temperature. This plot showed that the aluminum relaxation rate did increase suddenly below T_c, followed by a decrease at lower temperatures. In order to substantiate this quantitatively measurements of the relaxation rate were made, first using continuous wave saturation methods, and recently using a coherent pulsed nuclear induction spectrometer. These measurements have not yet been very successful in terms of the quality of the data, but we can say that there is an enhancement of the rate of about 30% just below T_c. This is less than the 100% enhancement found for pure bulk aluminum[12] and the 80% found for gallium.[13] Our result could be explained by quasi-particle level broadening or by an energy gap anisotropy. Thus a 14% broadening or anisotropy reduces the enhancement to 20%. Another possibility is that the energy gap itself is considerably reduced in these samples. Tunneling measurements would help answer this.

Resistance Measurements

One of the films was picked at random and put in the center of the stack with leads so that the resistance could be measured. The measuring current was low, and no effect on the measurements were seen with changes in the current. Figure 2 shows these results. The gradual change in R at large magnetic fields is perhaps the effect of fluxoids due to a perpendicular component of the field, as discussed by Tinkham.[15]

IV. DISCUSSION

The resistivity measurements indicate that the sample possibly was in some sort of mixed state, perhaps with fluxoids and a spatial varying energy gap. The NMR line shape did not change below T_c, and thus the distance in which the electron paramagnetic susceptibility changes must be less than the mean free path. If a mixed state is present in quantity then the domain size must be less than the mean free path, which is of the order of the coherence length and the film thickness.

In conclusion, the change in Knight shift observed is believed to be that for all the sample, but the influence of a mixed state with a spatially varying energy gap may be important.

ACKNOWLEDGMENTS

We wish to express our appreciation to Dr. Joachim Appel for many stimulating discussions, and to Professor W. D. Knight and Dr. P. H. Miller, Jr., for their encouragement in this work.

[15] M. Tinkham, Phys. Rev. 129, 2413 (1963).

HIGH FREQUENCY ABSORPTION

CHAIRMAN: *A. B. Pippard*

Surface Impedance of Superconductors

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1. INTRODUCTION

This paper is not concerned with the most recent measurements of the surface impedance of superconductors, but with the type of experiment fashionable ten years ago, measurements at a few kMc/sec or less on superconductors near the weak coupling limit, such as tin. There is a great range of such measurements, and at frequencies well below the gap frequency ($\omega \ll \epsilon_0/\hbar$) they are well described by the microscopic theory of surface impedance developed by Mattis and Bardeen (1958) and by Abrikosov *et al.* (1958). Our purpose is to look back at the small discrepancies between theory and experiment which exist and to try to discover their significance. In doing so, we shall use a number of the special limits of the theory, which are often more easily understood and applied than the general formulation. We give

first a resumé of the form of the theory, and then consider briefly the results for the surface impedance very near the transition, the penetration depth at low temperatures and the surface resistance at low temperatures, limiting our attention to one metal, tin.

2. THE FORM OF THE THEORY

2.1 The Surface Impedance Problem

The first task of the theory is to connect the surface impedance, which can be measured, with the current–field relation appropriate to the conductor, the simplest example of which is Ohm's law, $\mathbf{J} = \sigma\mathbf{E}$. We proceed by combining the current–field relation with Maxwell's equations, which yields a differential equation for the field that can be solved, using appropriate boundary conditions. In the case of Ohm's law, we obtain the distribution known as the classical skin effect, in which the field decays exponentially with depth, with change of phase, and we have for $\mathbf{E}$

$$\mathbf{E} = \mathbf{E}_0 e^{-iz/\delta}, \tag{1}$$

where the complex skin depth, δ, is given by

$$\delta = (1+i)(8\pi\omega\sigma)^{-\frac{1}{2}}. \tag{2}$$

The surface impedance, which is the ratio of the surface electric field and surface current, is easily obtained from the field distribution. For the classical skin effect it is given by

$$Z = 4\pi\omega\delta \tag{3}$$

and the surface resistance and reactance are equal.

