Concept of an "Inner Field" in the Theory of Superconductivity*

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1. INTRODUCTION

According to Landau and Lifshitz,[1] the properties of a second-order phase transition in a system may be expressed in terms of a set of order parameters that vary continuously with temperature and take nonzero values only below the transition point. This set of parameters normally constitutes a number of functions of position and we shall refer to it as an "inner field," as in the theory of ferromagnetism.

The aim of the present theory is to identify the inner field in a superconductor with the thermal average of some quantum-mechanical observable, and thereby to obtain a definitive characterization of the structure of the superconductive state. Once the identification is made, we are able to deduce the principal properties of superconductors (Meissner effect, persistent currents, flux quantization) from the symmetry properties of the inner field and the requirement that the structure of this field be thermodynamically stable. Thus we derive and interrelate the characteristic superconductive properties by means of a theory based solely on general properties of the inner field, without recourse to any approximative ansatz for the wave functions of the system. One would hope that the introduction of the unifying concept of inner field into the theory would also be useful in enabling one to calculate other dynamical properties of the superconductive phase.

Our theory is founded on the postulate that the inner field in a superconductor may be identified with the electron-pair correlation function,[2] first introduced by Gorkov.[3] The postulate itself is therefore satisfied by the ansatz of the Bardeen–Cooper–Schrieffer theory.[4] However, although one may regard this as providing strong support for the postulate, it should be stressed that our formulation does not invoke the BCS ansatz as such. In the context of the present theory, therefore, one may regard our basic postulate as constituting a well-defined first principle.

Our theory will be set out as follows. In Sec. 2 we extend the general LL theory slightly by formulating the requirement that the structure of an inner field be thermodynamically stable. This enables us to relate the continuity properties of the system, with respect to changes in external forces, to certain symmetry properties of the system. In Sec. 3 we formulate the relevant general properties of a quantum-mechanical model of a many-particle system, on which we base our treatment of the superconducting phase. In Sec. 4, we formulate our basic postulates that identify the inner field in terms of the model of Sec. 3. In Sec. 5, we show that these postulates, together with the general thermodynamic theory of Sec. 2, lead to the conclusion that the rotational properties of the inner field are governed by an integral quantum number, n. This quantization refers to the macroscopic properties of the system and ensures that n cannot change due to magnetic or dissipative forces. In the magnetic field-free case, $n = 0$ corresponds to a stable state, $n \neq 0$ to a metastable current-carrying state. The property that n does not change when a magnetic field is applied corresponds to a London type of rigidity[5] and thus ensures that the system exhibits a Meissner effect. If $n \neq 0$, this effect is accompanied by trapped flux, whose magnitude is quantized.

2. INNER FIELDS—STABILITY AND CONTINUITY

In the LL theory, the properties of a second-order transition are formulated in terms of a set of intensive

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[1] L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon Press, Inc., New York, 1958), p. 430.

[2] This is perhaps the simplest identification one could have guessed. For a theoretically acceptable inner field must be formed from a product of at least two fermion wave operators, since a single fermion wave could not build up to a macroscopic amplitude.

[3] L. P. Gorkov, Zh. Eksperim. i Teor. Fiz. **34**, 735 (1958) [English transl.: Soviet Phys.—JETP **7**, 505 (1958)]; Zh. Eksperim. i Teor. Fiz. **36**, 1918 (1959) [English transl. Soviet Phys.—JETP **9**, 1364 (1959)]. In the latter paper, Gorkov related his pair function to the Ginzburg–Landau order parameter by approximations equivalent to the BCS theory. He did not undertake to show, however, that the characteristic properties of superconductors arise as a consequence of general properties of this correlation function.

[4] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1958).

[5] F. London, *Superfluids* (John Wiley & Sons, Inc., New York, 1950), Vol. 1, p. 144.

thermodynamic variables $\alpha(=\alpha_1,\cdots,\alpha_n)$ and conjugate forces $\xi(=\xi_1,\cdots,\xi_n)$. At $\xi = 0$, the inner field α varies continuously with T, the temperature, and is nonzero only below the transition point T_c. Its properties may be derived from the thermodynamic potential

$$\Phi = \Phi(\alpha,\xi,\eta,T) = \Phi_0(\alpha,\eta,T) - \alpha\cdot\xi\,, \tag{1}$$

where $\alpha\cdot\xi = \sum \alpha_j\xi_j$ and η refers to forces other than ξ.

