

Erratum: The space groups of icosahedral quasicrystals and cubic, orthorhombic, monoclinic, and triclinic crystals [Rev. Mod. Phys. 64, 3 (1992)]

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Section II.J, "Real-space structure with a given space group" is incorrect from (2.26) to the end. The section is entirely independent of the rest of the paper and can be omitted without the need for any other deletions, additions, or corrections.

To make the point correctly, replace the text starting with the third sentence after Eq. (2.25) with:

Thus $\rho(\mathbf{k})$ as defined in (2.22) will indeed be the Fourier transform of a density characterized by a space group with the given phase functions, provided it has no point group symmetries not in G , and provided it does not vanish systematically on a set of wave vectors so large that the lattice is thinned out to a sublattice for which the space group has a different character. We can avoid both these pitfalls by taking the function $f(\mathbf{k})$ to be the Fourier transform of a function $\rho_0(\mathbf{r})$ with no symmetries whatever.

Because the density (2.22) is not gauge invariant, one can simplify its form by choosing a gauge in which the largest possible number of point group elements have zero phase functions. It follows from the group compatibility condition (2.11) that the set of g with $\Phi_g(\mathbf{k}) \equiv 0$ for all $\mathbf{k}$ is a subgroup, G_0 of G , and that for any h in G , the phase function $\Phi_{gh}(\mathbf{k})$ is the same for all g in G_0 , so that the phase functions are identical over each right coset of G_0 . In such a gauge (2.22) reduces to an expression of the same form, except that the sum now includes just one element from each right coset of G_0 , and the function $\rho_0(\mathbf{r})$ now has the symmetry of G_0 .

In the crystallographic case this expression has a direct geometrical interpretation in real space, since the phase function appearing in (2.22) can be expressed [using (2.18)] as

$$2\pi\Phi_h(\mathbf{k}) \equiv \mathbf{k} \cdot \sum_i \Phi_h(\mathbf{b}^{(i)})\mathbf{a}^{(i)}, \quad (2.26)$$

where the $\mathbf{a}^{(i)}$ are the basis for the real space direct lattice dual to the $\mathbf{b}^{(i)}$, satisfying the orthogonality relations (2.17). Equation (2.22) then describes an atomic basis given by an "atom" of density $\rho_0(\mathbf{r})$ and point group symmetry G_0 , together with its rotation through h and translations through $\sum_i \Phi_h(\mathbf{b}^{(i)})\mathbf{a}^{(i)}$ where one h (it does not matter which) is taken from each of the right cosets of G_0 in G , so that the number of atoms per primitive cell is the order of G divided by the order of G_0 .