captures their angular dependence. In the following we start by providing a general overview of dipole-dipole interactions (DDIs) and their properties, which dominate the interaction between molecules at long range.

For two generic dipoles oriented along the unit vectors $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$, the DDI is described by

$$V_{\text{dd}}^{\text{gen}}(\mathbf{r}) = \frac{C_3}{r^3} [(\hat{\mathbf{e}}_1 \cdot \hat{\mathbf{e}}_2) - 3(\hat{\mathbf{e}}_1 \cdot \hat{\mathbf{e}}_r)(\hat{\mathbf{e}}_2 \cdot \hat{\mathbf{e}}_r)], \quad (2)$$

where $\mathbf{r}$, which is often expressed in spherical coordinates as (r, θ, φ) , is the vector between the two centers of mass of dipoles 1 and 2, and $\hat{\mathbf{e}}_r = \mathbf{r}/r$ is the corresponding unit vector. The quantity $C_3 = C_{\text{dd}}/4\pi$ characterizes the strength of the DDI. For magnetically dipolar atoms, we have $C_{\text{dd}} = \mu_0 \mu_m^2$, where μ_0 is the vacuum permeability and μ_m is the magnitude of the atomic magnetic dipole moment. For electrically dipolar molecules, $C_{\text{dd}} = d^2/\epsilon_0$, where ϵ_0 is the vacuum permittivity and d is the molecular electric dipole moment.

If we now consider two dipoles that are oriented with respect to an external field along a unit vector $\hat{\mathbf{e}}_z$ taken as the quantization axis, Eq. (2) reduces to

$$V_{\text{dd}}(\mathbf{r}) = \frac{C_3}{r^3} (1 - 3 \cos^2 \theta). \quad (3)$$

In Eq. (3) θ denotes the angle between the quantization axis and $\mathbf{r}$, highlighting the anisotropic and long-range nature of the DDI.

The strength of DDI can be quantified by the *dipolar length*, in a manner similar to how the scattering length reflects the strength of short-range contact interactions. Various definitions of the dipolar length exist in the literature. In the context of scattering theory (Bohn, Cavagnero, and Ticknor, 2009), the dipolar length is often defined as

$a_d = C_{dd}\mu/(4\pi\hbar^2)$, where μ denotes the reduced mass. Another common definition is $\tilde{a}_d = 2a_d$ (Gao, 2010). In the context of dipolar many-body physics, $a_{dd} = C_{dd}M/(12\pi\hbar^2) = (2/3)a_d$, with the regular mass M , is defined such that the relative dipolar strength $\epsilon_{dd} = a_{dd}/a_s$ marks the instability of a homogeneous BEC against collapse for $\epsilon_{dd} \geq 1$ (Lahaye *et al.*, 2009; Chomaz *et al.*, 2023).

The corresponding characteristic interaction energy scale is set by the dipolar energy $E_d = C_{dd}/(4\pi a_d^3) = (\hbar^6/\mu^3)(4\pi/C_{dd})^2$ or $\tilde{E}_d = E_d/8$, which notably decreases as C_{dd} grows. This energy scale separates the threshold regime at low energies and the semiclassical regime at higher energies (Bohn, Cavagnero, and Ticknor, 2009). In the semiclassical regime, the elastic scattering cross section is temperature dependent and can be approximated as

$$\sigma_{\text{sc}} \approx \frac{8\pi a_d}{3k}, \quad (4)$$

with the thermally averaged collisional wave-vector $k = \sqrt{\pi M k_B T}/\hbar$ (Bohn, Cavagnero, and Ticknor, 2009; Bigagli *et al.*, 2023). In the threshold regime, the cross section becomes energy independent and for indistinguishable bosons is

$$\sigma_{\text{thr}} \approx \frac{32\pi}{45} a_d^2 + 8\pi a_s^2. \quad (5)$$

The dependence of the elastic cross section on the direction of incidence, which is relevant for cross-dimensional thermalization during evaporative cooling and for other out-of-equilibrium dynamics (Aikawa *et al.*, 2014; Valtolina *et al.*, 2020; Schindewolf *et al.*, 2022; Wang and Bohn, 2022, 2023, 2025; Bigagli *et al.*, 2023; Chen *et al.*, 2023; Lin *et al.*, 2023; Wang *et al.*, 2024), was described for the semiclassical regime by Bigagli *et al.* (2023), Lin *et al.* (2023), and Wang and Bohn (2024) and was calculated for the threshold regime by Bohn and Jin (2014), Sykes and Bohn (2015), and Wang and Bohn (2021).

B. Inducing dipoles in molecules

In the absence of external fields, gases of both magnetic atoms and molecules feature no laboratory-frame dipole moment. In this Colloquium we are concerned with dipolar molecules, which—unlike atoms—possess a permanent (i.e., intrinsic) molecule-frame electric dipole moment labeled d_0 in their vibrational ground state. A laboratory-frame dipole can be induced by applying a static electric field. Alternatively, oscillating electromagnetic fields such as microwaves (MWs) can coherently couple internal states to induce dipole moments.

Specifically, the induced laboratory-frame dipole moments are a result of coupling rotational states of opposing parity. For clarity we denote rotational states $|N, m_N\rangle$, where N is the rotational quantum number and m_N is its projection onto the quantization axis $\hat{e}_z$. In cases where electronic-spin degrees of freedom are relevant, one often also uses the coupled angular momentum quantum number J and projection m_J . Both notations appear in the literature, and we employ them accordingly whenever they are most suitable.

1. Static fields

More specifically, to realize dipoles oriented in the laboratory frame, as described in Eq. (3), a static electric field $\mathbf{E} = E\hat{e}_z$ can be applied to the molecules (Quémener and Bohn, 2011; Lassablière and Quémener, 2022). The field polarizes the molecules by hybridizing their bare rotational states. As a result, dressed rotational states $|\tilde{N}, m_N\rangle$, which carry a finite dipole moment along the field direction, are established. This induced dipole moment is defined as the expectation value of the electric dipole moment along the quantization axis $d_{\text{ind}} = \langle \tilde{N}, m_N | \mathbf{d}_0 \cdot \mathbf{e}_z | \tilde{N}, m_N \rangle$. Here, $C_{dd} = d_{\text{ind}}^2/\epsilon_0$, where $d_{\text{ind}} < d_0$ is now the induced dipole moment along the quantization axis.

Generally, the calculation of the induced dipole moment requires a full diagonalization of the Stark Hamiltonian. A straightforward approximation can be applied to Hund's case (b) states, which are commonly encountered in assembled alkali molecules. If the static electric field is weak enough so that $d_0 E \ll B_{\text{rot}}$ (B_{rot} is the rotational constant of the molecule), it is found that (Micheli *et al.*, 2007)

$$d_{\text{ind}} = \frac{d_0^2 E}{B_{\text{rot}}} \frac{3m_N^2/[N(N+1)] - 1}{(2N-1)(2N+3)}$$

for a given rotational state $|\tilde{N}, m_N\rangle$. Typically, $d_{\text{ind}} = d_0^2 E/3B_{\text{rot}}$ and $d_{\text{ind}} = -d_0^2 E/5B_{\text{rot}}$ for the two lowest rotational states $|\tilde{0}, 0\rangle$ and $|\tilde{1}, 0\rangle$.

2. Microwave fields

The interactions in MW-dressed molecules can significantly extend these simple bare dipolar interactions beyond those described by Eqs. (2) and (3) and can introduce additional tunable features into the collisional processes. As shown in Fig. 1, such dressing through a MW field $\mathbf{E}(t)$ mixes rotational states to induce the dipole moment $\mathbf{d}_{\text{ind}}(t) = \tilde{d}[\hat{e}_+(t) \cos \xi + \hat{e}_-(t) \sin \xi]$, where $\hat{e}_{\pm}(t)$ are basis vectors rotating with the field's frequency that describe positive and negative circularly polarized fields. For example, $\xi = (0, \pi/4)$ corresponds to a (σ^+, π) polarized MW. The resulting time-averaged interaction can be expressed as

$$V_{\text{dd}}^{\xi}(r, \theta, \varphi) = \frac{\tilde{d}^2}{4\pi\epsilon_0} \frac{3 \cos^2 \theta - 1 + 3 \sin(2\xi) \sin^2 \theta \cos(2\varphi)}{r^3}, \quad (6)$$

using $\tilde{d} = d_0/\sqrt{12[1 + (\Delta/\Omega)^2]}$ (Chen *et al.*, 2023), with $\Delta = \omega_{\text{MW}} - \omega_0$, where ω_{MW} is the MW frequency, ω_0 is the rotational transition frequency, and Ω is the Rabi frequency characterizing the coupling strength.

For a linearly polarized MW, we can define $\mathbf{E}(t) = E(t)\hat{e}_z$ such that $\hat{e}_z$ sets the quantization axis, in analogy with the aforementioned static-field case. The DDI can be then expressed by Eq. (3) using the effective dipole moment, which is $d_{\text{eff}} = d_0/\sqrt{6[1 + (\Delta/\Omega)^2]}$ when the MW couples $|0, 0\rangle$ and $|1, 0\rangle$ with $\Omega = (d_0/\sqrt{3})E/\hbar$ (Karman, Yan, and Zwierlein, 2022).

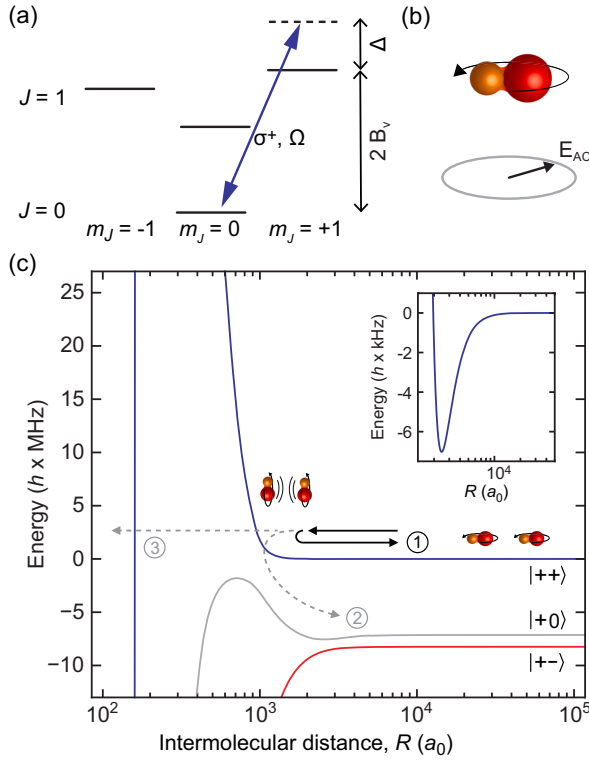


FIG. 1. Microwave shielding of a $1\Sigma^+$ molecule. (a) Lowest rotational levels of a $1\Sigma^+$ molecule. The states $|J, m_J\rangle = |0, 0\rangle$ and $|1, 1\rangle$ are split by an energy $2\hbar B_v$, where B_v is the rotational constant for states with a vibrational quantum number v , usually a few gigahertz. The MW field is blue detuned with respect to the resonance by an amount Δ . (b) Classically, a σ^+ field with circular polarization sets the molecular dipole spinning at the frequency of the MW field. (c) Potential energy curves of a pair of MW-dressed NaCs molecules approaching each other in the s -wave channel for $\Omega/(2\pi) = 4$ MHz and $\Delta/(2\pi) = 6$ MHz. Shown are the adiabatic potentials for $|++\rangle$ (solid blue line), $|+0\rangle$ (solid gray line), and $|+-\rangle$ (solid red line), which are the collisional channels based on the dressed states $|+\rangle$ and $|-\rangle$ and the spectator states $|0\rangle$ of the individual molecules; for spectroscopy of the dressed states, see Zhang *et al.* (2024) and Gu *et al.* (2025). Molecules (1) are reflected by the repulsive potential, (2) are lost to nonshielded states, or (3) reach short range. Inset: the shallow potential well in the intermolecular potential that can support bound states. Adapted from Bigagli *et al.*, 2023.

For a circularly polarized MW field, we can define the wave vector as the quantization axis. This procedure allows us to express the DDI as $-V_{dd}(\mathbf{r})$ using Eq. (3) with $d_{\text{eff}} = d_0/\sqrt{12[1 + (\Delta/\Omega)^2]}$ for the $|0, 0\rangle$ -to- $|1, \pm 1\rangle$ transition (Büchler *et al.*, 2007; Micheli *et al.*, 2007; Karman, Yan, and Zwierlein, 2022). Owing to the inverted sign of the DDI, this scenario is described as an *antidipolar* interaction. Specifically, in the antidipolar case, two circularly polarized dipoles experience a repulsive (attractive) DDI when approaching out of plane (in plane) of the rotating dipole, whereas in the regular dipolar case, two linearly polarized dipoles experience a repulsive (attractive) DDI under head-to-tail (side-by-side) configurations.

For arbitrary polarizations Eq. (6) can also be rearranged as a linear combination

$$V_{dd}^{\xi}(r, \theta, \varphi) = [1 - \sin(2\xi)]V_{dd}^{-} + \sin(2\xi)V_{dd}^{+} \quad (7)$$

of an antidipolar interaction $V_{dd}^{-} \propto -(1 - 3\cos^2\theta)$ with $d_{\text{eff}}^{-} = \bar{d}$ pointing along the MW wave vector and an orthogonal normal dipolar interaction $V_{dd}^{+} \propto (1 - 3\sin^2\theta\sin^2\varphi)$ with $d_{\text{eff}}^{+} = \sqrt{2}\bar{d}$. At the magic MW ellipticity $\xi_m = \sin^{-1}(1/3)/2 \approx 10^\circ$, we find antisymmetric interactions $V_{dd}^{\xi}(r, 0, \varphi) = -V_{dd}^{\xi}(r, \pi/2, \pi/2)$ along two axes and vanishing interactions $V_{dd}^{\xi}(r, \pi/2, 0) = 0$ along the third axis. We note that antidipolar interactions and limited tuning of the DDI can also be achieved for magnetic atoms via a continuous rotation of the magnetic field (Giovannazzi, Görlitz, and Pfau, 2002; Tang, Kao, Li, and Lev, 2018; Baillie and Blakie, 2020; Halder *et al.*, 2022).