The condensed phase is normally characterized by a broken symmetry. Thus, Φ_0 is invariant under a certain linear transformation group G,

$$\alpha \rightarrow L(\theta)\alpha\,, \qquad \theta \text{ variable}\,, \tag{2}$$

whereas any equilibrium field $\bar{\alpha}$ is variant under G. Consequently, the fields $L\bar{\alpha}$ correspond to a set of different equilibrium states.

The set of fields generated by operating on a given α with the different elements of G are equivalent with respect to intrinsic properties. We now postulate that the ordering in the condensed phase is such as to stabilize these properties. Accordingly, we assume that Φ_0 will increase if α is changed infinitesimally from an equilibrium value $\bar{\alpha}$ to $\bar{\alpha} + \Delta\alpha$, unless the latter field is intrinsically equivalent to $\bar{\alpha}$ (i.e. equal to $L\bar{\alpha}$). Thus

$$\Delta\Phi_0 \equiv \Phi_0(\bar{\alpha} + \Delta\alpha,\eta,T) - \Phi_0(\bar{\alpha},\eta,T) > 0\,, \tag{3}$$

unless $\bar{\alpha} + \Delta\alpha = L\bar{\alpha}$.

The invariance of the thermodynamic potential under G is destroyed by the application of a force $\xi = \xi_0$, since this introduces a G-variant term $-\alpha\cdot\xi_0$ into Φ. We shall suppose[6] G to be such that the full set of fields $\{\alpha\}$ may be generated from a subset $\{\alpha_0\}$, that forms a continuum, by operations

$$\alpha = L(\theta)\alpha_0\,, \tag{4}$$

where $\{\alpha_0\}$ is singled out by the property

$$\alpha_0\cdot\xi_0 > L(\theta)\alpha_0\cdot\xi_0\,, \qquad \text{for } L(\theta) \neq 1\,. \tag{5}$$

It follows now from (1)–(5) that Φ takes its least values for fields α that belong to $\{\alpha_0\}$; and that, if $\alpha = \bar{\alpha}_0$ represents an equilibrium inner field belonging to that set, then

$$\Delta\Phi \equiv \Phi(\bar{\alpha}_0 + \Delta\alpha,\xi,\eta,T) - \Phi(\bar{\alpha}_0,\xi,\eta,T) > 0 \tag{6}$$

for any infinitesimal $\Delta\alpha$. The exclusion of the possibility that $\Delta\Phi = 0$ is important here. The inequality (6) implies that $\alpha = \bar{\alpha}_0$ could correspond to either a metastable or a truly stable state. In the latter case, Φ takes its absolute minimum, not merely a local minimum, value at $\alpha = \bar{\alpha}_0$.

Consider now the response of the system to changes in η, for given ξ_0 and T. It will be assumed that we are dealing with a range of values in η for which changes in these forces do not lead to a phase transition. Thus Φ and its derivatives with respect to η will be "well-behaved." In this case, it may be shown that the condition (6) implies that an infinitesimal change $\Delta\eta$ in η will shift the equilibrium inner field $\bar{\alpha}$ from $\bar{\alpha}_0$ to another member of $\{\alpha_0\}$, namely $\bar{\alpha}_0 + \Delta\alpha_0$, where $\Delta\alpha_0 \rightarrow 0$ as $\Delta\eta \rightarrow 0$. Hence $\bar{\alpha}$ is a continuous function of η,

$$\bar{\alpha} = \bar{\alpha}_0(\eta)\,, \tag{7}$$

and belongs to $\{\alpha_0\}$ when η is in the range under consideration. This is true even if ξ_0 is infinitesimal, as we shall subsequently suppose to be the case. Similarly, for $\xi = \xi_0'$, a different infinitesimal force, there will be a set of equilibrium states[7] $\bar{\alpha} = \bar{\alpha}_0'(\eta)$, that vary continuously with η, and are related to $\bar{\alpha}_0$ by a one-to-one correspondence

$$\bar{\alpha}'(\eta) = L(\theta_0)\bar{\alpha}_0(\eta)\,. \tag{8}$$

3. QUANTUM-MECHANICAL MODEL

We base our treatment of the superconductive phase on a model whose Hamiltonian is of the form

