# Pi-Pi Correlations in $\bar{p}$-$p$ Annihilation*

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THE $\bar{p}$-$p$ annihilation process leading to pure pion production affords an opportunity to examine possible pion-pion interactions. An analysis of annihilation events in propane has been made. The events used were "hydrogenlike," i.e., events showing no visible evaporation prongs and with $N_{\pi^+}=N_{\pi^-}$. It was possible to show that the sample chosen in this manner consists of 85% pure $\bar{p}$H events.[1] Examined in particular were the distribution of angles between pions of like and unlike charge in the $\bar{p}$-$p$ c.m. system as well as the distribution of total energy $Q$ of pion pairs of like and unlike charge in their c.m. system.[2] The results showed that the distribution of the angles between like pion pairs is strikingly different from that of unlike pairs, namely, for like pions a predominance of small pion pair angles was observed.

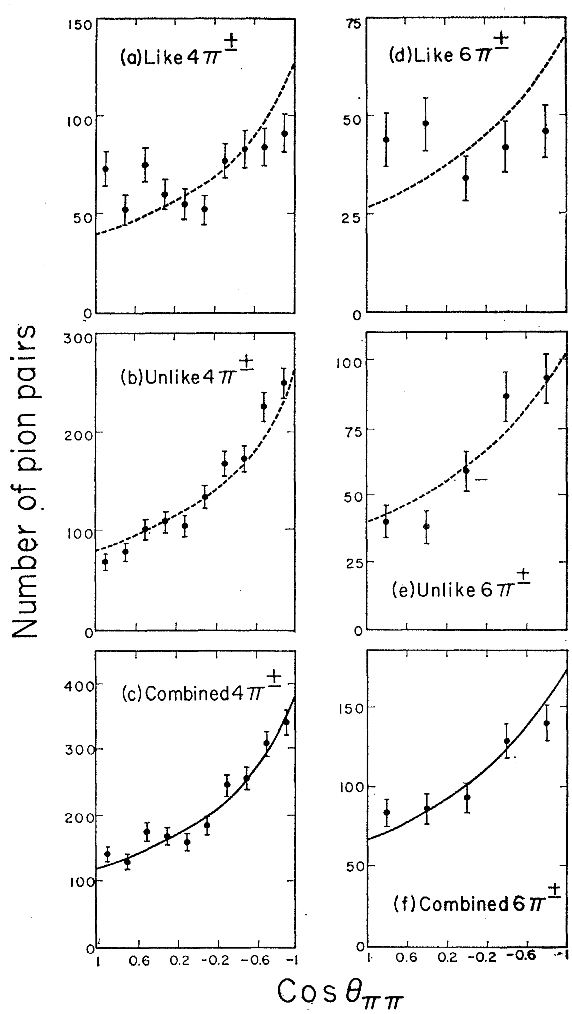


FIG. 1. Distribution of angles between pion pairs as a function of $\cos\theta_{\pi\pi}$. The curves correspond to the calculations on the Lorentz-invariant phase-space model.

Figure 1 gives the experimentally observed distribution in pion-pion angles together with calculations by Kalogeropoulos[3] based on the statistical model (SM). The experimental results have been expressed by a quantity $\gamma$ shown in Table I. The ratio $\gamma$ is defined as the ratio of the number of pion pair angles greater than 90° to the number less than 90°. The differences in the $Q$ distribution between like and unlike pairs were not as pronounced and of smaller statistical significance. The distributions for the two cases are given in Fig. 2.

TABLE I.

| | Number of pairs observed | $\gamma^{\text{exptl}}$ |
|---|---|---|
| Like | 752 | $1.23\pm0.10$ |
| Unlike | 1504 | $2.18\pm0.12$ |

In order to explain these results, a modification of the Fermi statistical model has been studied.[4] The modified model, in which Bose-Einstein (B-E) statistics were taken into account, reproduces the experimental angular distribution qualitatively with the interaction radius of 0.5~0.75 pion Compton wavelength as illustrated in Fig. 3 and Tables II and III. As was pointed out,[4] the B-E effect with this radius is not sufficient to yield the observed multiciplity. It was thus noted that additional dynamical effects could not be ruled out and might give rise to more accurate agreement with both the observed pion multiplicity and angular distributions.