C. Intermediate-range barrier and shielding

A central challenge in experiments with ultracold molecules is that they undergo lossy collisions when they reach short range. This loss can be the result of an exothermic chemical reaction channel, such as for KRb (Ni *et al.*, 2010; Ospelkaus, Ni, Wang *et al.*, 2010; Quémener and Julienne, 2012; Hu *et al.*, 2019) or CaF (Cheuk *et al.*, 2020; Sardar *et al.*, 2023). However, even molecules that are chemically stable with respect to two-body collisions (Żuchowski and Hutson, 2010; Śmiałkowski and Tomza, 2021; Tomza, 2021; Ladjimi and Tomza, 2024) can form sticky complexes at short range, which subsequently are subject to loss processes (Mayle *et al.*, 2013; Christianen, Zwierlein *et al.*, 2019; Gregory *et al.*, 2020; Liu *et al.*, 2020; Bause *et al.*, 2021, 2023; Gersema *et al.*, 2021). Harnessing the sizable long-range interaction of dipolar molecules opens a path to engineering a repulsive interparticle interaction that prevents colliding molecules from reaching short-range. However, for the bare dipole-dipole interaction, there is always at least one direction along which the anisotropic long-range interaction is attractive. Hence, additional potential barriers have to be erected to prevent the collapse of molecular ensembles. In the following we present several possible schemes to do so in order to shield molecules against collisional losses.

1. Confinement dc shielding

A first scheme is confinement shielding, which takes advantage of the anisotropy of the DDI. When a static electric field $\mathbf{E} = E\hat{e}_z$ is used to induce a dipole moment in ground-state molecules, a strong trapping potential in the $\hat{e}_z$ direction (for example, a one-dimensional optical lattice) can be used to effectively confine the molecules to a plane orthogonal to the electric-field axis (Büchler *et al.*, 2007; Micheli *et al.*, 2007). We assume that the angular trap frequency ω_z of the confinement is sufficiently strong and the temperature sufficiently low to limit the molecules to the motional ground state of this confinement. The spread of the molecular wave function in the confinement direction is then characterized by the harmonic oscillator length $a_{ho} = \sqrt{\hbar/\mu\omega_z}$. For $a_d \gg a_{ho}$ the oriented molecules mainly repel each other side by side at distances larger than a_{ho} , which renders the collisions effectively two dimensional (Ticknor, 2009; Quémener and Bohn, 2010). The

repulsive DDI shields the molecules from reaching the lossy short-range regime (Micheli *et al.*, 2010; Ticknor, 2010; de Miranda *et al.*, 2011; Julienne, Hanna, and Idziaszek, 2011; Quémener and Bohn, 2011; Frisch *et al.*, 2015; Quémener, Lepers, and Dulieu, 2015). For $a_d < a_{ho}$ the confinement alone is not sufficient to prevent attractive head-to-tail collisions when the molecules approach each other and the collision dynamics becomes more three dimensional. The exception is in the case of indistinguishable fermions, where the symmetrization principle of the wave function in confined geometries (de Miranda *et al.*, 2011; Quémener and Bohn, 2011) suppresses the head-to-tail component of the p -wave centrifugal barrier, leaving only the repulsive side-by-side collisions and hence supporting the shielding. The latter setting was used successfully by Valtolina *et al.* (2020) to create a degenerate Fermi gas of dipolar $^{40}\text{K}^{87}\text{Rb}$ molecules. As, however, confinement shielding is generally restricted to low-dimensional systems, it is useful to find shielding schemes that work in three dimensions.

2. Resonant dc shielding

A second scheme that achieves this goal is resonant dc-field shielding. This technique takes advantage of a Förster resonance between two scattering states that are described by the combined rotational states $|1\rangle \equiv |\tilde{1}, 0\rangle|\tilde{1}, 0\rangle$ and $|2\rangle \equiv (|\tilde{0}, 0\rangle|\tilde{2}, 0\rangle + |\tilde{2}, 0\rangle|\tilde{0}, 0\rangle)/\sqrt{2}$ (Avdeenkov, Kajita, and Bohn, 2006; Wang and Quémener, 2015; Quémener and Bohn, 2016; González-Martínez, Bohn, and Quémener, 2017; Matsuda *et al.*, 2020; Li *et al.*, 2021; Lassablière and Quémener, 2022; Mukherjee, Frye *et al.*, 2023; Mukherjee and Hutson, 2024; Mukherjee, Santos, and Hutson, 2025) of energy $\varepsilon_{|1\rangle}$ and $\varepsilon_{|2\rangle}$, as illustrated in Fig. 2. The notations for the scattering states $|1\rangle$ and $|2\rangle$ are used for simplicity. These two states are degenerate (i.e., $\varepsilon_{|1\rangle} = \varepsilon_{|2\rangle}$) around an electric field of $E^* \approx 3.25B_{\text{rot}}/d_0$ (González-Martínez, Bohn, and Quémener, 2017).

To understand the shielding mechanism at a glance, one can approach the problem as a two-level system (Li *et al.*, 2021; Lassablière and Quémener, 2022). In this case it can be shown that, to a good approximation, the DDI for the two scattering

states around the field E^* can be described by

$$V_{\text{dc}}^{\pm}(\mathbf{r}) \approx \frac{1}{2}[V_{\text{dd}}(\mathbf{r}, d_{11}^2 + d_{00}d_{22} + d_{02}^2) + \varepsilon_{|1\rangle} + \varepsilon_{|2\rangle}] \pm \frac{1}{2}\{[V_{\text{dd}}(\mathbf{r}, d_{11}^2 - d_{00}d_{22} - d_{02}^2) + \varepsilon_{|1\rangle} - \varepsilon_{|2\rangle}]^2 + 8[V_{\text{dd}}(\mathbf{r}, d_{10}d_{12})]^2\}^{1/2}, \quad (8)$$

where $V_{\text{dd}}(\mathbf{r}, D)$ corresponds to $V_{\text{dd}}(\mathbf{r})$ with $C_{\text{dd}} = D/\varepsilon_0$. One can define the transition dipole moment between states $|\tilde{N}, m_N\rangle$ and $|\tilde{N}', m'_N\rangle$ as $d_{NN'}^{m_N m'_N} = \langle \tilde{N}, m_N | \mathbf{d}_0 \cdot \mathbf{e}_z | \tilde{N}', m'_N \rangle$, which is a generalization of the induced dipole moment. The quantities $d_{NN'}$ used here are the transition dipole moments for projections $m_N = m'_N = 0$. The terms $V_{\text{dc}}^{\pm}$ are reminiscent of a Born-Oppenheimer-like picture (Micheli *et al.*, 2007; Lassablière and Quémener, 2022) where the two molecules collide on a unique potential energy surface (PES). The V_{dc}^+ (V_{dc}^-) PES is mostly repulsive (attractive). When $E > E^*$, the scattering state $|1\rangle$ is above $|2\rangle$ in energy and adiabatically follows the V_{dc}^+ PES to a good approximation. When V_{dc}^+ is averaged over the lowest partial waves $|L, m_L\rangle$, the overall effective radial potential becomes repulsive (Wang and Quémener, 2015; Lassablière and Quémener, 2022), thus providing a powerful shielding barrier, as shown in Fig. 2(c).

Mukherjee, Santos, and Hutson (2025) provided an effective interaction potential in second-order perturbation theory that has the form

$$V_{\text{dc, pert}}^+(r, \theta) = \frac{C_3(\theta)}{r^3} + \frac{C_6(\theta)}{r^6}, \quad (9)$$

with a repulsion dominant at short range and DDIs at long range. In Eq. (9) and throughout the Colloquium, the coefficient C_6 is not to be confused with the usual van der Waals terms, which mainly originate from couplings to higher electronic states or rovibrational states. Mukherjee, Santos, and Hutson fitted essential parameters such as $d_{NN'}$ and $\varepsilon_{|1\rangle} - \varepsilon_{|2\rangle}$ as functions of $d_0 E/B_{\text{rot}}$ with low-order polynomials to provide universal parameters that describe $V_{\text{dc, pert}}^+(r, \theta)$; see Table I of Mukherjee, Santos, and Hutson

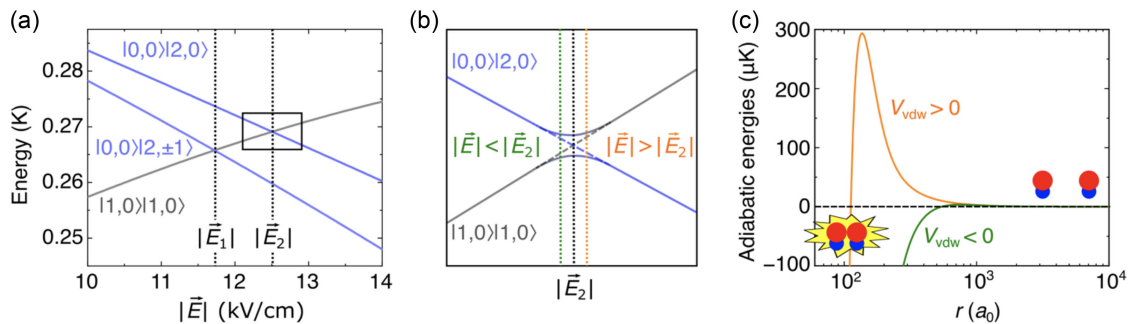


FIG. 2. Resonant static-field shielding. (a) Collision channels as a function of electric field. Shielding can be realized when channels cross. (b) Enlarged view of such a crossing at a field E_2 . Dipolar interactions lead to an avoided crossing between the channels. This avoided crossing leads to an effective van der Waals interaction. (c) Resulting adiabatic energies for two electric fields below (above) $|E_2|$. For $|E| > |E_2|$ a significant barrier that prevents the molecules from reaching short range is formed, thus leading to shielding. From Matsuda *et al.*, 2020, and shown here for KRb molecules.

(2025). Moreover, they also provided a more precise effective potential

$$V_{\text{dc,fc}}^+(r, \theta) = \frac{1}{2}(M_{11} + M_{22}) + \frac{1}{2}\sqrt{(M_{11} - M_{22})^2 + 4M_{12}^2} \quad (10)$$

using Van Vleck perturbation theory to build a fully coupled (fc) model. The matrix elements $M_{ij}(r, \theta)$, where i and j denote the states $|1\rangle$ and $|2\rangle$, include the elements that arise from the Van Vleck transformation, which can once again be expanded by universal polynomial parameters; see Table II of Mukherjee, Santos, and Hutson (2025).

Resonant dc shielding is typically efficient in the electric-field range of $3.25B_{\text{rot}}/d_0 \leq E \leq 3.8B_{\text{rot}}/d_0$. This relatively narrow window reflects a key limitation of the technique, namely, the restricted tunability of the interaction strength. Shielding is most effective around $E = E_s \approx 3.4B_{\text{rot}}/d_0$ (González-Martínez, Bohn, and Quémener, 2017) where a compromise is struck between maximizing the shielding barrier to avoid losses at short range and separating the states $|1\rangle$ and $|2\rangle$ to minimize state-changing collisions (Lassablière and Quémener, 2022; Mukherjee, Frye *et al.*, 2023; Mukherjee and Hutson, 2024). In general, it works best for

molecules with a high ratio of rotational constant to dipolar energy $\tilde{B}_{\text{rot}} = B_{\text{rot}}/\tilde{E}_d$ (González-Martínez, Bohn, and Quémener, 2017). While $\tilde{B}_{\text{rot}}$ is fairly small for KRb molecules, as shown in Table I, Matsuda *et al.* (2020) and Li *et al.* (2021) managed to apply resonant dc shielding to fermionic $^{40}\text{K}^{87}\text{Rb}$ molecules in 2D and 3D systems. To extend beyond shielding of a single internal state, Mukherjee and Hutson (2025) and Mukherjee, Hutson, and Hazzard (2025) suggested that resonant dc shielding is ideal to realize systems with $\text{SU}(N)$ symmetry by making use of the numerous hyperfine states in molecules.

3. Microwave and optical ac shielding

Using avoided crossings at an intermediate distance between attractive and repulsive scattering-state curves is not specific to resonant dc shielding but instead reflects a more general principle underlying various three-dimensional shielding techniques. Another example of such shielding is to use an ac electric field instead of a dc field. This third scheme takes advantage of the flexibility that an electromagnetic wave provides when dressing a molecule. The detuning Δ of the electromagnetic wave can vary depending on the

TABLE I. Field parameters for shielding of relevant bosonic molecular species in their electronic and vibrational ground state. The reduced rotational constant $\tilde{B} = B_{\text{rot}}/\tilde{E}_d$ is a measure for the effectiveness of the static electric-field shielding. At the resonance field E^* , the molecules in $|\tilde{1}, 0\rangle$ have the induced dipole moment d^* and the dipolar length a_{dd}^* . For microwave shielding we consider double MW shielding with fixed Ω_σ , Ω_π , and Δ_σ , while Δ_π^{min} and Δ_π^{max} mark the limits of a “stability window,” which is defined as region without a field-linked state. $a_{\text{dd}}^{\text{min}}$ and $a_{\text{dd}}^{\text{max}}$ are the corresponding dipolar lengths at these limits, where negative values denote antidipolar interactions. If no limits of Δ_π are stated, there is no field-linked state and $a_{\text{dd}}^{\text{min}}$ and $a_{\text{dd}}^{\text{max}}$ are given for $\Delta_\pi = 0$ and for pure circularly polarized MW shielding, respectively. For comparison, $a_{\text{dd}} = 130a_0$ for magnetic Dy atoms. The underlying molecular constants d_0 and B_{rot} are listed in the Appendix in Table II. Detailed calculations of two-body loss rates for several molecular species under double-microwave-shielding conditions were presented by Karman *et al.* (2025), who identified the general trend that heavier and more strongly dipolar molecules tend to exhibit lower loss rates. Experimental rates have been measured for NaCs, NaRb, and fermionic NaK (Schindewolf *et al.*, 2022; Shi *et al.*, 2025; Yuan *et al.*, 2025).

