The neutron interferometer as a device for illustrating the strange behavior of quantum systems

Daniel M. Greenberger

City College of the City University of New York, New York, New York, 10031

The neutron interferometer is a unique instrument that allows one to construct a neutron wave packet of macroscopic size, divide it into two components separated by centimeters, and then coherently recombine them. A number of experiments clearly showing the difference between quantum and classical theory have been performed with it, which are suitable for presentation in elementary quantum courses. This article presents a simple mathematical model of the interferometer, which can be used to illustrate clearly many of the surprising features of quantum systems. For example, one can describe an experiment to determine which component beam the neutron takes (an analog of the two-slit electron experiment). One can then trace in detail the loss of coherence of the wave function, rather than merely invoke the usual "handwaving" uncertainty arguments. The author discusses the effect of gravity on the neutron beam [the classic COW (Colella, Overhauser, and Werner) experiment], including a simple analysis in an accelerated reference frame, and its relation to the equivalence principle, the red shift, and the twin paradox. Also described are the effect of rotation of the neutron by 360° to change its phase, the effect on the wave function of measuring the absence of the particle from a beam ("Dicke's paradox"), and a realizable version of Wheeler's "delayed-choice" experiments, as well as their relation to the problem of "Schrödinger's cat." The treatment is suitable for bright undergraduates and first-year graduate students.

CONTENTS

I. INTRODUCTION—THE NEUTRON INTERFEROMETER

The neutron interferometer is a truly extraordinary piece of experimental equipment that permits one to perform in the laboratory a number of experiments that check the basic ideas of quantum mechanics. Until the advent of this device many of these experiments would have been described as *Gedankenexperimente,* that is, experiments which point up with particular clarity how the theory indicates particles ought to behave, but which would be so difficult to perform that one must be satisfied with considering them as "thought" experiments.

One of the most important attributes of the neutron interferometer is its conceptual simplicity, which allows one to understand the basic ideas of what is happening without going into a detailed technical description. Of course, as with any real device, there are a number of complicated phenomena taking place inside the interferometer, but to interpret the experiments we shall describe, it is not necessary to consider them. Instead we shall present a simplified model of the interferometer, in terms of which we can follow the neutron beam through it, and from which we can calculate all the results we need. With this model, one can present to students some of the more surprising properties of quantum-mechanical systems, in a way that is relatively easy both to understand and to calculate.

The simplest type of interferometer is constructed from a single crystal[1] [see Fig. 1(a)]. There are three slabs or "ears" cut from the crystal, with enough of the rest of the crystal left to offer structural support. The beam is split into two coherent sub-beams in the first ear, at point A, by Bragg scattering off the atomic planes perpendicular to the face of the crystal [the dotted lines in Fig. (1b)]. In the second ear the beams are redirected, at B and C, and

[1]An introductory overview of the subject, covering the more important experiments, is given in Werner, 1980. Another elementary account appears in Overhauser and Colella, 1980. One with more emphasis on the gravitational experiments is given by Greenberger and Overhauser, 1980. Two review articles that cover the detailed physics inside the interferometer, as well as a wide range of experiments, are Bonse and Graeff, 1977, and Rauch and Petraschek, 1979. A conference report covering the state of the art is Bonse and Rauch, 1979, while a detailed treatment of neutron optics can be found in Klein and Werner, 1983.

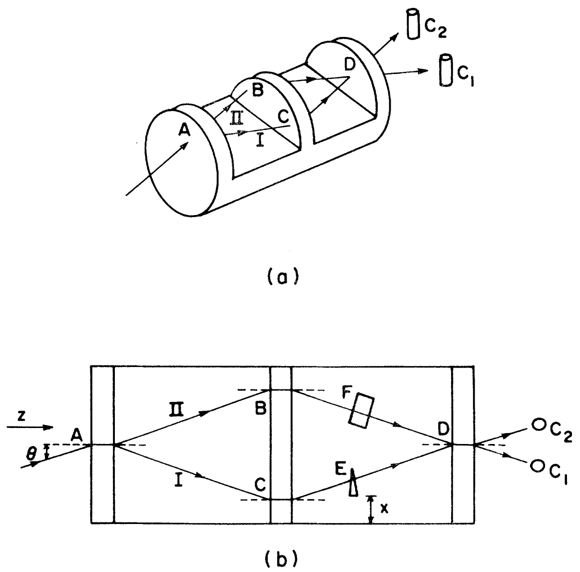

FIG. 1. The neutron interferometer. (a) In the most common type of interferometer three "ears" are cut from the same perfect crystal, ensuring coherence over the entire crystal (about 10 cm long). The incident beam is split at point A into two beams, I and II. These are redirected at B and C and recombined in the last ear at D. The relative phase at D determines the counting rates at the detectors $C1$ and $C2$. (b) The top view of the interferometer. The beam is split by Bragg scattering off the atomic planes perpendicular to the surface of the ear (the dashed lines). The relative phase of the beams I and II can be changed in a known way by inserting a wedge in one beam at E, and varying its thickness by displacing it (varying x). This can be used to determine the phase shift produced by an experiment performed at F.

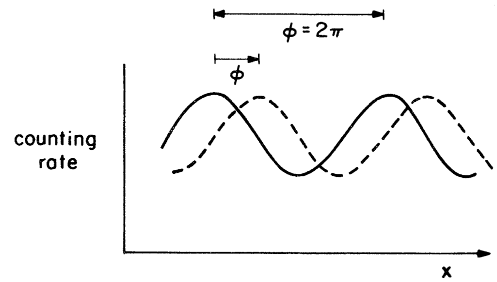

FIG. 2. The interference pattern. By moving the wedge at E in Fig. 1(b), the relative phase between beams I and II is varied. If the counting rate is a maximum in counter $C1$ (and a minimum in $C2$), then altering the relative phase by π will produce a minimum in $C1$ (maximum in $C2$). The counting rate can be plotted against the wedge displacement x, for either counter, yielding an interference pattern. If an experiment is performed at F, the same graph (dashed line) will reveal any extra phase shift ϕ due to the experiment. Since the beams are not totally coherent, the phase cancellation is not perfect, and the minimum intensity will not be zero.

in the third ear they interfere, at D, ultimately going into the two detectors $C1$ and $C2$. The relative counting rates in the detectors determine the relative phasing of the beams. For example, at one particular relative phase, all the neutrons may be entering the counter $C1$. If now the relative phase of the two beams is altered by 180°, corresponding to half a wavelength, then all the neutrons will now enter counter $C2$ (assuming the beams are totally coherent).

An interference pattern can be produced in the following manner. A triangular wedge of matter is placed in the beam I at point E. Because the index of refraction for neutrons will be slightly different from unity inside the matter, the wedge will cause a phase shift. If the counting rate in, for example, counter $C1$ is monitored, it will change as x is varied, because the different wedge thickness will produce a phase shift proportional to Δx. The counting rate can be plotted as a function of x, as in Fig. 2. Then if an experiment is to be performed which alters the phase of beam II at point F by ϕ, the counting rate of $C1$ can again be monitored as a function of x. It is then an easy matter to read off the phase ϕ from the two graphs (including its sign). With some care, the value of ϕ can be determined to within 1°.

The building of such a device became possible with the ability to grow perfect silicon crystals of about 10 cm in length.[2] Thus the atoms in the third ear of the crystal are lined up with those in the first ear, which is what makes coherence over such long distances possible. (Since the crystal spacing is about 1 Å, as is the neutron wavelength, this represents an alignment through about 10^9 λ.) The incident neutron beam need not be terribly well defined, as an extremely accurately defined beam is produced by the interferometer itself for those neutrons which have just the right direction and speed to be Bragg scattered off the planes perpendicular to the crystal face (called Laue scattering). Inside the crystal (see Fig. 3) the forward-scattered beam and backward-scattered beam form a standing wave, which propagates perpendicularly to the crystal face, and at the far end of the crystal, they separate and the forward beam becomes beam II, while the backward-scattered beam becomes beam I. For our purposes we do not need to know any of the details of this process.[3] (Originally, for x rays it was thought the beam would be rapidly absorbed, even at the Bragg angle. This anomalously large transmission process through the crystal is called the Borrmann effect.)

The Laue scattering off the first ear defines the beams I and II within an angular spread $\delta\theta \sim 10^{-6}$ rad (the "Bragg window"). Typical values of the interatomic spacing a are about $a \sim 1$ Å, with λ being given by the Bragg formula $\lambda = 2a \sin\theta$, where the Bragg angle θ is

[2]The first such interferometer was successfully operated for x rays by Bonse and Hart, 1965a, 1965b. It was first applied to neutrons by Rauch, Treimer, and Bonse, 1974.

[3]More of the details of the process are discussed in Greenberger and Overhauser, 1979. A more complete account is given in Squires, 1978, Chap. 6. Two very detailed accounts (for x rays) are James, 1963, and Batterman and Cole, 1964, where the Borrmann effect is especially emphasized.

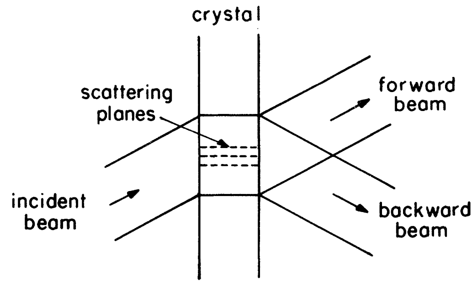

FIG. 3. The Borrmann effect. Inside one of the ears, the beam is Bragg scattered off the planes perpendicular to the crystal face (called Laue scattering). The overlap of scattered and forward wave causes a standing wave inside the ear, and propagation of the beam is perpendicular to the crystal face. This is called the Borrmann effect. At the far end the beam splits into a forward- and backward-traveling wave. No details of the scattering inside the crystal are needed for purposes of the discussion in this paper. (However, the phenomena are interesting in their own right. For example, the incident beam spreads into a region called the Borrmann fan, where the scattering is partially coherent. Inside the crystal there are two slightly different propagation vectors, and interference between them produces an effect called *pendellösung*. See footnote 3.)

typically $20°-30°$. The energy of the neutron beam is approximately $E \sim 0.02$ eV, and $\lambda \sim 1$ Å. Along the interferometer axis, one has $k_z = k\cos\theta$, $\delta k_z/k \sim \sin\theta\delta\theta$. So $\delta k_z \sim 2\pi(\sin\theta\delta\theta)/\lambda \sim 300$ and $\delta z \sim 1/\delta k_z \sim 0.03$ mm. This is the theoretical size of the wave packet that can be defined by the system, and coherence lengths of this order have actually been observed.[4]

One has here then, a system in which the neutron is really a macroscopic particle, large enough to be seen with the naked eye, if it were visible. Since its cross section is defined by the beam geometry to be about 1 cm^2, the wave packet is about the size and shape of a small postage stamp. The flux of a typical research reactor, such as the one at MIT, is of a magnitude to pass about one neutron per second coherently through the interferometer, so they rarely interact with each other.

Inside the interferometer, the neutron wave packet is split into two parts, one taking path I and the other path II. Thus we have a single particle whose wave function is divided into two coherent parts, separated by centimeters. This wave function has a group velocity of about 10^5 cm/sec, which is sufficiently slow that it is easy to control electronically any measurements that must be performed. (In contrast, to confine a photon within the interferometer would take better than nanosecond beam-chopping techniques, requiring state of the art electronics. And electron beams, which move not much slower than photons, can only be coherently separated by distances of small fractions of a millimeter, rather than by centimeters.)

One often describes two-slit experiments using electrons as an example of the wave nature of particles. One can then attempt to ascertain which slit the electron is located in by placing some kind of recording device in the neighborhood of one slit. In the act of recording, the device necessarily destroys the interference pattern. It is extremely difficult to calculate in detail the effect of the device on the electron wave function in order to show how the coherence is destroyed, and in fact we have never seen such a calculation. However, with our simple model for the neutron interferometer, we are able to set up an experiment to determine whether the neutron is in beam I or beam II. We can then calculate in full detail the effect of the experiment on the neutron wave function, follow the loss of coherence between the beams, and determine just how it is brought about. Thus the discussion of neutron coherence has certain real pedagogical advantages over that of the usual electron two-slit experiment. (A more involved but illustrative calculation, employing a Stern-Gerlach apparatus, was performed by Scully *et al.*, 1978.)

In Sec. II we shall present our model for the perfect interferometer, replacing the scattering planes by mirrors. In Sec. III we show how one can let one of the mirrors recoil, and thus tell which beam, I or II, the neutron is in. Then in Secs. IV and V we calculate in detail the effect this has on the coherence of the neutron beam. This part of the paper shows the uncertainty principle in action, and gives an explicit account of how the interaction between the system and measuring apparatus affects the results of an experiment, rather than just giving the usual "handwaving" arguments describing what these effects ought to be—although we do present these arguments also, in Sec. IV.

In Sec. VI we describe the effects of a small perturbation on the neutron beam, in order to be able to understand its behavior in a gravitational or magnetic field. In Sec. VII we discuss how, by keeping beams I and II at different heights, it has been possible to discern the effect of gravity on the phase of the wave function for the first time, and we discuss the significance this has for interpreting the "weak" equivalence principle. In Sec. VIII we show the equivalence between an analysis of the above experiment in the free fall system, and one in a static gravitational field in the laboratory, and relate this to the "strong" equivalence principle. In Sec. IX we use this to show how the gravitationally induced phase shift can be interpreted as the effect of the gravitational red shift on the relative rates of clocks moving with beam I or II, or alternatively how in the free fall system this phase shift can be interpreted as the "twin paradox" effect between the two beams.

In Sec. X we discuss how, by rotating one of the beams in a magnetic field, it has been possible to verify directly for the first time that the neutron wave function changes sign under a rotation of 360°. In Sec. XI we describe the effect of a measurement that detects the absence of a particle. This perplexing issue was recently raised by Dicke,

[4]See Shull, 1968. Some recent, beautiful experiments bearing on this were done by Kaiser *et al.*, 1983. The theory is discussed in Klein *et al.*, 1983.

but in the interferometer it must be faced head on. Here it comes up when we have to consider the effect on the wave function of a particle that passes through an absorber without being absorbed, and it clarifies some of the issues raised by Dicke. In Sec. XII we describe a beautiful neutron analog of a two-slit electron experiment, which has already been performed, and which clearly demonstrates both the particle and interference aspects of the neutron beam.

In Sec. XIII we present a discussion of Wheeler's "delayed-choice" experiments, and show how one could actually perform such an experiment with neutrons, without much difficulty. In Sec. XIV we discuss the issues involved in the "Schrödinger's cat" *Gedankenexperiment.* We point out that those features which are both interesting and unambiguous are present in the delayed-choice experiment, and so we suggest that discussions of the vague cat experiment be replaced by ones involving delayed-choice experiments. In Sec. XV we show that our discussion of the wave function, which has been presented so as to emphasize the difference between the classical and quantum theory, can be replaced (and thus usually simplified) by what has been called the "objective" interpretation (or "ensemble" interpretation) of the wave function. We conclude with an appendix which discusses the weak equivalence principle in greater detail, because the issues raised by the neutron gravity experiments are not discussed in the standard texts and are largely unknown even to research physicists.

It is our hope that these examples will show that the neutron interferometer is a device that clearly delineates both the wave and the particle nature of neutrons, in a manner which can be treated analytically. At the same time it presents us with an apparatus in which a particle is traveling by two widely separated but coherent paths, simultaneously. This situation, from a classical point of view, is mind-boggling. It therefore makes an ideal tool for illuminating the properties of wave mechanics under highly nonclassical conditions, but in a sufficiently simple manner as to be accessible to bright undergraduates and beginning graduate students.

II. A MODEL OF A PERFECT NEUTRON INTERFEROMETER

The simple model we should like to introduce for the neutron interferometer is the following. A neutron beam, of which one monochromatic component ($\mathbf{k}$) is drawn in Fig. 4(a), is scattered off a semitransparent mirror $a-a'$, at angle θ, and split into two equal components. One of these components, I, is reflected to $c-c'$, where it bounces off a fully reflecting mirror and then proceeds to a screen $d-d'$. The other component, II, equal in intensity, is transmitted to $b-b'$, where it is also fully reflected, and it proceeds to $d-d'$, where the two beams interfere. At the plane $d-d'$ the two beams are shown in Fig. 4(b), where they both have a maximum at points A and C, and are in phase here; but they both have a minimum at point B, and so are in phase here also. Thus the distance between points of constructive interference, AB, is

$$l=\lambda/(2\sin\theta)\ . \tag{2.1}$$

If beam I, approaching the screen at $d-d'$, has the wave function

$$\psi_{\mathrm{I}}'=e^{i\mathbf{k}_{\mathrm{I}}\cdot\mathbf{r}}=e^{-ik_x x}e^{ik_z z}\ , \tag{2.2}$$

and beam II has the wave function

$$\psi_{\mathrm{II}}'=e^{i\mathbf{k}_{\mathrm{II}}\cdot\mathbf{r}}=e^{ik_x x}e^{ik_z z}\ , \tag{2.3}$$

then the intensity at the screen $d-d'$ will be given by

$$\begin{aligned}\psi'&=\psi_{\mathrm{I}}'+\psi_{\mathrm{II}}'=e^{ik_z z}(e^{ik_x x}+e^{-k_x x})\\&=2e^{ik_z z}\cos k_x x\ ,\\|\psi'|^2&=4\cos^2 k_x x\ ,\end{aligned} \tag{2.4}$$

which gives a distance between maxima, l,

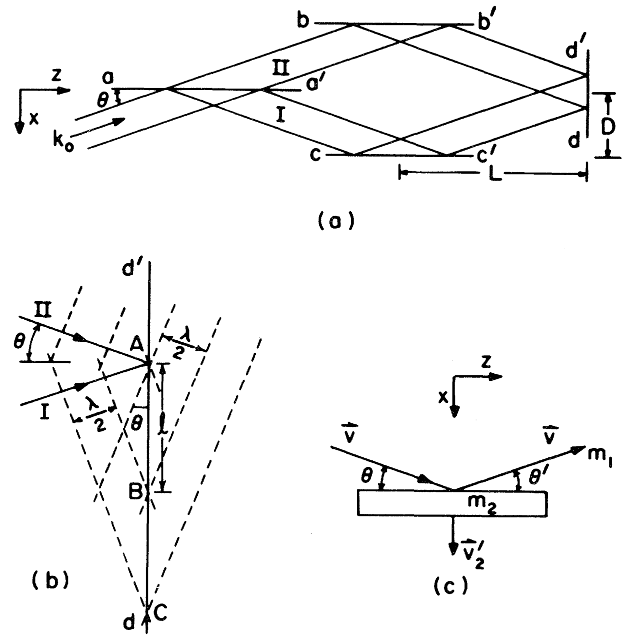

FIG. 4. Model for the interferometer. (a) Imagine the incident beam striking a semitransparent mirror $a-a'$. It is split into two beams, I and II, which are subsequently reflected by mirrors at $b-b'$ and $c-c'$. They are then recombined and the interference pattern observed on a screen $d-d'$. In describing various interferometer experiments we shall sometimes refer to this model, and sometimes to the interferometer, interchangeably. (b) If we slide along the screen $d-d'$, starting at a maximum, where the two waves I and II are in phase, then the next maximum will occur when one beam has lost half a wavelength, while the other has gained the same amount. Then the relative phase will have changed by 2π. This gives the distance between maxima along the screen as $l=\lambda/(2\sin\theta)$. (c) If the neutron has mass m_1, and the mirror $c-c'$ has a finite mass m_2, it can recoil. (We constrain it to the x direction.) The x origin is taken as the center of the undisplaced position of the mirror.

$$k_x l = \pi \; ,$$
$$l = \pi / k_{0x} = \pi / k_0 \sin\theta = \lambda / 2 \sin\theta \; , \tag{2.5}$$

as in Eq. (2.1).

