RMP Colloquia

This section, offered as an experiment beginning in January 1992, contains short articles intended to describe recent research of interest to a broad audience of physicsts. It will concentrate on research at the frontiers of physics, especially on concepts able to link many different subfields of physics. Responsibility for its contents and readability rests with the Advisory Committee on Colloquia, U. Fano, chair, Robert Cahn, S. Freedman, P. Parker, C. J. Pethick, and D. L. Stein. Prospective authors are encouraged to communicate with Professor Fano or one of the members of this committee.

Barrier interaction time in tunneling

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Over sixty years ago, it was suggested that there is a time associated with the passage of a particle under a tunneling barrier. The existence of such a time is now well accepted; in fact the time has been measured experimentally. There is no clear consensus, however, about the existence of a simple expression for this time, and the exact nature of that expression. The proposed expressions fall into three main classes. The authors argue that expressions based on following a feature of a wave packet through the barrier have little physical significance. A second class tries to identify a set of classical paths associated with the quantum-mechanical motion and then tries to average over these. This class is too diverse to permit assessment as a single entity. The third class invokes a physical clock involving degrees of freedom in addition to that involved in tunneling. This not only is a prescription for the derivation of expressions for the traversal time but also leads to a direct relationship to experiment.

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I. INTRODUCTION: THREE MAIN APPROACHES

The escape of a particle from a large vessel, through a narrow tube, can be characterized by two time scales. First of all there is the relatively long lifetime of the particle in the vessel, and then there is the typically much shorter time spent in the narrow tube. In the case of tunneling out of a well, through a barrier, every elementary quantum mechanics text tells us how to calculate the escape rate and the lifetime in the initial well. Since MacColl (1932) there have been repeated attempts to address the question: Is there a time analogous to the classical time spent in the escape tube, for the particle tunneling through a barrier? This subject has become very visible in the past decade and has been covered in a number of earlier reviews (Hauge and Støvneng, 1989; Büttiker, 1990; Leavens and Aers, 1990; Jauho, 1991, 1992; Jonson, 1991; Olkholovsky and Recami, 1992). This is a field with a diversity of viewpoints, without a clear consensus. A broad discussion in this field is, inevitably, an expression of a personal view, and that is characteristic of the cited reviews, as well as of this paper.

Many, though not all, of the approaches fit into three categories. First of all we can follow wave packets incident on the barrier, through the barrier. In this approach, typically, some feature of the incident wave packet and the comparable feature of the transmitted packet are identified. Most commonly the peak or the centroid are invoked, and a delay is calculated. This is, very likely, the most common approach and, in our opinion, the one that deserves least attention. Büttiker and Landauer (1982) pointed out that an incoming peak or centroid does not, in any obvious physically causative sense, turn into an outgoing peak or centroid.

The fact that arriving peaks do not turn into transmitted peaks was made more explicit by Landauer and Martin (1992) through analysis of a specific example. In this example a typical Gaussian minimal uncertainty wave packet is prepared, at a point far from the barrier, and sent toward it. Propagation of the packet is dispersive, and it is the high-energy components of the packet that reach the barrier first and, as a result of their higher energy, are transmitted more effectively than later portions. The relevant parameters can easily be chosen so that the transmitted packet comes almost entirely from the very front of the incident packet. Thus the peak, or centroid, of the transmitted packet can leave the barrier long before the peak, or centroid, of the incident packet has arrived, demonstrating the lack of a causal relationship.

The wave-packet approach is beset by other difficulties. As already suggested, the calculated delay depends on the details of the packet's construction. The packet can include components at an energy way above the barrier, transmitted most effectively, and the delay is then actually unconnected to tunneling. Simple barriers transmit higher-energy components more effectively; even if there are no components above the barrier, the emerging packet will have a higher mean velocity. Thus a simple barrier is an electron accelerator. If we measure a delay between two points, both far from the barrier and on opposite sides of the barrier, then insertion of a barrier can actually speed up the propagation (Hartman, 1962; Büttiker, 1983; Leavens and Aers, 1989a)!

Another wave-packet approach problem: electron wave-packet delays are not measured easily, and none of the proposed or performed experiments (Landauer, 1989) do that. Measuring the arrival time of a packet, at a point just ahead of the barrier, is a quantum mechanically invasive process, which cannot leave the packet unaltered. One might, in principle, have an apparatus far from the barrier, which launches carefully prepared packets, e.g., through opening and closing a reservoir gate. The packet's propagation toward the barrier can then be calculated, and we only need to measure the arrival time on the far side of the barrier. An analytical approach related to this possibility, and which avoids the need to identify an incoming peak or centroid, is due to Stevens (1980, 1983) and predates most of our other citations. This approach has been evaluated critically by Teranishi *et al.* (1987), Jauho and Jonson (1989), and Landauer (1991). The behavior of electrons contrasts with that of photons (Martin and Landauer, 1992). We can stuff many identical photons into a packet and sample only a small proportion to make a barrier arrival measurement.

We do not assert that *all* electron wave-packet analyses inevitably lead to an incorrect answer. Rather, we assert that it is the responsibility of wave-packet analysis proponents to provide a better physical justification. Simply restricting wave packets to a very narrow range does not help; that only gives the identification of a feature, such a peak, even less significance, as a result of the slow variation of the envelope.

There is a frequently expressed prejudice, mostly in casual conservation, that it is not allowable to talk about a traversal time for quantum-mechanical tunneling. That reaction is, very likely, rooted in the presumption that the time must be measured with clocks that determine the particle's passage past specific points. For example, Dumont and Marchioro (1993) state: "How much time does a tunneling particle spend under the barrier? Quantum mechanically [the] question, which seeks a 'dwell time' for the underbarrier region, has no meaning as it requires the simultaneous measurement of incompatible observables."

Earlier we mentioned three broad classes for analysis of traversal time. The two other classes will receive more detailed attention in separate sections. In one of these classes we determine a set of dynamic paths $x(t)$ and ask how long each path spends in the barrier. Then we, somehow, average over the set. The paths can be found through the Feynman path-integral formulation, through the Bohm approach, or through the Wigner distribution. These approaches naturally yield a distribution of times, not just a single time scale. Whether traversal time is best viewed as such a distribution, or as a single time scale indicator, is still an open question in the opinion of the authors.