Molecule	Static-field shielding				Microwave shielding $\Omega_\pi = \Omega_\sigma = 10 \times 2\pi \text{ MHz}$ $\Delta_\sigma = 0 \text{ MHz}$			
	$\tilde{B}$	E^* (kV/cm)	d^* (D)	a_{dd}^* (a_0)	$\Delta_\pi^{\text{min}}/2\pi$ (MHz)	$\Delta_\pi^{\text{max}}/2\pi$ (MHz)	$a_{\text{dd}}^{\text{min}}$ (a_0)	$a_{\text{dd}}^{\text{max}}$ (a_0)
$^7\text{Li}^{23}\text{Na}$	2.8×10^5	160.50	0.06	11			36	−71
$^6\text{Li}^{40}\text{K}$	2.5×10^9	16.60	0.48	979			3130	−6200
$^7\text{Li}^{85}\text{Rb}$	2.8×10^{10}	10.49	0.56	2686	0.11	9.9	8170	−10 800
$^7\text{Li}^{133}\text{Cs}$	3.1×10^{11}	6.59	0.77	7803	1.4	5.2	10 700	−15 400
$^{23}\text{Na}^{39}\text{K}$	8.4×10^8	6.72	0.38	855			2730	−5400
$^{23}\text{Na}^{87}\text{Rb}$	6.4×10^9	4.22	0.45	2075	0.19	11	6100	−8950
$^{23}\text{Na}^{133}\text{Cs}$	6.2×10^{10}	2.46	0.64	5950	1.3	5.5	9300	−13 300
$^{41}\text{K}^{87}\text{Rb}$	5.5×10^6	12.32	0.08	78			248	−492
$^{39}\text{K}^{133}\text{Cs}$	1.2×10^9	3.17	0.26	1096	0	20	3500	−6000
$^{87}\text{Rb}^{133}\text{Cs}$	2.6×10^8	2.58	0.17	608			1940	−3850
$^{39}\text{K}^{107}\text{Ag}$	7.1×10^{11}	1.52	1.19	19 428	1.9	4.1	16 400	−23 500
$^{87}\text{Rb}^{107}\text{Ag}$	1.2×10^{12}	0.78	1.26	29 133	2.1	3.8	18 500	−26 100
$^{133}\text{Cs}^{107}\text{Ag}$	2.2×10^{12}	0.53	1.37	42 025	2.3	3.5	19 700	−27 900
$^{87}\text{Rb}^{88}\text{Sr}$	3.3×10^8	2.36	0.21	725			2300	−4600
$^6\text{Li}^{53}\text{Cr}$	7.5×10^9	27.43	0.46	1184			3710	−7400
$^{40}\text{Ca}^{19}\text{F}$	4.1×10^9	21.59	0.43	1024			3270	−6500
$^{88}\text{Sr}^{19}\text{F}$	3.0×10^{10}	13.83	0.49	2409	0.34	9.8	6600	−9600
$^{138}\text{Ba}^{19}\text{F}$	5.6×10^{10}	13.18	0.44	2907	0.72	7.6	6500	−9400
$^{174}\text{Yb}^{19}\text{F}$	2.7×10^{11}	11.94	0.55	5438	1.3	5.5	8300	−11 900
$^{27}\text{Al}^{19}\text{F}$	1.9×10^8	70.26	0.21	195			620	−1230
$^{89}\text{Y}^{16}\text{O}$	1.2×10^{11}	16.60	0.63	3958	0.82	7.2	8380	−12 100

energy of the corresponding photon, and its Rabi frequency Ω related to the strength of the corresponding ac field and its polarization.

Many different methods along these lines have been theoretically proposed and investigated. These studies are typically based on either optical shielding, where a laser field excites electronic states, or microwave shielding, where a MW field dresses rotational states within the electronic ground state.

Although optical shielding (Suominen *et al.*, 1995; Napolitano, Weiner, and Julienne, 1997) has been extensively explored, it has thus far proven to be ineffective, primarily because of photon scattering and unwanted heating in a one-photon scheme (Xie *et al.*, 2020), and also because second-order couplings are too weak to generate a sufficiently repulsive shielding curve in a two-photon scheme (Karam *et al.*, 2023, 2025; Karam, 2024).

In contrast, microwave shielding has emerged as a more promising approach. Coupling rotational states within the electronic ground state enables strong and tunable interactions while minimizing losses. When a variety of theoretical proposals were followed (Büchler *et al.*, 2007; Micheli *et al.*, 2007; Gorshkov *et al.*, 2008; Cooper and Shlyapnikov, 2009; Huang *et al.*, 2012; Karman and Hutson, 2018, 2019; Lassablière and Quémener, 2018; Karman, 2020), microwave shielding was observed and implemented in a variety of experiments (Anderegg *et al.*, 2021; Bigagli *et al.*, 2023; Lin *et al.*, 2023) and has been a crucial tool for the realization of stable ultracold molecular gases (Schindewolf *et al.*, 2022; Bigagli *et al.*, 2024; Shi *et al.*, 2025).

To establish microwave shielding, a circularly polarized (i.e., $\xi = 0, \pi$ for σ^+, σ^- polarization) MW field is used that is blue detuned from the $|0, 0\rangle \rightarrow |\xi_+\rangle = \cos\xi|1, 1\rangle + \sin\xi|1, -1\rangle$ transition and transfers the molecules into a dressed state $|+\rangle = u|0, 0\rangle + v|\xi_+\rangle$; see Fig. 1(a). Here $u = \sqrt{(1 + \Delta/\Omega_{\text{eff}})/2}$, $v = \sqrt{(1 - \Delta/\Omega_{\text{eff}})/2}$, and the effective Rabi frequency $\Omega_{\text{eff}} = \sqrt{\Omega^2 + \Delta^2}$ (Deng *et al.*, 2023). The blue detuning ensures that the molecules collide in $|++\rangle$, the energetically highest state of a manifold of collisional dressed states; see Fig. 1(c). At long range Eq. (6) describes the interaction in this collisional channel. When the molecules approach along the attractive directions of $V_{\text{dd}}^\xi(r, \theta, \phi)$, they start coupling to repulsively interacting lower-lying collisional channels, which feature molecules in the other dressed state $|-\rangle$ or in the so-called spectator states $|0\rangle$, which are dark states for single-molecule MW dressing. The remaining inelastic processes originate either from tunneling into the short range or, more likely, from predissociation into lower-lying dressed states, as illustrated in Fig. 1(c). For moderate ellipticity ($\xi < \pi/6$), the interaction potential including the repulsive barrier has been approximated using perturbation theory as (Deng *et al.*, 2023; Xu and Chen, 2025)

$$V_{\text{MW}}(r, \theta, \varphi) \approx V_{\text{dd}}^\xi(r, \theta, \varphi) + \frac{C_6}{r^6} \sin^2 \theta \{1 - \sin^2(2\xi) \cos^2(2\varphi) + [1 - \sin(2\xi) \cos(2\varphi)]^2 \cos^2 \theta\}, \quad (11)$$

with $C_6 = d_0^4 / \{128\pi^2 \epsilon_0^2 \hbar \Omega [1 + (\Delta/\Omega)^2]^{3/2}\}$. The first term in Eq. (11) can again be rewritten using Eq. (6) in terms of a

dipolar C_3/r^3 term multiplied by an angular dependence as in Eq. (1). In contrast to resonant dc shielding, where the interaction parameters can be tuned only through E , the MW-shielding interaction potential is highly tunable through the MW-field parameters Ω , Δ , and ξ . Dutta, Mukherjee, and Hutson (2025) demonstrated that the elastic and inelastic scattering for various molecular species can universally be described after normalizing the involved energies by $\tilde{E}_d$ and the length scales by $R_3 = \tilde{a}_d$ as long as $\Omega, \Delta \ll B_{\text{rot}}/\hbar$. Consequently, molecules with large d_0 and large mass are, in principle, best suited for MW shielding.

Thus far, we have considered molecules in an electronic singlet ground state $^1\Sigma$ such as alkalis. In this case the coupling between the states that form the shielding potential typically dominates over the hyperfine structure splittings. However, for molecules that feature an unpaired electron, for example, laser-coolable molecules with a $^2\Sigma$ ground state, the fine and hyperfine structure in the ground state can have a comparable or dominating energy splitting. While the effects of the unpaired spins are minor for resonant dc shielding (Quémener and Bohn, 2016; González-Martínez, Bohn, and Quémener, 2017; Mukherjee, Frye *et al.*, 2023), they are more relevant for MW shielding. Applying a modest magnetic field can help suppress hyperfine-inelastic collisions (Karman and Hutson, 2018, 2019; Lassablière and Quémener, 2018). However, it can also break the singlet coupling between electronic and nuclear spin (Anderegg *et al.*, 2021), which can alter the hyperfine structure relevant for the shielding scheme. Selecting the spin-stretched transition with the largest energy can help to maintain a blue detuning with respect to other transitions (Karman and Hutson, 2018). Overall, these considerations highlight that the detailed internal spin structure of the molecules must be taken into account when designing microwave-shielding schemes.

4. Double microwave ac shielding

An even more tunable extension of the regular MW shielding is double microwave shielding (Deng *et al.*, 2025; Karman *et al.*, 2025). This technique features an extreme suppression of both inelastic two-body collisions and three-body recombination (see Sec. II.E), as well as a wide tuning range of dipolar interactions (Yuan *et al.*, 2025), which has been essential for the first realizations of BECs of dipolar molecules (Bigagli *et al.*, 2024; Shi *et al.*, 2025). Here a second blue-detuned MW field with a frequency that typically differs by at least $2\pi \times 2$ MHz from the first field is used. This second field is π polarized (i.e., orthogonal to the circular or elliptical field) and hence addresses the $|0, 0\rangle \rightarrow |1, 0\rangle$ transition, as illustrated in Fig. 3(e). In essence, it induces an effective regular dipolar interaction, which in itself does not provide a 3D shielding potential but can be used to cancel out the antidipolar long-range interaction induced by the circularly polarized shielding MW. The combination of the two fields maintains a shielding barrier at intermediate range while suppressing the long-range dipolar interaction; see Fig. 3(c). For $\xi = 0$ the coupling of the MW fields to the individual molecules is effectively a coupled three-level system where the eigenvectors can be expressed with the nontrivial Euler mixing angles α , β , and γ . Together with the

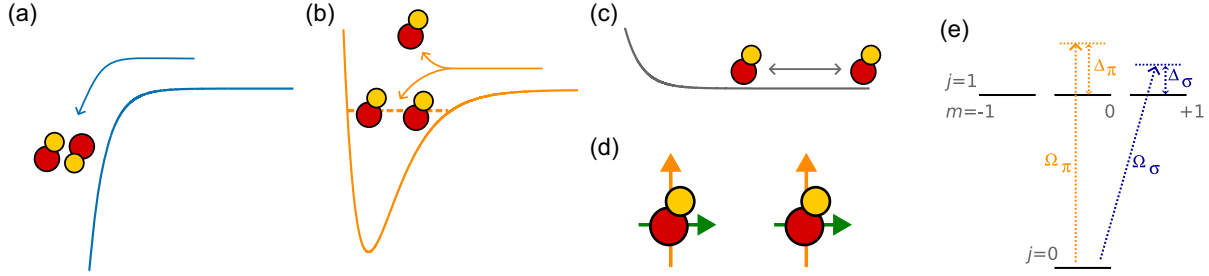


FIG. 3. Molecular collision dynamics and double MW shielding. (a) No shielding, where molecules interact by attractive van der Waals interactions and undergo loss at short range. (b) Single-field MW shielding, where molecules are shielded from two-body collisional loss but can undergo three-body recombination into field-linked bound states. (c) Double MW shielding, where the dipolar interaction between shielded molecules can be tuned or even compensated, resulting in a repulsive potential that does not support any bound states. (d) Illustration of long-range interactions between double-MW-shielded molecules, which can be thought of as the sum of independent interactions between laboratory-frame dipoles in the z direction (orange arrows) and, for finite ellipticity, laboratory-frame dipoles in the y direction (green arrows). The z direction is the propagation direction of the elliptically polarized microwave and the polarization direction of the linearly polarized microwave. The two dipolar interactions can be controlled using the MW-field intensities, frequencies, and ellipticities. (e) Double MW shielding is realized by simultaneously dressing the molecules with a π -polarized and a σ -polarized MW field with coupling strengths Ω_π and Ω_σ , which are blue detuned from a rotational transition with detunings Δ_π and Δ_σ , respectively. The lowest transition of $^1\Sigma^+$ molecules is shown. From Karman *et al.*, 2025.

coefficients w_0 , w_1 , and w_2 of Deng *et al.* (2025), an effective potential can be expressed using perturbation theory as

$$V_{\text{double MW}}(r, \theta) \approx \frac{C_6}{r^6} [w_2 \sin^4 \theta + w_1 \sin^2 \theta \cos^2 \theta + w_0 (3 \cos^2 \theta - 1)^2] + \frac{C_3}{r^3} (3 \cos^2 \theta - 1), \quad (12)$$

with $C_6 = d_0^4 / (4\epsilon_0^2)$ and

$$C_3 = \frac{d_0^2}{24\pi\epsilon_0} [3 \cos(2\beta) - 1] \cos^2 \alpha \sin^2 \alpha. \quad (13)$$

In Eq. (13) the angle α is a measure for the contribution of the excited rotational states to $|+\rangle$, while β is a measure for the contribution of the individual excited states ($\beta = 0$ for only $|1, \pm 1\rangle$ and $\beta = \pi/2$ for only $|1, 0\rangle$). Missing in this description are interactions from the exchange of photons between the two MW fields (in a Floquet picture). These additional couplings can play a significant role when the beat frequency between the MW frequencies matches the separation of the MW-dressed scattering channels and should be avoided (Karman *et al.*, 2025). Deviations from ideal σ and π polarization, which results in two orthogonal effective laboratory-frame dipole moments, as illustrated in Fig. 3(d), were also covered by Karman *et al.* (2025). Alternative possibilities for tuning the potentials have been proposed, either by addressing the π -polarized MW $N = 1 \rightarrow 2$ transition or by replacing it with a static electric field (Micheli *et al.*, 2007; Gorshkov *et al.*, 2008; Schmidt *et al.*, 2022; Xu and Chen, 2025).

D. Field-linked resonances and tunability

For magnetic atoms Feshbach resonances (Köhler, Góral, and Julienne, 2006; Chin *et al.*, 2010) provide a tool to tune

the scattering phase independently of the long-range DDI by coupling to a short-range bound state. This gives rise to an effective short-range contact interaction that, for ultracold bosons, is characterized by the s -wave scattering length a_s . The competition between the contact interaction and the DDI brings about the rich many-body physics that we discuss in Sec. III. Feshbach resonances have also been found for the special case of light NaLi molecules in their weakly dipolar triplet ground state (Park *et al.*, 2023) but are not widely available for polar molecules.

Even in the absence of accessible Feshbach resonances, the scattering potentials of the previously discussed shielding methods can naturally be deformed through the external-field parameters (for example, $|E|$ for resonant dc shielding and Ω , Δ , and ξ for MW shielding). This translates into tunability of the scattering phase shift and scattering length.