In view of the interest in the conjectured pion-pion resonance in the $T=1$, $J=1$[5] state, we have reexamined our data in the light of such resonance. The presence

* This work was done under the auspices of the U. S. Atomic Energy Commission.

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‡ Presented by William Chinowsky.

[1] S. Goldhaber, G. Goldhaber, W. M. Powell, and R. Silberberg, Phys. Rev. **121**, 1525 (1961).

[2] G. Goldhaber, W. B. Fowler, S. Goldhaber, T. F. Hoang, T. E. Kalogeropoulos, and W. M. Powell, Phys. Rev. Letters **3**, 181 (1959).

[3] T. Kalogeropoulos, thesis, UCRL 8677 (unpublished).

[4] G. Goldhaber, S. Goldhaber, W. Lee, and A. Pais, Phys. Rev. **120**, 300 (1960).

[5] W. R. Frazer and J. R. Fulco, Phys. Rev. Letters **2**, 365 (1959); Phys. Rev. **119**, 1420 (1960).

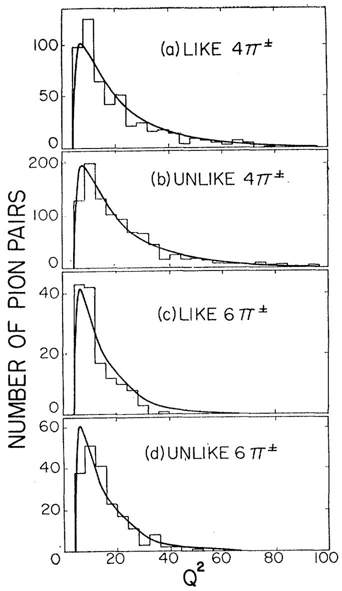



FIG. 2. Distribution of $Q^2$, the square of the total energy in the center of mass of the pion-pion system, in units of $\mu^2$. $Q^2=(w_1+w_2)^2-(\bar{p}_1+\bar{p}_2)^2$.

of such a resonance would lead to a correlation between unlike pions favoring small pair angles. This is not surprising since the proposed resonance implies a strong attraction between some of the unlike pairs. The experimental data, however, favor large pion-pair angles for unlike pions. Here we attempt to find phenomenologically by how much the proposed resonance might influence the angular distribution of the pions in the $\bar{p}+p\rightarrow\pi^++\pi^++\pi^-+\pi^-$ reaction.[6] We consider here the effect on the *unlike pairs* only. Since the effect of B-E statistics is rather small for the *unlike pairs*, we neglect B-E statistics in what follows.

At present, as in an earlier paper[4] on B-E statistics, we compute the distribution of pion-pair angles.[7] We use the Lorentz invariant statistical model and introduce an enhancement factor $|F_\pi|^2$ for each pion pair

TABLE II. Comparison between the experimental values ($N^{\pm}=4$) for $\gamma^l$ and $\gamma^u$ and the corresponding values derived by use of the B-E correlation functions for $\rho=0.5$ and 0.75. Also shown is the value for the usual Fermi SM. All the theoretical values have been averaged over the four-, five-, and six-pion distributions.

| | Exptl | $\rho=0.5$ | $\rho=0.75$ | av$^{SM}$ |
|---|---|---|---|---|
| Like | 1.23±0.10 | 1.41 | 1.38 | 1.80 |
| Unlike | 2.18±0.12 | 1.95 | 1.91 | |

[6] It should be noted that a possible complication due to a $T=0$, $J=1$ three-pion resonance [G. F. Chew, Phys. Rev. Letters 4, 142 (1960)] does not exist for the $2\pi^+2\pi^-$ case.