To represent the scattering of particles, we shall need a wave packet to represent the neutron. The monochromatic wave approaching the mirror $c-c'$

$$\psi_{\mathrm{I}} = e^{ik_x x} e^{ik_z z} \; , \tag{2.6}$$

and the scattered wave leaving it,

$$\psi_{\mathrm{I}}' = e^{-ik_x x} e^{ik_z z} \; , \tag{2.7}$$

will be connected by matching the boundary condition at the mirror, which is

$$\psi_{\mathrm{I}}(c-c') = 0 \; . \tag{2.8}$$

This boundary condition comes from solving the Schrödinger equation at the mirror, which we assume to be totally reflecting. So we use the potential

$$V(x) = \begin{cases} 0, & x \le 0 \\ \infty, & x > 0, \end{cases} \tag{2.9}$$

assuming that $x=0$ at the line $c-c'$ in Fig. 4, which leads to the boundary condition of Eq. (2.8), or equivalently

$$\psi_{\mathrm{I}}(x=0) = 0 \; . \tag{2.10}$$

This in turn allows us to match ψ_{I} and ψ_{I}', so

$$\psi_{\mathrm{I},k} = e^{ik_z z}(e^{ik_x x} - e^{-ik_x x}) \; . \tag{2.11}$$

We now make up the wave packet

$$\psi_{\mathrm{I}}(x,z) = \frac{1}{2\pi} \int dk_x dk_z a(k_x) b(k_z) \psi_{\mathrm{I},k}(x,z) \; . \tag{2.12}$$

We assume that the wave packet is very narrow in k space and centered about $\mathbf{k}_0 = \mathbf{i}k_{0x} + \mathbf{k}k_{0z}$, so that

$$\Delta k_x \ll k_{0x}, \;\; \Delta k_z \ll k_{0z} \; . \tag{2.13}$$

In position space the wave will be many wavelengths wide, so that in the region where it is nonzero it will approximate a plane wave, and

$$\Delta x \gg \lambda_x = \lambda/\sin\theta, \;\; \Delta z \gg \lambda_z = \lambda/\cos\theta \; . \tag{2.14}$$

The wave packet in the transverse direction, in k_x and x space, is shown in Fig. 5.

From a macroscopic point of view Δx will still be quite small, and we assume the packet is centered at a point $(-x_0)$ at time $(-t_0)$, such that

$$x_0 = v_x t_0 \; , \tag{2.15}$$

where v_x is the group velocity, so that it will be centered on the mirror at $t=0$. (We make similar assumptions concerning z.) Then the time dependence of the packet will be given by

$$\psi_{\mathrm{I}}(\mathbf{r},t) = \frac{1}{2\pi} \int dk_x dk_z a(k_x) b(k_z) \psi_{\mathrm{I},k}(x,z) e^{-i\omega t} \; . \tag{2.16}$$

Since the beam is narrow in k space, we may write

$$\omega = \frac{\hbar k^2}{2m} \approx \frac{\hbar k_0^2}{2m} + \frac{\hbar \mathbf{k}_0}{2m} \cdot (\mathbf{k} - \mathbf{k}_0)$$
$$= \omega_0 + \mathbf{v} \cdot (\mathbf{k} - \mathbf{k}_0) \; , \tag{2.17}$$

where $\mathbf{v} = \hbar \mathbf{k}_0 / m$ is the group velocity. Then the x integration gives, from Eqs. (2.11), (2.16), and (2.17),

$$\psi_{\mathrm{I}}(x,t) = e^{+i\omega_0 t}(2\pi)^{-1/2}$$
$$\times \int dk_x a(k_x)(e^{ikx(x-v_x t)} - e^{k_x(-k-v_x t)})$$
$$= e^{i\omega_0 t}[f(x-vt) - f(-(x+vt))]$$
$$= \psi_{\mathrm{I}}'(x,t) + \psi_{\mathrm{I}}''(x,t) \; . \tag{2.18}$$

This wave function represents the physical wave only in the region $x<0$, and it is shown in Fig. 5(c). It can be seen that the wave-packet description is that of a packet incident on the mirror from the left, for $t<0$, and of the reflected packet (which is upside down and reversed), for $t>0$. The full wave function is

$$\psi_{\mathrm{I}}(\mathbf{r},t) = g(z - v_2 t)\psi_{\mathrm{I}}(x,t) \; . \tag{2.19}$$

In the z direction the packet continues along, unaffected by the collision.

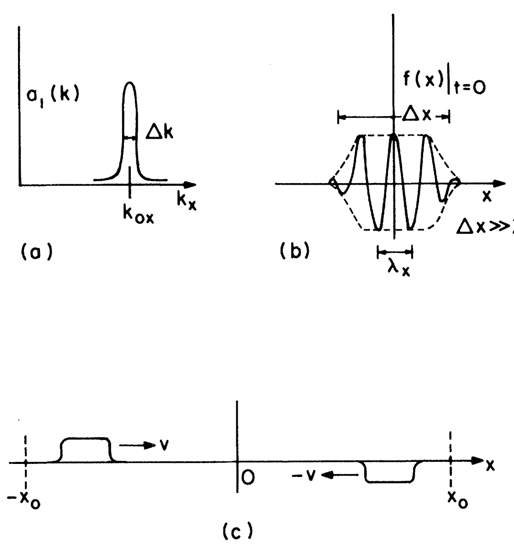

FIG. 5. The wave packet for the neutron. (a) In k_x space, the neutron is very narrow, centered at k_{0x}, with $\Delta k_x \ll k_{0x}$, so that the wave is approximately monotonic. In the z direction its motion is unaffected by the collision. (b) In x space, the wave packet will be relatively wide by microscopic standards, $\Delta x \gg \lambda_x \equiv 2\pi / k_{0x}$. But on a macroscopic scale it will still be close to a point particle. The wave will be centered at $x=0$ at $t=0$. (c) For the case of a rigid mirror at $c-c'$, with $m_2 = \infty$, the wave packet is initially centered at $-x_0$ at time $-t_0$. It approaches the mirror with speed $v = \hbar k_{0x}/m_1$. The image wave approaches from the other side, at speed $-v$, and has the opposite phase. After the collision, it becomes the reflected wave.

III. THE NEUTRON INTERFEROMETER WITH RECOIL

The interferometer we described above was perfect, in the sense that all three mirrors, $a-a'$, $b-b'$, and $c-c'$, were infinitely massive and so incapable of recoiling. In the act of recoiling they would absorb momentum from the incident beam, which would lose energy and so change frequency. Besides this, any intrinsic momentum uncertainty in the mirror will affect the recoiling particle. Thus, by recoiling, a mirror introduces incoherence into the problem. This recoil also provides a degree of freedom which enables one to perform experiments that probe the system. For example, if mirror $c-c'$ had a finite mass m_2, then when the neutron bounced off it one could observe the recoil and determine with certainty that the neutron had indeed taken the path I.

This type of experiment is very similar to the standard two-slit experiment, where one tries to determine which slit the particle takes. In that case, however, it is very difficult to create a device enabling one to calculate in detail the wave function of the total system. One can write down uncertainty relations that show quite definitively that a measurement to determine which slit the particle takes must necessarily destroy the two-slit interference pattern of the particle. But the detailed process by which this destruction takes place is itself fascinating, and provides insight into how the laws of quantum theory manifest themselves in actual physical situations. In our interferometer model, we can follow every step of the calculation and keep track of the coherence of the neutron beam.

Imagine, then, that in Fig. 4(a) the mirror $c-c'$ has finite mass m_2 and can recoil in the x direction. In the classical collision between the neutron, of mass m_1, moving with a component of velocity v_{1x} in the $+x$ direction, and the mirror, with $m_2 \gg m_1$, moving initially in the $+x$ direction with initial velocity v_2 (we shall eventually set $v_2=0$), momentum and energy conservation yield

$$\begin{aligned} &m_1 v_{1x} + m_2 v_2 = m_1 v'_{1x} + m_2 v'_2 \ , \\ &m_1 v_{1x}^2/2 + m_2 v_2^2/2 = m_1 (v'_{1x})^2/2 + m_2 (v'_2)^2/2 \ , \end{aligned} \tag{3.1}$$

where v'_{1x} and v'_2 are the final velocities of the masses m_1 and m_2. The z component of velocity of the neutron will be unaffected by the collision, and the mirror is taken to be so constructed that it can only recoil in the x direction.

Equations (3.1) yield the elasticity relation,

$$(v_{1x}-v_2) = -(v'_{1x}-v'_2) \ . \tag{3.2}$$

Together with the momentum equation, this gives

$$\begin{aligned} v'_{1x} &= -\left[\frac{m_2-m_1}{M}\right] v_{1x} + \frac{2m_2}{M} v_2 \ , \\ v'_2 &= \frac{2m_1}{M} v_{1x} + \left[\frac{m_2-m_1}{M}\right] v_2 \ , \end{aligned} \tag{3.3}$$

where we use

$$M = m_1 + m_2, \quad \mu = m_1 m_2 / M \ . \tag{3.4}$$

These equations can be inverted, because of the symmetry $v_{1x} \leftrightarrow v'_{1x}$ and $v_2 \leftrightarrow v'_2$, to give

$$\begin{aligned} v_{1x} &= -\left[\frac{m_2-m_1}{M}\right] v'_{1x} + \frac{2m_2}{M} v'_2 \ , \\ v_2 &= \frac{2m_1}{M} v'_{1x} + \left[\frac{m_2-m_1}{M}\right] v'_2 \ . \end{aligned} \tag{3.5}$$

The collision is shown in Fig. 4(c).

The quantum-mechanical solution to this problem is obtained by generalizing the one-particle problem in Sec. II. If one introduces center-of-mass coordinates

$$\begin{aligned} \mathbf{R} &= \frac{m_1}{M}\mathbf{r}_1 + \frac{m_2}{M}\mathbf{r}_2 \ , \\ \mathbf{r} &= \mathbf{r}_1 - \mathbf{r}_2 \ , \end{aligned} \tag{3.6}$$

and momenta

$$\begin{aligned} \mathbf{P} &= \mathbf{p}_1 + \mathbf{p}_2 \ , \\ \mathbf{p} &= u\dot{\mathbf{r}} = \frac{m_2}{M}\mathbf{p}_1 - \frac{m_1}{M}\mathbf{p}_2 \ , \end{aligned} \tag{3.7}$$

the one-particle problem solved previously can be interpreted as pertaining to the relative coordinate $\mathbf{r}=(x,y,z)$. Thus the potential given by Eq. (2.9) becomes

$$V(\mathbf{r}) = \begin{cases} 0, & x \le 0 \\ \infty, & x > 0 \ . \end{cases} \tag{3.8}$$

The boundary condition, Eq. (2.8) or (2.10), becomes

$$\psi_{\mathrm{I}}(x=0) = 0 \ , \tag{3.9}$$

where x is now the relative coordinate (x_1-x_2). Since the center-of-mass motion is free, the solution for ψ_{I}, Eq. (2.11), becomes

$$\psi_{\mathrm{I},\mathbf{K},\mathbf{k}}(\mathbf{R},\mathbf{r}) = e^{i\mathbf{K}\cdot\mathbf{R}} e^{ik_z z}(e^{ik_x x} - e^{-ik_x x}) \ , \tag{3.10}$$

where now the $\mathbf{k}$ and $\mathbf{K}$ are relative and center-of-mass momenta, and obey $\mathbf{p}=\hbar\mathbf{k}$ and $\mathbf{P}=\hbar\mathbf{K}$. If we suppress the z motion for convenience, we can express the x,X motion in terms of the variables x_1 and x_2, using Eqs. (3.6) and (3.7),

$$\begin{aligned} \psi_{\mathrm{I}} &= e^{iK_x X}(e^{ik_x x} - e^{-ik_x x}) \\ &= \exp\left[i(k_{1x}+k_{2x})\left[\frac{m_1}{M}x_1 + \frac{m_2}{M}x_2\right]\right](e^{i\rho} - e^{-i\rho}) \ , \\ \rho &= \left[\frac{m_2}{M}k_{1x} - \frac{m_1}{M}k_{2x}\right](x_1 - x_2) \ . \end{aligned} \tag{3.11}$$

Equation (3.11) can be further rewritten by factoring out the k_{1x} and k_{2x} dependence in each term to give (calling $k_{1x} \equiv k_1, k_{2x} \equiv k_2$)

$$\psi_{\mathrm{I}}=e^{ik_1x_1}e^{ik_2x_2}-e^{ik_1\alpha}e^{ik_2\beta}\,,$$

$$\alpha=\left[-\left[\frac{m_2-m_1}{M}\right]x_1+\frac{2m_2}{M}x_2\right]\,, \tag{3.12}$$

$$\beta=\left[\frac{2m_1}{M}x_1+\left[\frac{m_2-m_1}{M}\right]x_2\right]\,.$$

We can gain some insight into this wave function by noting that the first term is just the incident wave, while the second term must represent the reflected wave. In our problem the incident neutron beam x_1 will be represented by a wave packet which is very narrow in k space, centered about some value $k_1=k_{01}$, just as in the problem in Sec. II, where the interferometer was infinitely heavy. If for the moment we do not worry about the interpretation of the variable x_2 as representing an interferometer mirror, but think of it as representing a wave packet of a particle of mass m_2 with a narrow momentum spread Δk_2 centered about some momentum k_{02}, such that the two particles would classically collide at $x_1=x_2=0$ at time $t=0$, then we can set up the wave packet

$$\psi(x_1,x_2,t)=\frac{1}{2\pi}\int dk_1dk_2a_1(k_1)a_2(k_2)\times\psi_{\mathrm{I}}(x_1,x_2)e^{-i(\omega_1+\omega_2)t}\,. \tag{3.13}$$

For convenience we have suppressed the z dependence, but the x dependence yields to exactly the same analysis as that of Sec. II. Thus the first term $\exp(ik_1x_1+ik_2x_2)$ of the function ψ_{I} in Eq. (3.12) yields the result

$$\psi_{\mathrm{I}}'(x_1,x_2,t)=e^{i(\omega_1+\omega_2)t}f_1(x_1-v_1t)f_2(x_2-v_2t)\,, \tag{3.14}$$

where $v_1=\hbar k_{01}/m_1$, and $v_2=\hbar k_{02}/m_2$, and the phase factor, which corresponds to that in Eq. (2.18), does not affect the probability distribution.

The second term of Eq. (3.12), $\exp(ik_1\alpha+ik_2\beta)$, by the exact same analysis, gives

$$\psi_{\mathrm{I}}''(x_1,x_2,t)=-e^{i(\omega_1+\omega_2)t}f_1(\alpha-v_1t)f_2(\beta-v_2t)\,. \tag{3.15}$$

The first wave function ψ_{I} obviously represents the two wave packets approaching each other with velocities $v_1=x_1/t$ and $v_2=x_2/t$. The wave function ψ_{I}' represents the two recoil wave packets, moving with the velocities

$$v_1=\alpha/t,\quad v_2=\beta/t\,. \tag{3.16}$$

If we examine the defining equations, Eqs. (3.12), which give $\alpha(x_1,x_2)$ and $\beta(x_1,x_2)$, and in these equations denote x_1 and x_2 by x_1' and x_2' to denote their physical status as recoil positions, then we see that Eq. (3.16) is identical in content with Eq. (3.5), which says that the recoil wave packets are centered about the classical values of the parameters x_1' and x_2'.

Nevertheless, these packets are expressed in terms of α and β, and for an arbitrary wave packet are not factorable into functions of the form $u(x_1)v(x_2)$. We shall see in the next section that this is very important for identifying how the shape of the mirror wave packet before the collision determines to what extent the properties of the neutron can be measured accurately after the collision, in accordance with the uncertainty principle.

IV. AN EXPERIMENT TO DETERMINE WHICH PATH THE NEUTRON TAKES

If in Fig. 4 the mirror $c-c'$ has finite mass m_2, this can be used to determine which path the neutron takes. In this section we shall use the uncertainty principle to determine the extent to which this can be done without destroying the coherence of the neutron beam, and in the next section we perform a detailed calculation to verify the results.

Let us assume that we want to set up an experiment that will monitor the position of the mirror. When the neutron beam passes through the system, if it takes path I, it will strike the mirror $c-c'$, which will then recoil. Two ways to monitor this recoil are, first, to detect whether the mirror has drifted beyond a specific point, which it will do if it has recoiled, or, second, to monitor its velocity and detect the Doppler shift due to the recoil. The same criterion in momentum space will determine whether each of these two techniques can work.

In the first experiment, if we assume that the mirror is much more massive than the neutron, so $m_2\gg m_1$, then the mirror will pick up momentum $(p_{2x})\approx 2p_{1x}$ (that is, the neutron will almost reverse its velocity from v_{1x} to $-v_{1x}$, as in the case $m_2=\infty$),

$$(p_{2x})'\approx 2p_{1x}=2m_1v_{1x}\,. \tag{4.1}$$

In order for the recoil displacement of the mirror to be detectable, it must be greater than the natural spread Δx_2 in the mirror wave packet. In other words, the packet will naturally spread by

$$\Delta x_2\sim(\Delta x_2)_0+(\Delta v_2)t\,, \tag{4.2}$$

and we want to monitor the recoil

$$x_2'\sim v_{2x}'t\gg\Delta x_2\,. \tag{4.3}$$

[Strictly speaking, $(\Delta x)^2=(\Delta x)_0^2+(\Delta v)^2t^2$, but usually one term or the other dominates, and Eq. (4.2) is adequate.] If we assume that the velocity spread (Δv_2) is sufficiently limited so as to make the experiment feasible, then $\Delta x_2>\Delta v_2t$, so that $v_2'>\Delta v_2$, and

$$\Delta p_{2x}\ll p_{2x}'\approx 2p_{1x}=2p_1\sin\theta\,. \tag{4.4}$$

Similarly, in the case where we want to monitor the Doppler shift, we also want this shift, v_{2x}', to be greater than the natural spread of the packet, Δv_{2x}, so that Eq. (4.4) must also be satisfied.

If this is the case, then the initial spread in the x position of the mirror will be

$$(\Delta x_2)_0\sim h/\Delta p_{2x}\gg\frac{h}{2p_1\sin\theta}\sim\frac{\lambda}{2\sin\theta}\,. \tag{4.5}$$

But, from Eq. (2.1), this is exactly the distance l between maxima on the screen, so

$$\Delta x_2 \gg l \ , \tag{4.6}$$

and the position of the mirror itself must be so uncertain that it is not known to within the size of the interference period. Thus the instant the neutron collides with this fuzzy mirror, its phase coherence will be completely destroyed. This total loss of coherence is necessary, because one might otherwise have decided to move the screen up closer to mirror $c-c'$, in order to see the interference pattern before it was destroyed. This same consideration applies to the electron two-slit experiment. An attempt to measure which slit the electron passes through not only wipes out the interference pattern, but it must do so instantly, for the same reason.

If, instead, the mirror position uncertainty is restricted to a very small size, so that the interference pattern is not wiped out on contact, that is, if

$$(\Delta x_2)_0 = d \ll l = \lambda/(2\sin\theta) \ , \tag{4.7}$$

then

$$\Delta p_2 \sim h/\Delta x_2 \gg 2h\sin\theta/\lambda \sim 2p_1 \sin\theta \sim 2p_{1x} \ , \tag{4.8}$$

and the momentum uncertainty of the mirror will be greater than the total momentum of the neutron, thus making it impossible to measure the recoil of the mirror due to the neutron impact.

Since this latter experiment can no longer measure the mirror recoil, does it have to wipe out the interference pattern? If the mirror has an initial velocity v_2, the recoil neutron will pick up approximately $2v_2$ (since the relative velocity will be conserved), so for the neutron $\Delta v_{1x} \sim 2\Delta v_{2x}$. Then the neutron will lose its coherence in a time consistent with $\Delta x_1 \sim \lambda$, or

$$\begin{aligned} \Delta x_1 \sim \lambda \sim (\Delta v_{1x})'t \sim 2\Delta v_{2x} t \sim 2\Delta p_{2x} t/m_2 \\ \gg 4p_{1x} t/m_2 \sim 4(m_1/m_2) v_1 t \sim 4(m_1/m_2) x_1' \ . \end{aligned} \tag{4.9}$$

The neutron position uncertainty then grows linearly in time,[5] but whether or not it will lose coherence before striking the screen depends on (m_1/m_2). For example, when $m_2 = \infty$, there is no loss of coherence due to bouncing off the mirror (but of course we do not now know which beam the neutron is in).

V. A DETAILED CALCULATION OF THE LOSS OF COHERENCE IN THE BEAM

The calculation of Sec. III used the group velocity approximation, so that the wave packets did not spread in time. The purpose of this was to demonstrate that the trajectories of the particles followed those of their classical counterparts. While for the neutron this is still a good approximation when it scatters off the mirror, for the mirror itself it is not appropriate. If the mirror is initially at rest, $v_2 = k_2 = 0$, and we certainly cannot assume $\Delta k_2 \ll k_2$.