The third class, and our clear favorite, invokes a physical clock (Peres, 1980), which can be used to determine a time elapsed during tunneling. There is a wide choice for the physical degree of freedom constituting the clock. More generally, we can measure the effect of the clock on the tunneling particle or else analyze the change in clock variable. In the latter case, however, unless the clock is carried by the particle, it may be difficult to distinguish between the effect of particles that are eventually transmitted and those that are eventually reflected (Leavens, 1993b). Our primary interest in this paper will be in the transmitted particles. The clock can be an externally imposed time-dependent part of the Hamiltonian. Clocks can be chosen so as to be minimally invasive. Do all the possible clocks give us the same answer? No, they do not, but there is a wide range of overlap and similarity. The strength of the clock approach: It is not only a path to analysis, but also tells us how to do experiments.

Another frequent informal reaction: Do we need to bother about traversal time; is it an indispensable concept? Of course it is not. Physics is full of alternative viewpoints, e.g., Newton's equations versus Lagrange's equations. Quantum mechanics allows a Heisenberg formulation, the Schrödinger equation, the Feynman path-integral approach, plus a myriad of less widely accepted versions. Traversal time, like the uncertainty principle, is a useful conceptual tool. It does not permit us to find answers, unavailable in other ways. The concept can be avoided if it is not to your taste.

There is no copyright on the expressions *traversal time* and *tunneling time;* each author can choose an interpretation. If an investigator wants to associate it with the time required to write the Bardeen tunneling Hamiltonian on the blackboard, we cannot say that is wrong. We can only ask if this is a fruitful view, and we can ask if it is relatable to experiment. We can also ask whether it is a natural and analytic continuation of formulations that work when there is no ambiguity, e.g., classical motion over a barrier.

Attitudes toward traversal time are, as in any area of *uncertainty,* often tied to a particular background. Thus, for example, Ranfagni *et al.* (1993) tell us that "there is not a fairly clear definition of such a quantity." But their experiment and work is totally concerned with wave-packet following in microwave situations that are analogous to tunneling. Leavens (1993a, 1993b) tells us that traversal time is not "a meaningful concept *within conventional interpretations* of quantum mechanics." This is an investigator with a strong affinity for the Bohm viewpoint, and an investigator whose earlier papers emphasized a relation that we shall refute in Sec. IV. But that relationship is valid within the Bohm viewpoint.

Sokolovski and Connor (1993), on the other hand, tell us: "*Quantally the traversal time, like every other quantity, should be described by a wave function, i.e., the amplitude distribution for its possible values* τ." For a quantity to be interesting and useful, we do not believe that it has to be describable in that fashion. For example, spatial distributions of current near a scanning tunneling microscope tip, described by Lang (1986), are of considerable interest, even though they do not correspond to a clearly defined quantum-mechanical operator.

II. KINEMATIC PATHS

A. The Bohm view

This viewpoint, in relationship to traversal times, has been advocated and investigated particularly by Leavens and Aers (1993), and we refer the reader to that article for the complete citation trail. The Bohm formulation provides, on the one hand, an exact equivalent to the Schrödinger equation, and on the other hand, a departure point for interpretations of quantum mechanics. In the Bohm approach we write the wave function ψ in the form $Re^{iS/\hbar}$. R and S are real. S is the solution of the Hamilton-Jacobi equation for a potential which adds a *quantum potential*

$$Q=-(\hbar^2/2m)R^{-1}\partial^2 R/\partial x^2 \tag{2.1}$$

to the actual potential V. R^2, in turn, is determined by the density variation of a set of classical particles following the paths with velocity

$$v=m^{-1}\partial S/\partial x \ , \tag{2.2}$$

reminiscent of the WKB approximation. In the latter, however, we ignore the quantum potential. In the WKB approximation S is an exact solution of the classical Hamilton-Jacobi equation in the original potential, and can be complex. Once we have a set of classical paths, it is easy to ask how long they each spend in the barrier region. Note, however, that in the Bohm approach these are classical paths in a *modified* potential, not in the potential of the original problem.

The classical paths in the Bohm approach do not cross. Consider a very long wave packet, say, a day in duration, impinging on a very short (atomic dimensions) but high barrier, which transmits only about 10^{-11} of the incident carriers. Then, in the Bohm view, all of the transmitted packet will come from about the first microsecond of the incident packet. All of the later incident packet will be reflected. The prevailing common sense view: A day is long compared to any of the kinetic times easily associated with such a short barrier. We are dealing, during most of the day, with a steady-state problem, and 10^{-11} of each incident portion is transmitted. If we have two far apart and similar wave packets in succession, separated by a long interval, and incident on a very opaque barrier, then in the Bohm interpretation all the transmitted paths still come from the very first tip of the first packet (Leavens and Aers, 1993). Note that we have not, and cannot, disprove the views advocated by Leavens and Aers (1993), in accordance with the general remarks in Sec. I. We can only stress the unexpected features resulting from this view.

Nevertheless, the Bohm interpretation of quantum mechanics, as a result of two features, provides an unambiguous way to calculate traversal time. First, because it specifies a set of classical trajectories $x(t)$, the time spent under the barrier is specified by

$$t_{ab}^{cl}=\int_0^t dt'\theta_B(x(t')) \ , \tag{2.3}$$

where θ_B takes the value 1 when the position lies in the interval $[a,b]$ (the barrier region), and zero otherwise. Equation (2.3) will also be used subsequently in the Feynman and Wigner formulations of traversal time. Note that such an expression for measuring the time spent in a region makes no distinction between particles that are transmitted or reflected, but the selection of the path does makes this distinction. As pointed out above, however, in the Bohm approach there is a clear distinction (spatio-temporal separation) between these two types of trajectories. Thus there is no confusion here.

The second advantage of the Bohm approach: it gives positive-definite answers. All trajectories are weighted by the initial particle density, resulting in a positive measure for the traversal time. As we shall see in the next sections, this is not the case for the Feynman approach nor for the Wigner approach. When comparing the traversal time obtained with the Bohm view to the wave-packet scattering approach and the approaches that make use of a clock, both of the latter yield smaller answers than the Bohm traversal time, in the cases that have been examined. The case of ideal transmission, $T\sim 1$, is an exception.