If the potential well that forms between the attractive long-range potential and the shielding barrier is sufficiently deep, it can support long-range weakly bound states, so-called field-linked states; see Fig. 4(a). When these states emerge, they give rise to field-linked resonances at which the scattering length diverges; see Fig. 4(b). In contrast to Feshbach resonances, no second potential is required and the potential well exists only in the presence of the external field.

Originally, field-linked resonances and states were predicted for molecules that feature Lambda doubling in the presence of static electric fields, such as ^{16}OH (Avdeenkov and Bohn, 2002, 2003; Avdeenkov, Bortolotti, and Bohn, 2004), ^{16}OD (Avdeenkov and Bohn, 2005), and CaOH in a low-field-seeking state (Augustovičová and Bohn, 2019). Later, field-linked states were also predicted in the scenarios of MW shielding (Cooper and Shlyapnikov, 2009; Huang *et al.*, 2012; Lassablière and Quémener, 2018) and resonant dc shielding (Mukherjee and Hutson, 2024).

Experimentally, the first field-linked states were observed with MW-shielded fermionic Na^{40}K molecules by Chen *et al.* (2023, 2024) and Gu *et al.* (2025). While the depletion in the interaction potential V_{MW} of Eq. (11) was not deep enough to

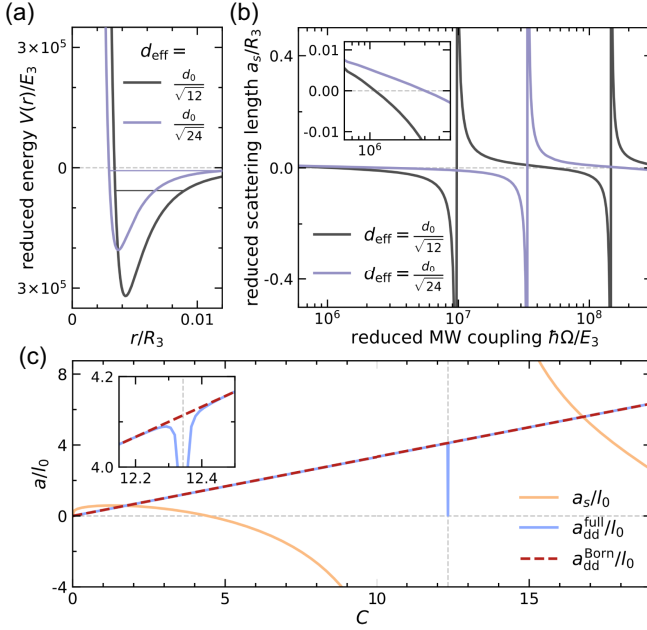


FIG. 4. Field-linked states and tunability of interactions with single MW shielding. (a) The adiabatic potential of the s -wave scattering channel as a function of the intermolecular distance r for $\hbar\Omega/E_3 = 7 \times 10^7$ with a resonant MW field (black line) and a MW detuning $\Delta = \Omega$ (purple line). The horizontal solid lines mark the energies of field-linked states. The energies are normalized by $E_3 = \tilde{E}_d$ and lengths by $R_3 = \tilde{a}_d$. (b) Universal tunability of the s -wave scattering length a_s for $\Delta = 0$ (black lines) and $\Delta = \Omega$ (purple lines) at a collision energy of $1000E_3$. Data from Dutta, Mukherjee, and Hutson, 2025. (c) Dependence of a_s and the dipolar length a_{dd} across the first field-linked resonance on the dimensionless interaction strength C from Eq. (16) for the effective potential of Eq. (11) with $\xi = 0$. The dashed red line and the solid blue line represent the Born approximation and the full calculation of a_{dd} , respectively, as discussed in Sec. III A. The two curves show excellent agreement except within a small region near the resonance (see the inset). Both lengths were scaled by $l_0 = (C_6/C_3)^{1/3}$.

support bound states for circular polarization ($\xi = 0$), turning to elliptical polarization closed the avoided crossing between the dressed states along one direction enough to support up to two field-linked states (Chen *et al.*, 2023, 2024). For MW-shielded bosonic molecules, field-linked states are even more accessible, as demonstrated by Stevenson *et al.* (2024), Yuan *et al.* (2025), and Zhang *et al.* (2026).

While for Feshbach resonances complex short-range physics determines the scattering properties, the size of the shielding barrier, which is typically on the order of $10^3 a_0$, limits the scattering physics to intermediate and long interparticle distances in shielded molecules.

On the one hand, this introduces challenges for the theoretical modeling of molecular gases when the size of the shielding core becomes comparable to the interparticle distance or dominates over the resulting scattering length, which is further discussed in Sec. III. On the other hand, ignoring short-range physics leads to a universal description of the particle interaction that depends only on the characteristics of the external field and on a few properties of the

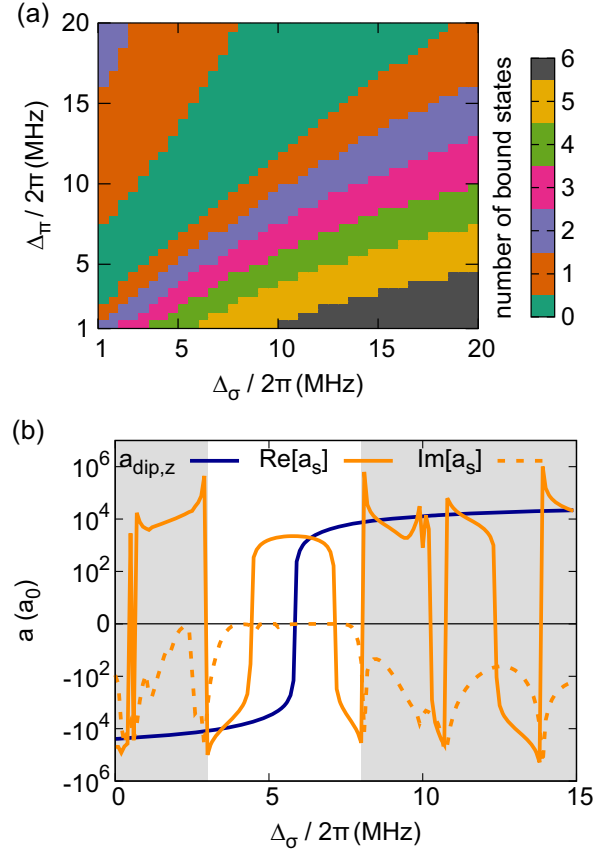


FIG. 5. Controlling the field-linked bound states with double MW shielding. (a) Number of field-linked states for NaCs molecules controlled by the detunings of the π and σ MW fields at $\Omega_\sigma = \Omega_\pi = 10 \times 2\pi$ MHz. (b) Scattering length and dipolar length calculated as a function of detunings in the double-MW-shielding scheme for NaCs. The calculation is performed at 100 nK for $\Omega_\sigma = 10 \times 2\pi$ MHz, $\Omega_\pi = 10 \times 2\pi$ MHz, and $\Delta_\pi = 10 \times 2\pi$ MHz. The results shown are for zero ellipticity. Data adapted from Karman *et al.*, 2025.

molecules, such as mass, dipole moment, and rotational constant. For single MW shielding, Dutta, Mukherjee, and Hutson (2025) demonstrated that the binding energies and scattering properties are universal when energies are normalized by the dipolar energy $E_3 = \tilde{E}_d$ and length scales are normalized by $R_3 = \tilde{a}_d$, as illustrated in Fig. 4. This universality is also applicable to double MW shielding (Karman *et al.*, 2025). Based on the effective potential given in Eq. (11), Ciardi *et al.* (2025) approximated for $\xi = 0$ that the first field-linked resonance appears at

$$\frac{M\Omega^2}{48\pi\epsilon_0\hbar^2} \left(\frac{8\pi\epsilon_0\hbar d_0^4}{3(\Omega^2 + \Delta^2)^{5/2}} \right)^{1/3} \approx 12.35, \quad (14)$$

as shown in Fig. 4(c). Universal expressions of the tunable binding energy and scattering length a_s were approximated by Li *et al.* (2026). Note that coupling to lower-lying dressed states increases near a field-linked resonance such that inelastic collisions are also enhanced. Chen *et al.* (2023) used this signature to map out the first field-linked resonances.

An example of a calculation for double MW shielding of NaCs is presented in Fig. 5. The resulting scattering length notably shows several zero crossings, allowing access to the full range of interactions, from attractive to noninteracting to repulsive, independent of a_{dd} .

E. Three-body collisions

While inelastic two-body collisions are enhanced near field-linked resonances, a more severe restriction of the stable parameter regime stems from three-body recombination. In parameter regimes that support field-linked states, two molecules can bind to a field-linked state, while a third molecule enables the exchange of binding and kinetic energy by providing momentum conservation (Stevenson *et al.*, 2024; Yuan *et al.*, 2025); see Fig. 3(b). Suppressing field-linked states to achieve stable shielding limits the effectively available dipole moment of bosonic molecules to around 1 D. Table I provides boundaries of the stable MW-shielding regime for various dipolar molecules. For species with smaller dipole moments and masses, single MW shielding is sufficient; the boundary Δ_π^{max} is omitted from Table I. In contrast, species with larger dipole moments and masses require double MW shielding to suppress the field-linked states that start to emerge in this case while simultaneously providing reasonable two-body shielding conditions. The stable parameter regime of NaCs is illustrated in Fig. 5. For resonant dc shielding of molecules with large dipole moments, the scarcity of tuning knobs makes it essentially impossible to suppress field-linked states while maintaining a decent ratio of elastic-to-inelastic two-body collisions, as calculated for NaCs, ^{39}KAg , and CsAg by Mukherjee and Hutson (2024). They also showed that NaRb provides a rare exception where the first field-linked state is accessible in a certain field regime that is embedded in a larger regime with favorable two-body shielding conditions.

If no field-linked states are supported, the role of three-body collisions remains an open question warranting further study. For double-MW-shielded NaCs molecules, inelastic loss appears to be limited by highly suppressed inelastic two-body collisions (Yuan *et al.*, 2025). To what extent the shielding barrier holds for three-body collisions has been discussed only in simplified scenarios for single MW shielding (Huang *et al.*, 2012) and resonant dc-field shielding (Lassablière, 2021). For bosons three-body collisions might enable coupling to dressed states that are antisymmetric under the exchange of two collision participants (Gorshkov *et al.*, 2008). This would assist inelastic transitions to lower-lying dressed states. However, Karman *et al.* (2025) illustrated that double MW shielding should suppress resonant transitions of this kind.

F. From few-body to many-body bound states

In analogy with magnetoassociation at atomic Feshbach resonances (Köhler, Góral, and Julienne, 2006; Chin *et al.*, 2010), field-linked resonances have enabled coherent *electro-association* (Quémener, Bohn, and Croft, 2023) of diatomic molecules into weakly bound states of tetramers, with the transfer efficiency depending on the motional states and quantum degeneracy (Chen *et al.*, 2024; Deng *et al.*, 2024). These states are macroscopic, with wave functions

that can spread from $10^3 a_0$ to $10^4 a_0$ (Chen *et al.*, 2023). The lifetime of the field-linked states is essentially limited by predissociation into the continuum of the lower-lying dressed scattering channels. Lifetimes of up to 8 ms were realized by Chen *et al.* (2024), but lifetimes beyond 10 s should be feasible for molecules with larger dipole moments (Deng *et al.*, 2024) and with double MW shielding. Field-linked molecules might be a promising initial state for transfer (Lepers *et al.*, 2013; Gacesa *et al.*, 2021) into tightly bound molecular states of tetramers (Byrd *et al.*, 2012; Christianen, Karman *et al.*, 2019; Yang, Zuo *et al.*, 2020; Sardar *et al.*, 2023), potentially through shielded electronically excited states (Mukherjee and Tomza, 2026).

It is likely that larger field-linked states composed of more than two molecules also exist (Wang, D’Incao, and Greene, 2011; Huang *et al.*, 2012; Shi, Wang, and Cui, 2026; Wang, Shi, and Cui, 2026). Such few-body bound states might bridge the gap to strongly correlated many-body states, which we now discuss in Sec. III.

III. MANY-BODY PHYSICS AND THEORY CHALLENGES

The long-range and anisotropic nature of dipolar interactions can give rise to a rich variety of exotic quantum phases in ultracold Bose gases. We begin by outlining the mean-field description of dipolar BECs, which successfully captures the essential physics of magnetic quantum gases (Lahaye *et al.*, 2009; Böttcher *et al.*, 2021; Chomaz *et al.*, 2023). We then address the limitations of this mean-field framework and its eventual breakdown as the dipolar interaction strength increases, driving the system into the strongly correlated regime. Finally, we discuss open questions and challenges in exploring this domain, along with first insights obtained from quantum Monte Carlo simulations.

A. Effective two-body interactions

In dilute and ultracold quantum gases, their interactions, for example, those discussed in Sec. II, can be described by pseudopotentials (Weidemüller and Zimmermann, 2003). This effective interaction is used in the later-discussed mean-field theory, in which the microscopic potential is replaced by the two-body T matrix at low energies, which provides an exact description of binary collisions, including bound-state resonances (Dalfvo *et al.*, 1999). For dipolar particles the scattering amplitude of low-energy collisions can be accurately captured by the effective potential (Marinescu and You, 1998; Yi and You, 2000, 2001)

$$V(\mathbf{r}) = \frac{4\pi\hbar^2 a_s}{M} \delta(\mathbf{r}) + V_3(\mathbf{r}), \quad (15)$$

which combines the contact interaction characterized by the s -wave scattering length a_s with the long-range dipole-dipole part of the interaction potential that scales as $\propto 1/r^3$ at large distances; see Eq. (1). The asymptotic form ($r \rightarrow \infty$) of all potentials presented in Sec. II.C can be expressed in this way; see Eqs. (8)–(12). The s -wave scattering length in Eq. (15) contains contributions from both the short-range part (the

$\propto C_6/r^6$ term) of the original interaction potential and the long-range dipole-dipole interactions (the $\propto C_3/r^3$ term). Contributions from higher-order partial-wave channels are, within the first Born approximation, well captured by $V_3(\mathbf{r})$, with small deviations occurring only in a narrow range around field-linked resonances; see Fig. 4(c). The accuracy of this simple pseudopotential was first numerically demonstrated for alkaline atoms with electric-field-induced dipole interactions (Marinescu and You, 1998; Yi and You, 2000, 2001) and later confirmed for magnetic dipolar atoms (Oldziejewski and Jachymski, 2016), as well as microwave-dressed molecules (Ciardi *et al.*, 2025; Xu and Chen, 2025; Cardinale, Bland, and Reimann, 2026) for the type of MW-dressed interactions discussed in Sec. II.