In fact, for our purposes it will suffice to take a plane wave for the neutron, of momentum k_0, and a Gaussian wave packet for the mirror, at time $t=0$, with initial width b. Then the mirror wave packet at $t=0$ will be

$$f(x_2) = e^{-x_2^2/2b^2}, \quad a_2(k_2) = Ae^{-b^2k_2^2/2} \ , \tag{5.1}$$

where A is the appropriate normalization factor. The total wave function will then be given by Eqs. (3.12) and (3.13), with $a_1(k_1) = \delta(k_1 - k_0)$ and $a_2(k_2)$ given by Eq. (5.1) above. Thus

$$\begin{aligned} \psi_{\mathrm{I}} = \int dk_2 A(e^{ik_0x_1} e^{ik_2x_2} - e^{ik_0\alpha} e^{ik_2\beta}) \\ \times e^{-b^2k_2^2/2} e^{-i(\omega_0+\omega_2)t} \ , \end{aligned} \tag{5.2}$$

where α, β are given by Eq. (3.12), and $\hbar\omega_0 = \hbar k_0^2/2m_1$. (Again, we are not interested in the z direction.) The first integral over k_2 gives, by completing the square,

$$\begin{aligned} \int dk_z \exp(-b^2k_2^2/2 - i\hbar k_2^2 t/2m_2 + ik_2x_2) \\ = B(t) e^{i\kappa x_2^2 t} e^{-x_2^2/2\Delta^2(t)} \ , \\ \Delta^2(t) = b^2 + \hbar^2 t^2/4m_2^2 b^2, \quad \kappa = \hbar/2m_2 b^2 \Delta^2(t) \ . \end{aligned} \tag{5.3}$$

The second integral, for ψ_{I}'', is identical, except that x_1 and x_2 are replaced by α and β. Here, $B(t)$ is a slowly varying amplitude factor, and $\exp(i\kappa x_2^2 t)$ is a phase factor. The second integral is the recoil wave packet, and the recoil wave function is given by

$$\psi_{\mathrm{I}}'' = B(t) e^{i\kappa\beta^2 t} e^{-\beta^2/2\Delta^2(t)} e^{i(k_0\alpha - \omega_0 t)} \ . \tag{5.4}$$

The Gaussian will be nonzero in the region approximately given by $\beta < \Delta(t)$.

If we call

$$m_1/m_2 = \varepsilon \ll 1 \ , \tag{5.5}$$

then from Eqs. (3.12).

$$\begin{aligned} \alpha \sim [-(1-2\varepsilon)x_1 + 2(1-\varepsilon)x_2] \sim [-x_1 + 2x_2] \ , \\ \beta \sim [(1-2\varepsilon)x_2 + 2\varepsilon x_1] \sim x_2 \ . \end{aligned} \tag{5.6}$$

By neglecting the ε terms in these equations we are ignoring the slight change in momentum of the neutron upon scattering. The $2x_2$ term in α then determines how the mirror recoil will affect the coherence of the neutron. By assuming $\beta \sim x_2$ and ignoring the $2\varepsilon x_1$ term we are neglecting the effect of the neutron on the coherence of the mirror. The mirror, however, is a macroscopic instrument, and we are not really concerned with its wave function; we shall ultimately integrate over all values of x_2.

[We should mention, for anyone who wants to carry out the calculation including all recoil effects consistently,

[5]We have described the neutron by a plane wave. By saying the uncertainty grows in time, we mean that if one makes a wave packet out of the plane-wave solutions and discusses their propagation from an initially narrow packet, this will be the time for which $\Delta x(t) \sim \lambda$.

that the way to proceed is to write $x_2=2(m_1/M)v_{1x}t+y$, from Eq. (3.3), assuming $v_2=0$. Here the first term is the classical center of the recoil wave packet, and the second term, y, determines the spread around this value, and ultimately is to be integrated over. This procedure allows one to keep track of all terms in ε.]

Equation (5.4) then becomes

$$\psi_{\rm I}'' \sim B(t)e^{i\kappa x_2^2 t}e^{-i\omega_0 t}e^{-ik_0(x_1-2x_2)}e^{-x_2^2/2\Delta^2} . \tag{5.7}$$

In this equation B is essentially constant, while the phase factor $\exp(i\kappa\beta^2 t)\sim 1$, since $\beta<\Delta$. The important elements in the wave function are the last two,

$$\psi_{\rm I}'' \sim Be^{i\gamma}e^{-ik_0(x_1-2x_2)}e^{-x_2^2/2\Delta^2} . \tag{5.8}$$

Equation (5.8) shows how the recoil of the mirror affects the coherence of the neutron. Instead of the recoil term $e^{-ik_0x_1}$, as in the case for m_2 infinite, there is now an extra term $e^{2ik_0x_2}$ which makes the phase more incoherent as Δx_2 increases, and Δx_2 is determined by the size of $\Delta(t)$, in the last term. The spread $\Delta(t)$ itself consists of two parts, the initial spread, b, which is Δx_2 at $t=0$, and, for later times, $\Delta(t)\sim \hbar t/m_2 b\sim \Delta p_2 t/m_2\sim \Delta v_2 t$, which is the uncertainty caused by the momentum spread. Thus $\Delta(t)$ increases in time, and one cannot eliminate it by making the wave packet narrow in either x_2 or p_2.

To see this in detail, first look at the infinite-mass case, for which the wave function at the screen, where beams I and II combine, will be

$$\begin{aligned} |\psi_0|^2 &= |\psi_{\rm II}'+\psi_{\rm I}'|^2 = |(e^{ik_0x_1}+e^{-ik_0x_1})|^2 \\ &= 4\cos^2 k_0x_1 = 2(1+\cos 2k_0x_1) . \end{aligned} \tag{5.9}$$

This interference pattern is shown in Fig. 6(a). The contrast of an interference pattern is defined by

$$C=(I_{\max}-I_{\min})/(I_{\max}+I_{\min}) . \tag{5.10}$$

For this case the contrast is 1, since $I_{\min}=0$.

For the finite-mass case, the wave function $\psi_{\rm II}''$, for the neutron taking beam II and not interacting with the mirror $c-c'$, is

$$\psi_{\rm II}''=Be^{ik_0x_1}e^{-x_2^2/2\Delta^2} , \tag{5.11}$$

while $\psi_{\rm I}''$ will be given by Eq. (5.8). Thus

$$\begin{aligned} |\psi|^2 &= |\psi_{\rm II}''+\psi_{\rm I}''|^2 \\ &= B^2|(e^{ik_0x_1}+e^{-ik_0x_1}e^{-2ik_0x_2})e^{-x_2^2/2\Delta^2}|^2 \\ &= 2B^2[1+\cos 2k_0(x_1+2x_2)]e^{-x_2^2/2\Delta^2} . \end{aligned} \tag{5.12}$$

The probability for finding the neutron at a point x_1 will then be

$$\begin{aligned} P(x_1) &= \int |\psi(x_1,x_2)|^2 dx_2 \\ &= 2(B')^2(1+e^{-4k_0^2\Delta^2(t)}\cos 2k_0x_1) , \end{aligned} \tag{5.13}$$

where again B' is a slowly varying amplitude factor.

This interference pattern is drawn for different times in Figs. 6(b)–6(d). The contrast in this case is given by

$$c=e^{-4k_0^2\Delta^2(t)} , \tag{5.14}$$

and vanishes in the characteristic time given by

$$\begin{aligned} &2k_0\Delta(t)\sim 1 , \quad (4\pi/\lambda)(\Delta v_2 t)\sim 1, \\ &\Delta v_2 t\sim \lambda/4\pi . \end{aligned} \tag{5.15}$$

In the case where $b\ll\lambda$, $\Delta(t)\sim\Delta v_2 t$, and Eq. (5.15) is essentially Eq. (4.9). In the case where $b>\lambda$, Δ is sufficiently large that even at $t=0$ there is little or no contrast.

We have seen that the wave packets for the reflected waves mix up the components of both x_1 and x_2. This has the effect of guaranteeing the classical motion for the center of the wave packets. It also provides the mechanism whereby the recoil affects the coherence of the wave functions in accordance with the uncertainty principle. In our example of a light neutron rebounding off a heavy mirror, the effect of the recoil is to introduce an extra phase factor into the neutron wave packet, which depends on the size of the mirror wave packet. Thus if the experiment is set up in such a manner as to be able to detect the mirror recoil and determine which beam the neutron takes, the neutron coherence will necessarily be destroyed.

It should be noted that in this experiment the interaction with the movable mirror induces the incoherent effects that wash out the interference pattern. It does not matter, therefore, whether the mirror is actually hooked up to a meter to measure the recoil or not—the interference pattern will be destroyed in either case. At any rate, there is certainly no need for the result to be transmitted to a conscious mind (or even transmitted). It is the uncontrolled interaction with a macroscopic object (which

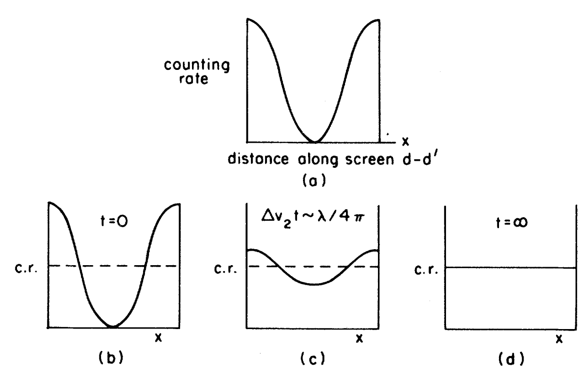

FIG. 6. Loss of coherence of the neutron. (a) In the case where the mirror $c-c'$ is rigid, $m_2=\infty$, the contrast of the interference pattern is $c=1$ for our idealized case. Of course for an actual wave the beams can never be totally coherent, and so $C<1$. (b) For a finite mass m_2, the neutron loses coherence as it rebounds from the collision, with the contrast given by $C=\exp[-4k_0^2\Delta^2(t)]$. As Δ grows the contrast decreases. It is drawn for the case Δ_0 very narrow, so $C(0)=1$. But for $\Delta_0\gg 1/2k_0\sim\lambda/4\pi$, $C\sim 0$, even for $t\sim 0$. (c) As t grows, Δ grows, and the contrast disappears, in a characteristic time t_0, given by $\Delta v_2 t_0\sim\lambda/4\pi$. (d) For $t\gg t_0$, the contrast approaches zero.

could serve as a detector) that destroys coherence.

Bohr frequently emphasized that the stage at which this interaction occurred was arbitrary but that it was ultimately necessary in order to make a macroscopic measurement, and for the reduction of the wave packet. It is sometimes possible, by exerting great care, to gather up all the parts of the wave function coherently and recombine them. In that case, however, the macroscopic measurement is never made, and the interaction is undone. For example, in our experiment, the mirror at C could be physically connected to the one at B, both having finite mass. Then, although the recoil of either mirror is uncontrollable, it will be transmitted to the other mirror in full and will not affect the relative motion of the neutron beams, and so the interference pattern will be preserved. But of course then we haven't any idea in which beam the neutron is located. Most physicists believe that the arbitrariness attending the reduction of the wave packet in no way implies the necessity of introducing a conscious mind into the analysis (but there are a few important exceptions[6]).

VI. THE EFFECT OF A SMALL PERTURBING FORCE ON THE BEAM

The neutron, as it passes through the interferometer between the ears, is essentially a free particle. However, it can be subject to a number of effects that will change the relative phase of the beams in various ways. For example, there is the effect of gravity. If the beams are at different heights, this will produce a phase shift between the beams. Another effect is that due to the rotation of the earth, which will shift the interferometer from under the beams. It is also possible to place a magnetic field in the region between the ears.

All of the effects mentioned are very small, but measurable, and it is possible to reduce them to an equivalent potential from which we can calculate the effective change in phase caused by them. In all these cases, the potential energy is of the order of 10^7 times smaller than the kinetic energy. The result is that in the WKB wave function, which consists of an amplitude factor (a function of the momentum) times a phase factor, the amplitude is essentially constant, and can be ignored. We can find the phase by a method which exploits this fact (Landau and Lifschitz, 1977).

If we assume that there exists a small perturbing force, which can be represented by a potential $V(x)$, and

$$V \ll T = p_0^2/2m = E \;, \tag{6.1}$$

then we can write the solution to the Schrödinger equation in the form

$$\psi = \psi_0 e^{i\alpha(x)} \;, \tag{6.2}$$

where α changes slightly within a distance of one wavelength, or $\delta\alpha = \nabla\alpha\cdot\delta\mathbf{x} \ll 2\pi$, for $\delta x \sim \lambda$, so

$$|\nabla\alpha| \ll 2\pi/\lambda = k_0 \;, \tag{6.3}$$

and ψ_0 is the solution to the Schrödinger equation with $V=0$. Then

$$\begin{aligned} &-(\hbar^2/2m)\nabla^2\psi + V\psi = E\psi \;, \\ &-(\hbar^2/2m)\nabla^2\psi_0 - i(\hbar^2/m)\nabla\alpha\cdot\nabla\psi_0 + (\hbar^2/2m)(\nabla^2\alpha)\psi_0 \\ &\qquad\qquad + V\psi_0 = E\psi_0 \;. \end{aligned} \tag{6.4}$$

We take ψ_0 to be a plane wave,

$$\psi_0 = e^{i\mathbf{k}_0\cdot\mathbf{r}} \;. \tag{6.5}$$

In this equation, the second term gives the small perturbation due to α, while the term in $\nabla^2\alpha$ is of even higher order, according to Eq. (6.3), and can be ignored. Thus

$$\nabla\alpha\cdot\mathbf{k}_0 = -(m/\hbar^2)V \;. \tag{6.6}$$

If we integrate α along the direction of $\mathbf{k}_0$, then

$$\alpha = -\int \frac{m}{\hbar^2 k_0} V\,dx = -\frac{1}{\hbar}\int V\frac{dx}{v_0} = -\frac{1}{\hbar}\int V\,dt \;, \tag{6.7}$$

where we have used $p_0 = mv_0 = \hbar k_0$. Thus we have an expression for ψ (which is very accurate in the interferometer, where $V \ll T$) of the form

$$\psi = \psi_0 \exp\left[-\frac{i}{\hbar}\int_P V\,dt\right] . \tag{6.8}$$

This phase is calculated by integrating along the path P of the unperturbed beam. An equivalent expression is given by

$$\begin{aligned} &p_0^2/2m = E, \quad \frac{(p_0+\delta p)^2}{2m} + V = E, \\ &v_0\delta p = -V, \quad -\int V\,dt = \int \delta\mathbf{p}\cdot d\mathbf{r} \;. \end{aligned} \tag{6.9}$$

Equation (6.8) will form the basis of our treatment of the effect caused by various perturbations on the system. One might expect there to be an extra effect, since one knows that the perturbation will cause the beam to bend. In calculating the phase shift, however, we have integrated along the unperturbed path. This is because any effects due to the bending of the beam are of higher order.

The reason that the bending of the beam does not affect the phase at a point along the motion can be seen by referring to Fig. 7. If a free particle wave function emanates from some point P, its wave fronts will be as shown. If the wave function is intercepted by a screen (which may be imaginary), the phase at point A will not

[6]See Wigner, 1961, reprinted in Wigner, 1967. We believe Professor Wigner would not disagree with our calculation above. The question lies in where the many coherent possible results suddenly become "reduced" to a single physical result. "Pure states" and "mixed states" do not evolve into each other by purely quantum-mechanical processes, and it is here where inherently nonquantum concepts may be introduced. See Wigner, 1963; Shimony, 1963.

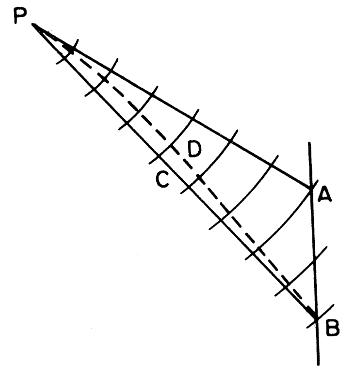

FIG. 7. The phase at a given point is unaffected by the bending of the beam. If a ray directed along the path *PA* is bent by a perturbing force into the path *PDB*, the phase at point *B* will be the same as the original phase along *PCB*, as the path length is the same, to first order. So the phase at *B* is unaffected by the bending of the beam, even though a different ray is now landing there.

be the same as that at *B*. Let us assume that a particular ray moves along the path *PA*. If now a perturbation is added to the system and the same ray shifts to the curved (dotted) path *PDB*, it is true that the phase at *B* will be different from that at *A*. But it always was different, even before the perturbation was added.

The important point is that the phase along the dotted path *PDB* is the same as that along the straight line *PCB*, and so the phase at point *B* has not changed, even though a different ray is now arriving there. (The length of *PDB* is the same to first order as that of *PCB*.) Of course, if the wavelength changes, the wave fronts will be distorted and the phase at a point will change, but that is exactly what Eqs. (6.8) and (6.9) take into account. There is no additional change in phase at a given point due to the bending of the beam. We shall need this result several times. (This result is discussed in greater detail in Greenberger and Overhauser, 1979.) It does not mean that there is no bending of the beam due to the force. In fact, the total effect at the screen caused by Eq. (6.8) integrated over the whole incident wave is that the entire interference pattern on the screen shifts by the amount of the classical motion. All these effects are included when one uses the prescription given by Eq. (6.8): to follow the phase along a single path from the incident point to a point on the screen, one uses Eq. (6.8) along the unperturbed path, and ignores any bending of the beam.

VII. THE GRAVITATIONAL PHASE SHIFT AND ITS SIGNIFICANCE

If the interferometer in Fig. 1 is rotated about the incident beam by an angle α, then one of the beams will be higher than the other. In the plane of the beams, the component of gravity will be

$$g_\alpha = g\sin\alpha \ , \tag{7.1}$$

and one can draw the diagram in Fig. 8(a). The difference in potential at time t is given by

$$V(P_2)-V(P_1)=mg_\alpha d = 2mg_\alpha x\sin\theta \ , \tag{7.2}$$

and the total phase difference will be

$$\begin{aligned}\Delta\phi &= -(1/\hbar)\int_{ACD}^{ABD} V\,dt \\ &= -(2/\hbar)\int_0^L 2mg_\alpha x\sin\theta(dx/v\cos\theta) \\ &= -mg_\alpha A/\hbar v \equiv -R \ ,\end{aligned} \tag{7.3}$$

where

$$A = 2L^2\tan\theta \tag{7.4}$$

is the area enclosed between the beams I and II.

In a typical neutron interferometer, the difference in height is of the order of 2 cm and the length $2L$ may be about 10 cm, so this phase shift is of the order of 100 rad (for $\alpha = 90°$, using $v \sim 10^5$ cm and $m \sim 10^{-24}$ g). Even with such a large height difference, the gravitational potential energy is only about 10^{-9} eV, which is extremely small compared to the kinetic energy of about $T \sim 10^{-2}$ eV. The question, therefore, arises as to how such a minute effect can be seen in the interferometer. The answer is that the distance over which the beam is coherent is so great. The wavelength is of the order of the interatomic spacing $a \sim 10^{-8}$ cm. The beam is coherent through the entire interferometer, $2L/\cos\theta$, or about $10^9\lambda$. Thus while the phase shift over one period, $\Delta\phi \sim V/T \sim 10^{-7}$, is minute, it nonetheless accumulates throughout the instrument so that overall $\Delta\phi \sim 100$ rad.

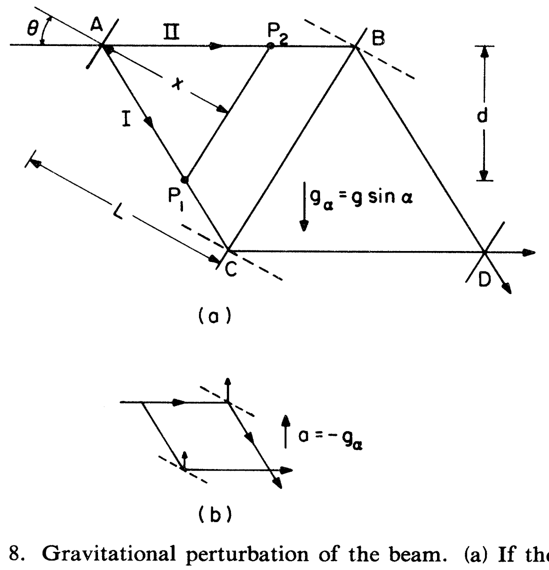

FIG. 8. Gravitational perturbation of the beam. (a) If the interferometer is rotated around the incident beam by an angle α, the beams will be at a different height, with an effective gravitational field $g_\alpha = g\sin\alpha$ in the interferometer plane. The difference in height between equivalent points along the paths of the two beams will be $d = 2x\sin\theta$, and the phase difference will be given by Eq. (7.3). (b) In the free fall system, the neutron beams will appear unaccelerated. However the interferometer scattering planes (or the mirrors $b-b', c-c'$) will appear to be accelerating upward, with the acceleration $a = g_\alpha$.

