

Cosmology at the millennium

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One hundred years ago we did not know how stars generate energy, the age of the Universe was thought to be only millions of years, and our Milky Way galaxy was the only galaxy known. Today, we know that we live in an evolving and expanding universe comprising billions of galaxies, all held together by dark matter. With the hot big-bang model we can trace the evolution of the Universe from the hot soup of quarks and leptons that existed a fraction of a second after the beginning, to the formation of galaxies a few billion years later, and finally to the Universe we see today 13 billion years after the big bang, with its clusters of galaxies, superclusters, voids, and great walls. The attractive force of gravity acting on tiny primeval inhomogeneities in the distribution of matter gave rise to all the structure seen today. A paradigm based upon deep connections between cosmology and elementary particle physics—inflation+cold dark matter—holds the promise of extending our understanding to an even more fundamental level and much earlier times, as well as shedding light on the unification of the forces and particles of Nature. As we enter the 21st century, a flood of observations is testing this paradigm. [S0034-6861(99)03102-5]

DEDICATION

This article is dedicated to the memory of a great cosmologist and a very dear friend, David N. Schramm, who, had he not died tragically in a plane crash, would have been a co-author of this review.

I. INTRODUCTION

One hundred years ago, we did not know how stars shine and we had only a rudimentary understanding of one galaxy, our own Milky Way. Our knowledge of the Universe—in both space and time—was scant: Most of it was as invisible as the world of the elementary particles.

Today, we know that we live in an evolving universe filled with billions of galaxies within our sphere of observation, and we have recently identified the epoch when galaxies first appeared. Cosmic structures from galaxies increasing in size to the Universe itself are held together by invisible matter whose presence is only known through its gravitational effects (the so-called dark matter).

The optical light we receive from the most distant galaxies takes us back to within a few billion years of the beginning. The microwave echo of the big bang discovered by Penzias and Wilson in 1964 is a snapshot of the Universe at 300 000 years, long before galaxies formed. Finally, the light elements D, ^3He , ^4He , and ^7Li were created by nuclear reactions even earlier and are relics of the first seconds. (The rest of the elements in the periodic table were created in stars and stellar explosions billions of years later.)

Crucial to the development of our understanding of the cosmos were advances in physics—atomic, quantum, nuclear, gravitational, and elementary-particle physics. The hot big-bang model, based upon Einstein's theory

of General Relativity and supplemented by the aforementioned microphysics, provides our quantitative understanding of the evolution of the Universe from a fraction a second after the beginning to the present, some 13 billion years later. It is so successful that for more than a decade it has been called the standard cosmology (see, e.g., Weinberg, 1972).

Beyond our current understanding, we are striving to answer fundamental questions and test bold ideas based on the connections between the inner space of the elementary particles and the deep outer space of cosmology. Is the ubiquitous dark matter that holds the Universe together composed of slowly moving elementary particles (called cold dark matter) left over from the earliest fiery moments? Does all the structure seen in the Universe today—from galaxies to superclusters and great walls—originate from quantum mechanical fluctuations occurring during a very early burst of expansion driven by vacuum energy (called “inflation”)? Is the Universe spatially flat as predicted by inflation? Does the absence of antimatter and the tiny ratio of matter to radiation (around one part in 10^{10}) involve forces operating in the early Universe that violate baryon-number conservation and matter-antimatter symmetry? Is inflation the dynamite of the big bang, and if not, what is? Is the expansion of the Universe today accelerating rather than slowing, due to the presence of vacuum energy or something even more mysterious?

Our ability to study the Universe has improved equally dramatically. One hundred years ago our window on the cosmos consisted of visible images taken on photographic plates using telescopes of aperture one meter or smaller. Today, arrays of charge-coupled devices have replaced photographic plates, improving photon collection efficiency one-hundred fold, and telescope apertures have grown tenfold. Together, they have in-

creased photon collection by a factor of 10^4 . Wavelength coverage has widened by a larger factor. We now view the Universe with eyes that are sensitive from radio waves of length 100 cm to gamma rays of energy up to 10^{12} eV, from neutrinos to cosmic-ray particles, and perhaps someday via dark-matter particles and gravitational radiation.

At all wavelengths advances in materials and device physics have spawned a new generation of low-noise, high-sensitivity detectors. Our new eyes have opened new windows, allowing us to see the Universe 300 000 years after the beginning, to detect the presence of black holes, neutron stars, and extra-solar planets, and to watch the birth of stars and galaxies. One hundred years ago the field of spectroscopy was in its infancy; today, spectra of stars and galaxies far too faint even to be seen then, are revealing the chemical composition and underlying physics of these objects. The advent of computers and their dramatic evolution in power (quadrupling every three years since the 1970s) has made it possible to handle the data flow from our new instruments as well as to analyze and to simulate the Universe.

This multitude of observations over the past decades has permitted cross checks of our basic model of the universe past as a denser, hotter environment in which structure forms via gravitational instability driven by dark matter. We stand on the firm foundation of the standard big-bang model, with compelling ideas motivated by observations and fundamental physics, as a flood of new observations looms. This is a very exciting time to be a cosmologist. Our late colleague David N. Schramm more than once proclaimed the beginning of a golden age, and we are inclined to agree with him.

II. FOUNDATIONS

There is now a substantial body of observations that support directly and indirectly the relativistic hot big-bang model for the expanding Universe. Equally important, there are no data that are inconsistent. This is no mean feat: The observations are sufficiently constraining that there is no alternative to the hot big bang consistent with all the data at hand. Reports in the popular press of the death of the big bang usually confuse detailed aspects of the theory that are still in a state of flux, such as models of dark matter or scenarios for large-scale structure formation, with the basic framework itself. There are indeed many open problems in cosmology, such as the age, size, and curvature of the Universe; the nature of the dark matter, and details of how large-scale structures form and how galaxies evolve—these issues are being addressed by a number of current observations. But the evidence that our Universe expanded from a dense hot phase roughly 13 billion years ago is now incontrovertible (see, e.g., Peebles *et al.*, 1991).

When studied with modern optical telescopes the sky is dominated by distant faint blue galaxies. To 30th magnitude per square arcsecond surface brightness (4×10^{-18} erg sec $^{-1}$ cm $^{-2}$ arcsec $^{-2}$ in 100 nm bandwidth at 450 nm wavelength, or about five photons per minute

per galaxy collected with a 4-meter mirror) there are about 50 billion galaxies over the sky. On scales less than around 100 Mpc galaxies are not distributed uniformly, but rather cluster in a hierarchical fashion. The correlation length for bright galaxies is $8h^{-1}$ Mpc (at this distance from a galaxy the probability of finding another galaxy is twice the average). (1 Mpc = 3.09×10^{24} cm $\approx$ 3 million light years, and $h = H_0/100$ km s $^{-1}$ Mpc $^{-1}$ is the dimensionless Hubble constant.)

About 10 percent of galaxies are found in clusters of galaxies, the largest of which contain thousands of galaxies. Like galaxies, clusters are gravitationally bound and no longer expanding. Fritz Zwicky was among the first to study clusters, and George Abell created the first systematic catalogue of clusters of galaxies in 1958; since then, some four thousand clusters have been identified (most discovered by optical images, but a significant number by the x rays emitted by the hot intracluster gas). Larger entities called superclusters are just now ceasing to expand and consist of several clusters. Our own supercluster was first identified in 1937 by Holmberg, and characterized by de Vaucouleurs in 1953. Other features in the distribution of galaxies in three-dimensional space have also been identified: regions devoid of bright galaxies of size roughly $30h^{-1}$ Mpc (simply called voids) and great walls of galaxies that stretch across a substantial fraction of the sky and appear to be separated by about $100h^{-1}$ Mpc. Figure 1 is a three panel summary of our knowledge of the large-scale structure of the Universe.

In the late 1920s Hubble established that the spectra of galaxies at greater distances were systematically shifted to longer wavelengths. The change in wavelength of a spectral line is expressed as the “redshift” of the observed feature,

$$1 + z \equiv \lambda_{\text{observed}} / \lambda_{\text{emitted}}. \quad (1)$$

Interpreting the redshift as a Doppler velocity, Hubble’s relationship can be written

$$z \approx H_0 d / c \quad (\text{for } z \ll 1). \quad (2)$$

The factor H_0 , now called the Hubble constant, is the expansion rate at the present epoch. Hubble’s measurements of H_0 began at 550 km sec $^{-1}$ Mpc $^{-1}$; a number of systematic errors were identified, and by the 1960s H_0 had dropped to 100 km s $^{-1}$ Mpc $^{-1}$. Over the last two decades controversy surrounded H_0 , with measurements clustered around 50 km s $^{-1}$ Mpc $^{-1}$ and 90 km s $^{-1}$ Mpc $^{-1}$. In the past two years or so, much progress has been made because of the calibration of standard candles by the Hubble Space Telescope (see, e.g., Filippenko and Riess *et al.*, 1998; Madore *et al.*, 1998), and there is now a general consensus that $H_0 = (67 \pm 10)$ km s $^{-1}$ Mpc $^{-1}$ (where ± 10 km s $^{-1}$ Mpc $^{-1}$ includes both statistical and systematic error; see Fig. 2). The inverse of the Hubble constant—the Hubble time—sets a timescale for the age of the Universe: $H_0^{-1} = (15 \pm 2)$ Gyr.

By now, through observations of a variety of phenomena from optical galaxies to radio galaxies, the cosmological interpretation of redshift is very well established.

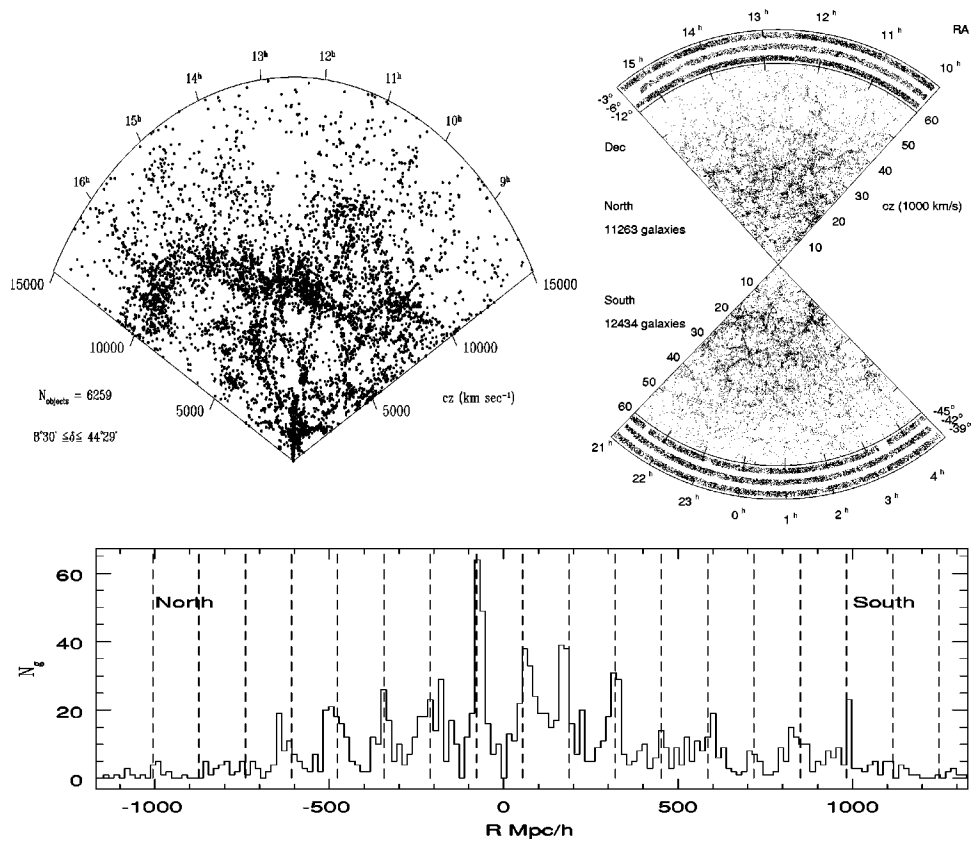


FIG. 1. Large-scale structure in the Universe as traced by bright galaxies: (upper left) The Great Wall, identified by Geller and Huchra (1989) (courtesy of E. Falco). This coherent object stretches across most of the sky; walls of galaxies are the largest known structures (see Oort, 1983). We are at the apex of the wedge, galaxies are placed at their “Hubble distances,” $d = H_0^{-1} z c$; note too, the regions devoid of galaxies (“voids”). (Upper right) Pie-diagram from the Las Campanas Redshift Survey (Shectman *et al.*, 1996). Note the structure on smaller length scales including voids and walls, which on larger scales dissolves into homogeneity. (Lower) Redshift-histogram from deep, pencil-beam surveys (Willmer, *et al.*, 1994; see also Broadhurst *et al.*, 1990; courtesy of T. Broadhurst). Each pencil beam covers only a square degree on the sky. The narrow width of the beam “distorts” the view of the Universe, making it appear more inhomogeneous. The large spikes spaced by about $100h^{-1}$ Mpc are believed to be great walls.