In general, the current–field relation will be *nonlocal*, however, and will take the form

$$\mathbf{J}(\mathbf{r}) = \int \frac{\mathbf{R}[\mathbf{R}\cdot\mathbf{A}(\mathbf{r}')]}{R^4} f(\mathbf{R})d\mathbf{r}'. \tag{4}$$

Here the current at $\mathbf{r}$ depends on an integral over the vector potential $\mathbf{A}$ at nearby points $\mathbf{r}'$. The range function $f(\mathbf{R})$ is the term characteristic of the metal. It shows how the effect of the distant part of the field drops off with distance ($\mathbf{R} = \mathbf{r} - \mathbf{r}'$), and, in anisotropic metals, how it varies with direction. In a normal metal it is proportional to the Gaussian radius of curvature of the Fermi surface, $\mathcal{R}$ (Pippard, 1961), and to $e^{-R/l}$, where l is the electronic mean free path; thus the current at a point is only seriously affected by the field within a mean free path of it. Combining Eq. (4) with Maxwell's equations leads to an integro-differential equation, which can be solved (Dingle, 1953). The surface impedance can be expressed as

$$Z = 4\pi^2 i\omega \Big/ \int_0^\infty \ln\left(1 + \frac{4\pi i\omega\sigma^q}{q^2}\right) dq. \tag{5}$$

Here σ^q is the "spectral conductivity"; it gives the response of the conductor to a Fourier component of the electric field, that is, $J^q = \sigma^q E^q$, and it is easily obtained from the current–field relation (4). Thus, given the form of $f(R)$ appropriate to a conductor, the apparatus exists for determining the surface impedance, Z.

2.2 The Range Function of Mattis and Bardeen

The theory of Mattis and Bardeen (1958) starts from the microscopic theory of Bardeen, Cooper, and Schrieffer (BCS, 1957) and deduces the range function appropriate to a superconductor. The theory is worked out for an isotropic metal, but if we extend it to the anisotropic Fermi surface, we may write the range factor as

$$f(\mathbf{R}) = I(\omega,\mathbf{R},T)\mathcal{R}e^{-R/l} \tag{6}$$

The new factor, $I(\omega,\mathbf{R},T)$, is a complicated integral, but it has the most important property that it is an explicit function of the energy gap parameter, ϵ_0. The theory can therefore be extended at once to include forms of $\epsilon_0(T)$ different from that suggested by BCS, and also to include anisotropy of the energy gap. This assumes that the *form* (but not the scale) of the density of states is as proposed by BCS, and that the concept of a direction-dependent energy gap is valid; these two assumptions are reasonable for a metal in the weak-coupling limit, and so long as the q of importance in the field structure are small compared to the Fermi surface.

The range factor drops off with distance in a roughly exponential manner, and its main properties are its value for $R = 0$, which measures the *strength* of the response to an applied field, and its *range*. At $R = 0$ the range factor is simply $\mathcal{R}I(\omega,0,T)$; its value in the normal metal is just $\mathcal{R}i\pi\hbar\omega$. The ratio of these two quantities is written

$$\frac{I(\omega,o,T)}{i\pi\hbar\omega} = \frac{\sigma_S}{\sigma_N} = \frac{\sigma_1}{\sigma_N} - \frac{i\sigma_2}{\sigma_N} \tag{7}$$

and σ_S/σ_N may often be treated as an effective conductivity ratio, its real and imaginary parts expressing the effective densities of normal electrons and superelectrons, respectively. So far as the range is concerned, the theory does not separate the range function into normal and superconducting parts, but it is sometimes possible to do so, and one finds that the normal current (which is carried by the excitations) has the same range as it does in the normal state, that is, the range of the normal current is lim-

ited by the mean free path only.[1] The range of the supercurrent, on the other hand, is limited even in the pure metal; the characteristic range is known as the coherence length, ξ. At zero frequency and absolute zero it is given by $\xi_0 = \hbar v_F/\pi\epsilon_0$, and it decreases slightly as the temperature rises. The range of the supercurrent is also limited by the mean free path.