The experiment that actually measured this effect was one of classic elegance and simplicity, and is usually called the COW experiment (after the initials of the participants, Colella, Overhauser, and Werner).[7] While the COW experiment is in principle very simple, and the resulting interference pattern very beautiful, as in any new experimental technique there were many experimental problems that had to be overcome. For example, as the crystal is rotated, it stretches elastically under its own weight, and this effect must be measured and compensated for. This was done by cleverly designing the interferometer so that it would also accept x rays. The effect of gravity on x rays is minute within the interferometer, and so they could be used to monitor the elastic deformation of the crystal.

The COW experiment was conceptually very important in the history of quantum theory, as it was the first experiment to measure the effect of a gravitational field on the wave function. Of course it would have been a great shock if a nonrelativistic gravitational field were to disobey the Schrödinger equation. But our highly successful experience with this equation comes from electromagnetic fields and nuclear forces, and until the COW experiment was performed there was no actual evidence that gravity was "just another force," so far as the Schrödinger equation was concerned.

In fact, there were good reasons to wonder whether the Schrödinger equation actually would work in a gravitational field (Greenberger, 1968), because the equivalence principle is quantum-mechanically quite different in nature from its classical counterpart. It was, therefore, an important milestone when the principle and the Schrödinger equation were experimentally verified. Classically, one differentiates the weak from the strong form of the equivalence principle.[8] In its weak form, the principle states that all bodies fall with the same acceleration in an external gravitational field. In its strong form, it states that being at rest in an external gravitational field is equivalent to being at rest in an accelerating coordinate system. Since we know how to make coordinate transformations in the absence of gravity, the strong form can then be used to predict the behavior of a particle in a gravitational field.

Classical experiments, like the Eotvos experiment,[9] have tested the weak equivalence principle very accurately. When we want to discuss the quantum implications of the principle, we can introduce a more specific version of the weak equivalence principle, the "geometrical" weak equivalence principle (Greenberger and Overhauser, 1980), which we like to introduce in the following way. In classical physics the motion of a particle placed in an external field can be summed up in very few parameters that depend on the particle itself, like its mass and charge. Nature could have been much more complicated, and this is a minor miracle. But if the particle is placed into an external gravitational field, with a given position and velocity, a major miracle takes place. No parameters at all are needed to describe the motion of the particle. All particles will have the same motion, which depends then only on the environment and not at all on the particle. Thus one sees that the motion depends only on the local properties of space, and this leads directly to the geometrical formulation of gravitational theory.

This statement, that in all aspects the motion of the particle depends only on the environment, when it is in an external gravitational field, is the geometrical weak equivalence principle. Surprisingly, this principle does *not* carry over into quantum theory. The wavelength of the particle, even for a free particle, does depend on the mass, $\lambda = h/p$, and so all interference phenomena are mass dependent. This leads to a mass dependence of most quantized quantities. For example, if a light particle of mass m is in a gravitational orbit, with $V = GMm/r$, around a heavy particle of mass M, the nth Bohr radius will be (where $e^2 \to GMm$) $r_n = \hbar^2 n^2 / GMm^2$. Thus one can tell the mass of the particle from a measurement of its radius. This mass dependence is built into the theory directly from the commutation rules, or in the Bohr limit,

$$\int p\, dq = nh, \quad \int v\, dq = nh/m \;, \tag{7.5}$$

where $v = p/m$ is the velocity, so that $\int v\, dq$, which should classically be independent of the mass, is instead a function of h/m. It is only in the limit of high n where a scaling in momentum space occurs, such that n is proportional to m, and the mass drops out (Greenberger, 1968). See the Appendix for details.

The reason for this unexpected occurrence is that quantum theory is formulated dynamically, in terms of the momentum p and position q, and so is intrinsically mass dependent. By dynamical, we mean here that the details of the structure of the system (such as its mass) enter into the description. In contrast, when one classically formulates the equivalence principle, one talks either of all particles falling together, so that the mass is irrelevant, or of kinematically accelerating the coordinate system, a procedure independent of the nature of the particles present. Thus everything relating to classical equivalence is kinematical and totally free of structural details. Even the description of the paths of the falling particles is formulated in terms of trajectories, involving knowledge of the position and velocity. Mass (and momentum) are irrelevant.

In the quantum case, not only does the mass enter into the description of the motion of propagating bodies, but when one makes a unitary transformation to an accelerating system, this too is a dynamical transformation, involving the mass. (See the next section.) All elements of

[7] Colella, Overhauser, and Werner, 1975; Overhauser and Colella, 1974. A more accurate experiment was subsequently performed; it is described and the conceptual difficulties discussed in Staudenmann *et al.*, 1980.

[8] See for example, Dicke, 1964. For the quantum distinction, see Greenberger, 1968, on which our Appendix is based.

[9] The classic experiment is that of Eotvos *et al.*, 1922. For more recent experiments, see Roll *et al.*, 1967; Braginsky and Panov, 1971.

the motion are intrinsically mass dependent in the quantum case. Thus the two concepts, quantum and classical equivalence, are formally incompatible, and this lends a distinctly nongeometrical cast to quantum theory. [Modern quantum gravity theories bring geometry back in via gauge theories, and thence fiber bundles. As a beautiful and distinct alternative, measurements of quantum phase interference effects can be used as a basis for formulating gravitational theories, which has been done by Anandan (1977, 1979). But the above dilemma is deeper than this, and remains.]

We also point out in the Appendix that the independence of the mass makes the equivalence principle an exact symmetry in the classical case of a point particle in an external gravitational field. But in the quantum case this becomes only an approximate symmetry, valid only in the high-n limit. This is very different from the usual situation, in which a classical symmetry assumes a much greater importance in the quantized version of the theory. Thus there were reasons for believing that the applicability of Schrödinger theory to gravitational forces was an open question. The fact that quantum theory does indeed work for gravity, which was shown by the COW experiment, was a legitimate victory for the theory and should be respected as such, even though no one really expected otherwise.[10]

Because of the general lack of knowledge concerning the issues involved, we have included an Appendix to the paper which treats the subject in greater detail.

VIII. THE MOTION IN THE FREE FALL REFERENCE FRAME

According to the strong principle of equivalence, a particle at rest in an accelerating coordinate system should be equivalent to a particle at rest in a gravitational field. The transformation from an inertial system to a rigidly accelerating system (i.e., where every point has the same velocity and acceleration) is given by

$$\mathbf{r}'=\mathbf{r}-\boldsymbol{\xi}(t)\ ,\qquad t'=t\ . \tag{8.1}$$

Under this transformation, which has been called the "extended Galilean transformation" (Rosen, 1972; Greenberger, 1979), the gradient transforms as

$$\nabla=\nabla'\ ,\qquad \frac{\partial}{\partial t}=\frac{\partial}{\partial t'}-\dot{\boldsymbol{\xi}}\cdot\nabla'\ . \tag{8.2}$$

Thus the Schrödinger equation becomes

$$-(\hbar^2/2m)\nabla'^2\psi+V\psi=i\hbar\left[\frac{\partial}{\partial t}-\dot{\boldsymbol{\xi}}\cdot\nabla'\right]\psi\ , \tag{8.3}$$

where, if V is taken with respect to some reference point $\mathbf{r}_0$,

$$V(\mathbf{r}'-\mathbf{r}_0')=V(\mathbf{r}-\mathbf{r}_0)\ . \tag{8.4}$$

If we write

$$\psi(\mathbf{r},t)=u(\mathbf{r}',t)e^{iS}\ ,\qquad S=(im/\hbar)\left[\dot{\boldsymbol{\xi}}\cdot\mathbf{r}'+\int^t\dot{\xi}^2dt/2\right]\ , \tag{8.5}$$

then u satisfies

$$-(\hbar^2/2m)\nabla'^2u+Vu+m\ddot{\boldsymbol{\xi}}\cdot\mathbf{r}'u=i\hbar\dot{u}\ , \tag{8.6}$$

corresponding to the effective gravitational force

$$F_g=-m\ddot{\xi}\ . \tag{8.7}$$

This is the form of the strong equivalence principle in quantum theory. The unitary transformation e^{iS} in Eq. (8.5) is not arbitrary, but corresponds to the appropriate canonical transformation in the classical case (see the appendix to Greenberger and Overhauser, 1979). Although the strong equivalence principle holds in this case, note that the factor S is mass dependent, so that the remarks we made in the preceding section apply here as well.

Using the connection, which is implied by Eq. (8.6), between the laboratory with its constant gravitational field and the free fall system, one can now analyze the COW experiment in the free fall system. In this system, one is falling with the neutron beam, which is now not accelerating but moving inertially. Instead, it is the interferometer which is accelerating upward and imparting a differential Doppler shift to the two beams I and II. In Fig. 8 the normal atomic planes off which the beam is scattered (the dotted lines in the figure) are approaching beam I at point C and receding from beam II at point B. The detailed analysis of this situation is quite tricky. It involves following the wave fronts as they bounce off an accelerating mirror, and we shall not reproduce it here (Greenberger and Overhauser, 1979), but the effect is to give the same phase shift as that of Eq. (7.3) in the laboratory system. We can, however, give a simpler argument to show that the phase shift is the same.

Imagine, as in Fig. 8(b), that there is no gravitational field, but instead the interferometer is accelerating upward, with acceleration g_α. Then after the beam I has traveled from A to C in time T, where

$$T=L/v\cos\theta\ , \tag{8.8}$$

the mirror at C has a velocity upward given by

$$u=g_\alpha T\ , \tag{8.9}$$

but only the component perpendicular to the mirror, $u\cos\theta$, will contribute to the recoil momentum of the neutron [Figs. 9(a) and 9(b)].

The component of velocity along the mirror,

[10]Greenberger, 1968, actually remarked about the standard theory, when applied to gravity, "... and until it proves in the laboratory its right to be taken seriously, we believe that the theory should be considered suspect." (Not until the COW experiment did the theory earn this right! See the Appendix.)

$v_1 = v\cos\theta$, will not change, but the neutron's perpendicular recoil velocity, $v_2 = v\sin\theta$, will pick up an extra component $2u\cos\theta$ perpendicular to the moving mirror [Eq. (3.3) for the case $m_2 \to \infty$]. Since $u \ll v$, the recoil velocity will be given by

$$\begin{aligned} v'^2 &= (v+\delta v)^2 = v^2 + 2v\delta v \\ &= (v\cos\theta)^2 + (v\sin\theta + 2u\cos\theta)^2 \\ &\approx v^2 + 4uv\sin\theta\cos\theta\,, \\ \delta v &= 2u\sin\theta\cos\theta\,. \end{aligned} \tag{8.10}$$

The effect this extra velocity will have on the phase at a given point on the screen will then be given by Eq. (6.9),

$$\begin{aligned} \delta\phi_{\mathrm{I}} &= (1/\hbar)\int \delta p\, dr = (m/\hbar)\int \delta v\, dr \\ &= (m/\hbar)(2u\sin\theta\cos\theta)(L/\cos\theta) \\ &= 2mg_\alpha L^2\tan\theta/\hbar v = R\,, \end{aligned} \tag{8.11}$$

using the explicit form for u, Eqs. (8.8) and (8.9). R is the phase shift calculated as due to gravity, Eq. (7.3).

Beam II will be slowed up by the same amount, since the mirror at B is accelerating away from the neutron, so

$$\delta\phi_{\mathrm{II-I}} = -2R\,. \tag{8.12}$$

These phase shifts for a point on the screen are again not affected by the bending of the beams.

There is one other effect that must be accounted for, and that is due to the fact that the screen itself, at D, is moving. If the pattern shifted by one fringe, but the screen moved over by one fringe, the pattern would appear unchanged on the screen. If the total movement of the screen is δa, and the distance between fringes is l, then the phase shift due to this effect is

$$\delta\phi_S = \pi(\delta a/l)\,, \tag{8.13}$$

since the distance between fringes corresponds to only half a period. But the distance l is given by Eq. (2.1) as $\lambda/2\sin\theta$, and δa is the total distance moved by the interferometer since the beam entered at point A. The total motion of the interferometer is vertical, and given by $\delta z = \frac{1}{2}g_\alpha(2T)^2$, but only the motion parallel to the screen will contribute to the shifting of the fringes, so

$$\delta a = \delta z\cos\theta\,, \tag{8.14}$$

as can be seen from Fig. 9(c), and

$$\begin{aligned} \delta\phi_S &= \pi(2\sin\theta/\lambda)[g_\alpha(2T)^2\cos\theta/2] \\ &= 2kg_\alpha\sin\theta\cos\theta(L/v\cos\theta)^2 \\ &= 2mg_\alpha L^2\tan\theta/\hbar v = R\,, \end{aligned} \tag{8.15}$$

where T is given by Eq. (8.8), and $k = mv/\hbar$. Finally,

$$\delta\phi_{\mathrm{total}} = \delta\phi_{\mathrm{II-I}} + \delta\phi_S = -R\,, \tag{8.16}$$

and the phase shift in the free fall system is the same as that due to gravity in the laboratory system.

This result has been used by Colella and Overhauser (1980) to argue that the COW experiment directly tests the strong equivalence principle. They point out that the COW experiment provided the direct experimental justification of the Schrödinger equation in the gravitational case. But we know from extensive experience that the free-particle Schrödinger equation works, and that one can describe the effects of acceleration by a unitary transformation. This part of the equivalence principle need not be checked directly by experiment, as the results are known. Hence the COW experiment checked the only unverified part of the equivalence, and the strong equivalence principle has been proven.

Since this argument was made, the effects of the acceleration of the interferometer have been directly checked experimentally, both by using the earth's rotation (Staudenmann *et al.*, 1980) and by artificially accelerating the interferometer up to 10 times the earth's rotation.[11] Thus the strong equivalence principle has been directly proven in quantum mechanics.

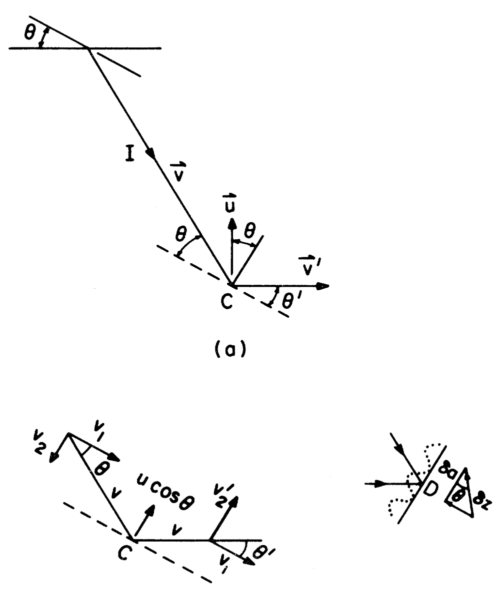

FIG. 9. The neutron motion in the free fall system. (a) In beam I, the atomic planes at C (or mirror $c-c'$) are moving upward with velocity u, and approaching the beam with velocity $u\cos\theta$. This imparts an extra component speed $2(u\cos\theta)$ to the reflected neutron in the normal direction. (b) The collision at C is shown in greater detail. (c) If the screen moves up by an amount δz, then it is displaced by $\delta a = \delta z\cos\theta$ relative to the interference pattern.

IX. THE RED SHIFT AND THE TWIN PARADOX

The classical theory is the limit of both general and special relativity. While it is certainly true that both of

[11]D. Atwood, MIT thesis, 1982, to be published; J. Arthur, MIT thesis, 1983, to be published. This work was performed at the neutron diffraction laboratory, MIT, under the direction of Professor C. G. Shull.

these theories reduce to classical mechanics for weak gravitational fields and low velocities, it would be much more to the point to say that in this limit these theories determine the structure of classical mechanics (Greenberger, 1982). Thus, while classical mechanics and nonrelativistic quantum theory predict the results of the COW experiment, and relativity is definitely not required to derive the appropriate gravitational phase shift, nonetheless relativity provides an insight into the nature of the effect that a purely classical treatment cannot provide.

If one travels along with a particle, the phase factor $e^{-iEt/\hbar}$ becomes

$$\psi \sim e^{-imc^2\tau/\hbar}, \tag{9.1}$$

where τ is the proper time along the path. In the laboratory system the proper time[12] is given by

$$d\tau = \left[g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} \right]^{1/2} dt \;. \tag{9.2}$$

To lowest order in a static gravitational potential ϕ, one has

$$\begin{aligned} &g_{00} \approx 1+2\phi/c^2, \;\; g_{ii} \approx -1/c^2 \;, \\ &g_{\mu\nu}=0, \;\; \mu \neq \nu \;, \end{aligned} \tag{9.3}$$

so that

$$\begin{aligned} d\tau &\approx (1+2\phi/c^2 - v^2/c^2)^{1/2} dt \\ &\approx (1+\phi/c^2 - v^2/2c^2) dt \;, \end{aligned} \tag{9.4}$$

and the action becomes

$$\begin{aligned} \exp\left[i \int L\, dt/\hbar \right] &= e^{-imc^2\tau/\hbar} \\ &\approx \exp\left[(i/\hbar)(-mc^2 t) \right. \\ &\qquad \left. + \int (mv^2/2 - m\phi) dt \right] \;. \end{aligned} \tag{9.5}$$

The rest-mass term, which leads to a rapid oscillation, is the same along any possible path the particle takes and so does not contribute to any nonrelativistic phase shifts, but the other contribution determines the form of the nonrelativistic Lagrangian,

$$L = T - V \;. \tag{9.6}$$

It also identifies the difference in proper time for two observers following each of the beams I and II, as a function of the laboratory time dt, in the form

$$\begin{aligned} &d\tau = (1+\phi/c^2 - v^2/2c^2) dt \;, \\ &\delta(d\tau) = d\tau_{\rm II} - d\tau_{\rm I} = \frac{\Delta\phi}{c^2} dt \;, \\ &\Delta\phi = (\phi_{\rm II} - \phi_{\rm I}) \;. \end{aligned} \tag{9.7}$$

[12]See any standard text on relativity, for example, Adler *et al.*, 1975, especially Chap. 4.

Each beam is deflected by the same small amount, so there is no significant contribution from δv. The only important contribution is from the overall difference in gravitational potential between the two beams, which is none other than the red shift. So the phase shift, $\exp(-i\int \Delta V\, dt/\hbar)$, is seen to be caused by the different rates at which a clock ticks along each of the two beams.

On the other hand, if one analyzes the problem from the free fall system, there is no gravitational field present. In this system, which is a true inertial system, the entire effect comes from the asymmetry between the beams caused by the different kick that each beam receives from the crystal at point B or C, in Fig. 8, as it recoils. (Remember that in this system the crystal is accelerating.) As to the significance of the effect, as seen in this system, the difference between proper time for the two beams is none other than the twin paradox effect in special relativity.