$$H = H_A + H_K \tag{9}$$

where H_A is the energy operator for an assembly of mutually interacting electrons in a given magnetic field, vector potential $\mathbf{A}(\mathbf{x})$, the energy per particle measured relative to the chemical potential; and where

$$H_K = \int [K(\mathbf{x}_1,\mathbf{x}_2)\chi(\mathbf{x}_1,\mathbf{x}_2) + \text{c.c.}]\, dv_1\, dv_2 \tag{10}$$

represents the interaction of the system with a classical source. Here, K is a single-valued c number and

$$\chi(\mathbf{x}_1,\mathbf{x}_2) = \psi_\uparrow(\mathbf{x}_1)\psi_\downarrow(\mathbf{x}_2) \tag{11}$$

$\psi_\uparrow$ and $\psi_\downarrow$ being the electron wave operators corresponding to two opposite spin directions. The coupling H_K is formally introduced into the theory for the purpose of "probing" the properties of H_A. In

[6] This supposition is valid if G is a gauge group of the first kind (as in our model of Secs. 3–5) or a group of reflections or rotations.

[7] It may be seen from (1) and (2) that, if G is discrete, $\alpha = \bar{\alpha}_0(\eta)$ will correspond to an equilibrium state even when $\xi = \xi_0'$. This state is metastable since Φ would be less if $\alpha = \bar{\alpha}_0'(\eta)$. A good example of such a metastable state would be that of an Ising ferromagnet (G = magnetization reflection group) in which the magnetization is opposed by a weak external field.

fact we shall be concerned only with properties of the model in the limit where $K \to 0$, so that it is unimportant whether the K coupling is realizable or not.

It suffices, for our purposes, to confine the theory to situations where H_A possesses a cylindrical symmetry. Accordingly, using cylindrical coordinates (r,ϕ,z), we stipulate that the system is bounded by a long cylinder, axis Oz, and that the magnetic field is $H(r)$, directed along Oz. Using a transverse gauge ($\operatorname{div} \mathbf{A} = 0$), we represent this field by a transversely directed vector potential $A(r)$.

The current density $\mathbf{j}(\mathbf{x})(= -\delta H/\delta \mathbf{A}(\mathbf{x}))$ and the Hamiltonian H will be invariant under transformations

$$\begin{aligned} \psi(r,\phi,Z) \equiv \psi(\mathbf{x}) &\to \psi(\mathbf{x})e^{i\nu\phi} , \\ A(r) &\to A(r) - (\nu ch/2\pi er) , \\ K(\mathbf{x}_1,\mathbf{x}_2) &\to K(\mathbf{x}_1,\mathbf{x}_2)e^{-i\nu(\phi_1+\phi_2)} , \end{aligned} \tag{12}$$

where ν is any integer and $(-e)$ the electronic charge. Also, H_A is invariant under

$$\psi \to \psi e^{\frac{1}{2}i\theta} \qquad (\theta \text{ independent of } \mathbf{x}) . \tag{13}$$

In view of the axial symmetry of H_A, the angular momentum M of the system about Oz will commute with H_A. Also M possesses the important property that

$$\exp\left(\frac{iM\phi}{\hbar}\right)\chi(\mathbf{x}_1,\mathbf{x}_2) \exp\left(-\frac{iM\phi}{\hbar}\right) = R(\phi)\chi(\mathbf{x}_1,\mathbf{x}_2) , \tag{14}$$

where $R(\phi)$ is the operator that generates rotations through the angle ϕ about Oz, i.e.

$$R(\phi)F(r_1,\phi_1,z_1;r_2,\phi_2,z_2) = F(r_1,\phi_1+\phi,z_1;r_2,\phi_2+\phi,z_2) \tag{15}$$

for any function F.