3. COHERENCE LENGTH NEAR THE TRANSITION

3.1 The Pippard Limit

The idea of the coherence length was introduced by Pippard (1953) before the advent of the microscopic theory. In the absence of any very strong evidence, he suggested that it might increase and become very large near T_c; we might argue loosely that since decreasing ϵ_0 at $T = 0$ increases ξ, we should expect the decrease of ϵ_0 with temperature to have the same effect. This is not what the theory predicts; as we noted above, it shows that ξ decreases slightly with temperature. As a first example of the comparison of theory and experiment, let us consider the experimental evidence for Pippard's conjecture. If, now, ξ becomes very large near T_c, then, for pure material, we approach what is known as the *Pippard limit*, that is ξ, $l \gg \delta$, where δ is the skin depth. In Eq. (4), $f(\mathbf{R})$ then varies so slowly with $\mathbf{R}$ that we may replace it by $f(0)$, and remove it from the integral. The situation is now as in the extreme anomalous limit ($l \gg \delta$) in pure normal metal, except that the conductivity has been changed in the ratio σ_S/σ_N. Now in that limit, the surface impedance varies as $\sigma^{-\frac{1}{3}}$, as we may see in a rough and ready way by remembering that the electrons are only effective in carrying current if they spend most of their lifetime before being scattered within the field; that is, within the skin depth. Only a fraction of order δ/l are moving in the right direction to do this, and so we have for the effective conductivity

$$\sigma' \propto \sigma\delta/l \tag{8}$$

and inserting this value in (2), we obtain

$$\delta \propto (\omega\sigma\delta/l)^{-\frac{1}{2}}, \quad \text{or } \delta \propto (\omega\sigma/l)^{-\frac{1}{3}}. \tag{9}$$

Thus, in the Pippard limit, we have

$$Z_S/Z_N = (\sigma_S/\sigma_N)^{-\frac{1}{3}} \tag{10}$$

[1] This is not quite true, since the group velocity is reduced near the Fermi surface in a superconductor, and the retardation terms, which describe the phase lag of contributions to the current from the distant parts of the field, are then more important and more complicated. These effects are not important at low frequencies.

and it is easy to construct the form of Z_S from the theoretical curves for σ_S/σ_N. In Fig. 1 we show the theoretical forms for σ_1/σ_N and σ_2/σ_N near T_c at 3kMc/sec. The rather unexpected rise in σ_1/σ_N below T_c is due to the increased density of states at the

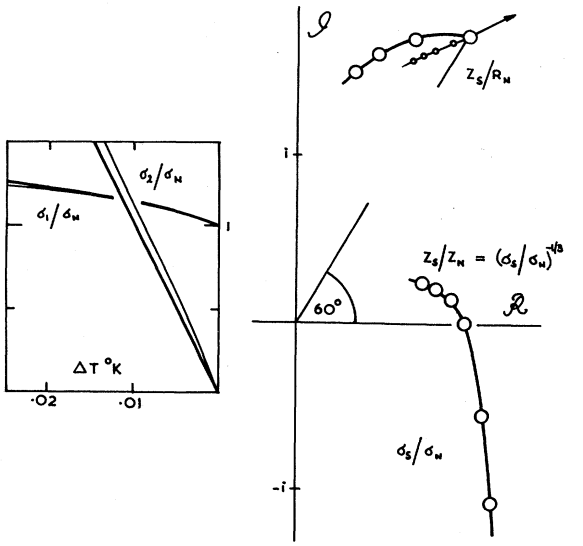

FIG. 1. Effective conductivity ratio and surface impedance of tin at 3 kMc/sec near T_c. The left-hand figure shows σ_1/σ_N and σ_2/σ_N as a function of temperature; heavy line: theory of Mattis and Bardeen; thin line: as determined from surface impedance of very impure samples. The right-hand figure shows the theoretical parameters plotted at intervals of 0.005°K in the complex plane, with the observed values of Z_S/R_N for a monocrystalline pure specimen (small circles). $X_N = \sqrt{3}\ R_N$ in the extreme anomalous limit. Arrow: direction of (dZ_s/dT) calculated at T_c for the theoretical value of ξ at zero frequency.