In the standard presentation of the twin paradox, one twin sits on the earth, while the other travels to a star at constant speed, quickly decelerates, and then comes back at constant speed. (One can imagine the deceleration as being caused by elastically bouncing off a mirror.) The difference between their elapsed times is due to the asymmetry produced by the deceleration of the one twin [see Fig. 10(a)]. Now imagine that both twins simultaneously head out to the star at slightly different speeds, one at speed v and the other at speed $v+\delta v$. Both twins now reverse their velocities simultaneously (by earth time) and head back toward earth, where they will arrive simultaneously by earth time [Fig. 10(b)]. Again there would be a

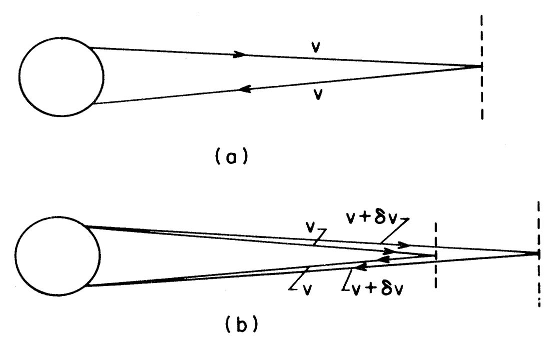

FIG. 10. The twin paradox. (a) One twin stays on the earth while the second moves out to a star at high speed v, turns around quickly (possibly by being reflected off a mirror), and returns at speed v. The twin who took the trip will be younger than the one who remained at home. The asymmetry in the problem is caused by the acceleration he received in turning around. (b) If both twins travel out, one at speed v, and the other at speed $v+\delta v$, and both turn around simultaneously (by earth time) and arrive home simultaneously, the one who traveled at higher speed will be younger. The asymmetry here is caused by the different accelerations received by the twins. There is a time difference even though one twin is traveling very slowly relative to the other (so that high speeds are not at all necessary). This is analogous to the different motions of beams I and II in the free fall system. In this system the phase shift can be attributed to the twin paradox effect.

time difference between them, caused this time by their different velocities. Note that in this case the speed differential δv can be very small, so that one twin, as seen by the other, has a very nonrelativistic velocity. In fact the speed of the faster twin, as seen by the slower one, is

$$\Delta v=\frac{(v+\delta v)-v}{1-v(v+\delta v)/c^2}\sim\frac{\delta v}{1-v^2/c^2}\ , \tag{9.8}$$

which can be kept small by keeping δv sufficiently small. The effect, therefore, does not depend on high velocities at all, but only on the asymmetry between the two systems (each receives a slightly different kick to reverse its velocity).

However, this latter case is very similar to that of the two neutron beams I and II in the interferometer, which separate at point A in Fig. 8, receive different kicks at B and C, and then recombine at point D. Thus in the free fall system the COW experiment is none other than the twin paradox, and the phase shift is caused by the difference in proper time between the two "twins," beam I and beam II, as measured by the velocity differential δv calculated in Sec. VIII.

We might mention here that this is really the "ultimate" twin paradox, because it is the very same neutron that is taking both paths and producing an interference pattern upon recombining. It is hard to imagine more identical twinning than that. As a final point, we note that the red shift and twin paradox are really two sides of the same coin, the description depending upon who is doing the describing, the gravitating or the free fall observer. Of course, as we noted earlier, this analysis does not "prove" relativity, since relativity is not needed at all to produce the phase shift in the COW experiment. What the analysis does is to provide an insight into the reason why the effect exists, even in the nonrelativistic limit.

X. THE SIGN CHANGE IN THE NEUTRON WAVE FUNCTION PRODUCED BY A 360° ROTATION

Another fundamental experiment that was first performed with the neutron interferometer was the direct verification that the wave function of the neutron changes sign when the neutron is rotated by 360°.[13] This is a basic result of quantum theory and is true for any fermion (e.g., the proton and electron). It had been indirectly verified every time a quantum-mechanical calculation concerning the neutron spin had been shown to work experimentally, so that nobody could seriously doubt the result. But an experiment involving the actual rotation of a neutron had not previously been done, and the outcome so defies one's classical intuition that it is an important experiment.

The appropriate way to describe a nonrelativistic neutron is as a column matrix (a spinor) describing the two directions of spin. A neutron that is spin up (down) along a particular direction $\hat{\mathbf{l}}$ will be an eigenfunction of the operator $\boldsymbol{\sigma}\cdot\hat{\mathbf{l}}$ such that

$$\begin{aligned}\boldsymbol{\sigma}\cdot\hat{\mathbf{l}}\psi&=+\psi\ ,\\ \boldsymbol{\sigma}\cdot\hat{\mathbf{l}}\psi&=-\psi\ ,\end{aligned} \tag{10.1}$$

for the spin-up and spin-down cases, respectively, where $\hat{\mathbf{l}}$ is a unit vector, and the σ_i are the standard (2×2) Pauli matrices,[14] obeying

$$\begin{aligned}&[\sigma_i,\sigma_j]=i\hbar\sigma_k,\quad (i,j,k\ \text{cyclic})\ ,\\ &\mathbf{S}=\hbar\boldsymbol{\sigma}/2\ ,\\ &(\boldsymbol{\sigma}\cdot\mathbf{a})(\boldsymbol{\sigma}\cdot\mathbf{b})=\mathbf{a}\cdot\mathbf{b}+i\boldsymbol{\sigma}\cdot(\mathbf{a}\times\mathbf{b})\ ,\end{aligned} \tag{10.2}$$

with $\mathbf{S}$ being the spin operator, S_z being diagonal and having eigenvalues $\pm\hbar/2$. In the last identity of Eq. (10.2), a and b are ordinary vectors, rather than quantum-mechanical operators (or if they are operators, they must commute).

A rotation of the neutron about an axis $\hat{\mathbf{n}}$, through an angle θ, is accomplished with the operator

$$\begin{aligned}\psi'&=U\psi=e^{-i\mathbf{S}\cdot\hat{\mathbf{n}}\theta/\hbar}\psi=e^{-i\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}\theta/2}\psi\\ &=[\cos\theta/2-i(\boldsymbol{\sigma}\cdot\hat{\mathbf{n}})\sin\theta/2]\psi\ .\end{aligned} \tag{10.3}$$

This can easily be seen by applying the infinitesimal rotation $\delta\boldsymbol{\omega}=\hat{\mathbf{n}}\delta\theta$, which gives

$$\begin{aligned}\psi'&=U\psi=U\boldsymbol{\sigma}\cdot\hat{\mathbf{l}}U^{-1}U\psi\\ &=(1-i\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}\delta\theta/2)\boldsymbol{\sigma}\cdot\hat{\mathbf{l}}(1+i\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}\delta\theta/2)\psi'\\ &=\boldsymbol{\sigma}\cdot(\hat{\mathbf{l}}+\delta\boldsymbol{\omega}\times\hat{\mathbf{l}})\psi'\ ,\end{aligned} \tag{10.4}$$

where we have kept only terms to first order in $\delta\theta$. This result shows that ψ' is spin up along the rotated direction $\hat{\mathbf{l}}'=\hat{\mathbf{l}}+\delta\boldsymbol{\omega}\times\hat{\mathbf{l}}$, if ψ was spin up along $\hat{\mathbf{l}}$, from Eq. (10.1).

A point particle with charge e moving in a circle in the x-y plane will have a magnetic moment $M=iA/c$ in cgs units, where i is its current (the total charge passing a given point in one second), $i=e\nu=e\omega/2\pi$, and $A=\pi r^2$, so that

$$\begin{aligned}&M=(e\omega/2\pi)(\pi r^2)/c=e(mr^2\omega)/2mc\ ,\\ &\mathbf{M}=(e/2mc)\mathbf{L}\ ,\end{aligned} \tag{10.5}$$

$\mathbf{L}$ being the orbital angular momentum. The Hamiltonian for the particle if it is placed in a uniform magnetic field $\mathbf{B}$ will be

$$H=-\mathbf{M}\cdot\mathbf{B}=-(e/2mc)\mathbf{L}\cdot\mathbf{B}\ . \tag{10.6}$$

If we use the Heisenberg equations of motion for an operator A,

$$\dot{A}=(i/\hbar)[H,A]\ , \tag{10.7}$$

---

[13]Werner *et al.*, 1975; Rauch *et al.*, 1975. This experiment has since been done using the Fresnel diffraction of cold neutrons (Klein and Opat, 1976).

[14]See any standard quantum text, for example, Merzbacher, 1970, Chap. 13.

and apply it to the operator **L**, which obeys the same cyclic commutation rules as the spin, Eq. (10.2), we get, for example,

$$\begin{aligned}\dot{L}_z &= (i/\hbar)(-e/2mc)[\Sigma L_j B_j, L_z] \\ &= (e/2mc)(B_y L_x - B_x L_y)\ , \\ \dot{\mathbf{L}} &= -(e/2mc)\mathbf{B}\times\mathbf{L}\ .\end{aligned} \tag{10.8}$$

This equation of motion, with $\dot{\mathbf{L}}$ perpendicular to **L** and **B**, represents precession[15] about the direction of B, with the Lamor frequency

$$\boldsymbol{\omega}_L = -\omega_L \hat{B} = -(eB/2mc)\hat{B}\ . \tag{10.9}$$

Similarly, any angular momentum vector can produce a magnetic moment if there is a charge distribution within the particle (even if the overall $Q=0$ for the particle). For a charge distribution, the difference from the case of a point charge, Eq. (10.5), is the presence of an extra factor g, a dimensionless factor which depends on the nature of the charge distribution. For the spinning neutron, we have

$$\mathbf{M} = (ge/2mc)\mathbf{S} = (g/2)\mu_0\boldsymbol{\sigma}\ , \tag{10.10}$$

where $\mu_0 = e\hbar/2mc$, the nuclear magneton, and $g=-1.91$ for the neutron. The Hamiltonian for the neutron is

$$H = -g\mu_0\boldsymbol{\sigma}\cdot\mathbf{B}/2\ , \tag{10.11}$$

and the motion of the neutron in the magnetic field will be given by

$$\begin{aligned}\psi(t) &= e^{-iHt/\hbar}\psi(0) \\ &= \exp[ig\mu_0\boldsymbol{\sigma}\cdot\mathbf{B}t/2\hbar]\psi(0)\ .\end{aligned} \tag{10.12}$$

Comparison with Eq. (10.3) shows that this is equivalent to a precession about the axis of **B**, by the amount

$$\theta = -g\mu_0 Bt/\hbar = -g\omega_L t\ . \tag{10.13}$$

Thus it can be seen that if the neutron is placed in a magnetic field of known magnitude, one can calculate precisely the angle of precession. However, according to Eq. (10.3), when the neutron has rotated through $\theta=2\pi$, the wave function will have become

$$\psi' = e^{-i\pi}\psi = -\psi\ , \tag{10.14}$$

and thus its phase should have changed by 180°. Only when the neutron has rotated through 4π, or 720°, will the phase have rotated back to its original value.[16]

According to classical physics, if one rotates the coordinate system one is using by 360°, it will certainly have no effect on the experiment. Alternatively, if one places the entire laboratory on a turntable, and then slowly rotates everything in the laboratory through 360°, again one would not expect this to have any effect on experiments done in the laboratory. But in this case, according to quantum mechanics, any fermions in the laboratory will have changed the sign of their wave function.

Of course, one could not see the effect of this change in quantum mechanics either, since all quantum expectation values are quadratic in the wave function and so will be unaffected. In fact, the initial phase of all particles in the laboratory is unknown, and it is only changes in phase that can be monitored by experiments. How, then, can this sign change be detected? Only by producing an interference between the rotated and unrotated state of the neutron could one hope to see this phase change due to the rotation. But in the neutron interferometer this is easy to do. All one must do is split the neutron into two beams, I and II, and rotate the neutron in one of them by one revolution. Then when the beams are recombined, the relative phase will have changed. Where before the interference pattern showed a maximum, it will now show a minimum. The experiment can be performed in a simple apparatus as in Fig. 11, where one of the beams passes through a uniform magnetic field of known magnitude on its way through the interferometer. The neutron beam need not even be polarized, as each direction of spin will be independently rotated through 2π. The experiment clearly shows that when the neutron is rotated an odd number of times, the phases are reversed.

By looking at Fig. 11, one sees that the experiment is not equivalent to rotating the entire laboratory. Only one of the beams is affected by the magnetic field, so only the neutron wave function on that side has been rotated. One makes the reasonable assumption that this corresponds to a true rotation of the neutron, on that side. At any rate, the wave functions for both beams are coherent, and the experiment shows that the relative phase between the beams has been reversed. And yet, all that has happened is that the neutron in one beam has been rotated by a full revolution and brought back to its initial orientation before the beams were recombined.

This is like being asked whether, while you were out,

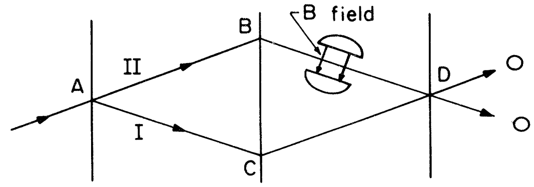

FIG. 11. Rotation of the neutron by 2π. If a uniform magnetic field is placed in one beam, the neutron can be made to precess by 2π. It can then be verified that under a 2π rotation the neutron wave function changes sign. Yet from a classical point of view, of course, a particle that has been rotated by 2π should be completely unaffected.

[15]This is covered in most modern physics texts. See also Goldstein, 1980, Sec. 5-9.

[16]The degree to which a magnetic field reproduces a true rotation has been discussed by Bernstein, 1979. See also Bernstein and Phillips, 1981. Byrne (1978) has raised the serious problem of whether the rotation is observable, which has been resolved by Zeilinger, 1981.

one of the objects on your desk was rotated by 360°. Classically there is no possible way to distinguish such an effect. But quantum mechanically, if the object were a neutron, it would stand out prominently. This is just one more quantum effect for which there is no hint that would allow one to anticipate it classically.

XI. MEASUREMENTS THAT DETECT THE ABSENCE OF A PARTICLE

Recently, Dicke (1981) raised the fascinating question of whether a measurement that is made to record the presence of a particle, but that fails to find it, reduces the wave packet. He calls it an "interaction-free measurement."[17] Consider the example of a particle in a box. Assume the particle is known only to be somewhere in the box, and a well-collimated and focused light beam is passed through part of the box, say the entire left half, with a high efficiency for striking the particle, if it is there. Then if the particle is in the left half it will scatter a photon and be detected. But if it is not there, the light beam will pass straight through.

After this measurement, if the particle is not seen, it is known to be on the right side of the box, so its center of mass has shifted. Also, since Δx has been decreased, Δp has therefore been increased, making its energy greater. There are two separate cases that must be considered here. The first case is that where the photon itself is always detected, right after it passes through the box. If it has been scattered, the electron is in that half of the box. If it has not been scattered, the photon will be detected in the forward direction. Then the electron is in the other half of the box, and its energy has been increased. But the wave function of the photon itself has not been changed, yielding an apparent paradox. (Where did the extra energy come from?) Dicke performs a model calculation, assuming a resonant absorption of the photon. He finds that there is an energy uncertainty produced by the experiment sufficient to guarantee that there is no energy violation.

The second case is where the photon, when it does not scatter the electron, is not immediately detected. Instead it continues to pass through the system and is still in a coherent state, only in this case its amplitude has been decreased. We shall refer to both cases as "Dicke's paradox." Dicke calculated the first case, but we shall be considering the second case, where the photon is not immediately counted, but passes through the system, because this is the case of relevance to the neutron interferometer.

In the first case, a photon is created and then detected, so that it makes sense to talk of the state of the individual photon. But in the second case one has an incident beam of photons, so the wave picture is appropriate, and one can say that the amplitude of this wave has been decreased. Because the possibility exists that there will be a collision between the electrons and the photons, some photons will be removed from the beam. Those photons which are not removed form a subset of the total incident beam, with decreased intensity. If these photons are subsequently allowed to interact with other matter, whether coherently or incoherently, their decreased amplitude will have to be taken into account.

This statement, while it may appear quite trivial as a quantum-mechanical result, is actually rather remarkable, and illustrates one further way in which quantum particles are different from their classical counterparts. For a wave it makes perfectly good sense to say that if the wave is split into two subsets, the amplitudes of each subset will be less than the total. But for a classical particle, if it is hurled at a target, it either strikes it or misses it. If it misses it, it should be totally unaffected. Only when it strikes the target (i.e., comes within the range of the potential) should its state be changed. There is no analog to the amplitude's being decreased. One might say that even classically the intensity of the beam is decreased, but this is a far cry from the quantum statement that the interference pattern will be changed for those particles which are not absorbed.

The quantum result is due to the fact that the wave function of a particle does not refer to the result of a single experiment performed on a particular particle. It refers to the probability of a given outcome in a large number of trials. (See Sec. XV.) The fact that the number of particles missing the target is less than the total number sent will be reflected in the wave-function amplitude's being decreased, even though each individual remaining particle may have its quantum numbers unchanged. Later, this beam may be coherently combined with another beam to form an interference pattern, and then the amplitude change can be converted into a phase change. (In the above experiment, the photon field loses intensity, and therefore energy, as the particle gains it. Thus indirectly there is an interaction, through probability conservation or unitarity.) This is an example of the intrinsic nonlocality of quantum mechanics, inherent in any wave theory.[18]

Another common example of this phenomenon is the optical theorem in scattering.[19] This theorem relates the imaginary part of the scattering amplitude in the forward direction to the total cross section. In words, it says that the intensity of the beam in the forward direction, which contains just those particles that missed the target, is decreased by the amount of particles scattered, and it is proved by using the conservation of probability. Again, it is a rather amazing theorem from a classical viewpoint,

[17]Similar considerations were previously raised by Renninger, 1960, and are discussed in Jammer, 1974.

[18]The nonlocal aspects of quantum theory are implicit, for example, in the Aharonov-Bohm effect (Aharonov and Bohm, 1959). These aspects are stated explicitly in their later papers, and emphasized in Greenberger, 1981.

[19]See any standard quantum text, for example, Merzbacher, 1970, Chap. 21.

because it says that even those particles which miss the target are affected by its presence (in a coherent way), in that the amplitude of the wave function is decreased.

This type of experiment, which measures the absence of a particle, actually comes up frequently with the neutron interferometer. Thus the physicists involved have had to take it into account, though they have not phrased it as provocatively as Dicke, nor given it the special emphasis he has. It enters here when an absorbing material is placed into one of the beams of the interferometer. Imagine a beam of neutrons which strikes an absorbing material [see Fig. 12(a)]. Inside the material, the neutron gets absorbed exponentially, with the penetration depth $1/\gamma$, so we can write the wave function emerging as

$$\psi = ae^{2i\delta}\psi_0, \quad a = e^{-\gamma L/2}, \tag{11.1}$$

where a is the amplitude of the partially absorbed wave and 2δ is any phase shift that has been induced by the absorber.

From a classical point of view, a neutron either passes through the absorber or gets absorbed. A neutron that passes through has not been materially affected by the passage, as 100% of the particle has passed through. (The absorber will also have an index of refraction $n \neq 1$. So it will affect the particles passing through it in this way. But this does not alter our classical argument, as once outside the absorber $n=1$ again, and there is no effect on the intensity of the beam. Of course quantum-mechanically absorption also produces dispersion, and the phase is shifted.) The absorber is a detector which detects some of the particles, namely those which are absorbed (and it can be arranged that they give a signal when they are absorbed—see Sec. XIII), while the rest pass through. We have here a case of Dicke's paradox, as those particles which pass through have had their wave function affected by having not been detected, and have a smaller amplitude.

We can see a further feature of Dicke's paradox here. If a particle is detected, its phase is destroyed, usually by an uncontrollable interaction with the detector, in this case the absorber. But if the particle's absence is detected, its phase is still coherent—only its amplitude has been changed. We can best see this by following the beam through the interferometer. If the absorber is placed in one of the beams [Fig. 12(b)], then at the last ear of the interferometer (or, in our example, at the screen) the wave function will be

$$\psi = e^{ikx} + ae^{2i\delta}e^{-ikx}. \tag{11.2}$$

This is to be contrasted with the beam if the absorber is absent,

$$\psi_0 = e^{ikx} + e^{-ikx}, \quad |\psi_0|^2 = 4\cos^2 kx, \tag{11.3}$$

and the contrast of the beam is $C=1$. The intensity for the partially absorbed case, Eq. (11.2), is

$$|\psi|^2 = (1-a)^2 + 4a\cos^2(kx+\delta). \tag{11.4}$$

The contrast in this case is

$$C = 2a/(1+a^2). \tag{11.5}$$

For complete absorption, $a=0$ and $C=0$. For no absorption, $a=1$ and $C=1$. So the partial absorption of the beam affects the contrast of the interference pattern. The greater the absorption of the beam, the more the contrast is affected, until there is only one beam left ($a=0$).

If the neutron is not absorbed, one cannot tell which side of the interferometer it traveled down, and so there is still an interference pattern. However, as more of beam II gets absorbed, it becomes less likely that the neutron reaching the screen was in this beam, and the contrast decreases. This absorption effect on the wave function has been verified several times.