The strength of interactions can then be parametrized by the contact-interaction strength $g_s = 4\pi\hbar^2 a_s/M$ and the dipole-dipole interaction strength $g_{dd} = 4\pi\hbar^2 a_{dd}/M = C_{dd}/3 \propto C_3$, such that the relative importance of dipole-dipole interactions is quantified by the parameter $\varepsilon_{dd} = g_{dd}/g_s = a_{dd}/a_s$. In the case of magnetic atoms, ε_{dd} can be tuned by varying the scattering length a_s via magnetic fields. For molecules such a tuning of a_s is also possible, as discussed in Secs. II.D and III.A, within the range constrained by the emergence of field-linked states and the corresponding losses; see Sec. II.E. Similarly, the dipolar length a_{dd} can also be controlled directly via external electric fields and microwave fields.

Notably, for a given shielding scheme and field configuration, the distance dependence of the generated interaction potentials discussed in Sec. II.C can also be quantified by the single dimensionless parameter

$$C = \frac{MC_3^{4/3}}{\hbar^2 C_6^{1/3}} \quad (16)$$

upon scaling with the characteristic length $\ell_0 = (C_6/C_3)^{1/3}$ and the characteristic energy $\mathcal{E}_0 = \hbar^2/(M\ell_0^2)$ (Ciardi *et al.*, 2025). The two length scales a_s/ℓ_0 and a_{dd}/ℓ_0 of the interaction are not independent parameters, as both depend on C .

The pseudopotential of Eq. (15) remains applicable in the dilute-gas limit of small densities, in which the typical interparticle spacing greatly exceeds the characteristic length of the molecular potential ℓ_0 , but it breaks down at higher densities, as we discuss in Sec. III.F. In this case it needs to be replaced with the full potentials that describe, in particular, the intermediate-range size of typical shielding cores. Further approximations that capture the corresponding effects, for example, in generalized mean-field descriptions, are a topic of ongoing research.

B. Extended Gross-Pitaevskii equation

A BEC forms when a dilute gas of bosonic particles is cooled to sufficiently low temperatures such that a macroscopic fraction of the particles occupies a single quantum state (Ketterle, Durfee, and Stamper-Kurn, 1999). In this regime the many-body system can be described by a macroscopic wave function, or order parameter, ψ , whose dynamics captures the collective behavior of the condensate.

Dilute and weakly interacting dipolar BECs are well described by mean-field theory in terms of the extended Gross-Pitaevskii equation (eGPE) (Wächtler and Santos, 2016a, 2016b),

$$i\hbar\partial_t\psi(\mathbf{r}) = \left(-\frac{\hbar^2\nabla^2}{2M} + V_{\text{ext}}(\mathbf{r}) + \int V(\mathbf{r}-\mathbf{r}')|\psi(\mathbf{r}')|^2 d^3r' \right. \\ \left. + \gamma_{\text{qf}}|\psi(\mathbf{r})|^3 - i\sum_{\alpha}\frac{\hbar L_{\alpha}}{2}|\psi(\mathbf{r})|^{2\alpha-2} \right)\psi(\mathbf{r}), \quad (17)$$

which governs the time evolution of this order parameter. Within its domain of validity, the eGPE is system agnostic and can be applied to both atoms and molecules.

In Eq. (17) $V_{\text{ext}}(\mathbf{r})$ denotes the confinement potential and typically takes the form of an anisotropic harmonic trap, and the two-body interaction is given by Eq. (15). The wave function ψ is normalized to the particle number $\mathcal{N} = \int d^3r|\psi|^2$.

Effects of quantum fluctuations can be included through the leading-order Lee-Huang-Yang (LHY) correction to the equation of state of a dipolar condensate (Schützhold *et al.*, 2006; Lima and Pelster, 2011, 2012) that describes interactions with excitations of the condensates (Lee, Huang, and Yang, 1957; Lee and Yang, 1957). For a homogeneous system with a constant condensate density $|\psi|^2$, the energy correction can be written as $\gamma_{\text{qf}}|\psi|^3$, which in trapped systems is typically applied locally within the local-density approximation (LDA). Here $\gamma_{\text{qf}} = (32/3\sqrt{\pi})g_s a_s^{3/2} Q_5(\varepsilon_{dd})$ and $Q_5(\varepsilon_{dd}) = \int_0^1 du (1 - \varepsilon_{dd} + 3\varepsilon_{dd}u^2)^{5/2}$ (Lima and Pelster, 2011, 2012). The function Q_5 is well approximated by its leading-order Taylor expansion $Q_5(\varepsilon_{dd}) \sim 1 + 3\varepsilon_{dd}^2/2$ for $\varepsilon_{dd} \lesssim 1$ (Bisset *et al.*, 2016). When ε_{dd} exceeds unity, long-wavelength excitations become unstable such that γ_{qf} acquires an imaginary part. This issue is often circumvented by introducing infrared momentum cutoffs (Bisset *et al.*, 2016; Wächtler and Santos, 2016b; He, Maucher, and Zhang, 2024) or simply by discarding the imaginary part. In the strongly dipolar regime, the imaginary contribution to γ_{qf} can become significant and render such simple solutions questionable, as we discuss in more detail in Sec. III.E.

Particle losses can be modeled in dynamical simulations via the last term in Eq. (17). For atomic gases the dominant loss channel stems from three-body collisions, where two particles can form a dimer, while the third particle absorbs the excess energy and momentum. The corresponding cubic density dependence of the loss rate (Moerdijk, Boesten, and Verhaar, 1996) is captured by the last term in Eq. (17), with $\alpha = 3$. The loss coefficient $L_3 > 0$ depends on the particle interactions (Fedichev, Reynolds, and Shlyapnikov, 1996; Köhler, 2002; Rem *et al.*, 2013) and assumes typical values of $1.3 \times 10^{-41} \text{ m}^6 \text{ s}^{-1}$ (Schmitt *et al.*, 2016) for ^{164}Dy , $1.5 \times 10^{-40} \text{ m}^6 \text{ s}^{-1}$ at $a_s = 94a_0$ (Böttcher, Schmidt *et al.*, 2019) for ^{162}Dy atoms. These numbers have significant influence on the achievable densities and observation times in experiments with atomic dipolar condensates.

In the context of MW shielding of molecules, the behavior of inelastic collisions differs significantly. As discussed in Sec. II.E, three-body loss primarily becomes significant in the regime where bound field-linked states are present. In the

absence of field-linked bound states, inelastic two-body loss can increase owing to stronger localization of the scattering wave function near the shielding barrier. This enhanced localization facilitates predissociation into lower-lying dressed states or tunneling to short range. Such losses can be accounted for via the last term in Eq. (17), with $\alpha = 2$. In addition, one-body losses ($\alpha = 1$) can arise from phase-noise-induced decoherence of the dressed states. Such losses stem from technical issues and can be suppressed in experiments, as we further discuss in Sec. IV.A.

C. Mean-field stability and geometry

For systems of dipolar atoms, stability on the mean-field level sets stringent requirements on confinement and interaction properties (Fischer, 2006), with the overall sum of all interactions in Eq. (17) always required to remain repulsive (Koch *et al.*, 2008; Lahaye *et al.*, 2008). Within the eGPE the inclusion of quantum fluctuations stabilizes the condensate at a density that decreases with increasing interaction strength. Interesting quantum phases typically emerge for $a_{\text{dd}} > a_s$. Realizing these phases requires sufficiently strong dipole-dipole interactions to maintain low densities and thereby suppress atomic three-body losses, which would otherwise lead to a dynamical collapse of the BEC, as observed in the first Cr BECs (Koch *et al.*, 2008; Lahaye *et al.*, 2008). Such strong dipolar interactions have become possible with quantum gases of Er and Dy atoms.

For dipolar molecules the tunability of the contact interaction via field-linked resonances (Lassablière and Quémener, 2018; Schmidt *et al.*, 2022; Chen *et al.*, 2023; Mukherjee and Hutson, 2024) can therefore be a valuable asset to stabilize a quantum degenerate ensemble. Even far from resonance, a_s can be tuned by a few hundred a_0 and sometimes features a zero crossing. With resonant static electric-field shielding (Mukherjee, Frye *et al.*, 2023), the calculated contact interaction can be attractive in the entire regime of effective shielding. In such a case, strong confinement in a pancake-shaped geometry, with the electric dipoles polarized perpendicular to this pancake, might be required for dipolar repulsion to compensate for the strong contact mean-field attraction.

D. Droplets and supersolids on the mean-field level

Supersolidity refers to a phase of matter that simultaneously exhibits superfluid transport and crystalline density order, implying the coexistence of off-diagonal long-range order and broken translational symmetry. The study of structure formation, supersolidity, and other quantum states in bosonic dipolar gases is historically motivated by the behavior of superfluid ^4He . Helium and dipolar BECs exhibit a similar roton excitation spectrum that introduces a natural length scale for structure formation and exotic superfluid properties (Landau, 1941; Feynman, 1957; Santos, Shlyapnikov, and Lewenstein, 2003; Chomaz *et al.*, 2018).

Under sufficiently high pressure, low-temperature helium forms a solid in which a so-called supersolid phase has been speculated to exist (Gross, 1958; Andreev and Lifshits, 1969; Chester, 1970; Leggett, 1970). This supersolid phase is based

on the appearance of defects in the form of vacancies or interstitials. If the energy gain from delocalization of defects is sufficient, they could exist in the ground state of an otherwise regular crystal and contribute to a finite superfluid flow (Andreev and Lifshits, 1969; Chester, 1970; Leggett, 1970). Despite extensive efforts (Balibar, 2010; Boninsegni and Prokof'ev, 2012; Kim and Chan, 2012), the observation of defect-induced supersolidity has remained elusive to this day, and theoretical signatures have been detected thus far only in the ground state of bosons with soft-core repulsion (Cinti *et al.*, 2014).

In dipolar quantum gases, a different type of supersolid state can emerge in the form of a periodic density wave or a regular array of superfluid droplets, as originally described by Gross (1958). While weakly interacting, the formed superfluid array of quantum droplets—each containing up to several hundred particles—breaks continuous translational symmetry, exhibits phononlike excitations, and thus shows the defining features of a supersolid phase. For ultracold atomic systems, the physics of such droplets and supersolids has been surveyed extensively in relatively recent reviews (Lahaye *et al.*, 2009; Böttcher *et al.*, 2021; Chomaz *et al.*, 2023; Mukherjee, Cardinale *et al.*, 2023; Recati and Stringari, 2023; Chomaz, 2026). In the following we highlight the key concepts that underpin droplet and supersolid formation in dipolar gases and discuss the potential of molecular BECs for further exploration of their properties.

In the absence of external confinement, the competition between the attractive part of the dipole-dipole interaction, short-range repulsion, and quantum fluctuations can give rise to the formation of stable self-bound solutions of the eGPE (Baillie *et al.*, 2016; Chomaz *et al.*, 2016; Saito, 2016; Schmitt *et al.*, 2016; Wächtler and Santos, 2016a); see Eq. (17). In the ground state of polarized dipoles, particles align along the polarization direction, minimizing the total energy and giving rise to an elongated droplet shape along the dipolar axis. The application of some external harmonic confinement will exert an energy penalty that eventually leads to a splitting into multiple droplets beyond a critical trap frequency or critical particle number. If the droplets overlap sufficiently to maintain global phase coherence against quantum and thermal fluctuations, the system can support a finite superfluid flow and thus forms a supersolid state. The phase transition to a density-modulated supersolid state has been studied under quasi-two-dimensional (Zhang, Maucher, and Pohl, 2019; Hertkorn *et al.*, 2021b; Zhang, Pohl, and Maucher, 2021; Bland *et al.*, 2022; Ripley, Baillie, and Blakie, 2023) and one-dimensional confinement (Roccuzzo and Ancilotto, 2019; Blakie *et al.*, 2020; Sánchez-Baena, Pohl, and Maucher, 2024) using the eGPE of Eq. (17). Depending on the parameters, the transition can be of first or second order (Zhang, Maucher, and Pohl, 2019). In the former case of a discontinuous first-order transition, the abrupt onset of strong density modulations often implies a direct transition from an unmodulated phase to an insulating droplet array without global phase coherence between the droplets, with only a small parameter region in which global phase coherence can be maintained. The existence of a second-order transition line in quasi-1D (Blakie *et al.*, 2020) and a critical point in quasi-2D geometries (Zhang, Maucher, and Pohl, 2019), however, provides a

substantial parameter regime in which supersolidity can occur and be maintained at finite temperatures. Experiments have demonstrated such supersolid phases in finite ensembles of up to several 10^4 magnetic atoms under quasi-1D (Böttcher, Schmidt *et al.*, 2019; Chomaz *et al.*, 2019; Tanzi, Lucioni *et al.*, 2019) and quasi-2D confinement (Norcia *et al.*, 2021). These observations of dipolar supersolids in atomic condensates have led to broad investigations of their properties and excitations (Santos, Shlyapnikov, and Lewenstein, 2003; Chomaz *et al.*, 2018; Guo *et al.*, 2019; Hertkorn *et al.*, 2019, 2021a, 2021b, 2024; Natale *et al.*, 2019; Tanzi, Rocuzzo *et al.*, 2019; Blakie, Baillie, and Pal, 2020; Hertkorn, Schmidt *et al.*, 2021; Poli *et al.*, 2021; Schmidt *et al.*, 2021; Tanzi *et al.*, 2021; Bland *et al.*, 2022; Norcia *et al.*, 2022; Blakie *et al.*, 2023; Bühler, Ilg, and Büchler, 2023; Ilg and Büchler, 2023; Sánchez-Baena *et al.*, 2023; Biagioni *et al.*, 2024; Poli *et al.*, 2024; Sánchez-Baena, Pohl, and Maucher, 2024; Šindik *et al.*, 2024; Blakie, 2025; Kirkby *et al.*, 2025; Senarath Yapa and Bland, 2025).