Fermi surface and the importance of coherence factors in the superconducting state. The right-hand figure shows σ_S/σ_N, Z_S/Z_N and Z_S/R_N plotted in the complex plane at intervals of 0.005°K. Some typical experimental results for a monocrystalline specimen are also shown. It is clear that the Pippard conjecture is quite wrong: there is no tendency for the experimental trace to approach the Pippard trace near T_c. In fact, it is not difficult, using Eq. (5) and allowing for the anisotropy of σ^q, to calculate the slope dZ/dT at T_c for the theoretical value of ξ. The result is shown in Fig. 1,[2] and is evidently in quite good agreement with what is observed.

3.2 Check on the Values of σ_S/σ_N

It is conceivable, however, that the Pippard conjecture is still correct, and that it is the theoretical values of σ_S/σ_N which are in error; we enquire whether there is any experimental method of checking them.

[2] Calculated using the value of ξ for zero frequency, instead of 3 kMc/sec, and therefore slightly inaccurately.

We may see that there is such a method by considering the *impure limit*, that is, the case of material so impure that we have $l \ll \xi,\delta$. In this case, both $\mathbf{A}(\mathbf{r}')$ and $I(\omega,\mathbf{R},T)$ vary so slowly in space compared to $e^{-R/l}$ that they may be treated as constant, and removed, in Eq. (4), from the integral, which then becomes independent of position. In fact, we have returned to a local relation,

$$\mathbf{J}(\mathbf{r}) = (\sigma_S/\sigma_N)\sigma\mathbf{E}(\mathbf{r}) , \tag{11}$$

and see at once that, from Eqs. (2) and (3),

$$Z_S/Z_N = (\sigma_S/\sigma_N)^{-\frac{1}{2}} . \tag{12}$$

It is thus extremely easy to deduce σ_S/σ_N from observations on very impure specimens; typical results are shown in Fig. 1.[3] It is clear that there is no serious error in σ_S/σ_N. There is a slight error, however, which is only just significant; the theory would fit the facts a little better if the temperature scale were decreased slightly. Since σ_S/σ_N is a function of $\hbar\omega$, kT and $\epsilon_0(T)$, but of these only $\epsilon_0(T)$ varies appreciably in this temperature range, and since $\epsilon_0(T)$ is parabolic near

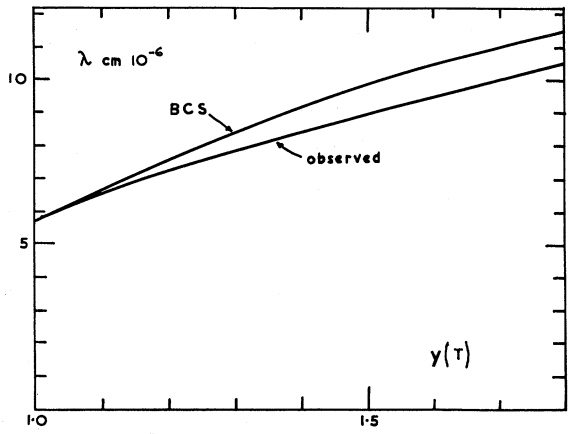

FIG. 2. Penetration depth in tin as a function of $y(T) = [1 - (T/T_c)^4]^{-\frac{1}{2}}$. [Schawlow and Devlin, 1959, and Waldram (to be published)].

T_c, we may treat this as an underestimate by the theory of $\epsilon_0(T)$ near T_c. It amounts to about 4%. This result is only just significant, but it agrees with the results at lower temperatures to which we now turn.

4. ERRORS IN THE PENETRATION DEPTH

The dc penetration depth may be measured in various ways. The most sensitive determinations have been the direct measurement at low frequency by Schawlow and Devlin (1959), and values obtained by the correction to zero frequency of the reactive part of the skin depth using Kronig–Kramers transforms and approximate surface resistance data, [Waldram (to be published)]. These agree sufficiently well to allow a useful comparison with the theory; the result is shown in Fig. 2 where λ is plotted against the conventional abscissa, $y(t) = (1 - t^4)^{-\frac{1}{2}}$. Only the second method is capable of giving λ apart from an additive constant, and even this is not reliable. There is however subsidiary evidence that the theory predicts $\lambda(0)$ correctly, as shown in the figure. At higher temperatures the theory decidedly overestimates $\lambda(T)$. In order to try to explain this overestimate, let us consider the form of the theory at zero frequency. Pippard (1953) originally suggested that the current density equation for $\omega = 0$ takes the form for an isotropic metal