Two recent interesting observations provide further evidence: numerous examples of high-redshift objects being gravitationally lensed by low redshift objects near the line of sight; and the fading of supernovae of type Ia, whose light curves are powered by the radioactive decay of Ni^{56} , and at high redshift exhibit time dilation by the predicted factor of $1+z$ (Leibundgut *et al.*, 1996).

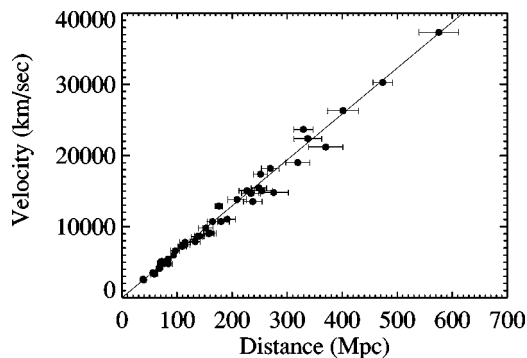


FIG. 2. Hubble diagram based upon distances to supernovae of type Ia (SNeIa). Note the linearity; the slope, or Hubble constant, $H_0 = 64 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (courtesy of A. Riess; see Filippenko and Riess, 1998).

An important consistency test of the standard cosmology is the congruence of the Hubble time with other independent determinations of the age of the Universe. (The product of the Hubble constant and the time back to the big bang, $H_0 t_0$, is expected to be between $2/3$ and 1 , depending upon the density of matter in the Universe; see Fig. 3). Since the discovery of the expansion, there have been occasions when the product $H_0 t_0$ far exceeded unity, indicating an inconsistency. Both H_0 and t_0 measurements have been plagued by systematic errors. Slowly, the situation has improved, and at present there is consistency within the uncertainties. Chaboyer *et al.* (1998) date the oldest globular stars at $11.5 \pm 1.3 \text{ Gyr}$; to obtain an estimate of the age of the Universe, another $1-2 \text{ Gyr}$ must be added to account for the time to the formation of the oldest globular clusters. Age estimates based upon abundance ratios of radioactive isotopes produced in stellar explosions, while dependent upon the time history of heavy-element nucleosynthesis in the Galaxy, provide a lower limit to the age of the Galaxy of 10 Gyr (Cowan *et al.*, 1991). Likewise, the age of the Galactic disk based upon the cooling of white dwarfs, $>9.5 \text{ Gyr}$, is also consistent with the

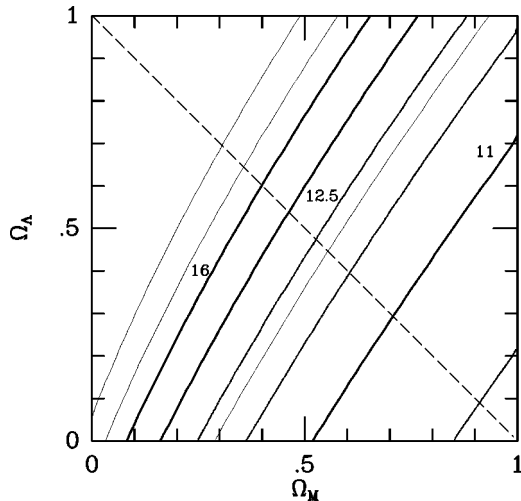


FIG. 3. Contours of constant time back to the big bang in the Ω_M - Ω_Λ plane. The three bold solid lines are for $h=0.65$; the light solid lines are for $h=0.7$; and the dotted lines are for $h=0.6$. The diagonal line corresponds to a flat Universe. Note, for $h\sim 0.65$ and $t_0\sim 13$ Gyr a flat universe is possible only if $\Omega_\Lambda\sim 0.6$; $\Omega_M=1$ is only possible if $t_0\sim 10$ Gyr and $h\sim 0.6$.

globular cluster age (Oswalt *et al.*, 1996). Recent type Ia supernova data yield an expansion age for the Universe of 14.0 ± 1.5 Gyr, including an estimate of systematic errors (Riess *et al.*, 1998).

Within the uncertainties, it is still possible that $H_0 t_0$ is slightly greater than one. This could either indicate a fundamental inconsistency or the presence of a cosmological constant (or something similar). A cosmological constant can lead to accelerated expansion and $H_0 t_0 > 1$. Recent measurements of the deceleration of the Universe, based upon the distances of high-redshift supernovae of type Ia (SNeIa), in fact show evidence for accelerated expansion; we will return to these interesting measurements later.

Another observational pillar of the big bang is the 2.73 K cosmic microwave background radiation (CMB) (see Wilkinson, 1999). The far infrared absolute spectrophotometer (FIRAS) on the Cosmic Background Explorer (COBE) satellite has probed the CMB to extraordinary precision (Mather *et al.*, 1990). The observed CMB spectrum is exquisitely Planckian: any deviations are smaller than 300 parts per million (Fixsen *et al.*, 1996), and the temperature is 2.7277 ± 0.002 K (see Fig. 4). The only viable explanation for such perfect black-body radiation is the hot, dense conditions that are predicted to exist at early times in the hot big-bang model. The CMB photons last scattered (with free electrons) when the Universe had cooled to a temperature of around 3000 K (around 300 000 years after the big bang), and ions and electrons combined to form neutral atoms. Since then the temperature decreased as $1+z$, with the expansion preserving the black body spectrum. The cosmological redshifting of the CMB temperature was confirmed by a measurement of a temperature of 7.4 ± 0.8 K at redshift 1.776 (Songaila *et al.*, 1994) and of 7.9 ± 1 K at redshift 1.973 (Ge *et al.*, 1997), based upon

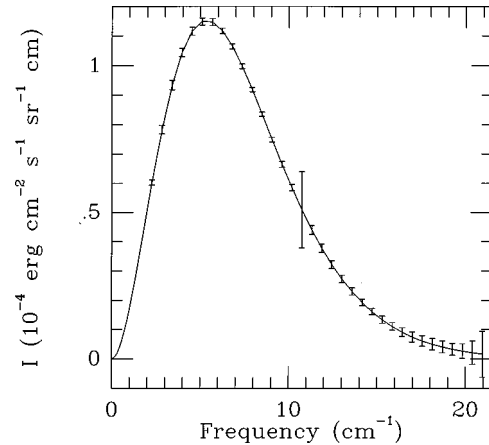


FIG. 4. Spectrum of the Cosmic Microwave Background Radiation as measured by the FIRAS instrument on COBE and a black-body curve for $T=2.7277$ K. Note, the error flags have been enlarged by a factor of 400. Any distortions from the Planck curve are less than 0.005% (see Fixsen *et al.*, 1996).

the population of hyperfine states in neutral carbon atoms bathed by the CMB.

The CMB is a snapshot of the Universe at 300 000 yrs. From the time of its discovery, its uniformity across the sky (isotropy) was scrutinized. The first anisotropy discovered was dipolar with an amplitude of about 3 mK, whose simplest interpretation is a velocity with respect to the cosmic rest frame. The FIRAS instrument on COBE has refined this measurement to high precision: the barycenter of the solar system moves at a velocity of 370 ± 0.5 km s $^{-1}$. Taking into account our motion around the center of the Galaxy, this translates to a motion of 620 ± 20 km s $^{-1}$ for our local group of galaxies. After almost thirty years of searching, firm evidence for primary anisotropy in the CMB, at the level of 30 μ K (or $\delta T/T \approx 10^{-5}$) on angular scales of 10° was found by the differential microwave radiometer (DMR) on COBE (see Fig. 5). The importance of this discovery was twofold. First, this is direct evidence that the Universe at early times was extremely smooth since density variations manifest themselves as temperature variations of the same magnitude. Second, the implied variations in the density were of the correct size to account for the structure that exists in the Universe today: According to the standard cosmology the structure seen today grew from small density inhomogeneities ($\delta\rho/\rho\sim 10^{-5}$) amplified by the attractive action of gravity over the past 13 Gyr.

The final current observational pillar of the standard cosmology is big-bang nucleosynthesis (BBN). When the Universe was seconds old and the temperature was around 1 MeV a sequence of nuclear reactions led to the production of the light elements D, ^3He , ^4He , and ^7Li . In the 1940s and early 1950s, Gamow and his collaborators suggested that nuclear reactions in the early Universe could account for the entire periodic table; as it turns out Coulomb barriers and the lack of stable nuclei with mass 5 and 8 prevent further nucleosynthesis. In any case, BBN is a powerful and very early test of the standard cosmology: the abundance pattern of the light

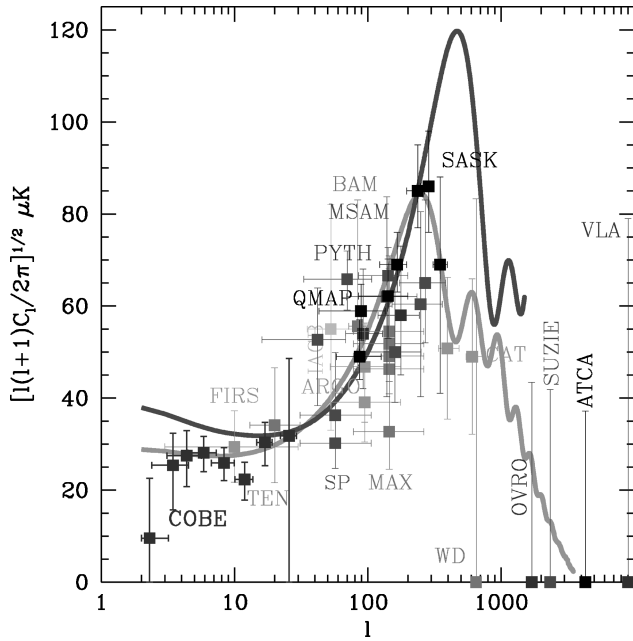


FIG. 5. Summary of current measurements of the power spectrum of CMB temperature variations across the sky against spherical harmonic number l for several experiments. The first acoustic peak is evident. The light curve, which is preferred by the data, is a flat universe ($\Omega_0=1$, $\Omega_M=0.35$), and the dark curve is for an open universe ($\Omega_0=0.3$) (courtesy of M. Tegmark).

elements predicted by BBN (see Fig. 6) is consistent with that seen in the most primitive samples of the cosmos. The abundance of deuterium is very sensitive to the density of baryons, and recent measurements of the deuterium abundance in clouds of hydrogen at high redshift (Burles and Tytler, 1998a, 1998b) have pinned down the baryon density to a precision of 10%.

As Schramm emphasized, BBN is also a powerful probe of fundamental physics. In 1977 he and his colleagues used BBN to place a limit to the number of neutrino species (Steigman, Schramm, and Gunn, 1977), $N_\nu < 7$, which, at the time, was very poorly constrained by laboratory experiments, $N_\nu \lesssim$ a few thousand. The limit is based upon the fact that the big-bang ^4He yield increases with N_ν ; see Fig. 7. In 1989, experiments done at $e^\pm$ colliders at CERN and SLAC determined that N_ν was equal to three, confirming the cosmological bound, which then stood at $N_\nu < 4$. Schramm used the BBN limit on N_ν to pique the interest of many particle physicists in cosmology, both as a heavenly laboratory and in its own right. This important cosmological constraint, and many others that followed, helped to establish the “inner space–outer space connection” that is now flourishing.