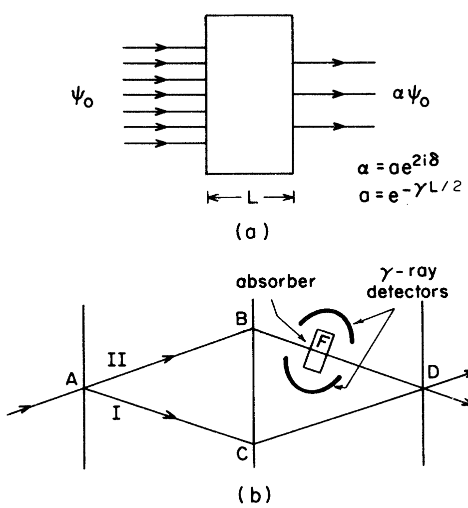

FIG. 12. The effect of a neutron absorber placed in the beam. (a) If the neutron passes through an absorber, its amplitude will be decreased (and it may be phase shifted). Classically there is no analog to the amplitude decrease, as these neutrons missed the target particles altogether. (b) If a cadmium absorber is placed in one beam of the interferometer, a γ ray will be emitted if the neutron is absorbed. Then the neutron was located in beam II. If the neutron is not absorbed, there will be an interference pattern on the screen at $d-d'$ (or at the counters), and the neutron must be "located" in both beams. This can be used to simulate either the Schrödinger cat or a delayed-choice experiment (see Secs. XIII and XIV).

XII. AN ELEGANT VERSION OF THE DOUBLE-SLIT EXPERIMENT

The standard double-slit diffraction experiment clearly shows the enormous contrast between quantum and classical measurement theory. In a typical experiment of this kind, a double slit is illuminated by an electron beam, and the diffracted beam is transmitted to a screen far from the plane of the slits. The coherent sum of the beams generated by the slits separately yields the standard double-slit interference pattern—with many maxima and minima. Of course each separate electron lands at some specific point along the screen. It is only as the result of counting many electrons that the double-slit pattern $|\psi_1+\psi_2|^2$ emerges.

Furthermore, if a device is placed in one of the beams, just beyond one of the slits, in order to ascertain which slit the electron "actually" took, then if it succeeds in this task it will necessarily destroy the interference pattern. This general result follows from the uncertainty principle.

A particularly pretty method for illustrating this effect is to imagine that the incident electron beam is completely polarized with its spin "up," along the $+z$ axis [see Fig. 13(a)]. Just behind slit II is a localized uniform magnetic field, which is of such a magnitude and length that it completely reverses the spin of particles in beam II, so that beam I will be spin up, while beam II will be spin down. Then a spin-sensitive detector is placed at some point along the screen. If this detector is set to measure only spin-up particles, it will see only those electrons coming from slit I. If it is set to detect only spin-down particles, it will see only those from slit II. In either case there will be no diffraction pattern.

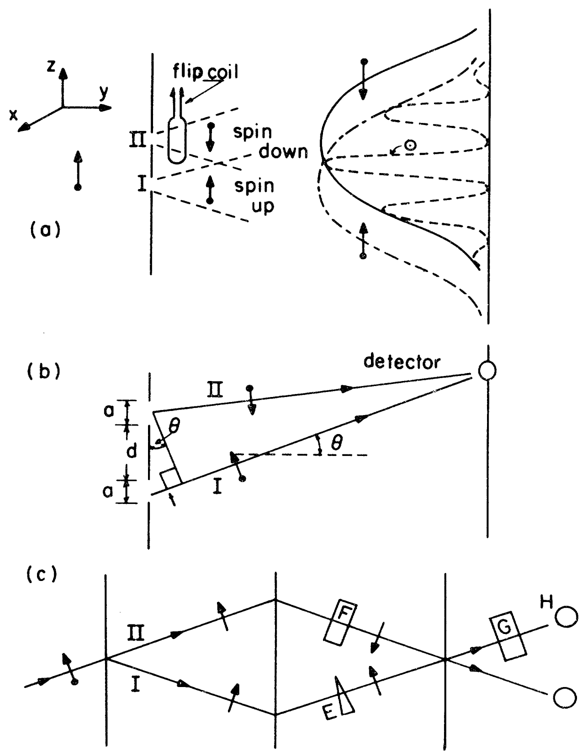

FIG. 13. An elegant double-slit experiment. (a) An electron beam polarized along the $+z$ axis is sent through a double slit. Immediately after slit II, a spin-flip coil rotates all the electrons in beam II to the spin-down position. A detector at the screen that detects only spin along the $+z$ axis will detect the single-slit pattern from beam I; if it detects along $-z$, it will detect the single-slit pattern from beam II; but if it is set to detect along the $+x$ direction, it will see an interference pattern between the beams, as it can no longer discern which beam the electron took. (b) The geometry for the spatial part of the phase relationship between the beams, assuming the distance to the detector is much greater than d. Every time t increases by λ, the phase difference β changes by 2π. (c) Simplified schematic for the neutron experiment of Summhammer *et al.*, 1982. The spin-flip coil is located at F. The polarization detector is at H. At G is a coil which can rotate the spin by a known amount, which is equivalent to rotating the detector, H. Again, a diffraction pattern is formed by inserting a wedge at E.

However, if it is set to detect polarization along the x axis, it will detect a coherent admixture of spin-up and spin-down states, and will therefore produce an interference pattern. If the slit width a is much less than the distance between the slits d [see Fig. 13(b)], then the phase β between the two beams I and II will be given by

$$\frac{\beta}{2\pi}=\frac{t}{\lambda},\quad \frac{t}{d}=\sin\theta\ ,$$
$$\beta=(2\pi d/\lambda)\sin\theta\ , \tag{12.1}$$

where θ is the angle from the normal between the slits to the detector. The wave function at the screen, for small θ, will be

$$\psi=\begin{bmatrix}\psi_1\\ \psi_2\end{bmatrix}=\frac{e^{i\kappa}}{\sqrt{2}}\begin{bmatrix}1\\ e^{i\beta}\end{bmatrix}$$
$$=\frac{e^{i(\kappa+\beta/2)}}{\sqrt{2}}\begin{bmatrix}e^{-i\beta/2}\\ e^{i\beta/2}\end{bmatrix}, \tag{12.2}$$

where κ is an irrelevant phase factor, and the $\sqrt{2}$ is for normalization. The spin functions corresponding to spin up and down along the x axis are

$$|\hat{x}\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix},\quad |-\hat{x}\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ -1\end{bmatrix}, \tag{12.3}$$

so that

$$\frac{1}{\sqrt{2}}\begin{bmatrix}e^{-i\phi}\\ e^{i\phi}\end{bmatrix}=\cos\phi\,|\hat{x}\rangle-i\sin\phi\,|-\hat{x}\rangle\ . \tag{12.4}$$

Therefore the detector, if it is located at some angle θ, in Fig. 13(b), and is sensitive only to spins along the $+x$ axis, will receive an intensity, from Eqs. (12.2) and (12.4),

$$|\langle\hat{x}|\psi\rangle|^2\sim\cos^2\beta/2=\cos^2(\pi d\sin\theta/\lambda)$$
$$\rightarrow\cos^2(\pi d\theta/\lambda)\ , \tag{12.5}$$

where the last expression holds for small θ. In this case the pattern goes from a maximum to a minimum as θ varies through

$$\Delta\theta=\lambda/d,\quad \lambda\ll d\ , \tag{12.6}$$

so the spatial diffraction pattern has been transformed into a spin-correlation one.

If the slit width a is not much less than the distance between slits d, then one can use the usual laws of elementary optics to get

$$\begin{bmatrix}\psi_1\\ \psi_2\end{bmatrix}=\frac{e^{i\kappa}}{\sqrt{2}}\frac{\sin\alpha/2}{\alpha/2}\begin{bmatrix}e^{-i(\alpha+\beta)/2}\\ e^{i(\alpha+\beta)/2}\end{bmatrix},$$
$$\alpha=(2\pi a/\lambda)\sin\theta,\quad \beta=(2\pi d/\lambda)\sin\theta\ , \tag{12.7}$$

where κ is again an irrelevant phase factor. This gives

$$|\langle \hat{x} \mid \psi\rangle|^2 \sim \frac{\sin^2\alpha/2}{(\alpha/2)^2}\cos^2(\alpha+\beta)/2 \quad (12.8)$$

so that the phase varies at the center of the pattern ($\alpha \ll 1$) as before (but with $d \rightarrow d+a$), while the entire pattern is modulated by the large single-slit diffraction envelope $(\sin^2\alpha/2)/(\alpha/2)^2$. In practice, it would be exceedingly difficult to perform this conceptually beautiful experiment with electrons, as one would have to localize the magnetic field over a very small region, since the beams are less than 1 mm apart.

However, the neutron interferometer analog of this experiment has already been performed by Summhammer *et al.* (1982), in an extremely elegant fashion. A simplified schematic diagram of their experiment is shown in Fig. 13(c). The incident beam was completely polarized, and the entire experiment was conducted in a Helmholtz coil so as to maintain a uniform magnetic field and thereby to continuously define the direction of polarization. At point F, a spin-flip coil was used to flip the direction of spin of beam II by 180°. A detector was located at H, which detected only spin up along the z axis. At position G, another localized magnetic field device was placed, which could rotate the spin of that beam by a known amount, which could be varied. The purpose of this was to make it unnecessary to rotate the detector (at H) itself, since this consisted of a bulky device plus a delicately aligned analyzer crystal. If the device at G rotated the spin by 180°, so that the crystal was detecting neutrons along the $-z$ direction, no interference pattern was seen. [Such a pattern could be detected as usual, by moving a wedge, as at point E in Fig. 1(b), through the beam.] This is because it would then be possible to know which beam, I or II, the neutron had taken.

However, when the device at G rotated the spin by 90°, so that the detector was effectively sensing the x direction, an interference pattern was recorded. In this case it was impossible to tell along which beam the neutron had come, and so the two beams could interfere. Prior to this experiment, many theoretical papers had been written which depended on the possibility of splitting a polarized beam and then coherently recombining it, usually by means of a double Stern-Gerlach device (see, for instance, Wigner, 1963). While this is a very easy experiment to perform with Pauli matrices, in practice the Summhammer *et al.* experiment is probably the first such experiment to have been done in a laboratory. At any rate, it is a classic demonstration of exactly what information can and cannot be obtained in a quantum two-slit diffraction experiment. (A simplified optical version of this experiment, in which individual photons are not detected, can be performed as a demonstration. See Henneberger and Zitter, 1983.)

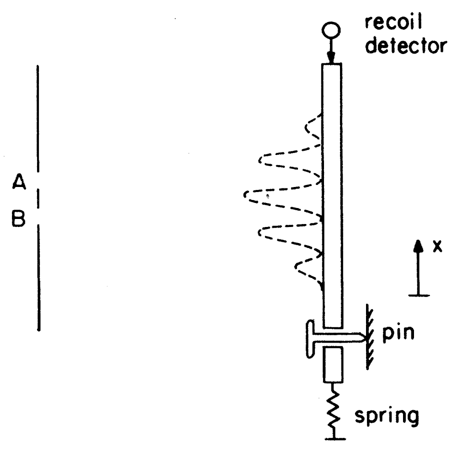

FIG. 14. Delayed-choice experiment. In a two-slit electron experiment, one can wait until the particle reaches the screen to determine which slit it passed through. If the screen is attached to a spring, its recoil can be detected. This will determine the particle momentum, and one can find out which slit it passed through, wiping out the interference pattern. However, one can place a pin through the screen, eliminating the recoil, and preserving the interference pattern. The choice of inserting the pin needn't be made until well after the electron passes the slits. An analog of this experiment can be set up with the neutron interferometer.

XIII. THE WHEELER DELAYED-CHOICE EXPERIMENTS

Wheeler[20] has described an extension of the standard electron double-slit experiment which is designed to emphasize the nonintuitive nature of quantum theory. A simple prototype of this kind of experiment is shown in Fig. 14. One performs a double-slit experiment with a particle, and a two-slit interference pattern appears on a screen behind the slits. Ordinarily, if one wanted to determine which slit the particle actually went through, one would place a momentum counter by one of the slits, A or B. This counter would click if the particle took that slit, but the interaction between particle and counter would necessarily destroy the interference pattern at the screen.

Wheeler's simple but ingenious modification is to make the screen itself part of the counter system. One way to do this (in theory, not practically) is to make the screen light and mobile, so one can measure its recoil, for example by holding it in place with an elastic spring. Then when the particle strikes the screen, its transverse recoil p_x can be accurately measured. This measurement, if sufficiently accurate, will by conservation of momentum be able to define the momentum of the particle, and thus one can determine which slit the particle took. The resulting Δx will be great enough to destroy the interference pattern.

Alternatively, one can insert a pin to hold the screen in place. This will prevent the screen from recoiling and

[20]A description of these experiments and Wheeler's thoughts on their significance can be found in Wheeler, 1978. The present status of such experiments, using photons, as well as many other topics, is discussed in Wheeler and Zurek, 1983.

give the characteristic interference pattern. There are, then, two alternatives, the first being to locate which slit the particle took, and the second being to preserve the interference pattern. What is so special about Wheeler's experiment is that one does not need to make up one's mind as to which measurement to make until just before the particle reaches the screen. Thus we have a situation where the particle has interacted with the screen at some time in the past, at which time it presumably passed through one slit or both. But which of these possibilities is realized is not actually determined until the experimenter decides which experiment to perform, and he can do this at a time long after the particle left the screen.

Quantum-mechanically, this experiment offers no surprises or difficulties in analysis, and there is nothing peculiar about the situation, since the actual experiment reduces the wave packet in the standard manner. Until the measurement at the screen is made, both possibilities are coherent.

But classically, the result is very paradoxical, and it looks as though the result of the experiment influences the behavior of the particle at an earlier time, when it passed through the slits. The result illustrates Wheeler's dictum that "No phenomenon is a phenomenon until it is an observed phenomenon." From his point of view, the experiment illustrates the strange, acausal nature of time itself, quantum-mechanically. In fact, in this experiment, the experimenter does not ever have to decide whether to keep the pin in or not. A computer can make the selection randomly at the last moment. So there is no "precognition" necessary here—the particle is not reading the mind of the experimenter—the strangeness of the result is inherent in quantum mechanics.

It is actually quite possible, without too much difficulty, to set up an experiment using the neutron interferometer that contains the essential features of the delayed-choice experiment.[21] Consider the experiment shown in Fig. 12(b). There an absorber is located in one side of the beam, in beam II. Let us assume that the absorber is sufficiently thick that half of the neutrons will be absorbed. If the absorber is cadmium, each time a neutron is absorbed the excited absorber will emit a γ ray. These γ rays can be detected with high efficiency, by placing the appropriate detectors around the absorber.

The neutron enters the interferometer at point A. At this point it splits up into beams I and II. However, if a γ ray from the absorber is detected, then the neutron will at that time be wholly in beam II, and the amplitude for being in beam I will abruptly drop to zero. Thus the detector will force the neutron into the state

$$\psi=\alpha u, \quad |\alpha| \sim 2^{-1/2} , \tag{13.1}$$

where it will be localized in a cadmium nucleus. On the other hand, if no γ ray is detected, the neutron will be in the superposition

$$\psi'=\beta\psi_{\rm II}+\psi_{\rm I}, \quad |\alpha|^2+|\beta|^2=1 , \tag{13.2}$$

and there will be an interference pattern formed on the screen (or by the counters $C1$ and $C2$).

From a classical point of view, the neutron enters the interferometer at point A and it is here that its future behavior is determined, as to whether it will have any component in beam I. But it is not until a γ ray is detected or not that this fact is experimentally determined.

Classically, the outcome of an earlier experiment should have produced a definitive result, whether we are aware of it or not, and not the superposition of several possible results, which is what we actually see. The neutron enters the interferometer at point A. At this point either the beam splits, and an interference pattern is recorded at the counters, or the particle takes beam II, and is captured by the absorber, emitting a γ ray. The "decision" whether or not to absorb the neutron is made by a cadmium atom in the absorber, long after the neutron has entered the interferometer and interacted with it at point A, where its subsequent motion presumably has been determined.

Once again, it looks classically as though an earlier decision is affected by a later one. Also, in this case, it is the random absorption by a cadmium atom that determines which possibility occurs, not the conscious decision of an experimenter. Again, this experiment is simple to interpret quantum-mechanically. It is only the classical desire to overdetermine the quantum possibilities that leads to a weird interpretation of the results. (The absorber at F could easily be arranged to be randomly inserted into the beam, after the neutron had passed point A, adding an even greater degree of randomness to the process, and possibly providing evidence against some forms of hidden variable theories. This has already been done in a beautiful experiment for the case of two-photon decays, by inserting essentially random polarizers into the beams. This experiment is also relevant to considerations of the Einstein-Podolsky-Rosen paradox. See Aspect *et al.*, 1982.)

In the delayed-choice experiment of Fig. 12(b), if the γ ray is detected, one has confirmation that the particle is in only one beam. On the other hand, when the γ ray is not detected, one has the interference pattern to show that the particle is "in both beams." We can therefore verify that a particle entering the apparatus at point A, but not being detected until point D or point F, is really not uniquely in one state or the other before the detection is made. This observation is made about 10^{-5} sec after the choice at A, which on a microscopic scale is a very long time. In this experiment, then, we actually cannot know which beam the particle took for this long period. The wave function in beam I has a finite amplitude, after the particle enters at A. But as soon as the γ ray is detected at the absorber, the probability for the wave packet to be in beam I drops abruptly to zero, this having been determined by an event happening several centimeters away!

We can describe the experiment in a slightly more quantitative way by introducing two states for the

[21]This experiment was first suggested by Greenberger *et al.*, 1983.

absorber-detector system: ξ, which represents the case where the detector has not recorded a γ ray, so that the neutron has passed through unabsorbed; and η, representing the case where the counter has clicked and the neutron has been absorbed. The two states ξ and η are orthogonal.

The detector, located at point F in Fig. 12, will have a state designated by $\phi(t)$. It starts out in the state

$$\phi(t)=\xi(t), \quad t<t_F \; . \tag{13.3}$$

However, after the neutron has reached it, if the amplitude for absorption is α, the wave function will become

$$\phi(t)=[\beta\xi(t)+\alpha\eta(t)]\delta_{i2}+\xi(t)\delta_{i1}, \quad t>t_F \; . \tag{13.4}$$

This equation says that if the neutron is in beam II, $\psi_i=\psi_2$, and there is a finite probability for the counter to click. If, on the other hand, the neutron is in beam I, $\psi_i=\psi_1$, then there will be no interaction between the neutron and detector. (The function η also has a random phase relative to ξ, which doesn't affect the following considerations.)

For the neutron, if it is in beam II, the initial state will be

$$\psi_2(t)=\psi_{\rm II}(t), \quad t<t_F \; . \tag{13.5}$$

($\psi_{\rm I,II}$ are the specific plane-wave states, or wave packets thereof, we have discussed.) If the neutron is absorbed, it will become localized to the state u of Eq. (13.1), and if it is not absorbed, it will remain in the state $\psi_{\rm II}$, so

$$\psi_2(t)=\begin{cases}\psi_{\rm II}(t), & \phi=\xi \\ u(t), & \phi=\eta \; , \quad t>t_F \; .\end{cases} \tag{13.6}$$

However, if the neutron is in beam I, far from the absorber at F, the wave function will be unaffected by the absorber,

$$\psi_1(t)=\psi_{\rm I}(t) \quad \text{for all } t \; . \tag{13.7}$$

The incident beam is one which has been equally split into beams I and II, and the counter has not yet been reached. So

$$\begin{aligned}\psi_{\rm total}(t)&=2^{-1/2}\psi_1(t)\xi(t)+2^{-1/2}\psi_2(t)\xi(t)\\ &=2^{-1/2}[\psi_{\rm I}(t)+\psi_{\rm II}(t)]\xi(t), \quad t<t_F.\end{aligned} \tag{13.8}$$

After it reaches the detector, the detector itself will evolve into the state given by Eq. (13.4). Therefore

$$\begin{aligned}\psi_{\rm total}(t)&=2^{-1/2}\psi_1(t)\xi(t)\\ &\quad+2^{-1/2}\psi_2(t)[\beta\xi(t)+\alpha\eta(t)], \quad t>t_F \; ,\end{aligned} \tag{13.9}$$

which according to Eqs. (13.6) and (13.7) becomes

$$\begin{aligned}\psi_{\rm total}(t)&=2^{-1/2}\psi_{\rm I}(t)\xi(t)+2^{-1/2}[\beta\psi_{\rm II}(t)\xi(t)+\alpha u(t)\eta(t)]\\ &=2^{-1/2}[\psi_{\rm I}(t)+\beta\psi_{\rm II}(t)]\xi(t)+2^{-1/2}\alpha u(t)\eta(t) \; ,\\ &\qquad t>t_F \; .\end{aligned} \tag{13.10}$$

The information contained in this equation is obviously equivalent to that in Eqs. (13.1) and (13.2), except that the extra $2^{1/2}$ represents a normalization before the beam enters the system at point A.