An eigenstate of the Hamiltonian defines both a particle flux j and a local density $\rho=\psi^*\psi$. These, in turn, define a local velocity through $j=\rho v$, and then a traversal time $\tau=\int dx/v$. This turns out to be the same velocity, $v=m^{-1}\partial S/\partial x$, as is obtained from the Bohm view, when applied to the eigenstate of the Hamiltonian (Jayannavar, 1987; Leavens and Aers, 1993). This expression was proposed, on the basis of the Bohm viewpoint by Spiller *et al.* (1990) and on the basis of a velocity derived from $j=\rho v$ by de Moura and de Albuquerque (1990) and by Cahay *et al.* (1992). It was pointed out by Jayannavar (1987), and by Leavens and Aers (1993), that the velocity under the barrier, evaluated this way, is associated with the combination of reflected and transmitted streams; it is inappropriate to assign this velocity simply to the transmitted stream. Jayannavar (1987) provides reasoning, based on physical plausibility, to extract a velocity applicable only to the transmitted stream, and this leads to an answer that agrees with our later Eq. (5.2). The Bohm trajectory velocity, for an energy eigenstate, always points in the direction of the particle flux. Consider the case of an approximate standing wave:

$$\psi=e^{ikx}+(1-\epsilon)e^{-ikx} \ , \tag{2.4}$$

with $\epsilon \ll 1$. The density ρ will be oscillatory, with places where the two counter-propagating waves almost cancel.

$v=j/\rho$ will be very large at these locations (Hagmann, 1993b). On the other hand, where ρ is large, v will be much smaller than $\hbar k/m$. This is a remarkable way to represent the obvious physical coexistence of two velocities $\pm\hbar k/m$ and brings any attempt to give physical significance to a Bohm velocity into question. Leavens and Aers (1993) solve this problem by ruling out the application of the Bohm approach to an eigenstate and limiting it to wave packets. When the incident, reflected, and transmitted packets are all well separated, we do not run into the difficulties emphasized via Eq. (2.4). But that still leaves the important parts of the process at the barrier.

B. Feynman path integrals

Another approach to the traversal time problem, which uses trajectories, follows naturally from the Feynman path-integral approach to quantum mechanics. We recall that this approach deals with continuous paths that are not necessarily differentiable in time. Sokolovski and Baskin (1987) applied this formulation to calculate a traversal time, using the measure of Eq. (2.3). Their way to calculate the traversal time is to average Eq. (2.3) with the weight $\exp[iS(x(t))/\hbar]$, where S is the action associated with the path $x(t)$, in a *functional* integration over all paths that start at position x_0 on the left side of the barrier and end at position x_1 after a time t on the opposite side:

$$\tau_{ab}=\frac{\int_{x(0)=x_0}^{x(t)=x_1} Dx(t')t_{ab}^{cl}(x(t'))e^{iS(x(t'))/\hbar}}{\int_{x(0)=x_0}^{x(t)=x_1} Dx(t')e^{iS(x(t'))/\hbar}} , \tag{2.5}$$

which can also be expressed as a functional derivative of the logarithm of the Feynman amplitude with respect to the potential:

$$\tau_{ab}=i\hbar\int_0^t dt'\int_B dx\frac{\delta\ln\langle x_1,t|x_0,0\rangle}{\delta V} . \tag{2.6}$$

Equation (2.5) has generated some controversy in the traversal time community, as it relies on a complex "probability amplitude distribution" (Hauge and Støvneng, 1989; Büttiker, 1990; Landauer, 1991; Jauho, 1992). The weighting of traversal time by $e^{iS/\hbar}$ is, after all, an unjustified step, and we return to this question later on in this section. In fact, Eq. (2.5) is analogous to an attempt to define an effective position for a particle with a wave function $\psi(x)$, via

$$x_{\rm eff}=\left[\int x\psi(x)dx\right]\Big/\int\psi(x)dx . \tag{2.7}$$

Obviously, if we repeatedly measure the particle position and average over the results, we would find

$$\langle x\rangle_{\rm Av}=\left[\int x|\psi^2(x)|dx\right]\Big/\int|\psi^2(x)|dx . \tag{2.8}$$

But if we avoid the measurements, leaving normal quantum interference intact, then the style of reasoning provided by Sokolovski and Connor (1993) leads us to Eq. (2.7).

To connect with the other traversal time definitions presented in this article, we need to restrict the energy of the tunneling particle to a narrow energy range, using asymptotic states to describe the initial and final states. The initial position must be chosen far away from the barrier, and the time to be measured must be proportionally long, to ensure a finite, well-defined velocity, which can in turn be related to the energy of the tunneling particle. Sokolovski and Connor (1990) adapted the path-integral approach to the wave-packet analysis, claiming that if the initial (final) state is described by the wave function ϕ_I (ϕ_T), the traversal time should be

$$\tau_{ab}=\langle\phi_T|t_{ab}^{cl}|\phi_I\rangle , \tag{2.9}$$

which implies summations over the initial and final position states, as in the definition of a *transition element* (Feynman and Hibbs, 1965).

The use of asymptotic states or wave packets can be avoided by restricting the Feynman path summation to paths that spend a specified time in the barrier region. Moreover, the energy of the tunneling particle can be determined by performing a Fourier transformation in time of the Feynman amplitude $\langle x_1,t|x_0,0\rangle$. For the square potential barrier, Fertig (1990) exploited the path decomposition of Auerbach and Kivelson (1985), introducing a propagator that corresponds to the amplitude for tunneling between two points on opposite sides of the barrier with initial energy E, summed over the Feynman trajectories that spend precisely an amount of time τ inside the barrier:

$$G_\tau(E,x_1,x_0)=\int_{-\infty}^{+\infty}dt e^{iEt/\hbar}\int_{x(0)=x_0}^{x(t)=x_1}Dx(t')\delta\left[\tau-\int_0^t dt'\theta_B(x(t'))\right]e^{iS(x(t'))/\hbar} . \tag{2.10}$$

Note that the initial and final values of x in the above expression specify whether the particle is reflected or transmitted. Equation (2.10) is a generalization of Fertig's probability amplitude distribution to arbitrary barriers.

Not surprisingly, for the square potential barrier, Fertig (1990) finds the same result as Sokolovski and Baskin (1987), expressing the average traversal time in terms of the derivative of the phase of the transmission coefficient, ϕ, and the derivative of the transmission probability, T, with respect to the potential height V_0:

$$\langle\tau\rangle=\frac{\int_0^\infty d\tau\,\tau G_\tau(E,x_2,x_1)}{\int_0^\infty d\tau\, G_\tau(E,x_2,x_1)}=-\frac{d\phi}{dV_0}+\frac{i}{2}\frac{d\ln T}{dV_0} . \tag{2.11}$$

Now, there is little doubt that the propagator $G_\tau(E,x_2,x_1)$ contains information about paths that traverse the barrier and spend a time τ in the process. But it is not clear what procedure is needed to calculate physical quantities with the Feynman approach. Proponents of this approach like to point out that quantities such as Eq. (2.5) can be related to "transition elements." Nevertheless, in their classical textbook on path integrals, Feynman and Hibbs (1965) do not provide any physical significance for the transition elements used to compute the properties of interest. Rather, an analogy with classical Brownian motion is pointed out, with the difference that the weight should be $\exp(iS/\hbar)$ for quantum systems. In this sense, the Feynman proponents go beyond Feynman's interpretation by giving a physical interpretation to Eq. (2.5). Equation (2.10) implies a distribution of traversal times that has a real and an imaginary component, with a normalization factor given by the inverse of the denominator in the ratio of integrals in Eq. (2.11). The propagator $G_\tau(E,x_1,x_0)$ is a complex quantity, but once integrated over τ from 0 to ∞, the imaginary contribution of this normalization factor averages out to zero. We do not see how to make the connection between such an object and the usual positive probability distribution.