III. THE STANDARD COSMOLOGY

Most of our present understanding of the Universe is concisely and beautifully summarized in the hot big-bang cosmological model (see, e.g., Weinberg, 1972;

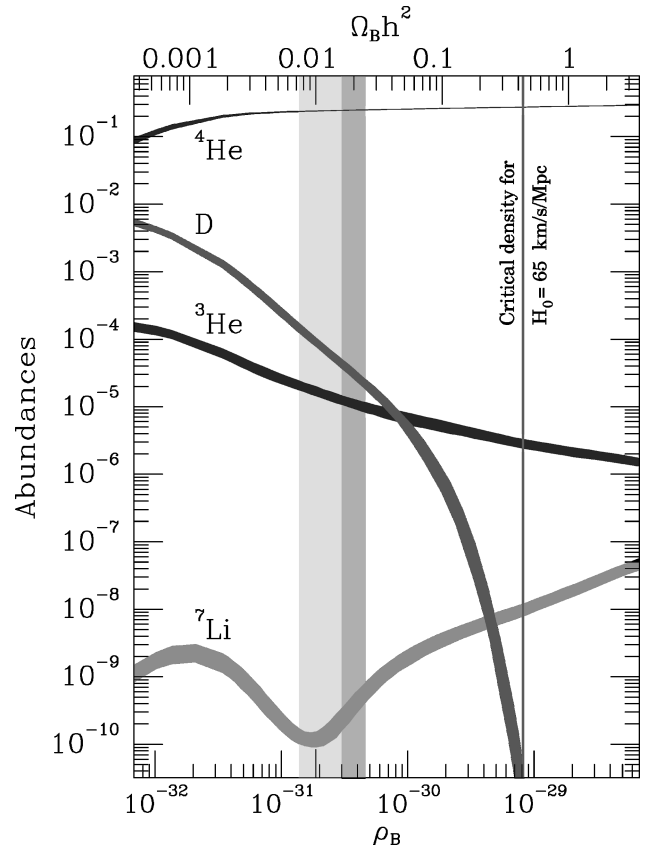


FIG. 6. Predicted abundances of ^4He , (mass fraction), D, ^3He , and ^7Li (relative to hydrogen) as a function of the baryon density. The broader band denotes the concordance interval based upon all four light elements. The narrower, darker band highlights the determination of the baryon density based upon a measurement of the primordial abundance of the most sensitive of these—deuterium (Burles and Tytler, 1998a, 1998b), which implies $\Omega_B h^2 = 0.02 \pm 0.002$.

Peebles, 1993). This mathematical description is based upon the isotropic and homogeneous Friedmann-Lemaître-Robertson-Walker (FLRW) solution of Einstein’s general relativity. The evolution of the Universe is embodied in the cosmic scale factor $R(t)$, which describes the scaling up of all physical distances in the Universe (separation of galaxies and wavelengths of photons). The conformal stretching of the wavelengths of photons accounts for the redshift of light from distant galaxies: the wavelength of the radiation we see today is larger by the factor $R(\text{now})/R(\text{then})$. Astronomers denote this factor by $1+z$, which means that an object at “redshift z ” emitted the light seen today when the Universe was a factor $1+z$ smaller. Normalizing the scale factor to unity today, $R_{\text{emission}} = 1/(1+z)$.

It is interesting to note that the assumption of isotropy and homogeneity was introduced by Einstein and others to simplify the mathematics; as it turns out, it is a remarkably accurate description at early times and today averaged over sufficiently large distances (greater than 100 Mpc or so).

The evolution of the scale factor is governed by the Friedmann equation for the expansion rate:

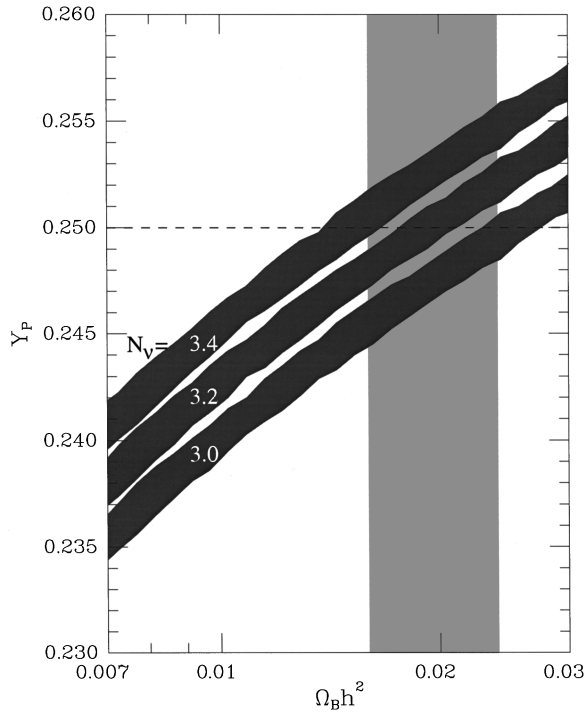


FIG. 7. The dependence of primordial ${}^4\text{He}$ production, relative to hydrogen, Y_P , on the number of light neutrino species. The vertical band denotes the baryon density inferred from the Burles-Tytler measurement of the primordial deuterium abundance (Burles and Tytler, 1998a, 1998b); using $Y_P < 0.25$, based upon current ${}^4\text{He}$ measurements, the BBN limit stands at $N_\nu < 3.4$ (from Schramm and Turner, 1998).

$$H^2 \equiv (\dot{R}/R)^2 = \frac{8\pi G\rho}{3} \pm \frac{1}{R_{\text{curv}}^2}, \quad (3)$$

where $\rho = \sum_i \rho_i$ is the total energy density from all components of mass energy, and R_{curv} is the spatial curvature radius, which grows as the scale factor, $R_{\text{curv}} \propto R(t)$. (Hereafter we shall set $c=1$.) As indicated by the $\pm$ sign in Eq. (3) there are actually three FLRW models; they differ in their spatial curvature: The plus sign applies to the negatively curved model, and the minus sign to the positively curved model. For the spatially flat model the curvature term is absent.

The energy density of a given component evolves according to

$$d\rho_i R^3 = -p_i dR^3, \quad (4)$$

where p_i is the pressure (e.g., $p_i \ll \rho_i$ for nonrelativistic matter or $p_i = \rho_i/3$ for ultrarelativistic particles and radiation). The energy density of matter decreases as R^{-3} , due to volume dilution. The energy density of radiation decreases more rapidly, as R^{-4} , the additional factor arising because the energy of a relativistic particle “redshifts” with the expansion, $E \propto 1/R(t)$. (This of course is equivalent to the wavelength of a photon growing as the scale factor.) This redshifting of the energy density of radiation by R^{-4} also implies that for black-body radiation, the temperature decreases as $T \propto R^{-1}$.

It is convenient to scale energy densities to the

critical density, $\rho_{\text{crit}} \equiv 3H_0^2/8\pi G = 1.88h^2 \times 10^{-29} \text{ g cm}^{-3} \approx 8.4 \times 10^{-30} \text{ g cm}^{-3}$ or approximately 5 protons per cubic meter,

$$\Omega_i \equiv \rho_i / \rho_{\text{crit}}, \quad (5)$$

$$\Omega_0 \equiv \sum_i \Omega_i, \quad (6)$$

$$R_{\text{curv}} = H_0^{-1} / |\Omega_0 - 1|^{1/2}. \quad (7)$$

Note that the critical-density universe ($\Omega_0 = 1$) is flat; the subcritical-density universe ($\Omega_0 < 1$) is negatively curved; and the supercritical-density universe ($\Omega_0 > 1$) is positively curved.

There are at least two components to the energy density: the photons in the 2.728 K cosmic microwave background radiation (number density $n_\gamma = 412 \text{ cm}^{-3}$); and ordinary matter in the formation of neutrons, protons, and associated electrons (referred to collectively as baryons). The theory of big-bang nucleosynthesis and the measured primordial abundance of deuterium imply that the mass density contributed by baryons is $\Omega_B = (0.02 \pm 0.002)h^{-2} \approx 0.05$. In addition, the weak interactions of neutrinos with electrons, positrons, and nucleons should have brought all three species of neutrinos into thermal equilibrium when the Universe was less than a second old, so that today there should be three cosmic seas of relic neutrinos of comparable abundance to the microwave photons, $n_\nu = \frac{3}{11}n_\gamma \approx 113 \text{ cm}^{-3}$ (per species). (BBN provides a nice check of this, because the yields depend sensitively upon the abundance of neutrinos.) Together, photons and neutrinos (assuming all three species are massless, or very light, $\ll 10^{-3} \text{ eV}$) contribute a very small energy density $\Omega_{\nu\gamma} = 4.17h^{-2} \times 10^{-5} \approx 10^{-4}$.

There is strong evidence for the existence of matter beyond the baryons, as dynamical measurements of the matter density indicate that it is at least 20% of the critical density ($\Omega_M > 0.2$), which is far more than ordinary matter can account for. The leading explanation for the additional matter is long-lived or stable elementary particles left over from the earliest moments (see Sec. V).

Finally, although it is now known that the mass density of the Universe in the form of dark matter exceeds 0.2 of the closure density, there are even more exotic possibilities for additional components to the mass-energy density, the simplest of which is Einstein’s cosmological constant. Seeking static solutions, Einstein introduced his infamous cosmological constant; after the discovery of the expansion by Hubble he discarded it. In the quantum world it is no longer optional: the cosmological constant represents the energy density of the quantum vacuum (Weinberg, 1989; Carroll *et al.*, 1992). Lorentz invariance implies the pressure associated with vacuum energy is $p_{\text{VAC}} = -\rho_{\text{VAC}}$, and this ensures that ρ_{VAC} remains constant as the Universe expands. Einstein’s cosmological constant appears as an additional term $\Lambda/3$ on the right-hand side of the Friedmann equation [Eq. (3)]; it is equivalent to a vacuum energy $\rho_{\text{VAC}} = \Lambda/8\pi G$.

All attempts to calculate the cosmological constant have been unsuccessful to say the very least: due to the zero-point energies the vacuum energy formally diverges (“the ultraviolet catastrophe”). Imposing a short wavelength cutoff corresponding to the weak scale ($\sim 10^{-17}$ cm) is of little help: $\Omega_{\text{VAC}} \sim 10^{55}$. The mystery of the cosmological constant is a fundamental one that is being attacked from both ends: Cosmologists are trying to measure it, and particle physicists are trying to understand why it is so small.

Because the different contributions to the energy density scale differently with the cosmic scale factor, the expansion of the Universe goes through qualitatively different phases. While today radiation and relativistic particles are not significant, at early times they dominated the energy, since their energy density depends most strongly on the scale factor (R^{-4} vs R^{-3} for matter). Only at late times does the curvature term ($\propto R^{-2}$) become important; for a negatively curved universe it becomes dominant. For a positively curved universe, the expansion halts when it cancels the matter density term and a contraction phase begins.

The presence of a cosmological constant, which is independent of scale factor, changes this a little. A flat or negatively curved universe ultimately enters an exponential expansion phase driven by the cosmological constant. This also occurs for a positively curved universe, provided the cosmological constant is large enough,

$$\Omega_{\Lambda} > 4\Omega_M \left\{ \cos \left[\frac{1}{3} \cos^{-1}(\Omega_M^{-1} - 1) + \frac{4\pi}{3} \right] \right\}^3. \quad (8)$$

If it is smaller than this, recollapse occurs. Einstein’s static universe obtains for $\rho_M = 2\rho_{\text{VAC}}$ and $R_{\text{curv}} = 1/\sqrt{8\pi G\rho_{\text{VAC}}}$.

The evolution of the Universe according to the standard hot big-bang model is summarized as follows:

- (1) *Radiation-dominated phase.* At times earlier than about 10 000 yrs, when the temperature exceeded $k_B T \gtrsim 3$ eV, the energy density in radiation and relativistic particles exceeded that in matter. The scale factor grew as $t^{1/2}$ and the temperature decreased as $k_B T \sim 1 \text{ MeV}(t/\text{sec})^{-1/2}$. At the earliest times, the energy in the Universe consists of radiation and seas of relativistic particle-antiparticle pairs. (When $k_B T \gg mc^2$ pair creation makes particle-antiparticle pairs as abundant as photons.) The standard model of particle physics, the $SU(3) \otimes SU(2) \otimes U(1)$ gauge theory of the strong, weak, and electromagnetic interactions, provides the microphysics input needed to go back to 10^{-11} sec when $k_B T \sim 300$ GeV. At this time the sea of relativistic particles includes six species of quarks and antiquarks (up, down, charm, strange, top, and bottom), six types of leptons and antileptons (electron, muon, and tauon, and their corresponding neutrinos), and twelve gauge bosons (photon, $W^\pm$, Z^0 , and eight gluons). When the temperature drops below the mass of a particle species, those particles and their antiparticles annihilate and disappear (e.g., $W^\pm$ and Z^0 dis-

appear when $k_B T \sim mc^2 \sim 90$ GeV). As the temperature fell below $k_B T \sim 200$ MeV, a phase transition occurred from a quark-gluon plasma to neutrons, protons, and pions, along with the leptons, antileptons, and photons. At a temperature of $k_B T \sim 100$ MeV, the muons and antimuons disappeared. When the temperature was around 1 MeV a sequence of events and nuclear reactions began that ultimately resulted in the synthesis of D, ^3He , ^4He , and ^7Li . During BBN, the last of the particle-antiparticle pairs, the electrons and positrons, annihilated.