In this experiment, the quantum-mechanical interpretation seems even more strange, from a classical viewpoint, because we are using neutrons. These classically are particles, and it is their nonclassical wavelike property, and the possibility of a coherent superposition, that causes the trouble. Or rather, it is this coupled with the strange properties of quantum measurement theory.

One can also perform a delayed-choice experiment with photons (Wheeler and Zurek, 1983). The same conclusions follow, but here the wavelike aspects of the behavior of the photons are quite classical. Also, to confine a photon wave packet to the small dimensions of a piece of laboratory equipment requires state-of-the-art electronics, making it a very difficult experiment.

Finally, it should be emphasized that the classical way of stating the location of the wave packet, as being "in beam I" or "in beam II" is inconsistent with quantum mechanics, as it presumes the existence of a hidden variable that determines which beam it is in. Furthermore, it is even misleading, when an interference pattern exists, to describe the particle as being "in both beams."

In fact, any discussion at all of the location of the particle assumes that "location" is a property of the particle. But we have seen that the result of an experiment to determine this "location" depends critically on what experiment is performed. Even long after the particle seems classically to have "chosen" which beam it is in, this is experimentally still a very open question. This point is a main result of the delayed-choice experiments.

Therefore, while one can write down a wave function from which to calculate the result of any experiment on the particle, it is really not proper to describe the particle itself as being "in" any combination of beams. One should always keep this in mind when interpreting descriptions of interferometer experiments.

XIV. SCHRÖDINGER'S CAT AS A DELAYED-CHOICE EXPERIMENT

Schrödinger proposed his famous *Gedankenexperiment* known as "Schrödinger's cat," in order to point up the implausible behavior of the quantum wave function (Schrödinger, 1935). From a classical viewpoint, there are two remarkable features to the experiment, which we shall discuss. One of these features has to do with the macroscopic nature of the cat, and this raises the question of whether it is appropriate to represent a complicated, living system by a wave function. Because of this many physicists do not like to discuss the experiment, as they feel it would be impossible to peform, even in principle. They see the underlying situation as ambiguous, at best, and as science fiction, at worst.

We shall point out that both remarkable features of the experiment are squarely faced in the delayed-choice experiments, and so they represent a version of "Schrödinger's cat" that can be unambiguously defined and even performed in the laboratory. We would therefore recommend that a discussion of the cat experiment in elementary courses be replaced by one involving delayed choice.

Schrödinger's experiment considers a cat in an isolated cage (Fig. 15). Inside the cage is a vial of poison gas and a trip-hammer, all cocked and set to fall and break the vial, killing the cat. The hammer is tripped by some atomic event, say the decay of a radioactive particle. If we wait long enough, until there is a 50% chance that the particle has decayed, then there is a 50% chance that the cat is alive. At this time the radioactive source is automatically removed. Then, according to Schrödinger, at subsequent times the cat is in a superposition of states, alive and dead, with equal weights. It is not until we open the box and look that we force it into an eigenstate, either alive or dead.

The two remarkable features of the experiment, from a classical point of view, are, first, that the experiment has

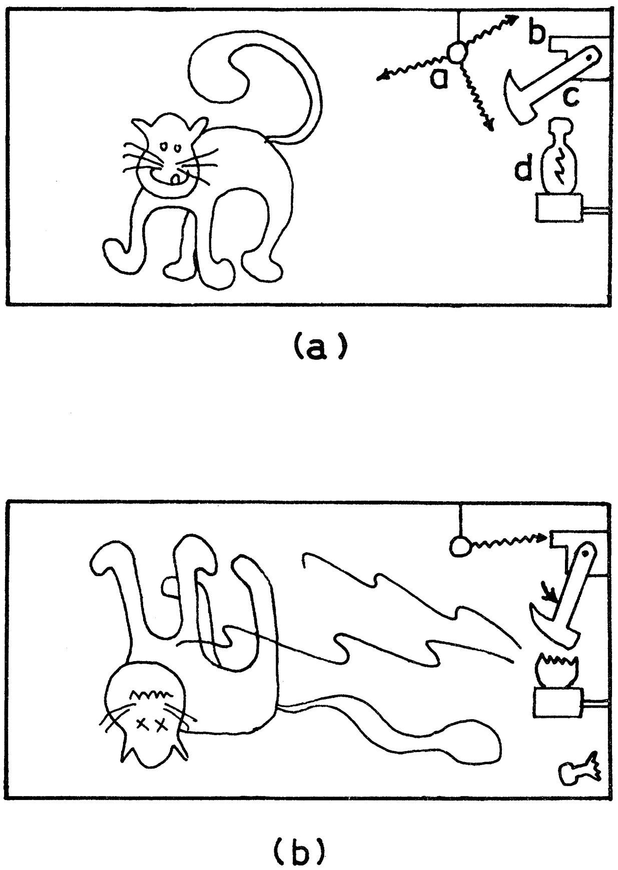

FIG. 15. Schrödinger's cat. A cat is placed in an isolated box with a radioactive source a and a detector for the decay b attached to the trip-hammer c. When the detector fires, the hammer breaks a vial of poison gas d, which kills the cat. If the cat can be represented by a coherent wave function, then until we open the box and observe it, it will be in a superposition of the states alive (a) and dead (b). The neutron in the interferometer can clearly be represented by a coherent wave function, and such a superposition of states brought about, so that an analog of this experiment can be performed [Fig. 12(b)].

already been completed, in the sense that the substance has either decayed or not. But from the viewpoint of quantum theory, until we look, the two states of live cat and dead cat are coherent and capable of interfering with each other. It is only the act of looking that completes the experiment and destroys the coherence, making it compatible with the classical possibilities. Second, the cat is a macroscopic object. It is hard enough to swallow quantum mechanics for microscopic objects, but a cat seems to be just too gross to be in a coherent superposition of states.

From the point of view of conventional quantum theory, there is indeed a serious objection to the use of a cat in this experiment, but it is not directly related to its being macroscopic in size. After all, today there are numerous macroscopic, coherent, quantum processes known, for example, superconductivity; it is not inconceivable that an experiment of this kind could be performed using the Josephson effect.

The real objection to using a cat has to do with coherence. Unlike the neutron, the cat has a very complex structure. Every time it twitches a muscle, it is sending uncontrollable vibration waves throughout its volume, creating and ultimately dissipating great numbers of phonons. Every cell in its body is continually undergoing a vast number of irreversible chemical reactions to support its large array of life processes, so on every possible level the cat is partaking in countless molecular events, almost none of which are coherent. Thus, even if there exists an overall wave function for a cat, its coherence time is close to zero, and there is no meaningful way to talk about superpositions of states. In short, cats are not very good coherent systems. (Even before the cat becomes involved, the release of the trip-hammer is an irreversible macroscopic event that destroys the coherence of the state, regardless of the coherence of the atomic event triggering it. The same is true for the spreading of the gas.)

Normally, by "macroscopic" one means a system of large size, of high mass, and of many degrees of freedom. But for any system, of any size, to maintain coherence, most of its degrees of freedom will have to be frozen out in some way, so that one can write a wave function for the few remaining ones. The neutron intrinsically contains only few degrees of freedom, which can be kept coherent. We pointed out in the Introduction that in the neutron interferometer the wave-packet coherence length may be as long as 0.1 mm, and overall the packet is about the size and shape of a postage stamp. Furthermore, this macroscopic object can be coherently separated by several centimeters and recombined. It is this combination of coherence and macroscopic size that makes the neutron especially suitable for performing an experiment of this type. Thus with neutrons we have a well-defined, even if somewhat limited, concept of "macroscopic" for use in such an experiment.

The delayed-choice experiment actually does reproduce the other remarkable feature of the cat experiment. The entry of the neutron into the interferometer at point A and its subsequent choice of which beam to take are the equivalent of the radioactive decay killing the cat or not. The ensuing situation, that there may or may not be a finite probability for finding the particle in beam I, is equivalent to the cat's being in a state partly alive, partly dead. It is not until this situation is later resolved by the γ-ray detector that the determination of whether the particle is partly in beam I is made, which is equivalent to the opening of the box and looking at the cat.

In this respect, our experiment is even superior to the Schrödinger one. In that experiment, one opens the box and sees the cat in one state or the other. But one has no proof that the cat was in a coherent state before one opened the box (and, of course, for a cat coherence is unattainable, as we have pointed out). But in our experiment one has the interference pattern to confirm that the particle is "in both beams."

We see, then, that the classically strange, acausal behavior of the cat experiment is fully reproduced in the delayed-choice experiments. The macroscopic nature of the wave function, which is very troublesome in the Schrödinger description, is well defined for the neutron, and gives rise to an experiment easily capable of being actually performed, even though the neutron is "macroscopic" only in a severely restricted sense. But the Schrödinger alternative produces too loose a description ever to be tested in any meaningful way.

XV. THE "OBJECTIVE" INTERPRETATION OF THE WAVE FUNCTION

In this paper we have usually interpreted the wave function as though it was the probability for an individual neutron. Since there is generally only one neutron in the interferometer at a time, this seems like a natural procedure. Such an interpretation also requires that when a measurement is made on a particular dynamical variable, the result must be an eigenvalue of the operator representing that variable, and the wave function is said to be "reduced" by the measurement to an eigenfunction of this operator. This reduction of the wave function is a very nonclassical type of behavior, and we have employed this interpretation specifically because it brings to the fore the conflict between the quantum and classical results. Thus, in the Wheeler delayed-choice experiment, the neutron seems to be in a state of "suspended animation" until an experimental determination is actually made from among the various possible states the system can have.

This is the interpretation of Von Neumann and Wigner[22] (although we do not go so far as to accept the intervention of a conscious mind). The reduction of the wave function can be thought of as being caused by the incoherent amplification process that necessarily accompanies any measurement, as the microscopic event is converted into the reading of a macroscopic meter. Thus, for example, when we calculated the recoil of the mirror, in Sec. V, we found that this recoil also caused a spread in the wave packet of the neutron, in order to guarantee the uncertainty principle. But although the wave functions of the neutron and mirror were both spread out, the various contributions were still coherent, having definite phase relationships to each other, so long as we were following solutions of the Schrödinger equation. It is only if the mirror recoil is actually measured by having it interact with some recording system, such as a beam of light or a pressure-sensitive detector, that these phase relationships will be broken.

During this period of interaction, the destruction of the phase is a statistical process, which results in an irreversible loss of information and the reduction of the wave function. It was often emphasized by Bohr that the exact pinpointing in time of this amplification process is not necessary, but that the process itself is crucial to the reduction of the wave packet. Wigner has emphasized that there is no way that one can follow the time development of a wave function, considered as a solution of the Schrödinger equation, and see it change from a pure state to a mixed state. Something beyond the quantum time development is taking place during a measurement. He has argued that at this stage one may have to introduce a conscious mind, although most physicists are content to accept the statistical amplification process as sufficient.

Once one is willing to accept this amplification process, however, there is an alternative interpretation of the wave function one can use, which Belinfante has called "objective" quantum theory. [The development of this approach is well explained in his book (Belinfante, 1975). See also Ballentine, 1970, and Hartle, 1968. Hartle points out that the one-particle wave function can be interpreted as providing "information" rather than objective knowledge. Classic references on the entire measurement problem can be found in the resource letter of DeWitt and Graham, 1971.]

According to this approach, one can only consider the wave function as describing an ensemble of particles. If one prepares many copies of the system identically, and repeats the experiment with each, then the wave function describes the probability of the various possible outcomes. With this interpretation, one need never face the strange situations we have described, and so this interpretation is generally more useful for the working physicist.

From this point of view, if one performs a delayed-choice experiment, say the double-slit experiment described in Sec. XIII, then it is irrelevant at what time one makes the choice of whether or not to pull the pin holding the screen. The wave function gives you the probability for seeing a result if the pin is in and the probability for seeing a result if the pin is out. The results for one particular neutron do not mean anything, even if only one neutron is in the system. The wave function only registers probabilities over large numbers of events. Simi-

[22]Both the adherents of this viewpoint and those of the "objective" viewpoint to be discussed claim theirs to be the "Copenhagen interpretation." Jammer 1974 claims that the Copenhagen interpretation can be stretched to include any point of view consistent with Bohr's complementary principle.

larly, the paradox of Schrödinger's cat becomes trivial. The wave function describes the probability for the experiment, if performed many times. It says nothing about a particular cat, so that one cannot think of the cat as being "half alive"—rather, one says that half of all the cats in the ensemble will be alive—and the same type of interpretation holds true for the neutron in the delayed-choice experiment.

For a beam of particles passing through an absorber, the reduced amplitude refers to the fact that only a certain percentage of the ensemble will pass through unabsorbed. Even for an experiment that looks like it applies to one particular particle, that particle must be thought of as merely one representative of an ensemble. For example, if a particle passes through a cloud chamber and leaves a track, one must think of ψ as applying to the small subensemble of all the particles entering the chamber that happened to take that particular path. (After the first two scatterings, the scattering amplitude will be strongly peaked in the forward direction, and all particles that produce the same two first scatterings will have closely similar tracks. And among these, we are interested in those members of the ensemble that took the identical path, to within the accuracy required.)

Finally, in this interpretation, when a measurement is made, as for example when an electron strikes a particular point on a screen, one just saves up the results of all the outcomes and determines probabilities for each outcome. One is not even required to consider the wave function as having been reduced. (However, the question still arises as to when the coherence between possible results is destroyed, even though one needn't explicitly refer the results to individual particles. This approach successfully sweeps the problem under a rug. It does not solve it.)

This interpretation, then, has the advantage that many of the "paradoxes" we mentioned do not have to be confronted. Furthermore, if the incident wave function itself is a mixture, rather than a pure state, as it usually is, then one can introduce a density matrix to describe the experiment throughout, and one is always dealing with probabilities of possible outcomes, rather than with wave functions.

Nonetheless, we have chosen an approach that deliberately confronts the specifically nonclassical features of quantum theory, rather than one which sidesteps them. The neutron interferometer is a device in which these features are easy to identify and analyze (and in many cases the experiments have already been done), so that they may be directly discussed in realistic situations and from several viewpoints, during an elementary course.

ACKNOWLEDGMENTS

The author would especially like to thank Professors Michael Horne, Albert Overhauser, and Abner Shimony for many valuable discussions. He has also profited significantly from discussions with Professors Yakir Aharonov, Herbert Bernstein, Roberto Colella, Robert Dicke, Anthony Klein, David Mermin, Clifford Shull, Laszlo Tisza, Samuel Werner, John Wheeler, and Anton Zeilinger, as well as Drs. John Arthur, Donald Atwood, and Husseyin Yilmaz. The author also owes a great debt of gratitude to Professor Shull for the wonderful hospitality extended to him by the MIT neutron diffraction laboratory during (and since) his recent sabbatical, as well as to Professors Herman Feshbach and Arthur Kerman and the MIT Center for Theoretical Physics. This research was supported in part by a grant from the City University of New York PSC-CUNY Research Award Program.

APPENDIX: THE WEAK EQUIVALENCE PRINCIPLE IN THE CLASSICAL LIMIT

In classical physics, the weak equivalence principle says that for a point particle moving in an external gravitational field, the mass of the particle does not enter the problem. In the quantum case this is not true, and yet the mass dependence must disappear in the classical limit! We shall see that there are two possible theories consistent with the classical limit. One is provided by conventional quantum theory, for which weak equivalence is approximate, and develops only in the classical limit. The other represents a theory which obeys weak equivalence even on the microscopic level.

In the classical limit, there is no possible way to choose between these alternatives, so that the very accurate classical Eotvos experiments provide no clue whatsoever as to the true state of affairs. Only an experiment that could see a quantum interference pattern due to gravity could prove that the conventional quantum theory is correct even in the presence of an external gravitational field and that gravity acts microscopically just like any other force placed into the Schrödinger equation. This is precisely what was accomplished by the COW experiment.

First we shall discuss the problem from the point of view of conventional quantum theory. If we imagine a particle placed in a one-dimensional (for convenience) external gravitational field with potential $m\phi(x)$, its classical Hamiltonian will be

$$\begin{aligned} H &= (\tfrac{1}{2})mv^2 + m\phi(x) \\ &= p^2/2m + m\phi = E \ . \end{aligned} \tag{A1}$$

Classically, the trajectory parameters $x(t)$ and $v(t)$ will be independent of m. This means that we can prepare a beam of particles, all of mass m_1, centered at x_1, v_1, with spreads $\Delta x_1, \Delta v_1$, which are released at time $t=0$, and determine their subsequent trajectories as a function of t. If we now prepare and release a beam of particles of mass m_2, but with the exact same distributions initially, then their subsequent time development will be identical to that for the case m_1. That is,

$$\begin{aligned} &f_i(t,m) \neq f(m) \ , \\ &f_i = x, v, \Delta x, \Delta v \ . \end{aligned} \tag{A2}$$

For classical point particles in an external field, this represents an exact, nonlocal symmetry of the theory. (All these functions are necessary to distinguish gravity from other forces. For example, one can devise a set of crossed electric and magnetic fields such that the forces cancel at the center of the beam, for any m. But this will not be true for the spread $\Delta x, \Delta v$ around the center.)

In phase space, the momentum will be proportional to the mass, as will the energy $[p = mv,\ E = m(v^2/2+\phi)]$, so

$$p_2 = Kp_1\ ,\quad E_2 = KE_1 \text{ for } K = m_2/m_1\ . \tag{A3}$$

Thus in phase space there is a scaling in p, which corresponds to particles of different mass having the same trajectories (see Fig. 16). The distribution for particles m_2 will take up $K\ (=m_2/m_1)$ times the area in phase space that the distribution for m_1 does.

To quantize this system, the Schrödinger equation becomes

$$\begin{aligned} H &= p^2/2m + m\phi \\ &\to -(\hbar^2/2m)\partial^2\psi/\partial x^2 + m\phi\psi\ . \end{aligned} \tag{A4}$$

The Bohr limit for this theory is

$$\int p\,dx = nh\ ,\quad \int v\,dx = nh/m\ . \tag{A5}$$

Thus trajectory parameters are quantized in units of (h/m). The Schrödinger equation can be written

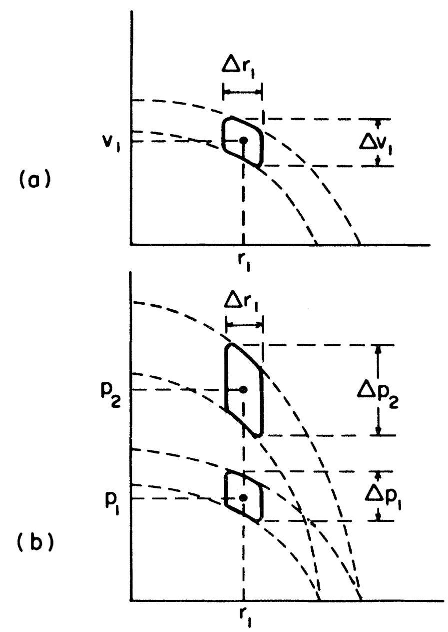

FIG. 16. The classical weak equivalence principle. (a) An ensemble of point particles released with a given distribution in r and v, in an external gravitational field, will follow the same trajectory in Boltzmann space (v vs r), regardless of the mass of the particles. This independence of m is an exact, nonlocal symmetry. (b) In phase space (p vs r), if the particles have mass $m_2 = Km_1$, the trajectory will be scaled in p by the amount K, so for the center of the packet, $p_2 = Kp_1$; for its spread, $\Delta p_2 = K\Delta p_1$; and for the area occupied, $A_2 = KA_1$.

$$\begin{aligned} &H\psi_n = m\varepsilon_n\psi_n\ , \\ &[-\tfrac{1}{2}(\hbar/m)^2\partial^2/\partial x^2 + \phi]\psi_n = \varepsilon_n\psi_n\ . \end{aligned} \tag{A6}$$

The parameter that enters this equation is $(\hbar/m)$, so we have

$$\psi_n = f(\hbar/m)\ ,\quad \varepsilon_n = f(\hbar/m)\ ,\quad E = mf(\hbar/m)\ . \tag{A7}$$

We see, then, that both in the Bohr limit and for the Schrödinger equation proper, the trajectory functions, which ought to be mass independent, are actually functions of (h/m). The energy, which ought to be linear in m, is a linear function times a function of (h/m).