$$\mathbf{J}(\mathbf{r}) = \frac{3}{4\pi\Lambda(T)}\int\frac{\mathbf{R}[\mathbf{R}\cdot\mathbf{A}(\mathbf{r}')]}{R^4}\,\frac{1}{\xi}\,e^{-R/\xi}e^{-R/l}d\mathbf{r}' , \tag{13}$$

and BCS were able to confirm this, with a slight modification to the form $e^{-R/\xi}$ which we can conveniently ignore. The range function has been split into two parts, and the term $\Lambda(T)$, the London parameter, which is a function of temperature, removed from the integral. If the equation fails in pure material it must be because one or both of $\Lambda(T)$ and ξ, the parameters which describe the form of the range function, are in error. Now we have already seen that ξ is not seriously in error near T_c and we can confirm this by considering the values of $\lambda(T)$ near T_c in impure tin. Near T_c, λ becomes very large, and, in Eq. (13), $\mathbf{A}(\mathbf{r}')$ varies so slowly with position that it may be treated as constant, and removed from the integral. In pure material, this situation is known as the *London limit*; the integral reduces to a constant, and we are left with the London equation, a local relation,

$$\mathbf{J}(\mathbf{r}) = (1/\Lambda)\mathbf{A}(\mathbf{r}) \tag{14}$$

for which the penetration depth is $\lambda_L = (\Lambda/4\pi)^{\frac{1}{2}}$. In impure material we obtain instead

$$\mathbf{J}(\mathbf{r}) = \frac{1}{\Lambda(1 + \xi/l)}\mathbf{A}(\mathbf{r}) \tag{15}$$

and the penetration depth is given by

$$\lambda = \lambda_L(1 + \xi/l)^{\frac{1}{2}} . \tag{16}$$

On plotting λ^2 for various impure samples near T_c against $1/l$, we obtain a straight line; the intercept on the $1/l$ axis gives an accurate value of ξ which agrees with the theoretical value within experimental error.

[3] The specimens actually used were only moderately impure, and a correction has been applied to the results plotted to allow for the finite value of l/ξ.

Now at absolute zero, $\Lambda(0)$ is a constant of the Fermi surface which can be measured; the fact that the theory predicts $\lambda(0)$ correctly therefore means that ξ_0 must also be correctly predicted. If, as we now see, ξ is correctly predicted at $T = 0$ and at T_c, and since

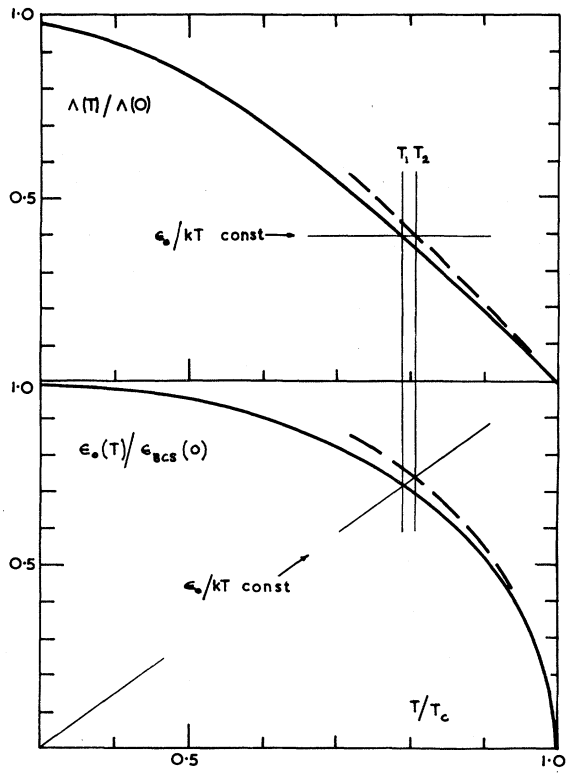

FIG. 3. Construction used to obtain the corrected form for $\epsilon_0(T)$ from the observed values of the London parameter. Heavy curves: theoretical. Dotted curves, upper figure: $\Lambda(T)/\Lambda(0)$ obtained from the observed form of $\lambda(T)$ with the assumption that the theory predicts ξ correctly; lower figure: the corresponding corrected form for $\epsilon_0(T)$. According to the theory, $(\Delta T)/\Delta(0)$ is a function of ϵ_0/kT, which therefore takes the same value at T_2 in the real metal as it does at T_1 in the theoretical model.