The Feynman approach is not the first approach to yield complex times. A few years prior to Sokolovski and Baskin (1987), Pollak and Miller (1984) and Pollak (1985) used the concept of flux-flux correlation functions, which had been exploited to extract characteristic times in classical chemical systems, to calculate an average tunneling time. The average of the imaginary part of the flux-flux correlation function leads to the "imaginary" time:

$$\tau_1 = \mathrm{Im}\left[-\frac{i\hbar}{t_E}\frac{\partial t_E}{\partial E}\right], \tag{2.12}$$

where t_E is the complex transmission coefficient. As we shall see below, in the section on clocks, the oscillatory-incident-amplitude approach and the generalized modulated-barrier approach (Martin and Landauer, 1993) yield characteristic times that also depend on energy derivatives of the transmission coefficient.

We are uneasy about the significance of a complex number for a traversal time. Quantum mechanics is full of complex numbers, but only in its intermediate steps. Results that are to be compared to experiment, as will be discussed in Sec. IV, are real. Indeed, Hänggi (1993), in a paper advocating the Feynman path approach and complex traversal times, states: "*Clearly, physical answers, . . . involves real-valued objects only; i.e., one has to take the absolute value of complex quantities, or its real part, etc.*" The real and imaginary parts of Eq. (2.11) are, respectively, the Larmor clock times that will be discussed in Eqs. (5.5) and (5.6). To justify their complex result, Sokolovski and Conner (1990) state that Eq. (2.9) corresponds to an "'off orbit' matrix element," in which the initial and final states ". . . do not lie on the same orbit in Hilbert space." It is unclear whether this concept sheds any light on the physical significance of complex traversal times. Other proponents of the Feynman approach (Hänggi, 1993) simply argue that the barrier interaction time problem is characterized by two time scales, the real and imaginary parts of Eq. (2.11), rather than the single time scale advocated in our introduction. But the spirit of the derivation does not lead us to expect an answer with two parts, requiring separate interpretation. In principle, real, positive time averages can be constructed by considering the square modulus of Eq. (2.10) as a probability distribution (Sokolovski and Connor, 1993), in analogy with Eq. (2.8).

C. Wigner distribution paths

The Wigner function provides yet another definition of the interaction time in tunneling, as it resembles to some extent a classical phase-space distribution function:

$$W(q,p,t) = \frac{1}{\pi\hbar}\int_{-\infty}^{+\infty} dx\, e^{-2ipx/\hbar}\psi^*(q-x,t)\psi(q+x,t)\ . \tag{2.13}$$

For a stationary barrier traversal problem, the Wigner function does not depend on time, and a set of trajectories in phase space can be determined by demanding that the Wigner function remain constant along these trajectories (Lee and Scully, 1983). These trajectories satisfy Hamilton's equation with a modified "quantum" potential $\tilde V$ defined by the relation

$$\partial_q \tilde V \partial_p W(q,p) = \frac{2}{\pi\hbar^3}\int_{-\infty}^{+\infty} dp'\, W(q,p')\int_0^\infty dx' \sin[2x'(p-p')/\hbar][V(q+x')-V(q-x')]\ , \tag{2.14}$$

where $V(x)$ is the potential associated with the barrier. The potential $\tilde V$ defined in Eq. (2.14) enables us to write a Liouville equation for the Wigner function. With the use of Eq. (2.3), the trajectories can be used to calculate a tunneling time for stationary scattering (Jensen and Buot, 1989), with each trajectory with initial conditions (q_0,p_0) weighted by the initial distribution $W(q_0,p_0)$. For the scattering of wave packets, Muga, Brouard, and Sala (1992) followed the analogy with classical distribution functions to compute the traversal time for parabolic barriers using Eq. (2.3).

The Wigner approach has limitations. Sala *et al.* (1993) have shown that in the general case the Wigner function does not satisfy Liouville's theorem globally: portions of phase space with a positive Wigner function may evolve into portions that bear *negative* weights, a problem that had been pointed out earlier (Lee and Scully, 1983). Wigner trajectories are well defined as long as the force $-\partial_x \tilde{V}$ associated with the Wigner potential has no singularities. However, if an initially well-behaved trajectory encounters such a singularity, the trajectory ceases to exist, which is at the origin of the violation of Liouville's theorem.

Before concluding this set of discussions on kinematic paths, we cite one more approach (Hagmann, 1992). Consider a particle tunneling through a rectangular barrier as *borrowing* an energy ΔE, to be able to cross the barrier with a real velocity, in time Δt. Then we can inquire about the minimum value of $\Delta E \Delta t$. That Δt agrees with our later Eq. (5.1). That is a pleasant coincidence, but it is not clear why the argument about borrowed energy has real validity.

III. RELATION OF DWELL TIME TO REFLECTION AND TRANSMISSION

In analogy to the traversal time τ_T, associated with the transmitted portion, we can equally associate a time τ_R with the reflected portion (Büttiker and Landauer, 1982; Büttiker, 1983). The literature also invokes a dwell time τ_D, most often defined as

$$\tau_D = \frac{1}{j}\int_B |\psi^2| dx \ . \tag{3.1}$$

Here j is the incident of flux and the integration extends over the barrier. Equation (3.1) gives us the time that the incident flux has to be turned on, to provide the accumulated particle storage in the barrier. It is also possible to define a time, denoted by τ_D', which simply averages over the weighted reflection and transmission times:

$$\tau_D' = T\tau_T + R\tau_R \ . \tag{3.2}$$

A remarkably common assertion, typically made without any attempt at justification, or even awareness that it is an assertion, is

$$\tau_D = \tau_D' \ . \tag{3.3}$$

This identity is accepted by Leavens and Aers (1987, 1990), Golub *et al.* (1990), Jonson (1991), and Jauho (1992), and many others, and is a principal point of focus for an earlier review (Hauge and Støvneng, 1989). The Bohm approach (Leavens and Aers, 1993) gives results that are consistent with Eq. (3.3).