The question naturally arises as to how this mass dependence disappears in the classical limit. The answer can be seen from Eq. (A5). If one particle of mass m_2 is K times heavier than another one of mass m_1 and it has the same trajectory, then its quantum state, n_2, must be K times that for the particle m_1, n_1. In that case,

$$\int v_2 dx = n_2 h/m_2 = Kn_1 h/Km_1 = n_1 h/m_1\ . \tag{A8}$$

Thus it is the K dependence that reproduces the classical scaling of the mass dependence. The same argument holds for the solutions to the Schrödinger equation, but only in the limit of high n. In that case the n dependence for high n follows from the WKB form for the wave function, and a similar scaling applies [see Eq. (A18)], and

$$\begin{aligned} E_2 &= m_2\varepsilon(n_2\hbar/m_2) \\ &= Km_1\varepsilon(n_1\hbar/m_1) = KE_1\ . \end{aligned} \tag{A9}$$

In other words, if the particle is K times heavier, its energy levels will be K times greater. In order for this scaling to be possible, one must be in the semiclassical domain, so that it is in fact possible to scale the quantum number n. For low-lying quantum states, the weak equivalence principle totally breaks down, as it must. For example, for a minimum wave packet,

$$\begin{aligned} &\Delta p\Delta x \sim \hbar\ , \\ &\Delta v\Delta x \sim \hbar/m\ , \end{aligned} \tag{A10}$$

and there is no way to have an identical velocity distribution for particles of different masses, if they also have an identical space distribution.

The reason why the scaling must break down for small n is that there is no natural mass scale (or equivalently, no fundamental length) in quantum theory, in terms of which such ratios can be universally expressed, and so weak equivalence breaks down for low-lying quantum states. Of course, within the spirit of quantum theory, one says that equivalence is formulated in terms of trajectories, while in quantum theory trajectories have no meaning for low-lying states.

If one has a wave packet centered about the state n_1 for a particle of mass m_1, then for a particle of mass m_2 $(=Km_1)$ the packet will be centered about the state

$n_2 = Kn_1$. Furthermore, if the contributing states to the packet for m_1 fill up a band of states Δn_1, then for particle 2 they will fill up the band

$$n_2 = Kn_1 \ , \quad \Delta n_2 = K\Delta n_1 \ . \tag{A11}$$

Within this band every state must contribute, so that the situation will be depicted by Fig. 17, where

$$\psi_1(m_1) = \sum_{N_1}^{N_2} a_n u_n \ , \quad \psi_2(m_2) = \sum_{KN_1}^{KN_2} b_n u_n \ , \quad \sum |a_n|^2 = 1 \ ,$$

$$N_1 \sim n_1 - \Delta n_1/2 \ , \quad N_2 \sim n_1 + \Delta n_1/2 \ , \tag{A12}$$

$$b_{Kn} = a_n/\sqrt{K} \ .$$

It is assumed here that the envelope of the a_n is a continuous function and all b_n between b_{Kn} and $b_{K(n+1)}$ are fitted to the slowly varying envelope, as in Fig. 17. Thus

$$\sum_{l=Kn}^{K(n+1)} |b_l|^2 \sim K\,|b_{Kn}|^2 \sim |a_n|^2 \ , \tag{A13}$$

which explains the normalization of the b_n, assuming that the a_n are normalized. For a continuous distribution, the relations (A12) take the form

$$\begin{aligned} &\psi_1 = \int a(k)u(k,x)dk \ , \\ &\int dx\, u^*(k,x)u(k',x) = \delta(k-k') \ , \\ &b(Kl) = a(l)/\sqrt{K} \ , \\ &\psi_2 = \int b(l)u(l,x)dl = \int \sqrt{K}a(k)u(Kk,x)dk \ . \end{aligned} \tag{A14}$$

The function ψ_2 (for particles of mass m_2) will be normalized if the function ψ_1 is.

We can give two arguments as to why every single one of the functions ψ_n within the envelope in Fig. 17 will contribute for the particle of mass m_2, rather than just the same number as for the particle m_1, but scaled up by the factor K. The first is that in the classical limit this distribution occupies K times as much phase space, and since each state occupies an area h of the phase space, this means that K times as many states contribute in this limit. The second argument has to do with the nature of matrix elements in the classical limit. (Our treatment is a simple extension of that in Landau and Lifschitz, 1977).

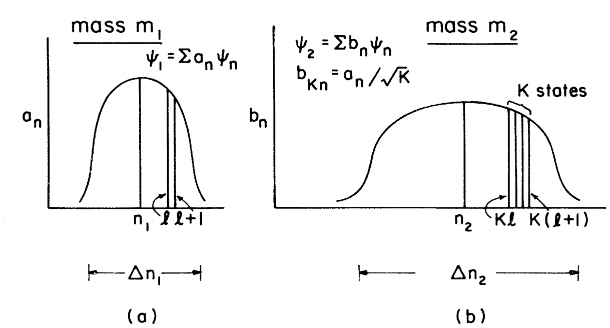

FIG. 17. The quantum equivalence principle. (a) A wave packet of states for a particle of mass m_1, centered about state n_1, with a spread Δn_1. (b) For a particle of mass $m_2 = Km_1$, the distribution will be scaled, in order to represent a beam with the same position and velocity. Thus $n_2 = Kn_1, \Delta n_2 = K\Delta n_1$. For two states, l and $l+1$, contributing in the m_1 case, all the K states between Kl and $K(l+1)$ will contribute with approximately equal amplitude. This scaling is possible only for high-n states, and weak equivalence holds only in the semiclassical limit.

In the high-n limit, we can use the WKB form for the wave function,

$$\psi_n = A[m/p(x)]^{1/2} \exp i \int^x p_n dx/\hbar \ . \tag{A15}$$

In this form, A is independent of the mass. In fact,

$$\begin{aligned} \int |\psi_n|^2 dx &\sim A^2 \int m\, dx/p(x) = A^2 \int dx/v \\ &= A^2 T_n = 1 \ , \end{aligned} \tag{A16}$$

so that $A \sim (T_n)^{-1/2}$, where T_n is the period of the particle. Then for two close-lying states, the momentum difference at a point is given by

$$\begin{aligned} &p_n^2/2m + m\phi(x) = E_n \ , \\ &p_n \Delta p/m = v_n \Delta p = \Delta E \ . \end{aligned} \tag{A17}$$

From this we see that for two states differing by Δn, where $\Delta n \ll n$,

$$\begin{aligned} \oint (p_{n+\Delta n} - p_n)dx &= \oint \Delta p\, dx = \Delta E \oint dx/v_n \\ &= \Delta E T_n = h\Delta n \ , \\ \Delta E &= h\Delta n/T_n = \hbar\omega_n \Delta n \ , \end{aligned} \tag{A18}$$

so that the energy levels near a given n are integrally spaced.

For any function of position $f(x)$, matrix elements are of the form

$$\langle n | f | n+l \rangle \sim A^2 m \int (dx/p_n) f \times \exp\left[i \int^x (p_{n+l} - p_n)dx/\hbar \right] \ , \tag{A19}$$

assuming $l \ll n$. Then

$$\begin{aligned} \exp\left[i \int^x \Delta p\, dx/\hbar \right] &= \exp\left[il\omega_n \int^{x(t)} dx/v_n \right] \\ &= \exp(il\omega_n t) \ , \end{aligned} \tag{A20}$$

so that

$$\langle n | f | n+l \rangle \sim (1/T_n) \oint dt\, f(x(t)) e^{il\omega_n t} \ . \tag{A21}$$

But this is just the classical Fourier component of the motion,

$$\begin{aligned} &f(x(t)) = \Sigma f_l e^{-il\omega t} \ , \\ &f_l = (1/T) \oint dt\, f(t) e^{il\omega t} \ , \\ &\langle n | f | n+l \rangle \to f_l \ . \end{aligned} \tag{A22}$$

We see that the Fourier components correspond to the matrix elements between neighboring states, for all neighboring states within the entire wave packet. The value of

n at the center of the packet, n_1, merely identifies the macroscopic orbit we are considering. So within conventional quantum theory, even though there is a gross scaling of the center of the wave packet, $n_2 = Kn_1$, and of its overall size, $\Delta n_2 = K\Delta n_1$, nonetheless the fine details of the macroscopic orbital motion depend on the overlapping of neighboring states. At this level there is no scaling. Also, all quantum interference effects, like low-lying effects, depend on the detail of the wave functions. They will thus be strongly mass dependent.

In the case of a nongravitational force, this same argument shows that the classical limit is also produced by the overlapping of neighboring matrix elements, as in Eq. (A22). However, in this case there is nothing corresponding to the scaling of the envelope of the wave packet, as there is no interest in relating the orbits of different particles with the same trajectory. We should emphasize that there is certainly nothing wrong with this entire viewpoint. Not only is it consistent, but the COW experiment shows that it corresponds to the reality of the situation. But the reason we have gone into such detail is to show that in the conventional quantum theory weak equivalence is a gross, approximate effect, valid only for high n. This is much weaker than the exact canceling of the mass in the classical theory, and seems to undermine the claim that it is a geometrical phenomenon. Equivalence is something that the quantum theory can certainly accommodate, but it must be tacked onto the theory. There is nothing "natural" about it.

Now there is an altogether different point of view one might take toward the equivalence principle. One might say that since the principle is exact classically, it should remain exact microscopically. Then it would be up to experiment to decide between the two possibilities. According to this viewpoint, we should be able to construct an alternate quantum scheme that worked in the case of a particle in an external gravitational field, in such a way as to preserve weak equivalence as an exact symmetry. One way to do this would be to introduce a new dynamical variable, the canonical velocity v_c, by

$$v_c = p/m \; . \tag{A23}$$

Then the Hamiltonian becomes

$$\begin{aligned} H &= p^2/2m + m\phi \rightarrow m(v_c^2/2+\phi) \\ &= m\mathscr{H}(x, v_c) \; . \end{aligned} \tag{A24}$$

The operator $\mathscr{H}$ is now classically independent of the mass, and so one could quantize the theory in terms of x and v_c, just as well as in terms of x and p. Considering only particles in an external gravity field, this approach is just as consistent.

Such a quantization scheme would necessarily involve introducing a fundamental length, such as by the commutation rule

$$[v_c, x] = c\lambda_0/i \; , \tag{A25}$$

where λ_0 is a length, and c is the speed of light. There is no point in pursuing this further, since we know from the COW experiment that it is unnecessary (at least in weak gravitational fields). But we did not know this until the COW experiment.

To summarize our argument, we have actually shown that conventional quantum theory is perfectly capable of describing the classical limit, and in fact by Ehrenfest's theorem, this limit reduces to $F = ma$ for expectation values. Therefore why should anyone be worried that the theory might not apply unchanged for gravity? This is the problem we have tried to discuss here, and we shall reiterate our concerns on this score:

(1) Quantum theory is dynamically formulated in terms of position and momentum. The equivalence principle is formulated in terms of identical trajectories for particles of different mass, and so the kinematical concepts of position and velocity enter.

(2) In the limit of low quantum numbers, trajectory parameters like radii, etc., will depend on the mass. So weak equivalence has no meaning for low-lying quantum states. Likewise, the concept of trajectory is not well defined here. On the other hand, in the classical theory, for a point particle in an external gravitational field, weak equivalence is an exact nonlocal symmetry (e.g., a light and heavy particle dropped simultaneously from the Leaning Tower of Pisa will follow identical trajectories all the way down).

(3) Normally, the significance of a classical symmetry is enhanced by quantum theory, not diminished. This case forms a striking exception.

(4) Interference effects are strongly mass dependent. Thus the geometric form of weak equivalence (which essentially says that it is an exact symmetry) breaks down. This gives a nongeometrical cast to the entire dynamical quantum approach, out of keeping with the spirit that classically leads to general relativity. (This nongeometrical flavor is also pointed out for other reasons in Weinberg, 1972.)

(5) As a corollary, we note that the form of the classical potential, $m\phi(x)$, is chosen so that the mass will cancel out of the problem. But since it does not do so quantum-mechanically, what is the justification for using this form for the gravitational potential? Only the argument that gravity is just another force, to be placed into the Schrödinger equation. However, classically, gravity is far from being "just another force," because of its geometrical implications.

(6) The classical limit is produced by matrix elements between neighboring states. In the case of equivalence, this must be implemented by a scaling procedure, so that the classical limit for gravity assumes a much wider context than that for other forces.

(7) This scaling breaks down for low quantum numbers. This is directly related to the lack of a fundamental length in the theory. If the theory did break down, this would be the ideal place to see evidence for a fundamental length. [One can make a length from the known constants ($\sim 10^{-32}$ cm). But this is no argument against new phenomena intervening. Historically, quantum effects unexpectedly undermined the scale set by the "classical

radius of the electron."]

(8) It is possible to construct an alternate quantum scheme that preserves weak equivalence microscopically. There is no classical way to decide between this and conventional quantum theory, such as by the Eotvos experiments. It is a matter for quantum interference experiments.

For all of these reasons it was at least proper to maintain an attitude of openmindedness concerning the applicability of conventional quantum theory to gravity. While no one expected the theory to break down, the correspondence limits for gravity and other forces are sufficiently different that the success of the conventional theory, primarily with electricity, could not be taken for granted as providing a sure guide to the gravitational case. The COW experiment was necessary to lay these doubts to rest. Of course, conventional expectations have occasionally been wrong before. That they were right in this case does not detract from the fundamental nature of the COW experiment.

REFERENCES

Adler, R. J., M. J. Bazin, and M. Schiffer, 1975, *Introduction to General Relativity* (McGraw-Hill, New York).
Aharonov, Y., and D. Bohm, 1959, Phys. Rev. **115**, 485.
Anandan, J., 1977, Phys. Rev. D **15**, 1448.
Anandan, J., 1979, Nuovo Cimento A **53**, 221.
Aspect, A., J. Dalibard, and G. Roger, 1982, Phys. Rev. Lett. **49**, 1804.
Ballentine, L. E., 1970, Rev. Mod. Phys. **42**, 358.
Batterman, B. W., and H. Cole, 1964, Rev. Mod. Phys. **36**, 681.
Belinfante, F. J., 1975, *Measurements and Time Reversal in Objective Quantum Theory* (Pergamon, Oxford).
Bernstein, H., 1979, in *Neutron Interferometry,* edited by U. Bonse and H. Rauch (Oxford University, Oxford), p. 231.
Bernstein, H., and A. V. Phillips, 1981, Sci. Am. **245** (July), p. 122.
Bonse, U., and W. Graeff, 1977, in *X-Ray Optics,* edited by H.-J. Queisser, Vol. 22 of *Topics in Applied Physics* (Springer, Berlin), p. 93.
Bonse, U., and M. Hart, 1965a, Appl. Phys. Lett. **6**, 155.
Bonse, U., and M. Hart, 1965b, Z. Phys. **188**, 154.
Bonse, U., and H. Rauch, 1979, Eds., *Neutron Interferometry* (Oxford University, Oxford).
Braginsky, V. B., and V. I. Panov, 1971, Zh. Eksp. Teor. Fiz. **61**, 873 [Sov. Phys.—JETP **34**, 464 (1971)].
Byrne, J., 1978, Nature **275**, 188.
Colella, R., A. W. Overhauser, and S. A. Werner, 1975, Phys. Rev. Lett. **34**, 1472.
d'Espagnat, B., 1976, *Conceptual Foundations of Quantum Mechanics,* 2nd ed. (Benjamin, New York).
DeWitt, B. S., and R. N. Graham, 1971, Am. J. Phys. **39**, 724.
Dicke, R. H., 1964, *The Theoretical Significance of Experimental Relativity* (Gordon and Breach, New York).
Dicke, R. H., 1981, Am. J. Phys. **49**, 925.
Eotvos, R. V., D. Pekar, and E. Fekete, 1922, Ann. Phys. **68**, 11.
Goldstein, H., 1980, *Classical Mechanics,* 2nd ed., Sec. 5-9 (McGraw-Hill, New York).
Greenberger, D. M., 1968, Ann. Phys. (N.Y.) **47**, 116.
Greenberger, D. M., 1979, Am. J. Phys. **47**, 35.
Greenberger, D. M., 1981, Phys. Rev. D **23**, 1460.
Greenberger, D. M., 1982, in *Physics as Natural Philosophy* (festschrift volume for L. Tisza), edited by A. Shimony and H. Feshbach (MIT, Cambridge, Mass.), p. 178.
Greenberger, D. M., M. Horne, C. Shull, and A. Zeilinger, 1983, in Proceedings of the International Symposium on Quantum Mechanics (Physical Society of Japan, Tokyo, in press).
Greenberger, D. M., and A. W. Overhauser, 1979, Rev. Mod. Phys. **51**, 43.
Greenberger, D. M., and A. W. Overhauser, 1980, Sci. Am. **242**, (May), 66.
Hartle, J. B., 1968, Am. J. Phys. **36**, 704.
Henneberger, W. C., and R. N. Zitter, 1983, Am. J. Phys. **51**, 464.
James, R. W., 1963, in *Solid State Physics,* edited by F. Seitz and D. Turnbull (Academic, New York), Vol. 15, p. 53.
Jammer, M., 1974, *The Philosophy of Quantum Mechanics* (Wiley, New York).
Kaiser, H., S. A. Werner, and E. A. George, 1983, Phys. Rev. Lett. **50**, 560.
Klein, A. G., and G. I. Opat, 1976, Phys. Rev. Lett. **37**, 238.
Klein, A. G., G. I. Opat, and W. A. Hamilton, 1983, Phys. Rev. Lett. **50**, 563.
Klein, A. G., and S. A. Werner, 1983, Rep. Prog. Phys. **46**, 259.
Landau, L. D., and E. M. Lifschitz, 1977, *Quantum Mechanics,* 3rd ed. (Addison-Wesley, Reading, Mass.).
Merzbacher, E., 1970, *Quantum Mechanics,* 2nd ed. (Wiley, New York).
Overhauser, A. W., and R. Colella, 1974, Phys. Rev. Lett. **33**, 1237.
Overhauser, A. W., and R. Colella, 1980, Am. Scientist **68**, (Jan.), 70.
Rauch, H., and D. Petrascheck, 1979, in *Neutron Diffraction,* Vol. 6 of *Topics in Current Physics* (Springer, Berlin), Chap. 9.
Rauch, H., W. Treimer, and U. Bonse, 1974, Phys. Lett. A **47**, 369.
Rauch, H., A. Zeilinger, G. Badurek, A. Wilfing, W. Bauspiess, and U. Bonse, 1975, Phys. Lett. A **54**, 425.
Renninger, M., 1960, Z. Phys. **158**, 417.
P. G. Roll, P. Kratkov, and R. H. Dicke, 1967, Ann. Phys. (N.Y.) **26**, 442.
Rosen, G., 1972, Am. J. Phys. **40**, 683.
Schrödinger, E., 1935, Naturwissenschaften **23**, 807.
Scully, M. O., R. Shea, and J. D. McMullen, 1978, Phys. Rep. **43**, 485.
Shimony, A., 1963, Am. J. Phys. **31**, 755.
Shull, C. G., 1968, Phys. Rev. Lett. **21**, 1585.
Squires, G. L., 1978, *Thermal Neutron Scattering* (Cambridge University, Cambridge), Chap. 6.
Staudenmann, J.-L., S. A. Werner, R. Colella, and A. W. Overhauser, 1980, Phys. Rev. A **21**, 1419.
Summhammer, J., G. Badurek, H. Rauch, and U. Kischko, 1982, Phys. Lett. A **90**, 110.
Weinberg, S., 1972, *Gravitation and Cosmology* (Wiley, New York).
Werner, S. A., 1980, Phys. Today **33** (Dec.), 24.
Werner, S. A., R. Colella, A. W. Overhauser, and C. F. Eagen, 1975, Phys. Rev. Lett. **35**, 1053.
Wheeler, J. A., 1978, in *Mathematical Foundations of Quantum Theory,* edited by A. R. Marlow (Academic, New York).
Wheeler, J. A., and W. H. Zurek, 1983, Eds., Quantum Theory

and Measurement (Princeton University, Princeton, in press).
Wigner, E. P., 1961, *Remarks on the Mind-Body Question,* in *The Scientist Speculates,* edited by I. J. Good (Heinemann, London/Basic Books, New York).
Wigner, E. P., 1963, Am. J. Phys. **31**, 6.
Wigner, E. P., 1967, *Symmetries and Reflections* (Indiana University, Bloomington), p. 171.
Zeilinger, A., 1981, Nature **294**, 544.