In two dimensions the interplay between the dipolar mean-field and quantum fluctuations can give rise to more structures than simple triangular lattices, such as honeycomb structures (Zhang, Maucher, and Pohl, 2019), stripe phases (Hertkorn, Schmidt *et al.*, 2021; Ripley, Baillie, and Blakie, 2023), and metastable ring-lattice phases (Zhang, Pohl, and Maucher, 2024), as illustrated in Figs. 6(c) and 6(d). In finite confined systems, these thermodynamic phases—which are predicted for homogeneous systems and the corresponding phase diagrams (Ripley, Baillie, and Blakie, 2023)—are reflected as finite-size realizations in a rich spectrum of morphologies ranging from droplet crystals, as well as stripe and labyrinth structures, to pumpkin- or cogwheel-shaped states of finite-size condensates (Hertkorn, Schmidt *et al.*, 2021; Zhang, Pohl, and Maucher, 2021).

Probing these states in experiments with magnetic atoms requires particle numbers in excess of 10^5 atoms for optimized trap geometries (Hertkorn, Schmidt *et al.*, 2021; Zhang, Pohl, and Maucher, 2021); see Fig. 6. Such large atom numbers are challenging to achieve in magnetic BECs (Norcia *et al.*, 2021; Jin *et al.*, 2023; Anich *et al.*, 2024), and the associated densities are typically so large that three-body losses become significant during the structure-formation dynamics (Zhang, Pohl, and Maucher, 2021).

This issue bears similarities to the previously discussed mere stabilization of dipolar droplets, which also requires strong dipoles that in atomic systems could be achieved only with Er and Dy quantum gases. In this context molecular gases with even stronger dipole-dipole interactions will therefore permit the various structural transitions of dipolar supersolids to be probed at lower particle numbers and densities, ensuring sufficient stability against losses. This opportunity is illustrated in Figs. 6(d) and 6(e), which shows that a fourfold increase of a_{dd} decreases the particle number required to observe structural transitions by an order of magnitude. More detailed discussions of the scaling behavior of trapped condensates with varying interaction strength were given by Góral, Rzażewski, and Pfau (2000), Hertkorn *et al.* (2021b), and Hertkorn, Schmidt *et al.* (2021). In short, the eGPE in dimensionless units is parametrized by dimensionless contact and dipolar interaction strengths $\tilde{g}_s \propto a_s N$ and

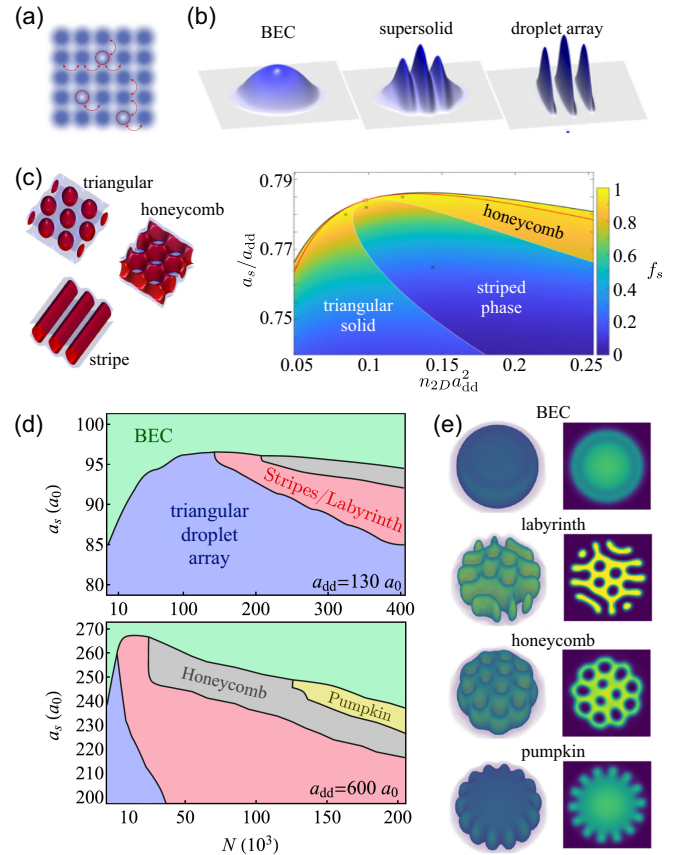


FIG. 6. Supersolids and patterned states of dipolar quantum gases. (a) Depiction of the concept of defect-induced supersolidity, as envisioned in helium, which is based on the spontaneous formation of vacancies (red circles) that can generate a finite superfluid response of an otherwise perfect crystal. From Balibar, 2010. (b) Graphic showing that in trapped dipolar gases, dipolar interactions can induce the spontaneous formation of periodic density modulations that realize a supersolid phase stabilized by quantum fluctuations. Adapted from Böttcher *et al.*, 2021. (c) Schematic showing that in quasi-2D geometries, such a supersolid can emerge in the form of stripes, triangular lattices, or honeycomb structures that can be explored by varying the different interaction strengths and the particle density. Adapted from Ripley, Baillie, and Blakie, 2023. In confined finite-size systems, these underlying structures give rise to (d) a rich phase diagram and (e) a broad spectrum of morphologies that become accessible at lower particle numbers for the stronger interactions available with ultracold molecules. (d) Adapted from Schmidt *et al.*, 2022. (e) Adapted from Hertkorn, Schmidt *et al.*, 2021.

$\tilde{g}_{dd} \propto a_{dd} N$, elucidating that for a larger a_{dd} , a given $\tilde{g}_{dd}$ is achieved for smaller N . Accordingly, the critical scattering length at which a phase transition occurs ($\epsilon_{dd} = \tilde{g}_{dd}/\tilde{g}_s \gtrsim 1$) shifts toward larger values. In addition, the richer angular structure of the molecular interactions can lead to modifications in the droplet formation and shape (Baillie, 2026). Eventually toward stronger dipolar interactions, the underlying mean-field theory in terms of the eGPE [see Eq. (17)] approaches its limit of validity, and other approaches for treating strongly correlated systems become necessary, which we discuss in Secs. III.E and III.F.

E. Limitations of mean-field theory

A fundamental limitation of the mean-field description based on the eGPE, given by Eq. (17), becomes clear upon inspecting the expression for the LHY energy correction

$$\gamma_{\text{qf}}|\psi|^3 = \sum_{\mathbf{k}} \tilde{V}(\mathbf{k})(|v_{\mathbf{k}}|^2 - u_{\mathbf{k}}v_{\mathbf{k}}^*), \quad (18)$$

which describes interactions between the condensate and excited Bogoliubov modes (Schützhold *et al.*, 2006; Lima and Pelster, 2011, 2012). In Eq. (18) $\tilde{V}(\mathbf{k})$ is the Fourier transform of the molecular interaction potential. When $\epsilon_{\text{dd}} > 1$, the homogeneous BEC ($\psi = \text{const}$) becomes unstable and gives way to supersolid patterns and quantum droplet solutions, as discussed in Sec. III.D, owing to the softening of the Bogoliubov modes, $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$. These soft modes generate complex contributions to the momentum sum in Eq. (18), which become problematic in the LDA treatment of the LHY correction used in the eGPE, Eq. (17). For the characteristic parameters of atomic BECs, the resulting imaginary part of γ_{qf} is typically small and often neglected or eliminated via small-momentum cutoffs of the sum in Eq. (18) (Bisset *et al.*, 2016; Wächtler and Santos, 2016b; Sánchez-Baena *et al.*, 2023; He, Maucher, and Zhang, 2024), as previously mentioned. Molecular condensates with stronger dipole-dipole interactions reach larger values of ϵ_{dd} for which the imaginary part of γ_{qf} becomes significant and even comparable to its real part (Langen *et al.*, 2025; Cardinale, Bland, and Reimann, 2026). Resolving this inconsistency will require an improved theoretical framework for dilute but highly dipolar condensates, with a self-consistent inclusion of quantum fluctuations beyond leading-order LHY corrections and beyond the local-density approximation.

The necessity to go beyond the common Hartree-Fock Bogoliubov framework (Dalfovo *et al.*, 1999) to account for quantum fluctuations may also have implications for effects at finite temperature. Like quantum fluctuations, thermal fluctuations were found to be significant in atomic BECs, as they affect the shape and phase boundary of quantum droplets (Aybar and Oktel, 2019) and even—counterintuitively—induce the formation of supersolid crystals upon heating the system (Sánchez-Baena *et al.*, 2023; Sánchez-Baena, Pohl, and Maucher, 2024; He *et al.*, 2025). Given the stronger interactions achievable in molecular BECs, and hence the larger fluctuations, understanding the effects of finite temperatures presents an important problem for ongoing and future work. Here the softening of long-wavelength modes appears to be even more problematic than in the treatment of quantum fluctuations at zero temperature because the thermal energy corrections comprise larger contributions from such excitation modes at longer wavelengths (Sánchez-Baena *et al.*, 2023, 2025).

Quantum and thermal fluctuations both lead to condensate depletion that becomes stronger for larger interactions. Recent calculations for MW-dressed molecular condensates predict a significant condensate depletion on the order of 20% for $T = 0$ and strong interactions (Jin *et al.*, 2025), which is much higher than in typical atomic BECs (Lopes *et al.*, 2017). Microwave dressing comes with an effective hard-core repulsion and the angular anisotropy of the interaction potential can

be modified compared to the standard dipole-dipole interaction (Baillie, 2026). As a result, it remains an open question as to what extent ϵ_{dd} alone provides a meaningful characterization of the interaction strength and the resulting condensate depletion in these systems.

Perhaps even more strikingly, the electric DDI strength between molecules can reach values beyond the typical domain of dilute quantum gases, where mean-field theory becomes inapplicable. Strongly correlated phases, such as dipolar Wigner crystals (Micheli *et al.*, 2007), may emerge at ultralow temperatures. Here the vast tunability of molecular interactions, where the shielding barrier and potential minima can be controlled on the length scales of typical particle spacings, presents a promising outlook for exploring exotic quantum phases in this strong-coupling regime.

F. Beyond mean field: Quantum Monte Carlo simulations

Quantum Monte Carlo (QMC) methods (Nightingale and Umrigar, 1998; Foulkes *et al.*, 2001; Becca and Sorella, 2017) offer a broad range of techniques for numerically exact studies of many-body problems. For the description of bosonic gases, path-integral Monte Carlo (PIMC) sampling (Ceperley, 1995; Boninsegni, Prokof'ev, and Svistunov, 2006; Yan and Blume, 2017) provides an efficient and well-developed approach to moderately sized systems in thermal equilibrium. Continuous-space PIMC algorithms were developed to study strongly correlated systems like superfluid or solid helium (Ceperley, 1995) and hence are particularly well suited for simulations of molecular condensates.

In the PIMC approach, the N -body density is approximated by its high-temperature limit, allowing one to evaluate the resulting high-dimensional path integral via Monte Carlo sampling, using the so-called worm algorithms (Boninsegni, Prokof'ev, and Svistunov, 2006) for an efficient inclusion of the bosonic exchange symmetry. With advanced discretization schemes such as the pair-product approximation (Pollock and Ceperley, 1984; Cao and Berne, 1992; Yan and Blume, 2015) and fourth-order expansions (Sakkos, Casulleras, and Boronat, 2009; Lindoy, Huang, and Jordan, 2018), accurate descriptions of hundreds of strongly interacting particles can be possible.

QMC simulations have been used to study strongly confined dipolar bosons in two spatial dimensions, where interactions are purely repulsive, as illustrated in Fig. 7(a). Such simulations reveal Wigner crystallization at high densities (Lozovik, Volkov, and Willander, 2004; Astrakharchik *et al.*, 2007; Büchler *et al.*, 2007; Mora, Parcollet, and Waintal, 2007; Jain, Cinti, and Boninsegni, 2011; Macia *et al.*, 2011) [see Figs. 7(b) and 7(c)], which can feature stripe symmetry for tilted dipoles (Macia, Boronat, and Mazzanti, 2014; Bombín, Boronat, and Mazzanti, 2017; Bombín, Mazzanti, and Boronat, 2019; Cinti and Boninsegni, 2019). Ensembles of magnetic atoms have been simulated using an additional hard wall (Nho and Landau, 2005; Saito, 2016) or a $V \propto r^{-6}$ or $V \propto r^{-12}$ repulsive potential to model atomic s -wave scattering. These simulations have confirmed the formation of quantum droplets (Macia *et al.*, 2016; Böttcher *et al.*, 2019; Boninsegni, 2021; Bombín, Mazzanti, and Boronat, 2024; Ghosh *et al.*,

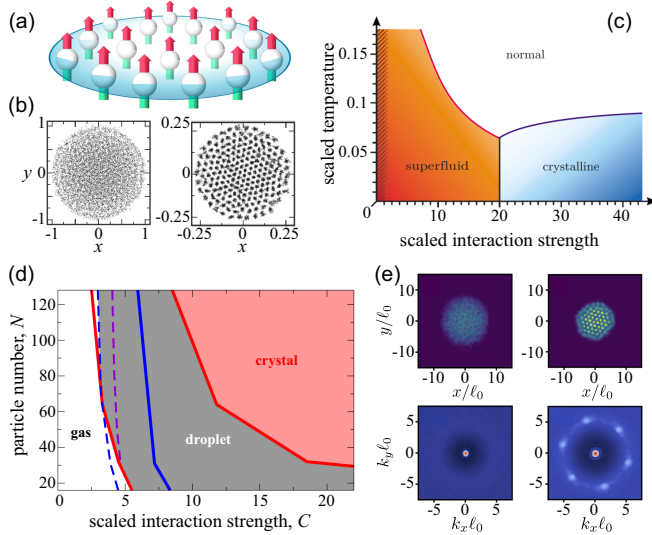


FIG. 7. Strongly interacting dipolar gases. (a) Confined two-dimensional layer of repulsive dipoles undergoing Wigner crystallization, as illustrated in (b) for a harmonically confined finite system. Adapted from Jain, Cinti, and Boninsegni, 2011. (c) Corresponding thermodynamic phase diagram exhibiting classical Wigner crystals as well as superfluid and normal fluid phases. Adapted from Micheli *et al.*, 2007. (d) Microwave-dressed molecules, with interactions described by Eq. (11), forming oblate self-confined quantum droplets, whose phase boundary has been determined with the eGPE (dashed blue line) (Langen *et al.*, 2025), variational calculations (dashed purple line) (Jin *et al.*, 2025), PIGS simulations (solid blue line) (Langen *et al.*, 2025), and PIMC simulations (solid red line) (Ciardi *et al.*, 2025). With increasing interactions the droplet undergoes a phase transition to a two-dimensional superfluid membrane and eventually forms a two-dimensional self-confined molecular van der Waals crystal (solid red line), as illustrated in (e). Adapted from Ciardi *et al.*, 2025.