The relation is questioned by Olkhovsky and Recami (1992), and by Brouard *et al.* (1983) for reasons similar to those we shall present here. Equation (3.3) is obviously applicable in the classical case, e.g., a diffusion process (Landauer and Büttiker, 1987), where inside the barrier the density can be decomposed into the sum of two contributions. One of these is due to the particles that are eventually transmitted and the other comes from the particles that are eventually turned around to the space from which they entered. We see no reason, *whatsoever,* why Eq. (3.3) can be expected to hold for the quantum mechanically coherent case. Note that Eqs. (3.1)–(3.3) imply

$$\int_B dx|\psi|^2 = jT\tau_T + jR\tau_R \ . \tag{3.4}$$

Thus the integrated density under the barrier consists of additive contributions from the transmitted and from the reflected beams. In quantum mechanics, however, we do not add particle densities; we add complex wave functions! [The interference effects, produced by the addition of wave functions rather than densities, can cancel under some fortuitous conditions. Such an approximate cancellation is discussed by Hauge and Støvneng (1989)]. Büttiker (1990), following a suggestion by Pippard (1982), calculated the diminution of transmitted particles in a barrier that absorbs, i.e., has an imaginary component to the barrier potential. The resulting reduction turns out to be proportional to the dwell time, rather than the traversal time (Büttiker, 1990; Golub *et al.*, 1990; Huang and Wang, 1991). The physical interpretation for this result: The absorption acts on *all* the particles in the barrier, they cannot—within the barrier—be separated into a transmitted component and a reflected component.

IV. EXPERIMENTS

The range of proposed and completed experiments, closely related to the notion of clocks in Sec. V, was reviewed by Landauer (1989). In this discussion we shall allude only to the two most significant of these, and to one additional experiment unrelated to clocks. In one of the clock-related experiments (Guéret *et al.*, 1988a, 1988b) the tunneling current through a long low heterojunction barrier is measured. On either side of the barrier we have degenerate electron gases. In these the electron is surrounded by an exchange-correlation hole. As the electron approaches the interface to the barrier, the hole has to spread, and after the electron enters the barrier, the spreading hole becomes the classical electrostatic image charge. If there is time for this spreading to take place, we can expect to find the customary image force component in the surface barrier. If, on the other hand, the electron moves too quickly away from the interface, then there is no time for the spreading. This leads to a larger force on the electron moving into the barrier, and therefore a larger barrier. That in turn implies a smaller tunneling current and smaller conductance. This dynamic screening was first analyzed in a pioneering discussion by Jonson (1980). The question was reexamined by Persson and Baratoff (1988) in the context of scanning tunneling microscope experiments, in which the tunneling electron interacts with virtual surface plasmon modes in order to "construct" the image charge potential. Following these ideas, Guéret *et al.* (1988a, 1988b) assume that the time scale of the surface spreading is related to a typical plasma frequency ω_p, and the speed of electron motion is related to the traversal time τ_T, which we shall specify in Eq. (5.2). Thus one might expect a transition in behavior, depending on the value of $\omega_p \tau_T$. This product can be varied by changing the concentration of carriers in the degenerate electron gas serving as electrode, and also by changing barrier width. Guéret *et al.* (1988a, 1988b), do find a transition at $\omega_p \tau_T \sim 4$. For higher values of $\omega_p \tau_T$, the measured conductance agrees with transmission as computed from the image barrier. At lower values of $\omega_p \tau_T$, a smaller conductance is observed.

Now for some caveats. First of all, data obtained from a range of separate samples are always open to the question: what unintended changes accompany the intended changes? Furthermore, while the experimental results agree with τ_T as stated in Eq. (5.2), the data are not really definitive enough to rule out alternative theories. Finally, Eq. (5.2) weights all the parts of the passage under the barrier equally. It is not obvious that this is the correct approach for calculating an effective barrier height. A more detailed analysis can be found in Jonson (1991).

The other significant experiment (Esteve *et al.*, 1989) relates to Josephson junctions under a fixed current bias. The energy as a function of θ, the phase difference across the junction, is

$$U = (J_0 \cos\theta - j\theta)\hbar/2e \ , \tag{4.1}$$

where j is the applied current and J_0 the maximum current the junction can carry without a voltage. This well-known tilted sinusoid has local minima if $|j| < J_0$, and the circuit can tunnel out of these states of local stability. For $j > 0$, tunneling will be to the right and, in accordance with $d\theta/dt = 2eV/\hbar$, a voltage will develop. This voltage causes energy loss in any normal resistance shunting the Josephson junction, and this in turn is known to reduce the tunneling rate (Leggett, 1992). In a well-characterized circuit, this reduction can be used to obtain information about the value of the shunting conductance. In the experiment of Esteve *et al.* (1989), the shunt consists of a transmission line mechanically variable in length, with a terminating resistor that does not match the characteristic impedance of the line and produces reflections. Now, if the traversal time is less than the round-trip time on the transmission line, then the energy extraction from the junction depends entirely on the transmission line parameters; the terminating resistor is not visible to the junction during tunneling. On the other hand, if the round-trip time on the line is less than the traversal time, then the losses will depend on both the parameters of the line and the value of the terminating resistance. Therefore, as the line length is varied, a transition in behavior should occur. Such a transition is seen, and the time involved agrees qualitatively with Eq. (5.2). Not enough data are available, over a range of parameters, to discriminate between various theories. Note, incidentally, that the role of the relationship between the length of a transmission line and the traversal time was first discussed by Chakravarty and Schmid (1986).

Now for qualifying comments. It is, perhaps, not clear whether the experiment measures the traversal time through the classically forbidden region, or a time associated with the departure from the interior of the initial well, where the particle is in its ground state. Devoret *et al.* (1992) call it a *latency time* for escape from the metastable state. We also note that our earlier theoretical discussions emphasized incident plane waves, whereas here we start in a bound state. Within the WKB approximation, we can view a bound state as composed of two counter-propagating waves, and the transmission is a consequence of the arriving incident wave. The phase relationship between the reflected wave and the incident wave, determined at the end of the well opposite to the tunneling barrier, determines the quantized bound states, but does not really, in any strong qualitative sense, control barrier transmission.

It is characteristic of the two experiments that we have discussed that they provide a clock sensitive to the particle's interaction time with the barrier. In both cases we have degrees of freedom, surface charges in one case, transmission line modes in the other, which may or may not have time to adjust to the particle's motion, while tunneling. This is, in a sense, our central point. The coupling to a clock provides, on the one hand, the analytical process for the evaluation of a traversal time and, on the other hand, the reason for interest in the traversal time. The experimental data, while consistent with our favorite candidate for traversal time, do not yet provide a clean selection, relative to other candidates. The experiments do, however, show that the traversal time is measurable, is of physical interest, and is a real (not complex) quantity.