[7] It is a peculiar circumstance that the distribution in pion-pair angles reflects on the distribution in $Q^2$ very closely. This fact is not obvious but can be explained as follows: The annihilation process releases a fixed amount of energy which is distributed, in the case under consideration here, among four or more pions in what appears to be essentially a statistical distribution in momentum. It thus turns out that only a narrow interval of pion-pair angles can contribute to $Q^2$ values in the region (8–12) $\mu^2$. These pion pair angles are small, i.e., well below 90°.

TABLE III. List of computed $(\gamma^l)_{av}$ and $(\gamma^u)_{av}$ values. The values for $\rho=0.5$ and 0.75 are repeated here for clarity.

| $\rho$ ($\hbar/\mu c$) | $(\gamma^l)_{av}$ | $(\gamma^u)_{av}$ |
|---|---|---|
| 0.3 | 1.57 | 1.91 |
| 0.5 | 1.41 | 1.95 |
| 0.75 | 1.38 | 1.91 |
| 1.0 | 1.44 | 1.87 |
| 2.0 | 1.66 | 1.79 |

in the $T=1$, $J=1$ state,[8] where $F_\pi$ is the pion electromagnetic form factor. We need to know further the probabilities for finding a given pair from the $\bar{p}-p$ annihilation process in the $T=1$ state and at the same time in the $J=1$ state. As far as the isotopic spin states are concerned, we follow a procedure recently described by Pais[9] which enables one to determine the relative abundances of the three following states of the four pions: (1) no pion pair in the $T=1$ state; (2) one pair of pions in the $T=1$ state; (3) two pairs of pions in the $T=1$ state. The statistical weights of these three states are 8/15, 12/5, and 2/3, respectively. These statistical weights determine the probability of finding pion pairs in the $T=1$ state. They do not, however, ensure that the pion pair is also in the $J=1$ state, the condition necessary for obtaining the conjectured resonance. To impose this additional condition, we have for the present introduced two factors $\alpha$ and $\beta$ which we treat as parameters. Here $\alpha$ corresponds to the weight factor that a pion pair in the $T=1$ state also be in the $J=1$ state, and $\beta$ corresponds to the weight factor that two pion pairs, both in $T=1$ states, also be in $J=1$ states.

On labeling pions $\pi^+$, $\pi^+$, $\pi^-$, and $\pi^-$ by 1, 2, 3, and 4,

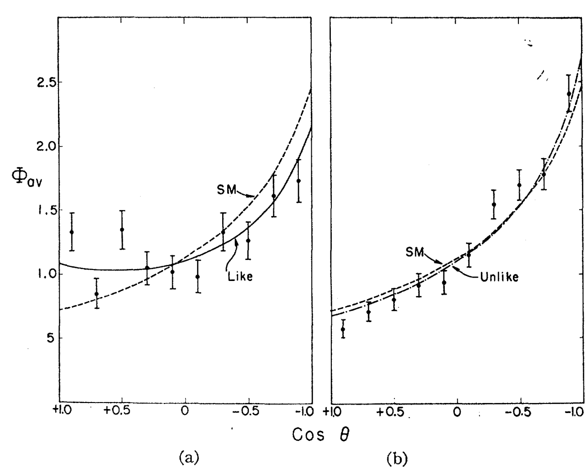



FIG. 3. The functions $\Phi_{av}(\cos\theta)$ computed at $\rho=0.75$ are compared with the experimental distribution of angles between pion pairs ($N_{\pi^{\pm}}=4$). (a) and (b) give the distributions for like and unlike pions, respectively. Also shown in each is the curve for $\Phi_{av}^{SM}(\cos\theta)$, the statistical distribution, without the effect of correlation functions. Here $\Phi_{av}$ represents an average of $\Phi_4$, $\Phi_5$, and $\Phi_6$, weighted according to the individual charge channels. The experimental data comes from reference 1.

[8] This approach was suggested to us by G. Chew.

[9] A. Pais, Ann. Phys. 9, 548 (1960).