it is much the same in these two limits, it is most unlikely that it is seriously in error elsewhere. We are thus forced to conclude that the ratio $\Lambda(T)/\Lambda(0)$ is in error. Now this ratio is a function of ϵ_0/kT only, in the theory, and we may interpret the error in $\lambda(T)$ at once as an error in $\epsilon_0(T)$; a simple construction for making the correction is shown in Fig. 3. The results are shown in Fig. 4, where they are compared with values of the energy gap deduced by a variety of methods. It will be seen that the low temperature values are not given precisely; this is because the theoretical curve of $\lambda(T)$ given by BCS with which we have compared the experimental results is itself not precise, and employs an interpolation between two forms of $\lambda(T)$, which were worked out with ξ held constant at its low and high temperature values. A similar correction to $\epsilon_0(T)$ is obtained from the penetration depth data for impure tin. The agreement with the other measurements of $\epsilon_0(T)$ is very satisfactory.

5. ERRORS IN SURFACE RESISTANCE AT LOW TEMPERATURES

5.1 Influence of λ on R_s

It has been known for some time that the theory underestimates the surface resistance of tin at low temperatures. Miller (1960) showed that the theory overestimated the surface resistance by about 20% for frequencies well below the gap frequency; I have recently found that roughly the same is true for a range of impure specimens at 3kMc/sec. It is natural to enquire how far the errors in $\lambda(T)$ noted above are responsible for this disagreement. We may see easily that R_s is strongly dependent on λ by remembering that at low temperatures there are very few excitations, and that the field structure will be dominated by the supercurrent. If the material is impure, so that the mean free path is short, the losses per unit volume are just σE^2. The volume over which the field acts is proportional to λ, and since curl $\mathbf{E} = \dot{\mathbf{B}}$, $\mathbf{E}$ is

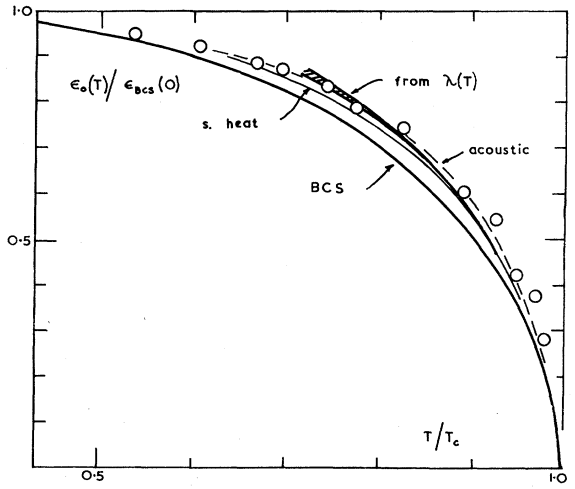

FIG. 4. Corrected form for $\epsilon_0(T)$ compared with the results of other measurements. Specific heat: Corak and Satterthwaite, 1956. Acoustic attenuation: Bohm and Morse, 1958. Circles: tunneling data of Townsend and Sutton, 1962.

proportional to the scale of the field structure for a given surface magnetic field, and hence to λ also. But for a given surface magnetic field, and hence a given surface current, the losses are proportional to the surface resistance, and we have that

$$R_s \propto \sigma\lambda^3 . \qquad (17)$$

For pure material, the effective conductivity is proportional to $\sigma\lambda/l$ [Eq. (8)], and we find that

$$R_s \propto \sigma\lambda^4/l \propto \frac{\sigma_1}{\sigma_N}\mathcal{R}\lambda^4 . \tag{18}$$

Clearly, in both cases the small overestimate of $\lambda(T)$ might well account for the larger overestimate of R_s, and calculations show that this is indeed so for the impure material.