A remarkable recent experiment measures the actual delay of a photon passing through an evanescent wave region (Steinberg *et al.*, 1993). A two-photon coincidence experiment is used to observe photon passage through the stop band of a filter constructed from a periodic array of layers. The insertion of the sample with the filter leads to a change in delay compensated by a change in path length for the vacuum comparison photon. This experiment demonstrated that the effective velocity in the middle of the stop band exceeds the velocity of light in vacuum. That underlines the point made in Sec. I that wavepacket following does not lead to meaningful causal velocity. Earlier measurements on electromagnetic wave delays, in evanescent regions (Ranfagni *et al.*, 1991; Enders and Nimtz, 1993), lead to similar results, but suffer from technical complexities related to the exact positions of microwave probes and to the effects of geometrical discontinuities not represented in the analysis of these experiments.

V. CLOCKS

Three version of clocks have been investigated in detail in the theoretical papers; undoubtedly many more can be proposed. Some of these additional possibilities are listed by Landauer (1991). The experimental approaches discussed in Sec. IV offer still further illustrations. Azbel (1992) has analyzed a process in which thermal activation plays a role in addition to tunneling, and the thermal activation rate can serve as a clock.

Büttiker and Landauer (1982) invoked an oscillatory barrier; the original static barrier was augmented by a small oscillation in the barrier height. The amplitude of the oscillation is kept small; the disturbance of the original kinetics can be made as small as desired. At very low modulation frequencies the incident particle sees a particular part of the modulation cycle. The particle sees an effectively static barrier; later parts of the incident wave see a slightly different barrier height. As we turn up the modulation frequency we eventually reach a range where an incident particle no longer sees a particular portion of the modulation cycle, but is affected by a substantial part of the cycle, or several cycles. The frequency at which this transition occurs, i.e., the frequency where we begin to deviate substantially from the adiabatic approximation, is an indication of the length of time that a particle interacts with the barrier. It is, of course, an approximate indication of a time scale. It is not the eigenvalue of a Hamiltonian, indicative of a precisely measurable value. As already indicated, this traversal time value may really be characteristic of a statistical distribution.

The possibility of comparing the speed of two simultaneous processes, as a clock for one of them, is well established in physics. One version (Zener, 1948) is particularly close to the analysis by Büttiker and Landauer (1982) and was a model for that. Consider an atom in a lattice that can jump between two positions. Now apply an oscillatory small stress favoring one of the two positions at a particular part of the cycle. At frequencies low compared to the jump rate, the atom will see a static stress and there will be a thermal equilibrium between the two sites. At frequencies high compared to the jump rate, the atom cannot respond to the oscillations. At intermediate stress frequencies the atomic adjustment will lag the stress, and the jump is a source of energy dissipation.

For an opaque rectangular barrier, the modulated-barrier approach yields (Büttiker and Landauer, 1982, 1985)

$$\tau = dm/\hbar\kappa \tag{5.1}$$

where d is the barrier length and $\hbar\kappa$ the magnitude of the imaginary momentum under the barrier. Particles with an energy just under the barrier are characterized by a long traversal time, like those with an energy just over the barrier. For a potential that allows the WKB approximation we have

$$\tau = \int_B dx \frac{m}{\hbar\kappa(x)} . \tag{5.2}$$

An expression of this sort is familiar as the *bounce* time, for motion along an imaginary time axis, but with rare exceptions (Leggett, 1980) is not given a physical significance in such discussions.

The case of a long region of free propagation, containing a much shorter barrier, has also been analyzed by Martin and Landauer (1993). In that case

$$\tau = \hbar \left| \left(\frac{\partial t_E}{\partial E} \right)^{-1} \frac{\partial^2 t_E}{\partial E^2} \right| . \tag{5.3}$$

Here t_E is the ratio of the wave function at the far end of the interval to that of the incident wave at the beginning. If the WKB approximation is applicable, Eq. (5.3) can be shown (Martin and Landauer, 1993) to reduce to

$$\tau = \sqrt{\tau_p^2 + \tau_B^2} , \tag{5.4}$$

where τ_p is the classical transit time for the regions where $E > V$, and τ_B is the time for barrier traversal given in Eq. (5.2).

The oscillating potential produces electrons that have gained or lost a quantum at the modulation frequency. We deviate from the adiabatic behavior when the transmission of these new components differs appreciably (in phase or in amplitude) from that at the original energy. Thus τ is a measure of the energy dependence of the transmission. The specific form of modulation we have invoked, varying the height of the potential, is an arbitrary choice. We could, instead, have moved the potential barrier back and forth, or could have expanded and contracted the barrier (Büttiker, 1984; Pimpale *et al.*, 1991). While the results given in Eqs. (5.1) and (5.2) have been somewhat controversial in the Western literature, the Russian literature has exhibited the opposite tendency. There the results are often treated as obvious, requiring no justification (Keldysh, 1964; Ivlev and Mel'nikov, 1985; Kagan and Prokof'ev, 1992). Indeed, the work by Keldysh (1964), as explained by Yücel and Andrei (1992), can be taken to be an early version of the modulated-potential-barrier approach. The expression *traversal time*, introduced by Büttiker and Landauer (1982), may not have been an optimum choice. It is too easily interpreted as a time *delay* in transmission, rather than an interaction time.

A second clock approach measures the precession of the spin of the tunneling particle due to a uniform infinitesimal magnetic field confined to the barrier region (Baz', 1966; Rybachenko, 1967). The incident spin is in the x direction, perpendicular both to the magnetic field (along the z axis) and to the direction of motion (along the y axis). The original scheme considered only the rotation of the spin in the plane defined by the direction of motion and the initial spin direction, and was later corrected by Büttiker (1983) for a rectangular potential barrier of height V_0 to include the spin rotation in three dimensions. Because of the Zeeman splitting, the component of the spin that is parallel to the field corresponds to a higher tunneling energy and tunnels preferentially. While tunneling, the spin acquires a component along the direction of motion as well as a component along the direction of the magnetic field:

$$\langle S_y \rangle = -\frac{\hbar^2 \omega_L}{2} \frac{\partial \phi}{\partial V_0} = -\frac{\hbar}{2} \omega_L \tau_y^L , \tag{5.5}$$