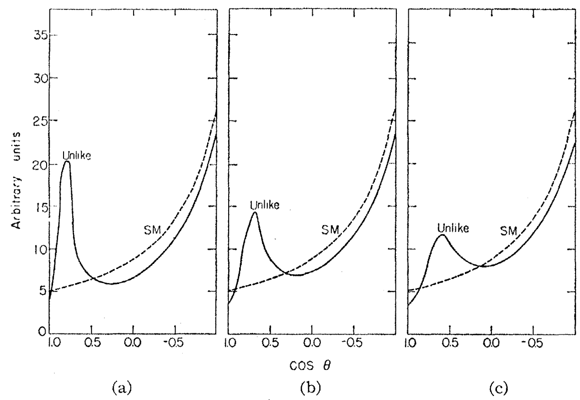



FIG. 4. Distribution functions of unlike pion pair angles for $\alpha=\beta=1$. (a)-(c) refer to $\nu_r=1.0$, $\nu_r=1.5$, and $\nu_r=2.0$, respectively.

respectively, the total rate $\Gamma$ of annihilation is

$$\Gamma=\text{const}\times\int\cdots\int\frac{d\bar{p}_1\cdots d\bar{p}_4}{w_1\cdots w_4}\delta(\bar{p}_1+\bar{p}_2+\bar{p}_3+\bar{p}_4)$$
$$\times\delta(w_1+w_2+w_3+w_4-W)\{(8/15)+(12/5)(1-\alpha)$$
$$+\tfrac{2}{3}(1-\beta)+\tfrac{1}{4}\cdot(12/5)\alpha[|F_\pi(Q_{13}^2)|^2+|F_\pi(Q_{14}^2)|^2$$
$$+|F_\pi(Q_{23}^2)|^2+|F_\pi(Q_{24}^2)|^2]+\tfrac{1}{2}\cdot\tfrac{2}{3}\beta[|F_\pi(Q_{13}^2)|^2$$
$$\times F_\pi(Q_{24}^2)|^2+|F_\pi(Q_{14}^2)|^2|F_\pi(Q_{23}^2)|^2]\}$$

where

$$Q_{ij}^2=(w_i+w_j)^2-(\bar{p}_i+\bar{p}_j)^2. \tag{1}$$

The total rate in Eq. (1) is given by the phase-space integral modified by an expression corresponding to the dynamical effects. The first term corresponds to states of type 1 in which no pions are in the $T=1$ state and hence no enhancement factors are involved. The second term corresponds to type 2 (i.e., one pion pair in the $T=1$ state). This breaks up into four parts corresponding to the four possible unlike pion pairs which must be considered. The third term corresponds to type 3 (i.e., two pion pairs in the $T=1$ state). The latter breaks up into two parts consisting of products of enhancement factors.

For the enhancement factors[5] we use the approximate form for convenience of computations:

$$|F_\pi(Q^2)|^2=\begin{cases}A\exp[-[Q^2-Q_0^2]^2/2\sigma^2]\\ 1\end{cases}$$

for

$$A\exp\left[-\frac{[Q^2-Q_0^2]^2}{2\sigma^2}\right]\begin{matrix}>1\\ \leq 1\end{matrix}$$

We neglect the crossing terms Re $[F_\pi^*(Q_{13}^2)F_\pi(Q_{14}^2)]$, $\cdots$, etc. We assume these to be negligible since the phase space is very small for the case of the two pion pairs 1, 3 and 1, 4 to be simultaneously in the resonance region.