- (2) *Matter-dominated phase.* When the temperature reached around $k_B T \sim 3$ eV, at a time of around 10 000 yrs the energy density in matter began to exceed that in radiation. At this time the Universe was about 10^{-4} of its present size and the cosmic-scale factor began to grow as $R(t) \propto t^{2/3}$. Once the Universe became matter dominated, primeval inhomogeneities in the density of matter (mostly dark matter), shown to be of size around $\delta\rho/\rho \sim 10^{-5}$ by COBE and other anisotropy experiments, began to grow under the attractive influence of gravity ($\delta\rho/\rho \propto R$). After 13 billion or so years of gravitational amplification, these tiny primeval density inhomogeneities developed into all the structure that we see in the Universe today: galaxies, clusters of the galaxies, superclusters, great walls, and voids. Shortly after matter domination begins, at a redshift $1+z \approx 1100$, photons in the Universe undergo their last-scattering off free electrons; last scattering is precipitated by the recombination of electrons and ions (mainly free protons), which occurs at a temperature of $k_B T \sim 0.3$ eV because neutral atoms are energetically favored. Before last-scattering, matter and radiation are tightly coupled; after last-scattering, matter and radiation are essentially decoupled.
- (3) *Curvature-dominated or cosmological constant dominated phase.* If the Universe is negatively curved and there is no cosmological constant, then when the size of the Universe is $\Omega_M/(1-\Omega_M) \sim \Omega_M$ times its present size the epoch of curvature domination begins (i.e., R_{curv}^{-2} becomes the dominant term on the right-hand side of the Friedmann equation). From this point forward the expansion no longer slows and $R(t) \propto t$ (free expansion). In the case of a cosmological constant and a flat universe, the cosmological constant becomes dominant when the size of the Universe is $[\Omega_M/(1-\Omega_M)]^{1/3}$. Thereafter, the scale factor grows exponentially. In either case, further growth of density inhomogeneities that are still linear ($\delta\rho/\rho < 1$) ceases. The structure that exists in the Universe is frozen in.

Finally, a comment on the expansion rate and the size of the “observable Universe.” The inverse of the expansion rate has units of time. The Hubble time, H^{-1} , corresponds to the time it takes for the scale factor to roughly double. For a matter-, radiation-, or curvature-

dominated universe, the age of the universe (time back to zero scale factor) is: $\frac{2}{3}H^{-1}$, $\frac{1}{2}H^{-1}$, and H^{-1} , respectively. The Hubble time also sets the size of the observable (or causally connected) universe: the distance to the “horizon,” which is equal to the distance that light could have traveled since time zero, is $2t=H^{-1}$ for a radiation-dominated universe and $3t=2H^{-1}$ for a matter-dominated universe. Paradoxically, although the size of the Universe goes to zero as one goes back to time zero, the expansion rate is larger, and so points separated by t are moving apart faster than light can catch up with them.

IV. INNER SPACE AND OUTER SPACE

The “hot” in the hot big-bang cosmology makes fundamental physics an inseparable part of the standard cosmology. The time-temperature relation, $k_B T \sim 1 \text{ MeV}(t/\text{sec})^{-1/2}$, implies that the physics of higher energies and shorter times is required to understand the Universe at earlier times: atomic physics at $t \sim 10^{13}$ sec, nuclear physics at $t \sim 1$ sec, and elementary-particle physics at $t < 10^{-5}$ sec. The standard cosmology model itself is based upon Einstein’s general relativity, which embodies our deepest and most accurate understanding of gravity.

The standard model of particle physics, which is a mathematical description of the strong, weak, and electromagnetic interactions based upon the $SU(3) \otimes SU(2) \otimes U(1)$ gauge theory, accounts for all known physics up to energies of about 300 GeV (Gaillard, Grannis, and Sciulli, 1999). It provides the input microphysics for the standard cosmology necessary to discuss events as early as 10^{-11} sec. It also provides a firm foundation for speculations about the Universe at even earlier times.

A key feature of the standard model of particle physics is asymptotic freedom: at high energies and short distances, the interactions between the fundamental constituents of matter—quarks and leptons—are perturbatively weak. This justifies approximating the early Universe as hot gas of noninteracting particles (dilute gas approximation) and opens the door to sensibly speculating about times as early as 10^{-43} sec, when the framework of general relativity becomes suspect, since quantum corrections to this classical description are expected to become important.

The importance of asymptotic freedom for early-Universe cosmology cannot be overstated. A little more than 25 years ago, before the advent of quarks and leptons and asymptotic freedom, cosmology hit a brick wall at 10^{-5} sec because extrapolation to early times was nonsensical. The problem was twofold: the finite size of nucleons and related particles and the exponential rise in the number of “elementary particles” with mass. At around 10^{-5} sec, nucleons would be overlapping, and with no understanding of the strong forces between them, together with the exponentially rising spectrum of particles, thermodynamics became ill defined at higher temperatures.

The standard model of particle physics has provided particle physicists with a reasonable foundation for speculating about physics at even shorter distances and higher energies. Their speculations have significant cosmological implications, and—conversely—cosmology holds the promise to test some of their speculations. The most promising particle physics ideas (see, e.g., Schwarz and Seiberg, 1999) and their cosmological implications are:

- (1) *Spontaneous symmetry breaking (SSB)*. A key idea, which is not fully tested, is that most of the underlying symmetry in a theory can be hidden because the vacuum state does not respect the full symmetry; this is known as spontaneous symmetry breaking and accounts for the carriers of the weak force, the $W^\pm$ and Z^0 bosons, being very massive. (Spontaneous symmetry breaking is seen in many systems, e.g., a ferromagnet at low temperatures: it is energetically favorable for the spins to align thereby breaking rotational symmetry.) In analogy to symmetry breaking in a ferromagnet, spontaneously broken symmetries are restored at high temperatures. Thus, it is likely that the Universe underwent a phase transition at around 10^{-11} sec when the symmetry of the electroweak theory was broken, $SU(2) \otimes U(1) \rightarrow U(1)$.
- (2) *Grand unification*. It is possible to unify the strong, weak, and electromagnetic interactions by a larger gauge group, e.g., $SU(5)$, $SO(10)$, or E_8 . The advantages are twofold: the three forces are described as different aspects of a more fundamental force with a single coupling constant, and the quarks and leptons are unified as they are placed in the same particle multiplets. If true, this would imply another stage of spontaneous symmetry breaking, $G \rightarrow SU(3) \otimes SU(2) \otimes U(1)$. In addition, grand unified theories (or GUTs), predict that baryon and lepton number are violated—so that the proton is unstable and neutrinos have mass—and that stable topological defects associated with SSB may exist, e.g., pointlike defects called magnetic monopoles, one-dimensional defects referred to as “cosmic” strings, and two-dimensional defects called domain walls. The cosmological implications of GUTs are manifold: neutrinos as a dark matter component, baryon and lepton number violation explaining the matter-antimatter asymmetry of the Universe, and SSB phase transitions producing topological defects that seed structure formation or a burst of tremendous expansion called inflation.
- (3) *Supersymmetry*. In an attempt to put bosons and fermions on the same footing, as well as to better understand the “hierarchy problem,” namely, the large gap between the weak scale (300 GeV) and the Planck scale (10^{19} GeV), particle theorists have postulated supersymmetry, the symmetry between fermions and bosons. (Supersymmetry also appears to have a role to play in understanding gravity.) Since the fundamental particles of the standard model of

particle physics cannot be classified as fermion-boson pairs, if correct, supersymmetry implies the existence of a superpartner for every known particle, with a typical mass of order 300 GeV. The lightest of these superpartners is usually stable and called “the neutralino.” The neutralino is an ideal dark matter candidate.

- (4) *Superstrings, supergravity, and M-theory.* The unification of gravity with the other forces of Nature has long been the holy grail of theorists. Over the past two decades there have been some significant advances: supergravity, an 11-dimensional version of general relativity with supersymmetry, which unifies gravity with the other forces; superstrings, a ten-dimensional theory of relativistic strings, which unifies gravity with the other forces in a self-consistent, finite theory; and M-theory, an ill-understood, “larger” theory that encompasses both superstring theory and supergravity theory. An obvious cosmological implication is the existence of additional spatial dimensions, which today must be “curled up” to escape notice, as well as the possibility of sensibly describing cosmology at times earlier than the Planck time.

Advances in fundamental physics have been crucial to advancing cosmology: e.g., general relativity led to the first self-consistent cosmological models; from nuclear physics came big-bang nucleosynthesis; and so on. The connection between fundamental physics and cosmology seems even stronger today and makes realistic the hope that much more of the evolution of the Universe will be explained by fundamental theory, rather than the *ad hoc* theory that dominated cosmology before the 1980s. Indeed, the most promising paradigm for extending the standard cosmology, inflation+cold dark matter, is deeply rooted in elementary particle physics.

V. DARK MATTER AND STRUCTURE FORMATION

As successful as the standard cosmology is, it leaves important questions about the origin and evolution of the Universe unanswered. To an optimist, these questions suggest that there is a grander cosmological theory, which encompasses the hot big-bang model and resolves these questions. It can easily be argued that the most pressing issues in cosmology are the quantity and composition of energy and matter in the Universe, and the origin and nature of the density perturbations that seeded all the structure in the Universe. Cosmology is poised for major progress on these two questions. Answering these questions will provide a window to see beyond the standard cosmology.

A. Dark matter and dark energy

Our knowledge of the mass and energy content of the Universe is still poor, but is improving rapidly (see Sadoulet, 1999). We can confidently say that most of the matter in the Universe is of unknown form and dark (see, e.g., Dekel, Burstein, and White, 1997; Bahcall

et al., 1993): Stars (and closely related material) contribute a tiny fraction of the critical density, $\Omega_{\text{lum}} = (0.003 \pm 0.001)h^{-1} \approx 0.004$, while the amount of matter known to be present from its gravitational effects contributes around ten times this amount, $\Omega_M = 0.35 \pm 0.07$ (this error flag is ours; it is meant to indicate 95% certainty that Ω_M is between 0.2 and 0.5). The gravity of dark matter is needed to hold together just about everything in the Universe—galaxies, clusters of galaxies, superclusters, and the Universe itself. A variety of methods for determining the amount of matter all seem to converge on $\Omega_M \sim 1/3$; they include measurements of the masses of clusters of galaxies and the peculiar motions of galaxies. Finally, the theory of big-bang nucleosynthesis and the recently measured primeval abundance of deuterium pin down the baryon density very precisely: $\Omega_B = (0.02 \pm 0.002)h^{-2} \approx 0.05$. The discrepancy between this number and dynamical measurements of the matter density is evidence for nonbaryonic dark matter.

Particle physics suggests three dark-matter candidates (Sadoulet, 1999): a 10^{-5} eV axion (Rosenberg, 1998); a 10 GeV–500 GeV neutralino (Jungman, Kamionkowski, and Griest, 1996); and a 30 eV neutrino. These three possibilities are highly motivated in two important senses: first, the axion and neutralino are predictions of fundamental theories that attempt to go beyond the standard model of particle physics, as are neutrino masses; and second, the relic abundances of the axion and neutralino turn out to be within a factor of ten of the critical density, and similarly for the neutrino—GUTs predict masses in the eV range, which is what is required to make neutrinos a significant contributor to the mass density.

Because measuring the masses of galaxy clusters has been key to defining the dark matter problems it is perhaps worth further discussion. Cluster masses can be estimated by three different techniques, which give consistent results. The first, which dates back to Fritz Zwicky (1935), uses the measured velocities of cluster galaxies and the virial theorem to determine the total mass (i.e., $KE_{\text{gal}} \approx |PE_{\text{gal}}|/2$). The second method uses the temperature of the hot x-ray emitting intracluster gas and the virial theorem to arrive at the total mass. The third and most direct method is using the gravitational lensing effects of the cluster on much more distant galaxies. Close to the cluster center, lensing is strong enough to produce multiple images; farther out, lensing distorts the shape of distant galaxies. The lensing method allows the cluster (surface) mass density to be mapped directly. An example of mapping the mass distribution of a cluster of galaxies is shown in Fig. 8.

