Errata and Note: Elementary Particles and Symmetry Principles

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THERE follows a list of errors which have been located in the paper as printed. We also correct an erroneous remark about the mathematics and try to make clear the rather abstruse concepts involved.

Page	*Paragraph*	*Instead of*	*Read*
481	next to last	Hill (1949)	Hill (1951)
482	3rd, near end	$\mathfrak{a}^\dagger = \mathfrak{b}, \mathfrak{b}^\dagger = \mathfrak{a}$	$\mathfrak{a} = \mathfrak{b}, \mathfrak{b}^\dagger = \mathfrak{a}^\dagger$
486	Table III, Sec. B, 1st column	for baryons	for leptons and baryons
487	Middle column, 3rd row from bottom of Table III	$\Psi = ({}^{p}{}_{n})$	$\Psi = \binom{p}{n}$
495	next to last	three two properties	these two properties
495	next to last	so "semisimple"	or "semisimple"
501	last, on right side of 1st	$\mathbf{A}^{a\prime}(\xi)$	$\mathbf{A}^{a}(\xi)$
501	last	$\mathbf{U} = \exp(-i\epsilon\mathbf{G})$	$\mathbf{U} = \exp(i\epsilon\mathbf{G})$
502	next to last, last equation	$-d\mathbf{Q}_T/dt = -i[\mathbf{H},\mathbf{QT}^T]$	$-d\mathbf{Q}_T/dt = -i[,\mathbf{H},\mathbf{Q}_T]$
504	4th from last, end	Nathans, 1959.)	Nathans, 1958.)
506	2nd	Schwever, 1955	Schweber, 1955
506	last	as assigned	be assigned
509	last, 9th line from bottom	meson-antineutrino	muon-antineutrino
509	Figure 5, upper right corner symbol AXIAL group	D_∞^h	C_∞^h
511	2nd	Telegdi and others, 1959	Telegdi and others, 1958
511	next to last, near end	$\bar{p}$ and n can then form.	p and $\bar{n}$ can then form.
512	ACKNOWLEDGMENTS	T. Linqvist	T. Lindqvist
513	Table VIII, last line of caption	$\mathbf{U}_k\, U_m$ equals im^2M_n	$[\mathbf{U}_k,\mathbf{U}_m]$ equals $im^2\mathbf{M}_n$
513	Table VIII, 1st column, last two rows of lower half should repeat those of upper half		
513	Table VIII, commutators in 4th column:		
	of $\mathbf{U}_0$ with $\mathbf{M}_m$	$-i(\mathfrak{M} \times \mathbf{P})$	0
	of $\mathbf{U}_k$ with $\mathbf{M}_m$	$i[\mathbf{U}_n\mathbf{P}_0 - (\mathfrak{M} \times \mathbf{P})_m\mathbf{P}_k]/\mathbf{P}_0$	$i\mathbf{U}_n$
	of $\mathbf{W}_k$ with $(\mathbf{W}_0\mathbf{P} - \mathbf{W}\mathbf{P}_0)_m$	$-i[m^2\mathbf{W}_n\delta_{km} + \mathbf{P}_k(\mathbf{W} \times \mathbf{P})_m]$	$-i[m^2\mathbf{W}_n + \mathbf{P}_k(\mathbf{W} \times \mathbf{P})_m]$
513	Table VIII, 5th column, commutator in next to last row, of $\mathbf{U}_k$ with $(\mathbf{W} \times \mathbf{P})_m$	$i[\mathbf{P}_k(\mathbf{W} \times \mathbf{P})_m - m^2\mathbf{W}_n\delta_{km}]$	$-i[\mathbf{P}_k(\mathbf{W} \times \mathbf{P})_m + m^2\mathbf{W}_n]$
513	Footnote 31, 3rd line from end	the centroidal moment	the orbital centroidal moment

Page	*Paragraph*	*Instead of*	*Read*
513	Eq. (I′)	$[\mathfrak{M}_{k},f'(\mathbf{P}_0)] = if(\mathbf{P}_0)\mathbf{P}_k$	$[\mathfrak{M}_{k},f(\mathbf{P}_0)] = if'(\mathbf{P}_0)\mathbf{P}_k$
516	line just below Eq. (VIII)	"where β' is the derivative of β with respect to $\mathbf{E}$" . . .	The same phrase should have appeared just after Eq. (IX)
517	bibliography (1935)		M. H. L. Pryce, Proc. Roy. Soc. (London) **A150,** 166 (1935)
517	bibliography (1948) after O. Klein reference		M. H. L. Pryce, Proc. Roy. Soc. (London) **A195,** 62 (1948)
518	bibliography (1958), after C. Bouchiat and L. Michel reference		M. T. Burgy, V. E. Krohn, T. B. Novey, G. R. Ringo, and V. L. Telegdi, Phys. Rev. Letters **1,** 324 (1958).
518	last	ADD:	(1960) J. M. Jauch, Helv. Phys. Acta **33,** 711 (1960) (1961) J. M. Jauch, CERN 61–14 Theory Division, 17 April (1961)

Finally the writer wishes to correct a conceptual error. Geographical isolation from references during the year in which the paper went to press caused the inclusion of two incorrect remarks on p. 497. One concerns Harish-Chandra's work and the other the properties of representations of the Poincaré group. The writer is very grateful to Professor George W. Mackey of the Mathematics Department of Harvard University for spontaneously taking the trouble to point out these errors, and for a helpful correspondence.