To obtain the distribution in pion-pair angles for unlike pions, we now compute the distribution for a particular unlike pair, i.e., (1,3). Then

$$\Phi^{\text{unlike}}(\cos\theta_{\pi\pi})$$
$$=\text{const}\times\int_\mu^{W''^2\geq 4\mu^2}\int_\mu dw_1dw_3\int_{-1}^1\int_{-1}^1 dz_1dz_2p_1p_3$$
$$\times\left(1-\frac{4\mu^2}{W''^2}\right)^{\frac{1}{2}}\{(8/15)+(12/5)(1-\alpha)+\tfrac{2}{3}(1-\beta)$$
$$+\tfrac{1}{4}(12/5)\alpha[|F_\pi(Q_{13}^2)|^2+|F_\pi(W''^2)|^2+|F_\pi(R^2)|^2$$
$$+|F_\pi(S^2)|^2]+\tfrac{1}{2}\tfrac{2}{3}\beta[|F_\pi(Q_{13}^2)|^2|F_\pi(W''^2)|^2$$
$$+|F_\pi(R^2)|^2|F_\pi(S^2)|^2]\}.$$

Here we have

$$W''^2=(W-w_1-w_3)^2-(\bar{p}_1+\bar{p}_3)^2$$
$$R^2=a_1+b_1p_1p_2'z_1+c_1p_3p_2'z_2$$
$$S^2=a_3+b_3p_1p_2'z_1+c_3p_3p_2'z_2$$
$$a_i=w_i(W-w_1-w_3)+\bar{p}_i\cdot(\bar{p}_1+\bar{p}_3)+2\mu^2$$
$$b_i=\epsilon_i\left\{\frac{2w_i}{W''}-\frac{2\bar{p}_i\cdot(\bar{p}_1+\bar{p}_3)}{W''(\bar{p}_1+\bar{p}_3)^2}[W''-(W-w_1-w_3)]+2\right\}$$
$$c_i=b_i-2,$$

where

$$\epsilon_1=+1,\quad \epsilon_3=-1,\quad p_2'=[(\tfrac{1}{2}W'')^2-\mu^2]^{\frac{1}{2}}; \tag{2}$$

$\mu$ is the mass of the pion.

We have carried out the calculations for the three different enhancement factors given by Frazer and Fulco,[5] corresponding to $\nu_r=1.0$, 1.5, and 2.0 in their notation. The corresponding constants are given in Table IV. Angular distributions were computed by the IBM 704 and the results are shown in Figs. 4 and 5. The angular distributions of unlike pion pairs have a rather sharp peak at small pion-pair angles, deviating strongly from that of the statistical model. One should also note that for $Q_0^2\approx10\mu^2$ the relative angular momentum of the pair is $\approx0.6$, thus a choice of $\alpha=\frac{1}{2}$ and $\beta=\frac{1}{4}$ are not unreasonable. An estimate based on the selection of all $J$=odd states also gives $\alpha\approx\frac{1}{2}$. The effect of the choice of $\alpha$ on the ratio $\gamma$ is illustrated in Fig. 6.

As antiprotons become available in larger numbers, experiments may be performed in which a sample of

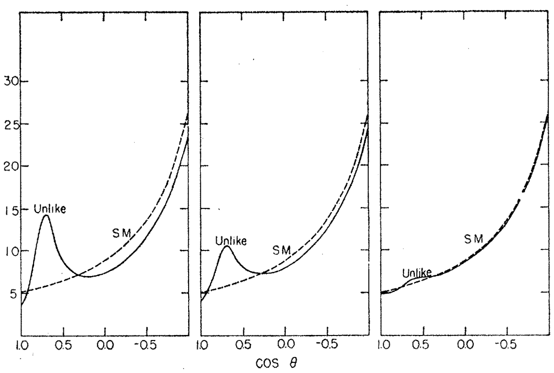
TABLE IV. The values of $A$'s, $\sigma$'s and $Q_0^2$'s corresponding to three different enhancement factors given by Frazer and Fulco.[a]

| | $\nu_r=1.0$ | $\nu_r=1.5$ | $\nu_r=2.0$ |
|---|---|---|---|
| $A$ | 29.2 | 16.5 | 11.7 |
| $\sigma$ | 1.3 | 2.3 | 3.45 |
| $Q_0^2$ | 8.1 | 10.4 | 12.5 |

[a] See reference 3.

pure $2\pi^+$, $2\pi^-$ annihilation events could be selected. In that case a direct comparison with our calculations would become possible and would allow a definite conclusion to be made on the presence or absence of the conjectured $T=1$, $J=1$[5] resonance. Since the experimental data on pion-pair angles[1] consists of a mixture of $4\pi$, $5\pi$, and $6\pi$ events, a direct quantitative comparison cannot be made at present.