Using clusters to estimate the mean mass density of the Universe requires a further assumption: that their mass-to-light ratio provides a good estimate for the mean mass-to-light ratio. This is because the mean mass density is determined by multiplying the mean luminosity density (which is reasonably well measured) by the inferred cluster mass-to-light ratio. Using this technique, Carlberg *et al.* (1996) find $\Omega_M = 0.19 \times 0.06$. If clusters

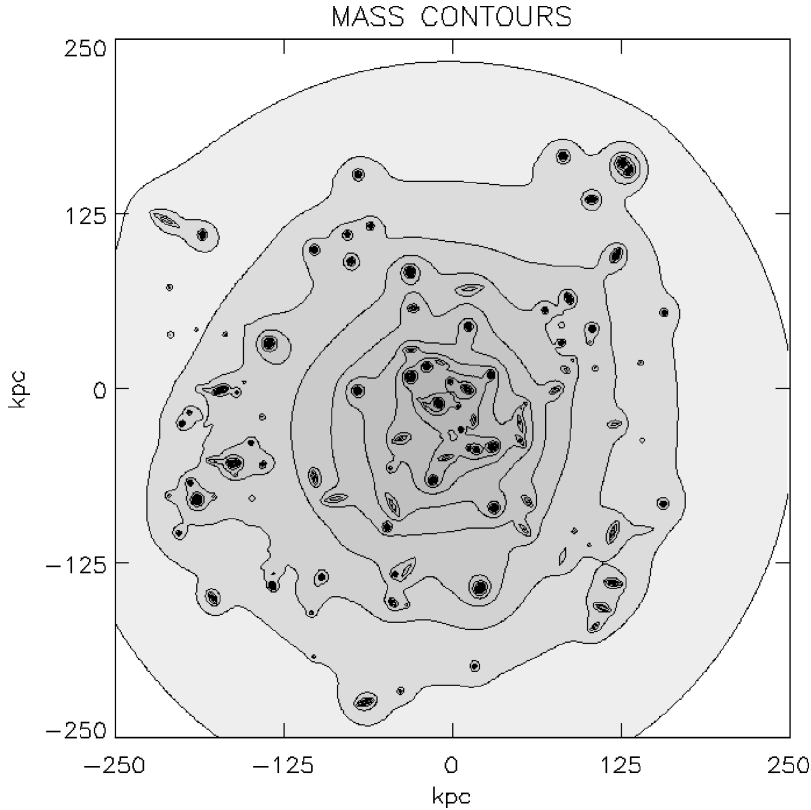


FIG. 8. The reconstructed total mass density in the cluster of galaxies 0024+1654 at redshift $z=0.39$, based on parametric inversion of the associated gravitational lens. Projected mass contours are spaced by $430 M_{\odot} \text{ pc}^{-2}$, with the outer contour at $1460 M_{\odot} \text{ pc}^{-2}$. Excluding dark mass concentrations centered on visible galaxies, more than 98% of the remaining mass is represented by a smooth concentration of dark matter centered near the brightest cluster galaxies, with a 50 kpc soft core (Tyson *et al.*, 1998).

have more luminosity per mass than average, this technique would underestimate Ω_M .

There is another way to estimate Ω_M , using clusters, based upon a different, more physically motivated assumption. X-ray measurements more easily determine the amount of hot, intracluster gas; and as it turns out, most of the baryonic mass in a cluster resides here rather than in the mass of individual galaxies (this fact is also confirmed by lensing measurements). Together with the total cluster mass, the ratio of baryonic mass to total mass can be determined; a compilation of the existing data give $M_B/M_{\text{tot}} = (0.07 \pm 0.007) h^{-3/2} \approx 0.15$ (Evrard, 1997, and references therein). Assuming that clusters provide a fair sample of matter in the Universe so that $\Omega_B/\Omega_M = M_B/M_{\text{tot}}$, the accurate BBN determination of Ω_B can be used to infer $\Omega_M = (0.3 \pm 0.05) h^{-1/2} \approx 0.4$. [A similar result for the cluster gas to total mass ratio is derived from cluster gas measurements based upon the distortion of the CMB spectrum due to CMB photons scattering off the hot cluster gas (Sunyaev-Zel'dovich effect); see Carlstrom, 1999.]

Two other measurements bear on the quantity and composition of energy and matter in the Universe. First, the pattern of anisotropy in the CMB depends upon the total energy density in the Universe (i.e., Ω_0) (see, e.g., Jungman, Kamionkowski, Kosowsky, and Spergel, 1996). The peak in the multipole power spectrum is

$l_{\text{peak}} \approx 200/\sqrt{\Omega_0}$. The current data, shown in Fig. 5, are consistent with $\Omega_0 \approx 1$, though $\Omega_0 \sim 0.3$ cannot be excluded. This together with the evidence that $\Omega_M \approx 0.3$ leaves room for a component of energy that does not clump, such as a cosmological constant.

The oldest approach to determining the total mass-energy density is through the deceleration parameter (Baum, 1957; Sandage, 1961), which quantifies the present slowing of the expansion due to gravity,

$$q_0 \equiv -\frac{(\ddot{R}/R)_0}{H_0^2} = \frac{\Omega_0}{2} [1 + 3p_0/\rho_0], \quad (9)$$

where subscript zero refers to quantities measured at the current epoch. Note, in a universe where the bulk of the matter is nonrelativistic ($p \ll \rho$), q_0 and Ω_0 differ only by a factor of two. The luminosity distance to an object at redshift $z \ll 1$ is related to q_0 ,

$$d_L H_0 = z + z^2(1 - q_0)/2 + \dots, \quad (10)$$

and thus accurate distance measurements can be used to determine q_0 . (The luminosity distance to an object is defined as that inferred from the inverse square law: $d_L \equiv \sqrt{\mathcal{L}/4\pi\mathcal{F}}$.)

Recently, two groups (the Supernova Cosmology Project and the High- z Supernova Team) using type Ia supernovae (SNeIa) as standard candles (objects of

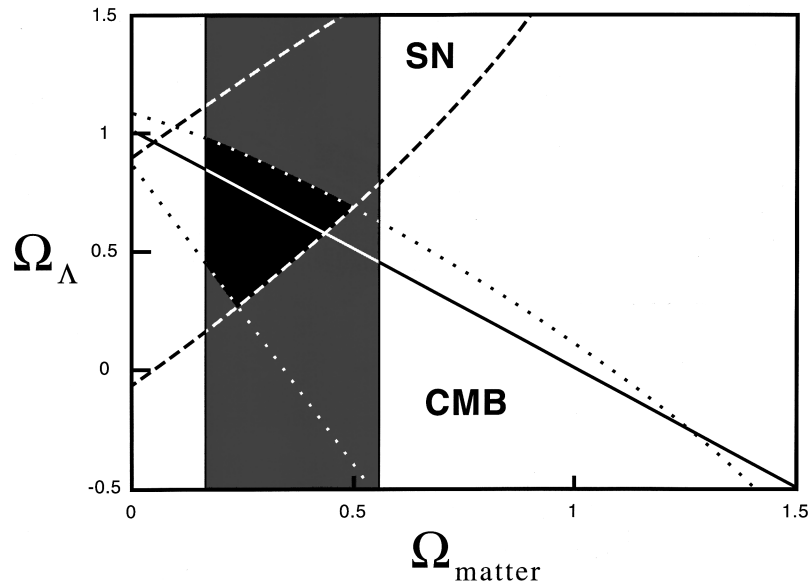


FIG. 9. Constraints in the Ω_Λ vs Ω_M plane. Three different types of observations are shown: SNe Ia measures of expansion acceleration (SN); the CMB observations of the location of the first acoustic peak (CMB); and determinations of the matter density, $\Omega_M = 0.35 \pm 0.07$ (dark vertical band). Diagonal line indicates a flat universe, $\Omega_M + \Omega_\Lambda = 1$; regions denote “ 3σ ” confidence. Darkest region denotes concordance region: $\Omega_\Lambda \sim 2/3$ and $\Omega_M \sim T/3$.

known $\mathcal{L}$) and assuming that their flux measurements (i.e., $\mathcal{F}$) were not contaminated by sample selection, evolution, or dust systematics, both conclude that the expansion of the Universe is accelerating rather than decelerating (i.e., $q_0 < 0$) (Schmidt *et al.*, 1998; Perlmutter *et al.*, 1998). If correct, this implies that much of the energy in the Universe is in an unknown component, with negative pressure, $p_X \leq -\rho_X/3$ (Garnavich *et al.*, 1998). The simplest explanation is a cosmological constant with $\Omega_\Lambda \sim 2/3$. [In fact, Eq. (10), which is deeply rooted in the history of cosmology, is not sufficiently accurate at the redshifts of the SNeIa being used, and the two groups compute $d_L \equiv (1+z)r(z)$ as a function of Ω_M and Ω_Λ and fit to the observations.]

Pulling this together, cosmologists for the first time have a plausible accounting of matter and energy in the Universe: stars contribute around 0.4% of the critical density, baryons contribute 5%, nonrelativistic particles of unknown type contribute 30%, and vacuum energy contributes 64%, for a total equaling the critical density (see Figs. 9 and 10). We should emphasize that plausible does not mean correct.

In addition to the fact that most of the matter and energy in the Universe is dark, most of the ordinary matter is dark (i.e., not in bright stars). The possibilities for the dark baryons include “dark stars” and diffuse hot or warm gas (recall, in clusters, most of the baryons are in hot, intracluster gas). Dark stars could take the form of faint, low-mass stars, failed stars (i.e., objects below the mass required for hydrogen burning, $M \lesssim 0.08M_\odot$), white dwarfs, neutron stars, or black holes.

Most of the mass of our own Milky Way galaxy is dark, existing in an extended halo (an approximately spherical distribution of matter with density falling as $1/r^2$). Unsuccessful searches for faint stars in our galaxy have eliminated them as a viable candidate, and theoret-

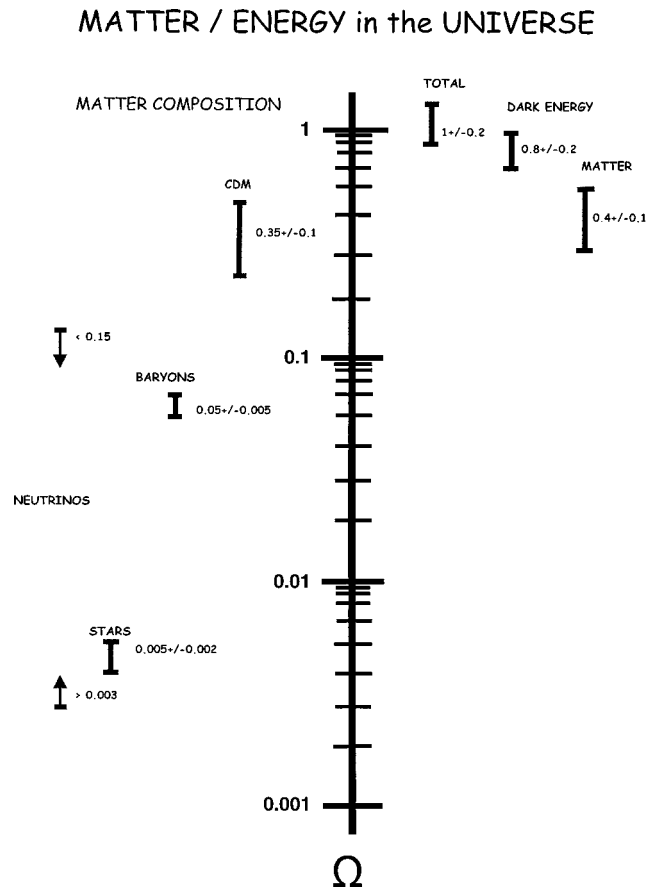


FIG. 10. Summary of matter/energy in the Universe. The right side refers to an overall accounting of matter and energy; the left refers to the composition of the matter component. The upper limit to mass density contributed by neutrinos is based upon the failure of the hot dark matter model and the lower limit follows from the SuperK evidence for neutrino oscillations.

ical arguments disfavor white dwarfs, black holes, and neutron stars—all should lead to the production of more heavy elements than are observed. Further, the measured rate of star formation indicates that only a fraction of the baryons have formed into bright, massive stars.

Experimental searches for dark stars in our own galaxy have been carried out using the gravitational microlensing technique: dark stars along the light-of-sight to nearby galaxies (e.g., the Large and Small Magellanic Clouds and Andromeda) can gravitationally lens the distant bright stars, causing a well-defined, temporary brightening (Paczynski, 1986). The results however are perplexing (see, e.g., Sadoulet, 1999). More than a dozen such brightenings of Large Magellanic Cloud (LMC) stars have been seen, suggesting that a significant fraction of our galaxy's halo exists in the form of half-solar mass white dwarfs. However, such a population of white dwarfs should be visible, and they have not been seen. Because of our imperfect knowledge of our own galaxy and the LMC it is possible that the lenses are not associated with the halo of our galaxy, but rather are low-mass stars in the LMC, in an intervening dwarf galaxy between us, or are actually in the disk of our galaxy, if the disk is warped enough to pass in front of the line to the LMC.

B. Structure formation and primeval inhomogeneity

The COBE detection of CMB anisotropy on angular scales of 10° was a major milestone (Smoot *et al.*, 1992), providing the first evidence for the fluctuations that seeded all the structure in the Universe and strong evidence for the gravitational instability picture for structure formation, as the size of the inhomogeneity was sufficient to explain the structure observed today. It also ushered in a powerful new probe of structure formation and dark matter. An early implication of COBE was galvanizing: nonbaryonic dark matter is required to explain the structure seen today. Because baryons are tightly coupled to photons in the Universe and thereby supported against gravitational collapse until after decoupling, larger amplitude density perturbations are required, which in turn lead to larger CMB temperature fluctuations than are observed.