The equilibrium angular momentum $\overline{M}$ will be related to the equilibrium values of the current density [$J(r)$, transversely directed] and the electron number density [$\overline{n}(r)$] by the equation

$$\frac{e}{mc}\overline{M} + \int\left[rJ(r) + \frac{e^2}{mc}r\overline{n}(r)A(r)\right] \, dv = 0 . \tag{16}$$

4. POSTULATES

We now postulate that the interelectronic forces are such as to bring about a second-order phase transition, at temperature T_c, that possesses the following properties: (I) The transition into the condensed phase is characterized by the onset of an inner field $\overline{\chi}$ (equals thermal average of χ) even in the limit $K \to 0$. (II) In cases where $A = 0$, the inner field $\overline{\chi}_0$, corresponding to the state of lowest free energy of the system, is invariant under rotations about Oz in the limit $K \to 0$. Thus, by (15),

$$R(\phi)\overline{\chi}_0(\mathbf{x}_1,\mathbf{x}_2) = \overline{\chi}_0(\mathbf{x}_1,\mathbf{x}_2) . \tag{17}$$

It follows from (I) that $\overline{\chi}$ and K may be identified with α and ξ of Sec. 2. Also, since H_A is invariant under transformations (14), it follows from (11) that the thermodynamic properties of H_A are invariant under the gauge group G,

$$\overline{\chi} \to L(\theta)\overline{\chi} \equiv \overline{\chi}e^{i\theta} . \tag{18}$$

It follows that the G-symmetry is broken in the condensed phase. Clearly, the above operator L corresponds to the L of Eq. (2).

By Eq. (11), the matrix element of χ between two states is zero unless the electron numbers for the states differ by 2. Therefore, the occurrence of an inner field $\overline{\chi}$ at $T = 0$, in the limit $K \to 0$, implies that the application of an infinitesimal K field removes the degeneracy of the ground states[8] Φ_N of H_A, for different particle numbers N, and leads to a linear combination

$$\Phi_g = \sum C_N\Phi_N \tag{19}$$

for the ground state of H. Hence, even for K infinitesimal,

$$\overline{\chi} = \textstyle\sum_N C_N^* C_{N+2}\langle\Phi_N|\chi|\Phi_{N+2}\rangle \quad \text{at} \quad T = 0 . \tag{20}$$

The coefficients C_N will be nonzero only in a range of N-values that is vanishingly small by comparison with the mean electronic number. Therefore, since the matrix elements $\langle\Phi_N|\chi|\Phi_{N+2}\rangle$ cannot vary significantly with N for such a narrow range, it follows from (20) that

$$\overline{\chi}(\mathbf{x}_1,\mathbf{x}_2) = \text{constant} \times \langle\Phi_N|\chi|\Phi_{N+2}\rangle \quad \text{at} \quad T = 0 . \tag{21}$$

5. DEDUCTIONS

Meissner Effect

The diamagnetic properties of any system are given by the relationship between the equilibrium current and the magnetic field in the system. This

[8] Here, "ground state" denotes a state whose energy will increase as a result of any microscopic processes, and is therefore applicable to both stable and metastable states. The density matrix for temperature T will be formed from the ground state and excitations based on that state. Clearly this set of states cannot comprise all the quantum states for the system if there are a number of different equilibrium states at the same temperature.

relationship may be shown[9,10] to take one of the following forms:

$$J(r) = -\int C(r,r')A(r') \quad dv', \text{ for Meissner effect,} \tag{22a}$$

or

$$J(r) = -\int D(r,r')w(r')dv', \quad \text{for normal diamagnetism}, \tag{22b}$$

where C and D are form factors, and w is the transverse (and only) part of curl $\mathbf{H}$. We shall show that our model satisfies (22a). This result will be seen to stem from the fact that certain rotational properties of the inner field (i.e. dependence of $\bar{\chi}$ on ϕ_1, and ϕ_2) are unchanged by the application of a magnetic field.

In order to show this, we investigate the dependence of $\bar{\chi}$ on A for given infinitesimal K. We note that, by the theory of Sec. 2, there will be equilibrium states for which $\bar{\chi}$ varies continuously with A and takes the value $\bar{\chi}_0$ (defined in Sec. 4) at $A = 0$. For these states, then, $\bar{\chi}$ is a well-defined functional of A. Similarly, if K is replaced by K', generated from K by a rotation [as in (15)]

$$K' = R(\phi)K,$$

then it follows from the axial symmetry of the system that there will be an equilibrium state $\bar{\chi}'$, related to $\bar{\chi}$ by

$$\bar{\chi}' = R(\phi)\bar{\chi}.$$

On the other hand, it follows from (8) and (18) that

$$\bar{\chi}' = \bar{\chi}e^{i\theta}.$$

Hence

$$R(\phi)\bar{\chi} = \bar{\chi}e^{i\theta},$$

where θ depends on ϕ. It may be shown from this equation and (15) that

$$R(\phi)\bar{\chi} = \bar{\chi}e^{in\phi}, \tag{23}$$

where the single-valuedness of $\bar{\chi}$ implies that n is an integer. Therefore, since $\bar{\chi}$ varies continuously with A and since $n = 0$ at $A = 0$ [Eq. (17)], it also follows that $n = 0$ when $A \neq 0$. Hence

$$R(\phi)\bar{\chi} = \bar{\chi}. \tag{24}$$

Thus, the inner field possesses a certain rotational rigidity in that n does not change when a magnetic field is applied.