The calculation for the four-pion case shows that the introduction of the $T=1$, $J=1$ resonance enhances the small angles in the pion-pair angular distribution for the *unlike* pions. It is hard to imagine a mechanism which would reverse this trend for the five- and six-pion cases. Thus, even at present we believe that it is very difficult to reconcile the conjectured resonance[5] (i.e., for values of $8\mu^2 \leq Q_0^2 \leq 12\mu^2$) with the available experimental data. If, on the other hand, values of $Q_0$ are increased or decreased by a substantial amount, then the values for $\alpha$ and $\beta$ as well as the available phase space decrease accordingly and the presence of such a resonance may not be detectable in the process under consideration.



(a) (b) (c)

FIG. 5. Distribution functions of unlike pion pair angles for $\nu_r=1.5$. (a)-(c) refer to $\alpha=1.0$, $\alpha=0.5$, and $\alpha=0.1$, respectively, and $\beta=\alpha^2$. Values shown in (b) are reasonable estimates for $Q_0^2 \approx 10\mu^2$.

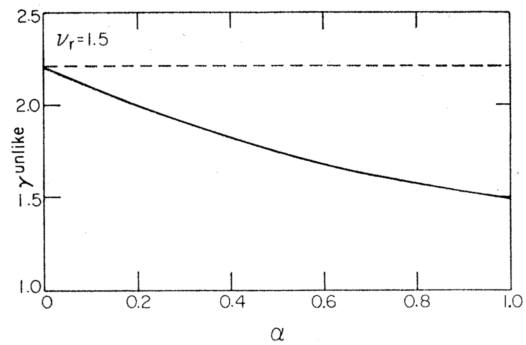



FIG. 6. Distribution of the ratio $\gamma^u$ as a function of $\alpha$ for $\nu_r=1.5$. The value $\alpha$ measures the amount of $J=1$ present in pairs with $T=1$. Here $\beta$ is taken as $\beta=\alpha^2$.

The idea of a strong $\pi$-$\pi$ resonance has been applied recently to the $\bar{p}$ annihilation process by Cerulus.[10] He introducted a $\pi$-$\pi$ isobar in the $T=1$ state into the statistical model. On the basis of this assumption he was able to explain the large pion multiplicity in the annihilation process for isobar masses of $3\mu\sim 4\mu$, i.e., $Q_0^2=(9\sim 16)\mu^2$. However, here again one would expect to observe a considerable enhancement at small pion-pair angles for the *unlike* pion distribution, which is contradictory to the experimental data.[1] It appears to us, thus, that the isobar model of Cerulus in its present form does not give a satisfactory explanation of the large pion multiplicity. If any dynamical $\pi$-$\pi$ effects are to be considered on the basis of the present data, they would rather have to be ascribed to the like pairs ($T=2$).[11] Dynamical effects between three or more pions *cannot* be excluded at present and should probably be investigated more closely.

[10] F. Cerulus, Nuovo cimento **14**, 827 (1959).

[11] A possibility of a strong attractive pion pion interaction in the $T=2$, $J=0$ state has been pointed out. See, for example, B. S. Thomas and W. G. Holliday. Phys. Rev. **115**, 1329 (1959); M. L. Good and W. G. Holliday. Phys. Rev. Letters **4**, 138 (1960); K. Chadan and S. Oneda, Phys. Rev. **119**, 1126 (1960).