2024) and droplet arrays (Cinti and Boninsegni, 2017; Cinti *et al.*, 2017; Kora and Boninsegni, 2019), as with predictions using mean-field calculations, but existing also in other parameter regimes. Notably, in such highly anisotropic self-assembled configurations, special care must be taken to handle metastable states (Ciardi *et al.*, 2025). In addition, in simulations of weakly interacting atomic condensates, QMC approaches often struggle to reach the required large particle numbers of several 10^3 atoms for a direct and accurate description of typical experimental conditions, whereby the length scale disparity between the short-range atomic repulsion and large interparticle spacings implies additional numerical challenges. Owing to the stronger interactions and typically smaller particle numbers in molecular condensates, together with the accurate knowledge of the MW-controlled interaction potentials, PIMC approaches indeed offer a well suited and powerful approach to explore the equilibrium quantum phases of molecular BECs.

Initial studies have explored the formation of self-bound droplets by MW-dressed condensates (Ciardi *et al.*, 2025; Langen *et al.*, 2025; Zhang *et al.*, 2025; Cardinale, Bland, and Reimann, 2026). For a single circularly polarized microwave field, as discussed in Sec. II.C, using the analytical expression

for the resulting interaction potential, the phase boundary between the droplet state and the dilute unconfined gas phase solely depends on the scaled interaction strength [Eq. (16)] and the number of molecules. Figure 7(d) compares the calculated phase boundary achieved in these recent studies, which were obtained from mean-field theory (dashed blue line) (Langen *et al.*, 2025), a variational approach (dashed purple line) (Jin *et al.*, 2025), path-integral ground-state (PIGS) simulations (solid blue line) (Langen *et al.*, 2025), and PIMC simulations (solid red line) (Ciardi *et al.*, 2025). While all current theoretical results show qualitatively similar behaviors, the predicted phase boundaries differ quantitatively. The PIGS simulations (Langen *et al.*, 2025) yield the largest critical interaction strength C for droplet formation, exceeding the results of PIMC simulations (Ciardi *et al.*, 2025) by about a factor of 2. The Monte Carlo results of Ciardi *et al.* (2025) are, however, close to the eGPE simulations reported in Langen *et al.* (2025). Both predict somewhat lower critical interaction strengths than the variational result of Jin *et al.* (2025). This is consistent with the expectation that variational theory provides an upper bound on the system's ground-state energy and should therefore tend to overestimate the critical interaction strength for droplet formation, in accordance with Ciardi *et al.* (2025). Further work and, in particular, accurate comparisons with experimental measurements will be necessary to better understand the detailed phenomenology of droplet formation by strongly interacting molecules.

Considering the similarities between the physics of ultra-cold molecular quantum gases and low-temperature superfluid helium, the peculiar and widely tunable shape of molecular interactions under MW dressing suggest new opportunities to explore exotic many-body phases. Indeed, recent QMC simulations of molecules in antidipolar configurations (Ciardi *et al.*, 2025) predict a new transition from the droplet phase to a self-bound superfluid membrane. With a further increase of the interaction strength, this self-confined two-dimensional superfluid, which consists of a single molecular layer, undergoes a crystallization transition to a self-confined molecular crystal, as illustrated in Fig. 7.

These recent findings offer an encouraging outlook for future explorations of exotic phases of strongly interacting bosons, such as heliumlike, defect-induced supersolid states that were predicted more than 50 years ago (Andreev and Lifshits, 1969) but thus far have remained elusive (Balibar, 2010). The combination of numerical simulations and experimental measurements of molecular BECs may also shed light on the physics of quantum fluctuations beyond LHY corrections and eGPE approaches and provide an understanding of patterned droplet phases under strong interactions, as recently observed in first experiments (Zhang *et al.*, 2026).

IV. EXPERIMENTAL PATH TO BECS

As outlined, molecular BECs offer a wide range of open questions for exploration. The first molecular BEC was realized with NaCs in 2023 (Bigagli *et al.*, 2024), followed by NaRb in 2025 (Shi *et al.*, 2025) using similar techniques. In the following we summarize the central advances that were necessary to achieve these results, addressing both the resolved and the present technical challenges.

A. Implementation and quality of shielding

Having discussed the physics of shielding techniques in Sec. II, we now focus on their implementation. Such implementations are crucial for the efficient cooling of a molecular sample and need to be specifically chosen and adapted to the species and experimental situation at hand.

For example, when molecules are confined in optical dipole traps that either are near resonant or have particularly high intensities, tensor light shifts can be present in the rotationally excited states. The polarization of the trapping light can then compete as a quantization axis with the magnetic and shielding fields (Anderegg *et al.*, 2021; Zhang *et al.*, 2024). To mitigate such effects, effective shielding requires sufficiently strong shielding fields to outweigh other couplings.

Generally, the techniques discussed in Sec. II.C for shielding and tuning interactions rely on relatively strong static or MW electric fields. Generating such fields with sufficient stability and tunability can be experimentally challenging—especially given the importance of preserving as much as possible the optical access available in the vacuum environments of ultracold-molecule experiments. Field inhomogeneities can degrade trap confinement (Gempel *et al.*, 2016; Covey, 2017; Lassablière and Quémener, 2018; Valtolina *et al.*, 2020) or perturb molecular trajectories in time-of-flight measurements. In the following we discuss in more detail the corresponding considerations and technical constraints relevant to both static and MW fields.

1. Static-field shielding

For both confinement dc shielding and resonant dc shielding, large static electric fields of many kV/cm are required, and they are challenging to realize. Static electric fields are typically applied with either electric-field rods or transparent glass plate electrodes coated with indium tin oxide (ITO) (Gempel *et al.*, 2016; Covey, 2017). A thin ITO layer features high transparency in the visible and near-infrared regime and thereby provides good optical access. However, finite absorption limits the intensity of optical dipole traps that pass through ITO electrodes.

High-voltage electrodes near insulator materials can cause charges emitted by an electrode to trigger secondary electron emission avalanches (Ziemba, 2021) or radiation hazards (West *et al.*, 2017). Remaining charges on the insulator lead to unintended dc Stark shifts of the molecules, which impede the precise manipulation of the internal molecular states. Therefore, it is of paramount importance to avoid field enhancement at the edges of electrodes and at surface imperfections (Covey, 2017).

Stability of static electric fields on the parts-per-million level is needed for reliable internal state manipulation. This can be realized by state-of-the-art high-voltage servos (Shaw, 2015; Covey, 2017) but comes at the cost of reducing the speed at which electric fields can be varied, which in turn limits the speed at which dipolar interactions can be tuned.

2. MW shielding

MW shielding circumvents a number of the technical challenges associated with dc shielding, and implementation

into existing experimental setups is relatively straightforward. MW shielding itself still requires relatively strong microwave fields. For the MW-field generation, various types of antennas can be used, depending on the wavelength and accessibility of the experimental setup. Thus far, individual helical antennas (Schindewolf *et al.*, 2022), helical antenna arrays (Anderegg *et al.*, 2021), dual-feed waveguide antennas (Chen *et al.*, 2023; Lin *et al.*, 2023), and combinations of full-wave loop antennas (Bigagli *et al.*, 2023; Yuan *et al.*, 2023; Shi *et al.*, 2025) have been used to generate circularly polarized MWs, while individual loop antennas can add linearly polarized MWs (Bigagli *et al.*, 2024; Shi *et al.*, 2025).

However, waveguide and loop antennas have poor directionality. To realize high field strengths, it is thus desirable to place the molecules in the near field of these antennas. Helical antennas that are operated in axial mode, in contrast, have enhanced directionality, which scales with the number of turns of their helix (Kraus, 1949). They are particularly well suited for short MW wavelengths corresponding to rotational transitions of molecules with large rotational constants (such as CaF, AlF, and YO). MW-field focusing through curved mirrors is another option if the wavelength is much shorter than the metallic openings and structures surrounding the molecules. At long wavelengths corresponding to molecules with smaller rotational constants (such as RbCs and RbSr), the antenna designs discussed thus far will likely become too large. In this case high field strengths might be achievable in the near field of electrodes that surround the molecules. In-vacuum MW cavities (Dunseith *et al.*, 2015) could provide another way to maximize field strengths.

For MW shielding the induced dipoles of the molecules are synced by the MW field. However, phase noise at detunings comparable to Ω_{eff} can couple the dressed states $|+\rangle$ and $|-\rangle$. This effectively leads to decoherence of the rotational superposition and results in unshielded inelastic collisions. While the inelastic loss mechanism is a two-body process, the time signature is dominated by one-body decoherence with a decay rate of $\Omega^2 S_\phi(\Omega_{\text{eff}})/2$, where $S_\phi(\Omega_{\text{eff}})$ is the single-sideband phase noise (Anderegg *et al.*, 2021; Chen, 2023). To reach one-body lifetimes of more than 10 s, one typically requires $S_\phi(\Omega_{\text{eff}}) < -170$ dBc/Hz. Given that state-of-the-art MW sources typically have phase-noise levels of -160 dBc/Hz at a detuning of 10 MHz and that Johnson-Nyquist noise is about -174 dBm/Hz at room temperature, it has become an established method to use MW bandpass filters in combination with low-gain high-power amplifiers as the output stage (Lin *et al.*, 2023; Bigagli *et al.*, 2024; Biswas *et al.*, 2026).

While less critical, amplitude noise can in principle also couple MW-dressed states by modulating Ω_{eff} through Ω . Finite detunings can suppress this coupling.

Karman *et al.* (2025) studied the effect of typical imperfections of the MW polarization in the context of double MW shielding. The imperfections prevent full compensation of the dipolar interaction in double MW shielding. In the compensation regime, the effect of finite ellipticity dominates, while a typical finite tilt of the π field with respect to the σ field can be neglected.

B. Making molecular BECs

For a trapped gas in a 3D harmonic trap, Bose-Einstein condensation occurs when the phase-space density, $\rho = \lambda_{\text{dB}}^3 n_0$, exceeds $\zeta(3) \approx 1.202$ (Dalfvo *et al.*, 1999; Ketterle, Durfee, and Stamper-Kurn, 1999). Here λ_{dB} is the thermal de Broglie wavelength in a gas and n_0 is the peak density. To meet this condition, temperatures need to reach the ultracold regime while maintaining sufficiently high particle densities. In general, molecules can be brought to ultracold temperatures ($\sim 1 \mu\text{K}$) by association (Danzl *et al.*, 2008; Ni *et al.*, 2008) or direct laser cooling (Fitch and Tarbutt, 2021; Langen *et al.*, 2024). BECs of molecules require further cooling to temperatures in the 10 nK regime. Despite important progress in laser cooling, dipolar molecular BECs have thus far been produced only by association. We now summarize the procedure.

To generate dense molecular gases in the microkelvin regime, the first clouds of two atomic species are laser cooled, sympathetically cooled, and then transferred into diatomic molecules by magnetoassociation (Köhler, Góral, and Julienne, 2006; Chin *et al.*, 2010). In magnetoassociation atom pairs are adiabatically transferred from the scattering continuum into a weakly bound molecular state via an interspecies magnetic Feshbach resonance. The resulting molecules—so-called Feshbach molecules—typically have a binding energy of about 0.1–10 MHz and a negligible permanent dipole moment. Feshbach molecules are then transferred to the rovibrational ground state through a coherent two-photon process known as stimulated Raman adiabatic passage (STIRAP) (Stwalley, 2004; Vitanov *et al.*, 2017). The power of the association technique is that the resulting molecules inherit the cold temperature and high phase-space density from the atoms. The process results in ultracold gases of ground-state molecules with binding energies of hundreds of terahertz and a significant permanent dipole moment, typically at temperatures of about 1 μK .

Association from overlapping atomic BECs (Wang *et al.*, 2016; Warner *et al.*, 2021; Lam *et al.*, 2022) or from dual single-shell Mott insulators in an optical lattice (Reichsöllner *et al.*, 2017) may appear natural for realizing even colder molecular samples but have been relatively inefficient thus far. In practice, it has proven advantageous to associate molecules from near-degenerate thermal atomic mixtures, thereby maximizing the number of molecules in the resulting samples, which is typically around 10^4 – 10^5 molecules at a temperature of about 1 μK , corresponding to a phase-space density of about 10^{-3} (Schindewolf *et al.*, 2022; Bigagli *et al.*, 2023; Lin *et al.*, 2023). This approach, which has also been adopted by the NaCs and NaRb BEC experiments, relies on efficient evaporative cooling of the molecular gas that is enabled by collisional shielding. In evaporative cooling the hottest molecules in a thermal sample are selectively removed. When the sample rethermalizes through collisions, the temperature will then be reduced.

The development of double MW shielding was key to realizing the first molecular BEC via evaporative cooling of NaCs molecules (Bigagli *et al.*, 2024; Deng *et al.*, 2025; Karman *et al.*, 2025). This technique also enabled Bose-Einstein condensation of NaRb (Shi *et al.*, 2025) and is

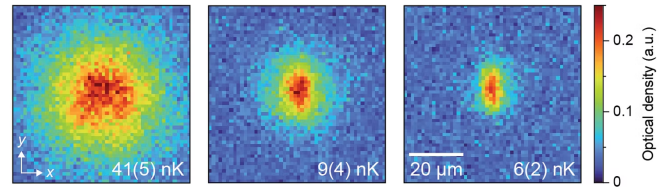


FIG. 8. Creation of the first molecular BEC. Transition from a thermal gas to a BEC of NaCs molecules (progressing from left to right). Similar results have recently also been obtained for NaRb (Shi *et al.*, 2025). Adapted from Bigagli *et al.*, 2024.

expected to be adaptable to a broad range of molecules (Karman *et al.*, 2025). Double MW shielding provides high suppression of inelastic losses while maintaining sufficiently strong interactions for efficient thermalization.

As quantum degeneracy is approached, the momentum distribution of an ultracold gas changes dramatically and develops a characteristic peak structure (Ketterle, Durfee, and Stamper-Kurn, 1999). To begin, as the phase-space density approaches 1, the bosonic enhancement becomes significant and the profile changes from a Gaussian to a Bose-enhanced Gaussian. Once a BEC emerges, the momentum distribution becomes bimodal, with the molecules in the BEC occupying the interaction-broadened lowest momentum state in the trap, and the remaining thermal molecules follow a Bose-enhanced profile. As evaporation proceeds, the thermal fraction of molecules is reduced until a quasipure BEC remains (Bigagli *et al.*, 2024; Zhang *et al.*, 2026). Figure 8 shows this evolution of the momentum distribution for the first molecular BEC.