Consider now the properties of $\bar{\chi}$ when $T = 0$. By Eqs. (14), (20), and (24),

$$\langle\Phi_N|e^{iM\phi}\chi e^{-iM\phi}|\Phi_{N+2}\rangle = \langle\Phi_N|\chi|\Phi_{N+2}\rangle.$$

Consequently the angular momenta for the states Φ_N, Φ_{N+2} are equal, which means that $\overline{M}$ does not vary with N at $T = 0$.

It follows from Eq. (16) that, if ΔJ and $\Delta\bar{n}$ denote changes in J and $\bar{n}$ due to the addition of ΔN particles, then

$$\int\left[r\Delta J(r) + \frac{e^2}{mc}r\Delta\bar{n}(r)A(r)\right] \quad dv = 0. \tag{25}$$

This relation does not depend on the form of $A(r)$. We now examine its consequences by considering a special case, where the cylinder has a hole ($r \leqslant a$) and where the magnetic field is zero in the body but not in the hole, i.e.

$$A = A_0(r) = \frac{f}{r}, \ f \text{ constant}. \tag{26}$$

It should be stressed that there is no *a priori* reason for supposing that such a field does not affect the system (cf. Aharonov–Bohm effect[11]). Also, it is important to note[10] that the question of whether the field A_0 were rendered unrealistic by requirements of the Maxwell equations is quite irrelevant to our purpose of studying the structure of the current–field relation. On substituting (26) into (25) we obtain

$$\int r\Delta J(r) \quad dv + \frac{e^2 f}{mc}\Delta N = 0.$$

This implies that $\Delta J \neq 0$ for $A = A_0$, which immediately rules out (22b), since $w = 0$ when $A = A_0$. We therefore conclude that, for general A, (22a) is valid, i.e., that the system exhibits the Meissner effect at $T = 0$.

This in turn implies a Meissner effect for all $T < T_c$, for an assumption of a Meissner effect in a temperature range ($0 < T_B < T_c$) and a normal diamagnetism in (T_B,T_c) would imply a discontinuity in the free energy at T_B, in the presence of certain magnetic fields. This implies infinite entropy at T_B, and so constitutes a *reductio ad absurdum* of the supposition that $T_B < T_c$. Hence we conclude that the system exhibits a Meissner effect for all $T < T_c$.

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[9] M. R. Schafroth, Helv. Phys. Acta **24**, 645 (1951).

[10] This relation represents the response of the system to a given magnetic field, and its form is independent of Maxwell's equations. The latter are required to interrelate the actual field with the applied one. On the other hand, they are irrelevant to the form of equation (22). Consequently, one may derive properties of such current field relations by calculating the response of the system to formally defined "fields" which really (i.e. in view of Maxwell's equations) could not enter the body.

[11] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959).

Superfluidity, Flux Quantization

It follows from (22a) and invariance of J and H under (12) that there are equilibrium states for which $n(=2\nu)$ is even and nonzero in (23) and where

$$J(r) = -\int C(r,r')\left[A(r') + \frac{nch}{4\pi er'}\right] dv' . \quad (27)$$

Putting $A = 0$ in (23), we see that the system possesses metastable current-carrying states. Their stability may be seen to depend on the quantization of n in much the same manner as the stability of states, cited in footnote 7, depended on the discreteness of G.