Two key issues are the character and origin of the inhomogeneity and the quantity and composition of matter, discussed above. It is expected that there is a spectrum of fluctuations, described by its Fourier decomposition into plane waves. In addition, there are two generic types of inhomogeneity: curvature perturbations, fluctuations in the local curvature of the Universe, which by the equivalence principle, affect all components of the energy density alike; and isocurvature perturbations, which as their name indicates are not ingrained in the curvature but arise as pressure perturbations caused by local changes in the equation of state of matter and energy in the Universe.

The two most promising ideas for the fundamental origin of the primeval inhomogeneity are quantum fluctuations that become curvature fluctuations during infla-

tion (Hawking, 1982; Guth and Pi, 1982; Starobinskii, 1982; Bardeen, Steinhardt, and Turner, 1983) and topological defects (such as cosmic strings) that are produced during a cosmological phase transition (see, e.g., Vilenkin and Shellard, 1994). The inflation scenario will be discussed in detail later on. Topological defects produced in a cosmological symmetry-breaking phase transition around 10^{-36} sec generate isocurvature fluctuations: the conversion of energy from radiation to defects leads to a pressure perturbation that propagates outward and ultimately leads to a density inhomogeneity. The defect scenario is currently disfavored by measurements of CMB anisotropy (Allen *et al.*, 1997; Pen *et al.*, 1997).

One graphic indicator of the progress being made on the large-scale structure problem is the number of viable models: the flood of data has trimmed the field to one or possibly two models. A few years ago the defect model was a leading contender; and another, more phenomenological model put forth by Peebles was also in the running (Peebles, 1987). Peebles' model dispensed with nonbaryonic dark matter, assumed $\Omega_B = \Omega_0 \sim 0.2$, and posited local variations in the distribution of baryons (isocurvature perturbations) of unknown origin. Its demise was CMB anisotropy: it predicted too much anisotropy on small angular scales. The one clearly viable model is cold dark matter + inflation, which is discussed below. The challenge to theorists is to make sure that at least one model remains viable as the quantity and quality of data improve.

VI. MORE FUNDAMENTAL QUESTIONS

Beyond the questions involving dark matter and structure formation, there is a set of more fundamental questions, ranging from the matter-antimatter asymmetry in the Universe to the origin of the expansion itself. For these questions there are attractive ideas, mainly rooted in the physics of the early Universe, which remain to be developed, suggesting that a more fundamental understanding of our Universe is possible.

Baryon-lepton asymmetry. While the laws of physics are very nearly matter-antimatter symmetric, the Universe is not. On scales as large as clusters of galaxies there is no evidence for antimatter. In the context of the hot big bang, a symmetric universe would be even more puzzling: at early times ($t \ll 10^{-5}$ sec) matter-antimatter pairs would be as abundant as photons, but as the Universe cooled matter and antimatter would annihilate until nucleons and antinucleons were too rare to find one another. This would result in only trace amounts of matter and antimatter, a few nucleons and antinucleons per 10^{18} photons, compared to the observed nucleon to photon ratio: $\eta \equiv n_N/n_\gamma = (5 \pm 0.5) \times 10^{-10}$.

In order to avoid the annihilation catastrophe the early Universe must possess a slight excess of matter over antimatter, i.e., a small net baryon number: $n_B/n_\gamma \equiv n_b/n_\gamma - n_{\bar{b}}/n_\gamma = \eta = 5 \times 10^{-10}$. Such an initial condition for the Universe seems as odd as having to assume the ^4He mass fraction is 25%. (Charge neutrality re-

quires a similar excess of electrons over positrons; because lepton number can be hidden in the three neutrino species, it is not possible to say that the total lepton asymmetry is comparable to the baryon asymmetry.)

A framework for understanding the origin of the baryon asymmetry of the Universe was put forth in a prescient paper by Sakharov in 1967: baryon-number violating and matter-antimatter symmetry violating interactions occurring in a state of nonequilibrium allow a small, net baryon number to develop. If the idea of baryogenesis is correct, the explanation of the baryon asymmetry is not unlike that of the primeval ${}^4\text{He}$ abundance (produced by nonequilibrium nuclear reactions). The key elements of baryogenesis are all in place: baryon number is violated in the standard model of particle physics (by subtle quantum mechanical effects) and in GUTs; matter-antimatter symmetry is known to be violated by a small amount in the neutral Kaon system (CP violation at the level of 10^{-3}); and maintaining thermal equilibrium in the expanding and cooling Universe depends upon whether or not particle interactions proceed rapidly compared to the expansion rate. The details of baryogenesis have not been worked out, and may involve grand unification physics, but the basic idea is very compelling (see, e.g., Kolb and Turner, 1990).

The heat of the big bang. The entropy associated with the CMB and three neutrino seas is enormous: within the observable Universe, 10^{88} in units of k_B (the number of nucleons is 10 orders of magnitude smaller). Where did all the heat come from? As we discuss in the next section, inflation may provide the answer.

Origin of the smoothness and flatness. On large scales today and at very early times the Universe is very smooth. (The appearance of inhomogeneity today does belie a smooth beginning as gravity drives the growth of fluctuations.) Since the particle horizon at last-scattering (when matter and radiation decoupled) corresponds to an angle of only 1° on the sky, the smoothness could not have arisen via causal physics. (Within the isotropic and homogeneous FLRW model no explanation is required of course.)

In a sense emphasized first by Dicke and Peebles (1979) and later by Guth (1982), the Universe is very flat. Since Ω_0 is not drastically different from unity, the curvature radius of the Universe is comparable to the Hubble radius. During a matter or radiation dominated phase the curvature radius decreases relative to the Hubble radius. This implies that at earlier times it was even larger than the Hubble radius, and that Ω was even closer to one: $|\Omega - 1| < 10^{-16}$ at 1 sec. To arrive at the Universe we see today, the Universe must have begun very flat (and thus, must have been expanding very close to the critical expansion rate).

The flatness and smoothness problems are not indicative of any inconsistency of the standard model, but they do require special initial conditions. As stated by Collins and Hawking (1973), the set of initial conditions that evolve to a universe qualitatively similar to ours is of

measure zero. While not *required* by observational data, the inflation model addresses both the smoothness and flatness problems.

Origin of the big bang, expansion, and all that. In naming the big-bang theory Hoyle tried to call attention to the colossal big-bang event, which, in the context of general relativity corresponds to the creation of matter, space, and time from a space-time singularity. In its success, the big-bang theory is a theory of the events *following* the big-bang singularity. In the context of general relativity the big-bang event requires no further explanation (it is consistent with “St. Augustine’s principle,” since time is created along with space, there is no *before* the big bang). However, many if not most physicists believe that general relativity, which is a classical theory, is not applicable any earlier than 10^{-43} sec because quantum corrections should become very significant, and further, that a quantum theory of gravity will eliminate the big-bang singularity allowing the “before the big-bang question” to be addressed. As we will discuss, inflation addresses the big-bang question too.

VII. BEYOND THE STANDARD MODEL: INFLATION+COLD DARK MATTER

The 1980s were ripe with interesting ideas about the early Universe inspired by speculations about the unification of the forces and particles of Nature (see, e.g., Kolb and Turner, 1990): relic elementary particles as the dark matter; topological defects as the seeds for structure formation; baryon number violation and C, CP violation as the origin of the baryon asymmetry of the Universe (baryogenesis); and inflation. From all this, a compelling paradigm for extending the standard cosmology has evolved: inflation+cold dark matter. It is bold and expansive and is being tested by a flood of observations. It may even be correct!

The story begins with a brief period of tremendous expansion—a factor of greater than 10^{27} growth in the scale factor in 10^{-32} sec. The precise details of this “inflationary phase” are not understood, but in most models the exponential expansion is driven by the (potential) energy of a scalar field initially displaced from the minimum of its potential energy curve. Inflation blows up a small, subhorizon-sized portion of the Universe to a size much greater than that of the observable Universe today. Because this subhorizon-sized region was causally connected before inflation, it can be expected to be smooth—including the very small portion of it that is our observable part of the Universe. Likewise, because our Hubble volume is but a small part of the region that inflated, it looks flat, regardless of the initial curvature of the region that inflated, $R_{\text{curv}} \gg H_0^{-1}$, which via the Friedmann equation implies that $\Omega_0 = 1$.

It is while this scalar field responsible for inflation rolls slowly down its potential that the exponential expansion takes place (Albrecht and Steinhardt, 1982; Linde, 1982). As the field reaches the minimum of the potential energy curve, it overshoots and oscillates about it: the potential energy of the scalar field has been

converted to coherent scalar field oscillations (equivalently, a condensate of zero momentum scalar-field particles). Eventually, these particles decay into lighter particles which thermalize, thereby explaining the tremendous heat content of the Universe and ultimately the photons in the CMB (Albrecht *et al.*, 1982).

Quantum mechanical fluctuations arise in such a scalar field that drives inflation; they are on truly microscopic scales ($\lesssim 10^{-23}$ cm). However, they are stretched in size by the tremendous expansion during inflation to astrophysical scales. Because the energy density associated with the scalar field depends upon its value (through the scalar field potential energy), these fluctuations also correspond to energy density perturbations, and they are imprinted upon the Universe as perturbations in the local curvature. Quantum mechanical fluctuations in the space-time metric give rise to a stochastic, low-frequency background of gravitational waves.

The equivalence principle holds that local acceleration cannot be distinguished from gravity; from this it follows that curvature perturbations ultimately become density perturbations in all species—photons, neutrinos, baryons, and particle dark matter. The shape of the spectrum of perturbations is nearly scale invariant. [Such a form for the spectrum was first discussed by Harrison (1970). Zel'dovich, who appreciated the merits of such a spectrum early on, emphasized its importance for structure formation.] Scale-invariant refers to the fact that the perturbations in the gravitational potential have the same amplitude on all length scales (which is not the same as the density perturbations having the same amplitude). When the wavelength of a given mode crosses inside the horizon ($\lambda = H^{-1}$), the amplitude of the density perturbation on that scale is equal to the perturbation in the gravitational potential.

The overall amplitude (or normalization) depends very much upon the specific model of inflation (of which there are many). Once the overall normalization is set, the shape fixes the level of inhomogeneity on all scales. The detection of anisotropy on the scale of 10° by COBE in 1992 and the subsequent refinement of that measurement with the full four-year data set permitted the accurate (10%) normalization of the inflationary spectrum of density perturbations; soon, the term COBE-normalized became a part of the cosmological vernacular.

On to the cold dark matter (CDM) part: inflation predicts a flat Universe (total energy density equal to the critical density). Since ordinary matter (baryons) contributes only about 5% of the critical density there must be something else. The leading candidate is elementary particles remaining from the earliest moments of particle democracy. Generically, they fall into two classes—fast moving, or hot dark matter; and slowly moving, or cold dark matter (see Sadoulet, 1999). Neutrinos of mass 30 eV or so are the prime example of hot dark matter—they move quickly because they were once in thermal equilibrium and are very light. Axions and neutralinos are examples of cold dark matter. Neutralinos move slowly because they too were once in thermal equilib-

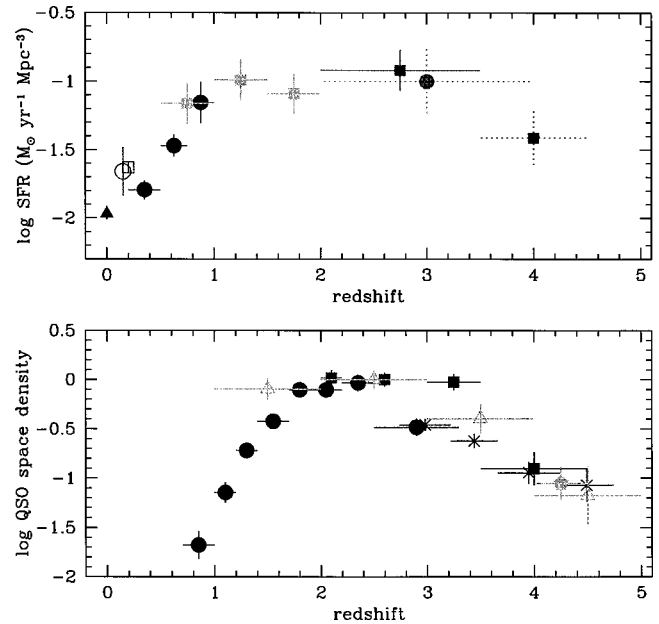


FIG. 11. Top: Star formation rate in galaxies is plotted vs redshift. Bottom: The number density of quasistellar objects (galaxies with accreting black holes) vs redshift. The points have been corrected for dust and the relative space density of QSOs (from Madau, 1999).

rium and they are very heavy. Axions are extremely light but were never in thermal equilibrium (having been produced very, very cold).