5.2 Anisotropy of the Energy Gap

If we examine pure material, the same is partly true, but certain complications arise. By a similar argument to that used above, applied to the normal state, we can easily show that, for pure material,

$$\frac{R_S}{R_N} \propto \frac{\sigma_1}{\sigma_N}\left(\frac{\lambda}{\delta}\right)^4 . \tag{19}$$

A natural way to test this relation is to use *measured* values of λ and δ in conjunction with the measured surface resistance ratio, R_S/R_N. Since the zero of the measured values of λ is uncertain, however, this is best done by plotting λ/δ against $[R_S\sigma_N/R_N\sigma_1]^{\frac{1}{4}}$, using the theoretical form for σ_1/σ_N.[4] We should obtain a straight line, and, incidentally, any zero error in λ will be detectable as a nonzero intercept on the λ axis. This indirect measurement has been useful as a check on the absolute values of $\lambda(0)$. The plot is shown for three monocrystalline specimens of differens orientations at 3 kMc/sec in Fig. 5 (the heavy lines). At the higher temperatures the theory works well. We obtain good straight lines all having the same slope. At the lowest temperatures, there are marked deviations, which are clearly anisotropic. Now, since all the other quantities in Eq. (19) are *measured*, we must conclude that the deviations in the plot are caused by using wrong values of σ_1/σ_N,[5] and that the true values of this quantity are themselves anisotropic. We must also remember that the normal current is carried almost entirely by excitations moving nearly parallel to the surface in pure material, as we showed in Sec. 2.1, and that σ_1/σ_N refers to them. In particular, it will reflect any local anisotropy of the energy gap which they see; if we can recalculate σ_1/σ_N for the local gap, and insert the new values in Eq. (19), the deviations from linearity in Fig. 5 should disappear. We show in Fig. 5 plots corresponding to various simple scalings of the BCS energy gap parameter,

$$\epsilon_0(T) = \alpha\epsilon_{\mathrm{BCS}}(T) . \tag{20}$$

The values of α are given in the figure. In each case, one of the scalings gives a straight-line plot, and these straight lines then have the same slopes; we can thus have reasonable confidence that our analysis is correct, and that we may take the scalings as a measure of the local value of $\epsilon_0(T)$. This method of measuring the anisotropy of the energy gap is very much *in embryo*, and the conclusions must be treated as highly tentative. We give, however, a rough idea of the variation with direction so far revealed in the perspective drawing of Fig. 5; these conclusions agree quite well with data obtained from acoustic attenuation work (Morse, Olsen, and Gavenda, 1959, and Bezuglyi *et al.*, 1960).

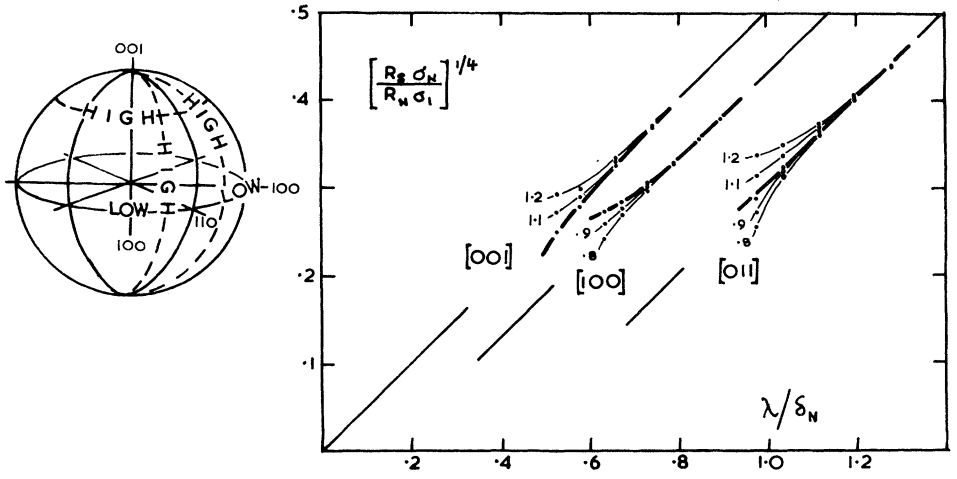

FIG. 5. Plot of $[R_S\sigma_N/R_N\sigma_1]^{\frac{1}{4}}$ against λ/δ_N for monocrystalline samples of pure tin at 3 kMc/sec. The thin lines show the same quantity, but with σ_1/σ_N calculated for a simple scaling of the BCS energy gap; the scaling factors are shown. The perspective figure shows approximately how the energy gap varies with the direction of motion of the corresponding excitation.