For $A \neq 0$, Eq. (27) is the current–field relation for cases where the system is disturbed from a metastable state by a magnetic field. It implies the existence of trapped fluxoid[12]

$$F = nch/2e . \quad (28)$$

So far, we have established (27) and (28) only for even n. This restriction arises only from our method of using the transformation (12) to generate fields for which $n = 2\nu$ from those with $n = 0$. In fact, one may deduce (27) and (28) directly from (23), which we derived earlier without involving (12). In that earlier derivation[13] of (23) there is no restriction of n to even numbers. Therefore, since there appears no *a priori* reason why we should impose such a restriction, i.e., why states with n odd cannot be prepared, we shall assume the validity of (23), (27), and (28) for all integral n. In this case, the quantum of flux is seen to be $hc/2e$, as observed experimentally.[14]

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[12] F. London, Ref. 5, p. 150.

[13] That derivation is general, and independent of the condition that $\bar{x} = \bar{x}_0$ at $A = 0$. The latter condition refers to a special set of fields.

[14] B. S. Deaver, Jr. and W. M. Fairbank, Phys. Rev. Letters **7**, 43 (1961).

Spatial Dependence of Pair Correlation Functions in Nonhomogeneous Superconductors*

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It has been shown by Gor'kov[1] that the theory of superconductivity can be derived in terms of differential equations relating the single-particle Green's functions $G(x,x') = -iT\langle\psi(x)\psi^\dagger(x')\rangle$ and the pair amplitude $F(x,x') = T\langle\psi(x)\psi(x')\rangle$. As the pairing is implicit in this formalism, the method is useful when treating problems in which it is not convenient to work with the explicit single-particle states. In this paper we develop a form of the theory appropriate to the investigation of superconducting contacts.

The Gor'kov equations, Fourier transformed in the relative time coordinate and written without spin indices, are[1,2]

$$\begin{aligned}(\omega - h_1)G(\mathbf{r},\mathbf{r}',\omega) &= G_0^{-1}(\mathbf{r},\bar{\mathbf{r}},\omega)G(\bar{\mathbf{r}},\mathbf{r}',\omega)\\ &= \delta^3(\mathbf{r}-\mathbf{r}') + i\Delta(\mathbf{r})F^*(\mathbf{r},\mathbf{r}',\omega) , \quad (1)\end{aligned}$$

$$\begin{aligned}(-\omega - h_1)F^*(\mathbf{r},\mathbf{r}',\omega) &= G_0^{-1}(\bar{\mathbf{r}},\mathbf{r}, -\omega)F^*(\bar{\mathbf{r}},\mathbf{r}',\omega)\\ &= i\Delta^*(\mathbf{r})G(\mathbf{r},\mathbf{r}',\omega) , \quad (2)\end{aligned}$$

where h_1 is the one-particle Hamiltonian, G_0 is the Green's function in the absence of two-body interactions, a bar over a quantity signifies integration over all space, and

$$\begin{aligned}\Delta(\mathbf{r}) &\equiv V(\mathbf{r})F(\mathbf{r},\mathbf{r},t=0)\\ &= (2\pi)^{-1}V(\mathbf{r})\int_{-\infty}^{\infty} F(\mathbf{r},\mathbf{r},\omega)d\omega . \quad (3)\end{aligned}$$

Equations (1) and (2) can be expanded in powers of Δ as follows:

Letting

$$f(\mathbf{r},\mathbf{r}',\omega) \equiv f^{(1)}(\mathbf{r},\mathbf{r}',\omega) \equiv G_0(\mathbf{r},\bar{\mathbf{r}},\omega)i\Delta(\bar{\mathbf{r}})G_0(\mathbf{r}',\bar{\mathbf{r}}, -\omega) , \quad (4)$$

$$f^{(n)}(\mathbf{r},\mathbf{r}',\omega) \equiv f^{(n-1)}(\mathbf{r},\bar{\mathbf{r}},\omega)i\Delta^*(\bar{\mathbf{r}})f(\bar{\mathbf{r}},\mathbf{r}',\omega) , \quad (5)$$

we then have

$$F(\mathbf{r},\mathbf{r}',\omega) = \sum_{n=1}^{\infty} f^{(n)}(\mathbf{r},\mathbf{r}',\omega) . \quad (6)$$

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[1] L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. **34**, 735 (1958) [English transl.: Soviet Phys.—JETP **7**, 505 (1958)].

[2] L. P. Kadanoff and P. C. Martin, Phys. Rev. **124**, 670 (1961); V. Ambegaokar and L. P. Kadanoff, in *The Many-Body Problem* (W. A. Benjamin, Inc., New York, 1962), p. 66.