If most of the matter is hot, then structure in the Universe forms from the top down: large things, like superclusters, form first, and fragment into smaller objects such as galaxies. This is because fast moving neutrinos smooth out density perturbations on small scales by moving from regions of high density into regions of low density (Landau damping or collisionless phase mixing). Observations very clearly indicate that galaxies formed at redshifts $z \sim 2-4$ (see Fig. 11), before superclusters which are just forming today. So hot dark matter is out, at least as a major component of the dark matter (White, Frenk, and Davis, 1983). This leaves cold dark matter.

Cold dark matter particles cannot move far enough to damp perturbations on small scales, and structure then forms from the bottom up: galaxies, followed by clusters of galaxies, and so on (see, e.g., Blumenthal *et al.*, 1984). For COBE-normalized cold dark matter we can be even more specific. The bulk of galaxies should form around redshifts $z \sim 2-4$, just as the observations now indicate.

At present, the cold dark matter+inflation scenario looks very promising—it is consistent with a large body of observations: measurements of the anisotropy of the CMB, redshift surveys of the distribution of matter today, deep probes of the Universe (such as the Hubble Deep Field), and more (see Liddle and Lyth, 1993, and Fig. 11). While the evidence is by no means definitive, and has hardly begun to discriminate between different inflationary models and versions of CDM, we can say that the data favor a flat Universe, almost scale-invariant

density perturbations, and cold dark matter with a small admixture of baryons.

VIII. PRECISION COSMOLOGY

The COBE DMR measurement of CMB anisotropy on the 10° angular scale and determination of the primeval deuterium abundance served to mark the beginning of a new era of precision cosmology. Overnight, COBE changed the study of large-scale structure: for theories like inflation and defects that specify the shape of the spectrum of density perturbations, the COBE measurement fixed the level of inhomogeneity on all scales to an accuracy of around 10%. Likewise, the measurement of the primeval deuterium abundance led to a 10% determination of the baryon density.

Within the next few years, an avalanche of data, driven by advances in technology, promises definitive independent observations of the geometry, mass distribution and composition, and detailed structure of the Universe. In a radical departure from its history, cosmology is becoming an exact science. These new observations span the wavelength range from microwave to gamma rays and beyond, and utilize techniques as varied as CMB microwave interferometry, faint supernova photometry and spectroscopy, gravitational lensing, and massive photometric and spectroscopic surveys of millions of galaxies.

The COBE measurement of CMB anisotropy on angular scales from around 10° to 100° yielded a precise determination of the amplitude of mass fluctuations on very large scales, 10^3 Mpc– 10^4 Mpc. A host of experiments will view the CMB with much higher angular resolution and more precision than COBE, culminating in the two satellite experiments, NASA's MAP and ESA's Planck Surveyor, which will map the full sky to an angular resolution of 0.1° . In so doing, the mass distribution in the Universe at a simpler time, before non-linear structures had formed, will be determined on scales from 10^4 Mpc down to 10 Mpc. (Temperature fluctuations on angular scale θ arise from density fluctuations on length scales $L \sim 100h^{-1}$ Mpc [θ/deg]; fluctuations on scales ~ 1 Mpc give rise to galaxies, on scales ~ 10 Mpc give rise to clusters, and on scales ~ 100 Mpc give rise to great walls.)

The multipole power spectrum of CMB temperature fluctuations has a rich structure and encodes a wealth of information about the Universe. The peaks in the power spectrum are caused by baryon-photon oscillations, which are driven by the gravitational force of the dark matter. Since decoupling is essentially instantaneous, different Fourier modes are caught at different phases, which is reflected in multiple spectrum of anisotropy (see Fig. 5). The existence of the first peak is evident. If the satellite missions are as successful as cosmologists hope, and if foregrounds (e.g., diffuse emission from our own galaxy and extragalactic point sources) are not a serious problem, it should be possible to use the measured multipole power spectrum to determine Ω_0 and many other cosmological parameters (e.g., $\Omega_B h^2$, h , n ,

level of gravitational waves, Ω_Λ , and Ω_ν) to precision of few percent or better in some cases (see Wilkinson, 1999).

Another impressive map is in the works. The present three-dimensional structure of the local Universe will also be mapped to unprecedented precision in the next few years by the Sloan Digital Sky Survey (SDSS) (see Gunn *et al.*, 1998), which will obtain the redshifts of a million galaxies over 25% of the northern sky out to redshift $z \sim 0.1$, and the Two-degree Field Survey (2dF), which will collect 250 000 redshifts in many 2° patches of the southern sky (Colless, 1998). These surveys will cover around 0.1% of the observable Universe, and more importantly, will map structure out to scales of about $500h^{-1}$ Mpc, well beyond the size of the largest structures known. This should be large enough to provide a typical sample of the Universe. The two maps—CMB snapshot of the Universe at 300 000 yrs and the SDSS map of the Universe today—when used together have enormous leverage to test cosmological models and determine cosmological parameters.

Several projects are underway to map smaller, more distant parts of the Universe to study the “recent” evolution of galaxies and structure. Using a new large spectrograph, the 10-meter Keck telescope will begin to map galaxies in smaller fields on the sky out to redshifts of 4 or so. Ultimately, the Next Generation Space Telescope, which is likely to have an 8-meter mirror and capability in the infrared (most of the light of high-redshift galaxies has been shifted into the infrared) will probe the first generation of stars and galaxies.

Much of our current understanding of the Universe is based on the assumption that light traces mass, because telescopes detect light and not mass. There is some evidence that light is not a terribly “biased” tracer of mass, at least on the scales of galaxies. However, it would be a convenient accident if the mass-to-light ratio were universal. It is possible that there is a lot of undiscovered matter, perhaps even enough to bring Ω_M to unity, associated with dim galaxies or other mass concentrations that are not correlated with bright galaxies.

Gravitational lensing is a powerful means of measuring cosmic mass overdensities in the linear regime directly (see, e.g., Blandford and Narayan, 1992; Tyson, 1993): dark matter overdensities at moderate redshift ($z \sim 0.2$ – 0.5) systematically distort background galaxy images (referred to as weak gravitational lensing). A typical random one-square-degree patch of the sky contains a million faint high-redshift galaxies. Using these galaxies, weak gravitational lensing may be used to map the intervening dark matter overdensities directly. This technique has been used to map known or suspected mass concentrations in clusters of galaxies over redshifts $0.1 < z < 0.8$ (see Fig. 8 and Clowe *et al.*, 1998) and only recently has been applied to random fields. Large mosaics of CCDs make this kind of direct mass survey possible, and results from these surveys are expected in the coming years.

Crucial to taking advantage of the advances in our understanding of the distribution of matter in the Universe and the formation of galaxies are the numerical

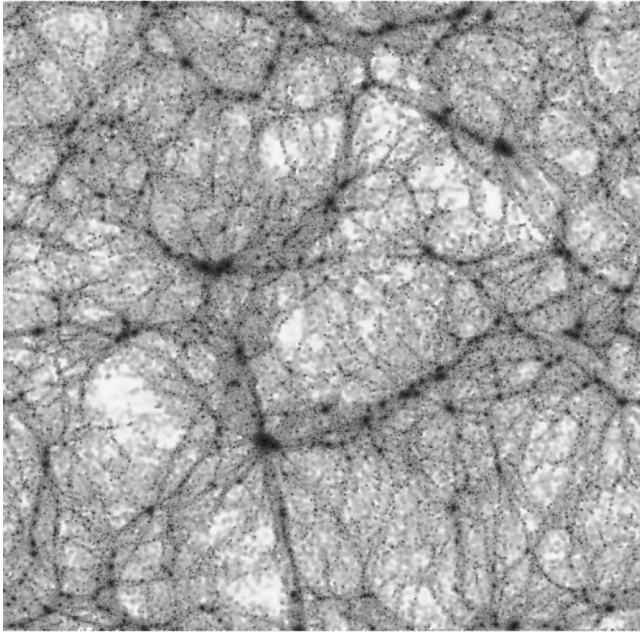


FIG. 12. One of the largest simulations of the development of structure in the Universe (from Virgo Collaboration, 1998). Shown here is projected mass in a Λ CDM simulation with 256^3 particles and $\Omega_M=0.3$, $240 \text{ Mpc } h^{-1}$ on a side. The map shown in Fig. 8 would correspond to a window 0.5 Mpc across, centered on one of the minor mass concentrations.

simulations that link theory with observation. Simulations now involve billions of particles, allowing a dynamical range of a factor of one thousand (see Fig. 12). Many simulations now involve not only gravity, but the hydrodynamics of the baryons. Advances in computing have been crucial.

Impressive progress has been made toward measuring the cosmological parameters H_0 , q_0 , and t_0 , and more progress is on the horizon. A 5% or better measurement of the Hubble constant on scales that are a substantial fraction of the distance across the Universe may be within our grasp. Techniques that do not rely upon phenomenological standard candles are beginning to play an important role. The time delay of a flare event seen in the multiple images of a lensed quasar is related only to the redshifts of the lens and quasar, the lens magnification, the angular separation of the quasar images, and the Hubble constant. Thanks to a recent flare, an accurate time delay between the two images of the gravitationally lensed quasar Q0957+561 has been reliably determined, but the lens itself must be mapped before H_0 is precisely determined (Kundic *et al.*, 1997). This technique is being applied to other lensed quasar systems as well. The pattern of CMB anisotropy has great potential to accurately determine H_0 . Another technique (Sunyaev-Zel'dovich, or SZ), which uses the small distortion of the CMB when viewed through a cluster containing hot gas (due to Compton up-scattering of CMB photons), has begun to produce reliable numbers (Birkinshaw, 1998).

Currently, the largest gap in our knowledge of the mass content of the Universe is identifying the bulk of

the matter density, $\Omega_\gamma = \Omega_M - \Omega_B \sim 0.3$. The most compelling idea is that this component consists of relic elementary particles, such as neutralinos, axions, or neutrinos. If such particles comprise most of the dark matter, then they should account for most of the dark matter in the halo of our own galaxy and have a local mass density of around $10^{-24} \text{ g cm}^{-3}$. Several laboratory experiments are currently running with sufficient sensitivity to search directly for neutralinos of mass $10 \text{ GeV} - 500 \text{ GeV}$ and cross section that is motivated by the minimal supersymmetric standard model. While the supersymmetric parameter space spans more than three orders of magnitude in cross section, even greater sensitivities are expected in the near future. These experiments involve high-sensitivity, low-background detectors designed to detect the small (order keV) recoil energy when a neutralino elastically scatters off a nucleus in the detector; the small rates (less than one scattering per day per kg of detector) add to the challenge (Sadoulet, 1999).

An axion detector has achieved sufficient sensitivity to detect halo axions, and is searching the mass range $10^{-6} \text{ eV} - 10^{-5} \text{ eV}$ where axions would contribute significantly to the mass density. This detector, based upon the conversion of axions to photons in strong magnetic field, consists of a hi-Q cavity immersed in a 7 Tesla magnetic field and is operating with a sensitivity of 10^{-23} W in the GHz frequency range. Within five years it is hoped that the entire theoretically favored mass range will be explored (Rosenberg, 1998).

While light neutrinos are no longer favored by cosmologists for the dark matter, as they would lead to structure in the Universe that is not consistent with what we see today, because of their large numbers, 113 cm^{-3} , they could be an important component of mass density even if only one species has a tiny mass:

$$\Omega_\nu = \sum_i (m_{\nu_i}/90h^2 \text{ eV}),$$

$$\Omega_\nu/\Omega_{\text{lum}} \approx \sum_i (m_{\nu_i}/0.2 \text{ eV}),$$

$$\Omega_\nu/\Omega_B \approx \sum_i (m_{\nu_i}/2 \text{ eV}). \quad (11)$$

Even with a mass as small as one eV neutrinos would make an imprint on the structure of the Universe that is potentially detectable.

Particle theorists strongly favor the idea that neutrinos have small, but nonzero mass, and the see-saw mechanism can explain why their masses are so much smaller than the other quarks and leptons: $m_\nu \sim m_{q,l}^2/M$ where $M \sim 10^{10} \text{ GeV} - 10^{15} \text{ GeV}$ is the very large mass of the right-handed partner(s) of the usual left-handed neutrinos (see, e.g., Schwarz and Seiberg, 1999). Because neutrino masses are a fundamental prediction of unified field theories, much effort is directed at probing neutrino masses. The majority of experiments now involve looking for the oscillation of one neutrino species into another, which is only possible if neutrinos have

mass. These experiments are carried out at accelerators, at nuclear reactors, and in large-underground detectors such as Super-Kamiokande (Super-K) and the SNO facility.