[4] To be consistent, σ_1/σ_N should be calculated for the *corrected* form of $\epsilon_0(T)$ deduced in Sec. 4; in practice, this correction has a negligible effect on the plots of Fig. 5.

[5] Since $R_N \propto \delta^4$, the Gaussian radius of curvature, and hence the anisotropy of the Fermi surface, are not involved in Eq. (19).

6. CONCLUSIONS

We have found that it is possible to explain all the discrepancies which have been noted here in terms of three things: anisotropy of the Fermi surface, anisotropy of the energy gap, and inexactness of the BCS form for $\epsilon_0(T)$. This has necessarily been a brief survey, but more detailed examination of all the data leads to the same conclusion. The inclusion of anisotropy in the theory is a natural extension, which has always been recognized as necessary. The slight failure of the BCS form for $\epsilon_0(T)$ might be thought to be more serious, since, in the weak coupling limit, the BCS form for the density of states, on which the thermodynamic argument for the variation of ϵ_0 with temperature is based, should be a good approximation. It seems, however, that the apparent discrepancy may be a mirage. We must remember that tin has several pieces in its Fermi surface, and that these may show different energy gaps, even in the same direction. If this is so, it will affect both the thermodynamic argument leading to $\epsilon_0(T)$, and the apparent form of the London parameter as a function of temperature. If this is the real cause of the discrepancy, we may conclude, finally, that the microscopic theory of the surface impedance of superconducting tin for frequencies below about 10 kMc/sec appears to be entirely successful.

I should like to thank the Conference Committee for making it possible for me to give this paper in person.

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Discussion 29

SCHRIEFFER: In regard to the temperature dependence of the energy gap, I would like to point out that if one includes temperature dependent damping effects then the transition temperature will be lowered somewhat. For a given value of the zero temperature energy gap, the gap vs temperature will essentially curve more sharply toward zero at the transition temperature; it will be bowed out somewhat from our original simple calculation. Perhaps this might have some relevance to what you were saying.

J. R. WALDRAM, *Royal Society Mond Laboratory:* I think there is another possibility. It seems to me quite possible that in tin you may have different energy gaps on different pieces of the Fermi surface. It has a complex Fermi surface. If you take a straightforward model with different energy gaps on different pieces of the Fermi surface you can also get quite good agreement with the data. I don't know how relevant this is.

MAXWELL: You showed a slide comparing the BCS form of $\lambda(t)$ and the experimentally measured values. I'm curious to know how you fixed the zero for the experimental curve.

WALDRAM: It's a long detailed question so I think perhaps it would be simpler if we discussed it privately. There are three different lines of evidence which give a value for the zero of the experimental curve. None of them are very reliable. This is certainly a point which I skipped over.

T. A. BUCHHOLD, *General Electric Advanced Technology Laboratory:* We have made a number of loss measurements however at relatively low frequencies, 30 to 1000 cps. What we have found is something which cannot be expressed as a constant resistance. In most cases it was found for constant frequency that the losses vary as the 4th or the 5th power of the applied flux density. Investigations on niobium, lead, and tin gave relatively similar results. As a function of frequency the losses are in most cases linear. We are coming to the conclusions that all of these losses are due to imperfections of the surface. I doubt very much that one could express them by theory.

KEESOM: We measured the specific heat of lead and there was some trouble below 1°. One explanation is that there could be in addition to the normal energy gap in certain directions, a very small energy gap in other directions. I wonder if you would be able to measure it, or is that a difficult metal to measure?

WALDRAM: I wouldn't care to attempt to estimate the size of another energy gap, if it were present. If somebody told me that a given second energy gap was present, I could tell him whether it fitted my data but the reverse procedure is highly inaccurate. For instance, I can fit the tin data very well with a very small contribution of an energy gap about double the usual one. But there are a lot of other combinations which would do just as well.