C. Probing BECs

The momentum distribution in these experiments is measured using absorption imaging after time-of-flight expansion (Bigagli *et al.*, 2024; Shi *et al.*, 2025). Absorption imaging of molecules comes with additional challenges compared to atoms.

First, during expansion, the molecules must remain in the collisionally shielded state. The density in the BEC before expansion is so high that the entire BEC would be lost within about 1 ms if it is unshielded. Only at the end of the expansion does the density become much lower and this condition is relaxed.

Second, for molecules like NaCs and NaRb, direct detection is not possible and the molecules must be dissociated into their atomic components for detection. It is important for dissociation to happen rapidly at the end of time-of-flight expansion and without introducing additional energy to prevent a distortion of the measured momentum distribution, which would lead to inaccuracies in thermometry (Lam *et al.*, 2022). Reverse STIRAP from the molecular ground state directly into the continuum of unpaired atomic states fulfills these requirements. It allows dissociation within about 10 μs and does not add extra energy—in contrast to reverse STIRAP to a weakly bound molecular state followed by magnetic dissociation.

Finally, the need for reverse STIRAP at the end of time-of-flight expansion requires the STIRAP lasers to hit the

molecular cloud after a substantial amount of free fall (up to about 1 mm). This can be achieved either by implementing vertical STIRAP beams (Bigagli *et al.*, 2023, 2024) propagating in the direction of gravity or by dynamically changing the pointing of horizontally propagating STIRAP beams during time of flight (Shi *et al.*, 2025). In addition, the shielding fields need to be uniform. Gradients could lead to a force on the cloud during time-of-flight expansion and push it out of the STIRAP beams; curvature could lead to a distortion of the momentum distribution. These experimental techniques have been shown to allow for detection efficiencies of over 40% after 25 ms of time-of-flight expansion (Zhang *et al.*, 2026).

The available detection techniques for a molecular BEC impact the range of physics that can be explored. Early experiments of atomic BECs relied almost exclusively on absorption imaging after time-of-flight expansion (Ketterle, Durfee, and Stamper-Kurn, 1999), naturally steering investigations toward phenomena that are accessible with this technique—such as collective excitations, superfluidity, interference, and thermodynamics. At this point molecular BECs are similarly poised for studies of bulk properties that explore dipolar quantum gases and quantum liquids with strong and highly tunable interactions. The recent observation of strongly interacting droplet phases in a BEC of NaCs molecules by Zhang *et al.* (2026) is a first example. We expect that detection methods can be further expanded to high-resolution *in situ* imaging. The relevant techniques have been developed and are in wide use in the context of atomic quantum gases. They should be applicable with straightforward modifications to the detection of molecular quantum gases after reverse STIRAP, as previously discussed (Rosenberg *et al.*, 2022; Christakis *et al.*, 2023; Mortlock *et al.*, 2025). They also appear to be even more readily implementable in laser-coolable molecules that can scatter many photons for efficient fluorescence detection (Anderegg *et al.*, 2019), although BECs of such species have not yet been realized.

D. Larger BECs

As the creation of molecular BECs requires a complex multistage process—involving evaporative cooling of ultracold atomic gases followed by molecular association and evaporation of molecules—the sizes of molecular BECs are naturally smaller than typical atomic BECs. To date, NaCs BECs of up to 2000 molecules (Bigagli *et al.*, 2024; Zhang *et al.*, 2026) and NaRb BECs with about 600 molecules (Shi *et al.*, 2025) have been reported, whereas atomic condensates routinely reach 10^5 atoms. While relatively small molecular BECs already offer intriguing scientific opportunities—on both the experimental and theoretical sides—an increase of BEC size in experiments will be highly desirable.

The main avenues for reaching larger molecular condensates are (1) larger molecular samples at the beginning of evaporative cooling and (2) molecular evaporation with higher efficiency. Larger samples of ultracold molecules can be achieved by increasing the size of the initial atomic samples and attaining a higher efficiency of magnetoassociation. However, it is not straightforward to reap the benefits: increasing the size of the initial atomic samples typically

also raises the atomic density, which can reduce the pairing efficiency and even decrease the final number of molecules. This challenge can be mitigated by using atomic density profiles that are more uniform, for example, by reducing the geometric-mean trap frequency in the harmonic trap $\bar{\omega}$, as demonstrated for NaCs (Lam *et al.*, 2022), or by employing box traps (Navon, Smith, and Hadzibabic, 2021), as in the case of fermionic NaK (Bause *et al.*, 2021).

To enhance the evaporative cooling of molecules, further improvement of the ratio of elastic (“good”) to inelastic (“bad”) collisions, γ , is highly desirable. For unshielded molecules $\gamma \approx 1$, which prevents efficient evaporative cooling. As discussed in Sec. II, shielding both increases the elastic cross section σ_{el} and reduces the inelastic cross section. For highly polar molecules such as NaCs and NaRb, shielded samples reside in the hydrodynamic regime, which occurs when a trapped gas undergoes many scattering events within one trap-oscillation period, i.e., when $\sigma_{\text{el}}, n_0 > \bar{\omega}$ (Ma *et al.*, 2003; Schindewolf *et al.*, 2022; Bigagli *et al.*, 2023). In this regime the thermalization rate saturates at the geometric-mean trap frequency $\bar{\omega}$, and further increases in the elastic rate have little effect. This shifts the focus toward minimizing the inelastic processes.

There are a number of inelastic channels that should be suppressed to achieve efficient evaporation and long-lived degenerate molecular gases. The first channel involves off-resonant scattering of the trapping light (Grimm, Weidemüller, and Ovchinnikov, 2000). Absorption of a single photon from the optical dipole trap can change the internal state of a molecule, resulting in loss. Even if the molecule remains trapped, it will typically be lost in a subsequent collision. This fundamental process generally limits the one-body lifetime of a collisionally shielded gas to approximately 10 s.

The second channel involves microwave phase noise, which can flip molecules into unshielded dressed states (Anderegg *et al.*, 2021; Schindewolf *et al.*, 2022). This process has been widely observed in recent shielding experiments (Chen, 2023; Lin *et al.*, 2023; Bigagli *et al.*, 2024; Shi *et al.*, 2025; Yuan *et al.*, 2025; Biswas *et al.*, 2026; Zhang *et al.*, 2026) and has been effectively mitigated via the use of narrow microwave filter cavities. These cavities dramatically reduce phase noise, yielding lifetimes that exceed 50 s.

The third channel involves two-body collisional loss. While shielding strongly suppresses two-body loss, some residual loss remains. It has been shown that the loss rate coefficients can be suppressed to $\beta_{2\text{B}} \approx 10^{-14} \text{ cm}^3/\text{s}$ ($\beta_{2\text{B}} \approx 10^{-13} \text{ cm}^3/\text{s}$) for NaCs (NaRb) (Shi *et al.*, 2025; Yuan *et al.*, 2025). At typical BEC densities of 10^{12} cm^{-3} , this corresponds to lifetimes of about 100 s (10 s). These numbers are already impressive, corresponding to an improvement by at least 3 orders of magnitude compared to the unshielded case. Further improvements are predicted when molecules with large dipole moments, such as KAg and CsAg, are used; see Table I. In that case two-body loss can be suppressed by another 1 or 2 orders of magnitude (Karman *et al.*, 2025).

The fourth channel involves three-body recombination (Stevenson *et al.*, 2024). The current understanding is that double MW shielding leads to three-body recombination rates

that are consistent with zero (Yuan *et al.*, 2025) when it is used in the regime where field-linked bound states are absent.

Thus far, molecular evaporation efficiencies α of ≈ 2.5 have been reached (Toprakci *et al.*, 2025; Zhang *et al.*, 2026), which is close to the evaporation efficiencies reached in typical atomic quantum gas experiments. Here $\alpha = -\ln d\rho / \ln dN$ and ρ is the phase-space density. Thus, for every order of magnitude lost in number, the molecules gain around 2.5 orders of magnitude in phase-space density. In addition, laser-cooled molecules have now taken initial steps into the collisional regime, providing an additional platform for future exploration (Bureau *et al.*, 2024; Jorapur *et al.*, 2024). The best atomic evaporation experiments achieve $\alpha = 4$ (Olson, Niffenegger, and Chen, 2013). If inelastic decay can be further reduced—which appears to be within experimental reach—and molecule experiments achieve $\alpha = 4$, this would increase the final particle numbers in BECs by an order of magnitude. Such improvements, together with advances in pairing efficiency and trap design, have realistically brought molecular BECs with more than 10^4 molecules into reach. At this point the field of molecular BECs is moving from first proof-of-principle realizations to a regime in which strongly dipolar condensates can systematically address open questions on dipolar many-body physics and associated novel states of matter.

V. CONCLUSION

The realization of strongly dipolar molecular BECs marks a new era in the study of quantum matter with dipolar interactions. In these systems the interplay between precisely engineered shielding schemes and tunable interactions enables a controlled exploration of dipolar interactions beyond the paradigms established with magnetic atoms. The successful creation of stable, shielded molecular condensates demonstrates that losses can be suppressed to the point where genuine many-body physics becomes accessible, opening the door to the further exploration of novel quantum phases such as dipolar supersolids and droplets. Together with other recent developments such as the realization of quantum gas microscopy with molecules (Rosenberg *et al.*, 2022; Christakis *et al.*, 2023; Mortlock *et al.*, 2025), long-lived rotational coherence (Hepworth *et al.*, 2025), Floquet engineering of spin models (Miller *et al.*, 2024), quantum gates (Bao *et al.*, 2023; Holland, Lu, and Cheuk, 2023; Picard *et al.*, 2025; Ruttle *et al.*, 2025), and schemes for the implementation of synthetic gauge fields (Xu *et al.*, 2025) and $SU(N)$ symmetry (Mukherjee and Hutson, 2025; Mukherjee, Hutson, and Hazzard, 2025), these advances establish molecular BECs

as a uniquely versatile platform for probing fundamental questions in dipolar quantum matter ranging from few-body physics to the emergence of collective phenomena in systems dominated by anisotropic, long-range interactions.

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APPENDIX: MOLECULE PARAMETERS

The internal dipole moment and the rotational constant in the lowest vibrational state of the electronic ground state are given in Table II for the various molecular species.

TABLE II. Parameters of relevant bosonic molecular species. The values are given for the electronic and vibrational ground states if not otherwise specified.

Molecule	d_0 (D)	B_{rot}/h (GHz)	Reference(s)
$^7\text{Li}^{23}\text{Na}$	0.4513(4)	11.220 01(18)	Graff, Dagdigan, and Wharton (1972) and Fellows (1991) ^a
$^6\text{Li}^{40}\text{K}$	3.3992(15)	8.742(3)	Dagdigan and Wharton (1972), Bednarska, Kowalczyk, and Jastrzebski (1996), and Yang, Botsi <i>et al.</i> (2020) ^a
$^7\text{Li}^{85}\text{Rb}$	3.9810(22)	6.4695(9)	Dagdigan and Wharton (1972) and Dutta <i>et al.</i> (2011) ^a

(Table continued)

TABLE II. (Continued)

Molecule	d_0 (D)	B_{rot}/h (GHz)	Reference(s)
$^7\text{Li}^{133}\text{Cs}$	5.5(2) ^b	5.617 43	Staanum <i>et al.</i> (2007) and Deiglmayr <i>et al.</i> (2010)
$^{23}\text{Na}^{39}\text{K}$	2.7358(14)	2.848 1659(5)	Dagdigian and Wharton (1972) and Yamada and Hirota (1992) ^a
$^{23}\text{Na}^{87}\text{Rb}$	3.2(1)	2.089 6628(4)	Guo <i>et al.</i> (2016, 2018)
$^{23}\text{Na}^{133}\text{Cs}$	4.55(3)	1.735 616	Dagdigian and Wharton (1972), Docenko <i>et al.</i> (2004), and Stevenson <i>et al.</i> (2023) ^a
$^{41}\text{K}^{87}\text{Rb}$	0.574(17)	1.095 362(5) ^c	Ni (2009) and Ospelkaus, Ni, Quémener <i>et al.</i> (2010)
$^{39}\text{K}^{133}\text{Cs}$	1.86	0.912 68(18)	Ladjimi and Tomza (2024) and Zamarski <i>et al.</i> (2025)
$^{87}\text{Rb}^{133}\text{Cs}$	1.225(3)(8)	0.490 173 994(45)	Molony <i>et al.</i> (2014) and Gregory <i>et al.</i> (2016)
$^{39}\text{K}^{107}\text{Ag}$	8.50	2.000 ^d	Śmiałkowski and Tomza (2021)
$^{87}\text{Rb}^{107}\text{Ag}$	9.03	1.092 ^{c,d}	Śmiałkowski and Tomza (2021)
$^{133}\text{Cs}^{107}\text{Ag}$	9.75	0.807 ^d	Śmiałkowski and Tomza (2021)
$^{87}\text{Rb}^{88}\text{Sr}$	1.50	0.548 ^{c,d}	Ladjimi and Tomza (2024)
$^6\text{Li}^{53}\text{Cr}$	3.3(2)	14.02(4)	Finelli <i>et al.</i> (2024)
$^{40}\text{Ca}^{19}\text{F}$	3.07(7)	10.267 5373(3)	Childs <i>et al.</i> (1984) and Kaledin <i>et al.</i> (1999)
$^{88}\text{Sr}^{19}\text{F}$	3.4963(6)(35)	7.487 5989(8)	Ernst <i>et al.</i> (1985) and Colarusso <i>et al.</i> (1996)
$^{138}\text{Ba}^{19}\text{F}$	3.1702(15)(32)	6.473 958 42(24)	Ernst, Kändler, and Törring (1986) and Preston <i>et al.</i> (2026)
$^{174}\text{Yb}^{19}\text{F}$	3.91(4)	7.233 8007(10)	Sauer, Wang, and Hinds (1996)
$^{27}\text{Al}^{19}\text{F}$	1.515(4)	16.488 3548(3)	Truppe <i>et al.</i> (2019)
$^{89}\text{Y}^{16}\text{O}$	4.524(7)	11.631 900(6)	Suenram <i>et al.</i> (1990) and Hoeft and Törring (1993)

^a d_0 is derived using B_0 from one of the more recent publications.^bIn the second excited vibrational state.^cDerived from another isotopologue through the ratio of reduced masses.^dAt the equilibrium bond length.

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