Super-K detects neutrinos from the sun and those produced in the earth's atmosphere by cosmic-ray interactions. For several years now the solar-neutrino data has shown evidence for neutrino oscillations, corresponding to a neutrino mass-difference squared of around 10^{-5} eV^2 or 10^{-10} eV^2 , too small to be of cosmological interest (unless two neutrino species are nearly degenerate in mass). The Super-K Collaboration recently announced evidence for neutrino oscillations based upon the atmospheric neutrino data. Their results, which indicate a mass difference squared of around $10^{-3} - 10^{-2} \text{ eV}^2$ (Fukuda *et al.*, 1998) and imply at least one neutrino has a mass of order 0.1 eV or larger, are much more interesting cosmologically. Over the next decade particle physicists will pursue neutrino mass with a host of new experiments, characterized by very long baselines (neutrino source and detector separated by hundreds of kilometers) and should clarify the situation.

A. Testing inflation+CDM in the precision era

As we look forward to the abundance (avalanche) of high-quality observations that will test inflation+CDM, we have to make sure the predictions of the theory match the precision of the data. In so doing, CDM+inflation becomes a theory with ten or more parameters. For cosmologists, this is a bit daunting, as it may seem that a ten-parameter theory can be made to fit any set of observations. This will not be the case when one has the quality and quantity of data that are coming. The standard model of particle physics offers an excellent example: it is a 19-parameter theory, and because of the high quality of data from experiments at high-energy accelerators and other facilities it has been rigorously tested, with parameters measured to a precision of better than 1% in some cases.

In fact, the ten parameters of CDM+inflation are an opportunity rather than a curse: Because the parameters depend upon the underlying inflationary model and fundamental aspects of the Universe, we have the very real possibility of learning much about the Universe, inflation, and perhaps fundamental physics. The ten parameters can be split into two groups: cosmological and dark-matter.

1. Cosmological parameters

- (1) h , the Hubble constant in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$.
- (2) $\Omega_B h^2$, the baryon density.
- (3) n , the power-law index of the scalar density perturbations. CMB measurements indicate $n = 1.1 \pm 0.2$; $n = 1$ corresponds to scale-invariant density perturbations. Several popular inflationary models predict $n \approx 0.95$; range of predictions runs from 0.7 to 1.2.

- (4) $dn/d \ln k$, “running” of the scalar index with comoving scale ($k = \text{wave number}$). Inflationary models predict a value of $\mathcal{O}(\pm 10^{-3})$ or smaller.
- (5) S , the overall amplitude squared of density perturbations, quantified by their contribution to the variance of the quadrupole CMB anisotropy.
- (6) T , the overall amplitude squared of gravitational waves, quantified by their contribution to the variance of the quadrupole CMB anisotropy. Note, the COBE normalization determines $T + S$ (see below).
- (7) n_T , the power-law index of the gravitational wave spectrum. Scale invariance corresponds to $n_T = 0$; for inflation, n_T is given by $-\frac{1}{2}(T/S)$.

2. Dark-matter parameters

- (1) Ω_ν , the fraction of critical density in neutrinos ($= \sum_i m_{\nu_i} / 90 h^2$). While the hot dark matter theory of structure formation is not viable, it is possible that a small fraction of the matter density exists in the form of neutrinos.
- (2) Ω_X , the fraction of critical density in a smooth component of unknown composition and negative pressure ($w_X \lesssim -0.3$); the simplest example is a cosmological constant ($w_X = -1$).
- (3) g_* , the quantity that counts the number of ultrarelativistic degrees of freedom (at late times). The standard cosmology/standard model of particle physics predicts $g_* = 3.3626$ [photons in the CMB + three massless neutrino species with temperature $(4/11)^{1/3}$ times that of the photons]. The amount of radiation controls when the Universe became matter dominated and thus affects the present spectrum of density fluctuations.

The parameters involving density and gravitational-wave perturbations depend directly upon the inflationary potential. In particular, they can be expressed in terms of the potential and its first two derivatives (see, e.g., Lidsey *et al.*, 1997):

$$S \equiv \frac{5 \langle |a_{2m}|^2 \rangle}{4\pi} \approx 2.2 \frac{V_* / m_{\text{Pl}}^4}{(m_{\text{Pl}} V'_* / V_*)^2}, \quad (12)$$

$$n - 1 = -\frac{1}{8\pi} \left(\frac{m_{\text{Pl}} V'_*}{V_*} \right)^2 + \frac{m_{\text{Pl}}}{4\pi} \left(\frac{m_{\text{Pl}} V''_*}{V_*} \right)', \quad (13)$$

$$T \equiv \frac{5 \langle |a_{2m}|^2 \rangle}{4\pi} = 0.61 (V_* / m_{\text{Pl}}^4), \quad (14)$$

where $V(\phi)$ is the inflationary potential, prime denotes $d/d\phi$, and V_* is the value of the scalar potential when the present horizon scale crossed outside the horizon during inflation.

As particle physicists can testify, testing a ten (or more) parameter theory is a long, but potentially rewarding process. To begin, one has to test the basic tenets and consistency of the underlying theory. Only then can one proceed to take full advantage of the data to precisely measure parameters of the theory. The importance of establishing a theoretical framework is illus-

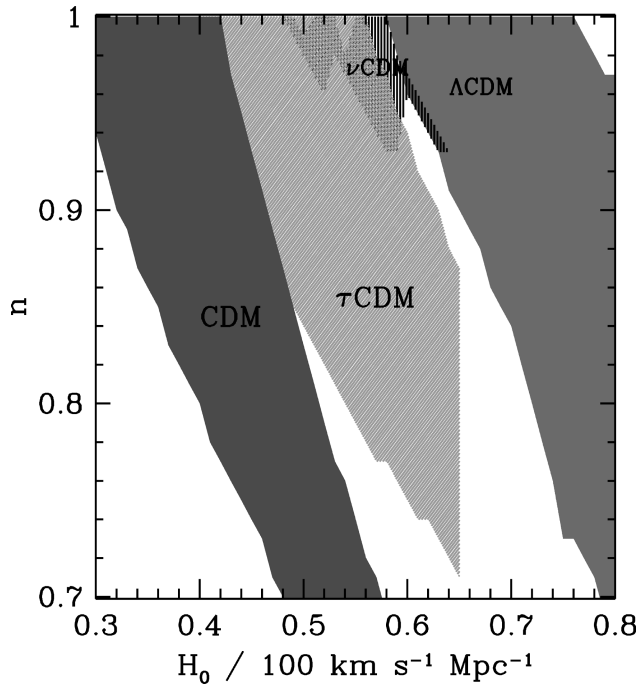


FIG. 13. Acceptable cosmological parameters for different CDM models, as are characterized by their invisible matter content: simple CDM (CDM), CDM plus cosmological constant (Λ CDM), CDM plus some hot dark matter (ν CDM), and CDM plus added relativistic particles (τ CDM) (from Dodelson *et al.*, 1996).

trated by measurements of the number of light neutrino species derived from the decay width of the Z^0 boson: $N_\nu = 3.07 \pm 0.12$ (not assuming the correctness of the standard model); $N_\nu = 2.994 \pm 0.012$ (assuming the correctness of the standard model).

In the present case, the putative theoretical framework is inflation+CDM, and its basic tenets are a flat, critical density Universe, a nearly scale-invariant spectrum of Gaussian density perturbations, and stochastic background of gravitational waves. The first two predictions are much more amenable to testing, by a combination of CMB anisotropy and large-scale structure measurements. For example, a flat universe with Gaussian curvature perturbations implies a multipole power spectrum of well-defined acoustic peaks, beginning at $l \approx 200$ (see Fig. 5). In addition, there are consistency tests: comparison of the precise BBN determination of the baryon density with that derived from CMB anisotropy; an accounting of the dark matter and dark energy by gravitational lensing; SNe1a measurements of acceleration, and comparison of the different determinations of the Hubble constant. Once the correctness and consistency of inflation+CDM has been verified—assuming it is—one can zero in on the remaining parameters (subset of the list above) and hope to determine them with precision.

B. Present status of inflation+CDM

A useful way to organize the different CDM models is by their dark-matter content; within each CDM family,

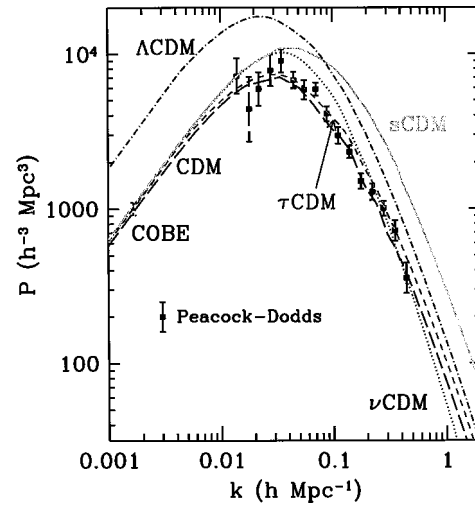


FIG. 14. The power spectrum of fluctuations today, as traced by bright galaxies (light), as derived from redshift surveys assuming light traces mass (Peacock and Dodds, 1994). The curves correspond to the predictions of various cold dark matter models. The relationship between the power spectrum and CMB anisotropy in a Λ CDM model is different, and in fact, the Λ CDM model shown is COBE normalized.

the cosmological parameters can still vary: sCDM (for simple), only CDM and baryons; τ CDM: in addition to CDM and baryons additional radiation (e.g., produced by the decay of an unstable massive tau neutrino); ν CDM: CDM, baryons, and a dash of hot dark matter (e.g., $\Omega_\nu = 0.2$); and Λ CDM: CDM, baryons, and a cosmological constant (e.g., $\Omega_\Lambda = 0.6$). In all these models, the total energy density sums to the critical energy density; in all but Λ CDM, $\Omega_M = 1$.

Figure 13 summarizes the viability of these different CDM models, based upon CBR measurements and current determinations of the present power spectrum of fluctuations (derived from redshift surveys; see Fig. 14). sCDM is only viable for low values of the Hubble constant (less than $55 \text{ km s}^{-1} \text{ Mpc}^{-1}$) and/or significant tilt (deviation from scale invariance); the region of viability for τ CDM is similar to sCDM, but shifted to larger values of the Hubble constant (as large as $65 \text{ km s}^{-1} \text{ Mpc}^{-1}$). ν CDM has an island of viability around $H_0 \approx 60 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $n \approx 0.95$. Λ CDM can tolerate the largest values of the Hubble constant.

Considering other relevant data too—e.g., age of the Universe, determinations of Ω_M , measurements of the Hubble constant, and limits to Ω_Λ — Λ CDM emerges as the “best-fit CDM model” (see, e.g., Krauss and Turner, 1995; Ostriker and Steinhardt, 1995). Moreover, its “key signature,” $q_0 \sim -0.5$, may have been confirmed. Given the possible systematic uncertainties in the SNe1a data and other measurements, it is premature to conclude that Λ CDM is anything but the model at which to take aim.

IX. THE NEXT HUNDRED YEARS

The progress in cosmology over the last hundred years has been stunning. With the hot big-bang cosmol-

ogy we can trace the history of the Universe to within a fraction of a second of the beginning. Beyond the standard cosmology, we have promising ideas, rooted in fundamental theory, about how to extend our understanding to even earlier times addressing more profound questions, e.g., inflation+cold dark matter. While it remains to be seen whether or not the expansion is accelerating, it is a fact that our knowledge of the Universe is accelerating, driven by new observational results. Cosmology seems to be in the midst of a golden age; within ten years we may have a cosmological theory that explains almost all the fundamental features of the Universe—the smoothness and flatness, the heat of the CMB, the baryon asymmetry, and the origin of structure.

There are still larger questions to be answered and to be asked. What is the global topology of the Universe? Did the Universe begin with more than four dimensions? Is inflation the dynamite of the big bang, and were there other such big bangs? Are there cosmological signatures of the quantum gravity epoch?

It is difficult—and dangerous—to speculate where cosmology will go in the next twenty years, let alone the next hundred. One can never predict the serendipitous discovery that radically transforms our understanding. In an age of expensive, complex and highly focused experiments, we must be especially vigilant and keep an open mind. And it can be argued that the two most important discoveries in cosmology—the expansion and the CMB—were unexpected. In astrophysics, it is usually a safe bet that things are more complicated than expected. But then again, Einstein's ansatz of large-scale homogeneity and isotropy—made to make the equations of general relativity tractable—turned out to be a remarkably good description of the Universe.

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