The last sentence of the first paragraph, p. 497, should be replaced by the following corrected statement and amplification:

"For connected semisimple[1] Lie groups one has the theorem (Harish-Chandra 1953, 1954) that all representations are of 'type I.'[2] A formal definition of a type I representation is that it is *a discrete direct sum of 'multiplicity-free representations;'* where multiplicity freeness of a representation L means that the set of all its commutors—its *commutant* L'—is Abelian.[3] It follows that a type I representation has a canonical 'discrete-continuous' decomposition, i.e., as a direct sum of integrals, each an integral of inequivalent reps none repeating. Alternatively to this discrete-continuous decomposition, every type I representation has an essentially unique 'continuous-discrete decomposition' into a direct integral over reps, each possibly occurring with some positive integral multiplicity.[2] We might ask if uniqueness of decomposition into reps is a distinguishing characteristic of type I representations. It seems never to have been proved that no non-type I representation has a unique decomposition. But it is very likely the case. If so, then type I would be distinguished from the

[1] The condition of semisimplicity cannot be left out because examples of connected Lie groups with non-type I representations are known.

[2] The original classification of any unitary representation $x \rightarrow L_x$ into types I, II, and III by Murray and von Neumann was in terms of the structure of the smallest 'weakly closed' ring of operators containing all the operators L_x being represented ('weakly closed' means, closed in the weak topology). This ring is a direct integral (in an essentially unique way) of basic building-block subrings called "factors." A subring is a factor if every commutor in the subring is a multiple of the identity. Corresponding to Schur's lemma, every rep is associated with a factor. The converse however is not true; e.g., a factor may not be a rep, i.e., it may be associated with nontrivial commutors outside the subring. In this case, *if it is discretely decomposable into reps,* the decomposition is into a multiple or discrete direct sum of equivalent reps; when this occurs we have the type I case. More generally, there are non-type I factors; these cannot be associated with a discrete direct sum of reps. When a non-type I factor representation is decomposed into a direct integral of reps one always gets inequivalent reps appearing, and the decomposition appears to be in all known cases nonunique. The theory shows that the type of the factor representations determines the type of any representation in which they appear.

[3] The abstract definition of a multiplicity-free representation L requires that any two bounded linear operators T and S which commute with all members of L necessarily commute with each other. Professor Mackey, who favors the definition of 'Type I-ness' in terms of the condition of multiplicity-freeness, remarks in his letter: "When L is a discrete direct sum of irreducibles this condition is equivalent to stating that no two summands are equivalent. The point of the abstract definition is that it does not involve talking about the decomposition of $L \cdots$. It is in this sense (although with a different formulation of the definition) that Harish-Chandra showed every semisimple Lie group to be of Type I."

other two Murray-von Neumann types of infinite unitary representations in that it alone would retain (like the finite-dimensional case which it includes) a certain uniqueness of decomposition into reps—uniqueness but not necessarily discreteness since it may involve either a continuous-discrete sum or a discrete-continuous sum.

"For physics we may add the following comments: The concept of multiplicity-freeness enables us to define quite generally the conditions on a set of observables such that, when the observables are measured, the quantum mechanical state is completely specified. In other words the concept of 'multiplicity freeness' permits us to state more concisely a general definition which Jauch (1960, 1961) has given of 'a complete set of commuting observables.' This definition applies equally well to operators with continuous as with discrete spectra—in contrast with the usual definition of a complete commuting set, requiring that there occur only nondegenerate common eigenvalues. Suppose L is the set of all bounded functions generated by a set of commuting observables. Then L is the smallest 'weakly closed' algebra[2] containing the original set. By a theorem of von Neumann (1929) any such 'smallest weakly closed algebra' is equal to its *double commutant*, i.e., the commutant of its commutant:

$$L'' = L\,.$$

In the particular case we are considering, since L is generated by commuting observables it is Abelian. If L is also multiplicity free, i.e., its commutant L' Abelian, then

$$L \subseteq L' \quad L' \subseteq L'' \quad \therefore L' = L$$

and L is said to be *maximal Abelian*. Under these conditions the original set of generators of L is said to form a *complete set*. Restating: *A commuting set of operators generates with all its bounded functions an Abelian algebra. If this algebra is multiplicity free* (i.e., *its commutant is Abelian*) *the original set is complete.* One sees that multiplicity freeness, or type I-ness, guarantees the existence of a complete set of commuting observables. Jauch (1960, 1961) has shown that the converse is also true: Type I-ness of the algebra of observables is a necessary and sufficient condition that a complete set of commuting observables exist, i.e., that the theory not exhibit innumerable and irremovable 'hidden variable' features."

Correspondingly the last sentence of the second paragraph, p. 497, should be replaced by:

"Thus for example, the Poincaré group is of type I. This follows from the relation of the Poincaré group to the Lorentz group and from a general theorem of Mackey (1952) about unitary representations of group extensions."

(1929) J. v. Neumann, Math. Ann. **102**, 370 (1929).
(1936) F. J. Murray and J. v. Neumann, Ann. Math. **37**, 116 (1936).
(1952) G. W. Mackey, Ann. Math. **55**, 101 (1952); **58**, 193 (1953).
(1953, 1954) Harish-Chandra, Trans. Amer. Math. Soc. **75**, 185 (1953); **76**, 26 (1954).
(1960, 1961) J. M. Jauch Helv. Phys. Acta **33**, 711 (1960); CERN 61-14 Theory Division 17 April (1961).

Erratum: Simple Groups and Strong Interaction Symmetries

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The fourth author's name should be B. W. Lee as above instead of W. Lee as printed.