$$\langle S_z \rangle = -\frac{\hbar^2 \omega_L}{2} \frac{\partial \log T^{1/2}}{\partial V_0} = \frac{\hbar}{2} \omega_L \tau_z^L , \tag{5.6}$$

where ϕ is the phase of the transmission coefficient, ω_L is the Larmor frequency, and $\tau_{y,z}^L$ are defined by these equations. These two time scales were later found to correspond to the real and imaginary parts of the complex tunneling time obtained from the Feynman approach (Sokolovski and Baskin, 1987). Büttiker (1983) argued that a total tunneling time should be specified by measuring the *total* rotation of the spin in three dimensions. With the help of Eqs. (5.5) and (5.6), he then identified the total traversal time by

$$\tau_T^L = \sqrt{(\tau_y^L)^2 + (\tau_z^L)^2} . \tag{5.7}$$

Note the similarity to Eq. (5.4). The reasoning leading to the tunneling time of Eq. (5.7) has been questioned by some authors (Jauho, 1991; Landauer, 1991). Nevertheless, it provides a suggestive way to combine two characteristic time scales associated with the Larmor clock. A generalization of the Larmor clock analysis to arbitrary potential barriers was proposed by Leavens and Aers (1987). In this definition of the traversal time, the derivatives in Eqs. (5.5) and (5.6) are replaced by derivatives with respect to the *average* height of the barrier, keeping the spatial variation in barrier potential fixed.

Our third clock approach involves an oscillatory incident amplitude; it was proposed by Büttiker and Landauer (1985) and analyzed by Martin and Landauer (1993). The oscillation is produced by two interfering incident waves,

$$\psi_i(x,t) = e^{ikx - iE(k)t/\hbar} + e^{i(k+\Delta k)x - iE(k+\Delta k)t/\hbar} . \tag{5.8}$$

The emerging wave is

$$\psi_t(x,t) = t_E e^{ik(x-L) - iE(k)t/\hbar} + t_{E+\Delta E} e^{i(k+\Delta k)(x-L) - iE(k+\Delta k)t/\hbar} . \tag{5.9}$$

Note that we are using the convention invoked earlier in this section, in which t_E is the ratio of the transmitted wave function to the incident one. The transmitted wave faithfully reproduces the incident wave, if t_E and $t_{E+\Delta E}$ are close to each other. The departure from adiabaticity is measured by the energy scale on which t_E varies appreciably. That, in turn, determines an oscillation frequency for the amplitude of the interfering incident waves, and thus a time scale. This yields

$$\begin{aligned} \tau &= \hbar |t_E^{-1}| \, |dt_E/dE| \\ &= \sqrt{(d\phi/dE)^2 + T^{-1}(dT^{1/2}/dE)^2} , \end{aligned} \tag{5.10}$$

where $t_E = T^{1/2} e^{i\phi}$. Both T and ϕ are real. Note the similarity to Eq. (5.4). This approach can, perhaps, be made more convincing by superimposing two incident oscillatory waves of the form of Eq. (5.8), but with opposite spins. The two modulation envelopes are chosen to be $\pi/2$ out of phase:

$$\begin{aligned} |\psi\rangle(x,t) = {} & [e^{ikx - iE(k)t/\hbar} + e^{i(k+\Delta k)x - iE(k+\Delta k)t/\hbar}] |\uparrow\rangle \\ & + [e^{ikx - iE(k)t/\hbar} - e^{i(k+\Delta k)x - iE(k+\Delta k)t/\hbar}] |\downarrow\rangle , \end{aligned} \tag{5.11}$$

so that the total incident-particle current is uniform in time:

$$J_{\text{charge}} \equiv \frac{J_\uparrow + J_\downarrow}{2} = \frac{\hbar}{m}(k + \Delta k/2) , \tag{5.12}$$

but bears an oscillatory spin:

$$\begin{aligned} J_{\text{spin}} &\equiv \frac{J_\uparrow - J_\downarrow}{2} \\ &= \frac{2\hbar}{m} \left[k + \frac{1}{2}\Delta k \right] \cos \left[x\Delta k - t \frac{\Delta k}{\hbar} \frac{\partial E}{\partial k} \right] . \end{aligned} \tag{5.13}$$

Thus the particle carries its own clock. Is the emerging spin appreciably different from the incident spin?

The modulated-amplitude approach shares one of the shortcomings of the wave-packet approach; it is not very directly relatable to experiment. The incident spin cannot be measured noninvasively; the oscillatory spin must be prepared in a known way at a source. An additional problem with the incident-oscillatory-spin approach arises from the fact that it involves two different energies. It does not give us a clean way to assign a traversal time to a *particular* energy. In other ways, however, the approach seems more clearly founded than the modulated-potential-barrier approach or the Larmor clock. In contrast to the modulated barrier, we do not need arbitrary choices regarding the aspect of the barrier that is to be modulated. Furthermore, this incident-spin clock approach is immediately applicable to all potentials. In contrast to the Larmor clock approach, the incident-spin clock approach does not require indirect analogies to two-level systems for the extraction of a time scale.

The three clock methods we have discussed give closely related answers, in many cases identical answers. The modulated-barrier approach and the incident-spin clock pose different physical questions and we cannot expect the answer to be identical in all cases. These similarities and differences have been discussed extensively in the detailed literature (Leavens and Aers, 1987, 1988; Büttiker, 1990; Landauer, 1991; Hagmann, 1993a; Martin and Landauer, 1993), and there is little purpose in detailed analytical comparisons here.

The notion of a clock as auxiliary degrees of freedom, coupled to the tunneling process, is closely related to the effect of an *environment* on the tunneling process as studied in many papers, of which we can only cite a small sample (Hedegård, 1987; Karlsson, 1989; Eckern and Schmid, 1992; Kagan and Leggett, 1992; Leggett, 1992). A primitive connection between traversal time and the reduction of tunneling observed in the presence of friction (Leggett, 1992) was already discussed by Büttiker and Landauer (1982). More detailed and authoritative connections, however, remain to be made.

VI. MISCELLANEOUS COMMENTS

A. Periodic structures

In a one-dimensional periodic structure of length L, within an allowed zone, we have

$$t_E = e^{ikL} M \tag{6.1}$$

where k is the Bloch wave vector and M represents terms due to the matching processes at the ends. For large L the primary energy dependence comes from k, not M, and Eqs. (5.10) and (6.1) yield

$$\tau_T \sim \hbar |t_E^{-1} dt_E / dE| \cong L(\hbar\, dk/dE) = L/v_g \ , \tag{6.2}$$

where v_g is the usual in-band group velocity. Now dk/dE varies within the band, but its typical order of magnitude is $\Delta k/\Delta E$, where we can take Δk and ΔE to characterize a half-band, say, the part with positive k. Then ΔE is the bandwidth and $\Delta k = (\pi/a)$ where a is a unit cell. Therefore

$$\tau_T = \hbar L \frac{\pi}{a} \frac{1}{\Delta E} = N \frac{\pi\hbar}{\Delta E} \ , \tag{6.3}$$

where $N = L/a$, the number of unit cells. $\pi\hbar/\Delta E$ is the time determined by the bandwidth, i.e., the level broadening determined by the coupling between adjacent cells in a tight-binding model. This closely relates to analyses of traversal time for resonant tunneling, in which we have only a single well instead of a long chain (Leavens and Aers, 1989b; Büttiker, 1990; Jiang, 1990; Sheng and Sinkkonen, 1990; Björksten *et al.*, 1991; Jauho, 1991; Muga and Cruz, 1992; Pastawski, 1992; and many others). In the tight-binding limit we note that ΔE, related to the lifetime in a well, determines τ_T. The details of the barrier shape are irrelevant. It is the time of motion back and forth within the well, between tunneling events, that counts.

B. Anomalously short traversal times

This field contains a number of allusions to barrier traversal times that seem excessively short, corresponding to superluminal or even negative velocities. The easiest way to obtain these is to follow peaks or centroids, which permit a transmitted peak to depart before the incident one has arrived. This has been observed experimentally, in the case of an absorbing medium, for photons (Chu and Wong, 1982), as well as for evanescent waves (Steinberg *et al.*, 1993). As emphasized in Sec. I, no physical significance can be attached to delays based on peak motion. Another physically transparent way to achieve large velocities uses the effective electron acceleration of simple barriers that transmit higher energies more effectively. Thus insertion of a barrier in a region that includes a long stretch for the transmitted wave can give arbitrarily large reductions in almost any sensible definition of a time associated with traversal of the large region.

A less trivial problem occurs when we view Eq. (5.10), $\tau = \hbar |t_E^{-1}(\partial t_E/\partial E)|$, in a number of situations other than the simple rectangular barrier. These are situations in which t becomes relatively insensitive to energy in some energy region. Consider, first, a periodic potential sandwiched between regions of constant potential on either side. Near the middle of a forbidden band the exponential decay due to Bragg reflections reaches an extremum. Furthermore, the decaying wave, in the forbidden region, is characterized only by a cell-to-cell decay and perhaps a sign change, but no phase shift. Thus $\partial t_E/\partial E$, to the extent it results from the Floquet factor, vanishes. There will be additional contributions to τ due to matching at the interfaces. But for an opaque region these will be independent of length and can correspond to arbitrarily large velocities if the periodic region is made sufficiently long. We can get similar behavior from other physical situations. Consider a set of torsion-coupled pendulums, a system giving rise to sine-Gordon solitons. For small excursions about the ground state this system is governed by a dispersion equation

$$\omega^2 = \omega_0^2 + c^2 k^2 \ , \tag{6.4}$$

where ω_0 is the angular pendulum frequency and c the transverse velocity along the coupling chain, without a restoring force on the pendulums. For $\omega < \omega_0$ we have evanescent waves. At $\omega = 0$, $dk/d\omega$ vanishes, again corresponding to values of τ that can correspond to arbitrarily large velocities. The velocities can be larger than the velocity of light, but also larger than the velocity c of Eq. (6.4), i.e., larger than the velocity of transverse waves in the torsion-coupled chain without a pendulum restoring force. Finally, if we have an evanescent electromagnetic wave (frustrated internal reflection), as discussed by Martin and Landauer (1992), for suitable parameter choices, anomalous velocities can be obtained. Anomalously large electromagnetic wave velocities have also been discussed by Band (1988) and Bosanac (1993). The relationship of these discussions, if any, to ours, is not clear.

In the Feynman approach to the interaction time of tunneling, Fertig (1990) attributes the appearance of anomalously short times to the fact that the propagator of Eq. (2.10) oscillates rapidly when the potential barrier height V_0 is large. However, this does not necessarily imply that the summation over trajectories is dominated by a subset of Feynman paths that cross the barrier at a superluminal velocity. Rather, in the opaque regime, there seems to be a strong interference effect between different Feynman paths within the barrier, which gives rise to short complex averaged times in the sense of the definition of Eq. (2.5).

It should be pointed out that the paradox of anomalously short traversal times is not limited to the nonrelativistic Schrödinger formulation. Leavens and Aers (1989b) calculated the Larmor clock times using the Dirac equation, for both double barriers and single barriers. While the double barrier case gives the satisfactory result that the velocity associated with the tunneling process is smaller than the velocity of light, for a simple barrier the opposite remains true, despite the use of the Dirac formulation to describe the dynamics. A simple physical resolution of the appearance of these anomalously short times is still missing. We can, of course, take refuge in the fact that Eq. (5.10) is *not* a propagation delay, and therefore causality is not clearly contradicted. Nevertheless, this remains an uncomfortable situation.

C. Kinetic delay in establishing tunneling current

A tunneling barrier is associated with a capacitance as well as a resistance, defining an RC time constant. Is our traversal time simply related to this? No, and we shall explain that.

As discussed in Sec. II, in connection with the modulated barrier approach, the traversal time represents a response time to a change in barrier height. It is *not* a response time to a change in voltage *across* the barrier. The latter has been discussed by Ivlev and Mel'nikov (1992), Tien and Gordon (1963), and Tucker (1981). It is likely that these two response times will be similar for simple barriers. Both of these response times measure the length of time taken for adjustment in the tunneling current, following a change in the potential profile. In particular, the response time to a voltage change across the barrier measures a lag in current, following a voltage. That is inductive behavior (Fu and Dudley, 1993). Thus the actual tunneling current behavior can be characterized approximately by an inductance L, in series with the tunneling resistance and with L/R equalling the response time. It is this L, R series combination that is in parallel with the geometrical capacitance.

VII. CONCLUSION AND OVERVIEW

The physics literature has, by 1993, largely accepted the fact that there is a time scale associated with the duration of the tunneling process. In fact, it has been measured. But there still is a lack of consensus on the existence of a unique and simple expression for this time scale and on the exact nature of that expression. We have emphasized a viewpoint which uses a physical clock, involving a degree, or degrees, of freedom in addition to the one related to motion under the barrier. The reactions of the readers are likely to mirror the spread that exists in the literature, and we do not necessarily expect the reader to accept our viewpoint in its entirety.

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