## DISCUSSION

**L. W. Alvarez**, *University of California, Berkeley, California:* This is probably not a fair question to ask Dr. Chinowsky, but perhaps Dr. Pais could answer it. This is an experiment in which one finds an angular distribution in complete agreement with a "reasonable" annihilation radius, but one finds that the multiplicity is in quite strong disagreement with a reasonable radius. Then seeing this situation, one looks for an explanation to make the angular distribution agree with what would originally have been thought to be an unreasonable annihilation radius. It would seem to me to be more natural to say, "The disagreement is with the multiplicity and let us see whether we cannot find something that brings the multiplicity into agreement with a more reasonable radius." I do not understand the rationale of finding something that is very reasonable and something that is unreasonable and then looking for an explanation to bring the reasonable thing into agreement with an unreasonable set of conditions.

**W. Chinowsky:** No matter what is done, it must be recognized that $\pi$ mesons are bosons and effects of the Bose statistics must be included in any event. When this was done by Pais *et al.*, they found that the effect was to produce the kind of angular correlations observed. I do not think there is a complete explanation of both the multiplicity and the angular distributions.

**S. Mandelstam,** *University of Birmingham, Birmingham, England:* With reference to this pion-pion resonance, work done at Berkeley and also at CERN on the pion-nucleon problem also seems to indicate that if such a pion-pion resonance does exist, its energy would be higher than originally thought by Frazer and Fulco. The resonance would be about $4\frac{1}{2}$-pion masses which is certainly higher than even the highest resonance shown on your curves. And since this curve was going down with increased energy of the resonance, this would seem to indicate again, if such resonance did exist, its energy would be higher than Fraser and Fulco thought. However, before coming to such a conclusion, if two pions can interact in a resonant state, there must also be a similar interaction between three pions, any two of which are in the resonance state at the same time, which would complicate things quite a lot. Has any attempt been made in this calculation to take this fact into account? Looking at it off hand, I do not really see how one could get such angular correlations, though if such a calculation was being done I would be interested to hear.

**W. Chinowsky:** I think the answer is no. I do not see how one can get two pions of like charges to have smaller angles. You need an attraction between pairs of pi mesons of like charges, effectively.

**S. Mandelstam:** Yes, I certainly agree with that. I am just at the moment really interested in the question as to whether this includes the possibility of a resonance. Now these are the tentative conclusions according to the calculations made by Chew and myself (these are tentative—I am certainly not prepared to stand by them): It looks as though we may need an attraction in the $T=0$ state in order to fit in with the resonance of the $T=1$ state.

REVIEWS OF MODERN PHYSICS VOLUME 33, NUMBER 3 JULY, 1961

# Some Considerations concerning Final-State Interactions and the Reaction $K_2^0+p\rightarrow$ Lambda$^0$ + Pi$^+$ + Pi$^0$

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## INTRODUCTION

THERE has been considerable interest recently concerning the investigation of the momentum correlations among the several particles produced in nuclear reactions with the hope of obtaining information concerning the existence of excited states through final state interactions. This discussion is concerned with some general and some specific problems of this nature. The paper is divided into four sections: Sec. I, an experiment designed to study the interactions of $K_2^0$ mesons with protons, Sec. II, an analysis of the effect of symmetries and angular momentum barriers, particularly as applied to the reaction $K_2^0+p\rightarrow\Lambda^0+\pi^++\pi^0$, Sec. III, a discussion of properties of resonant final-state interactions, and Sec. IV, experimental results concerning the reaction $K_2^0+p\rightarrow\Lambda^0+\pi^++\pi^0$, and their interpretation.

## I. EXPERIMENTAL DESIGN AND PROCEDURE

The basic experimental design[1] can be discussed with reference to Fig. 1. The external proton beam from the Brookhaven cosmotron was brought to a focus by three 8-in. aperture quadrupole magnets and then refocused by a 12-in. quadrupole magnet pair, to form an image about 3 cm high by $1\frac{1}{2}$ cm wide on a steel target. The target density was about 80 g/cm$^2$, which is about two-thirds of a mean free path. Operation at the second focus was desirable for logistic reasons; there was better crane coverage near the second focus, and a target could be inserted at the first focus conveniently and used for a parallel experiment. Mesons produced in the steel target at an angle of about 4° from the proton beam passed through two 12-in. quadrupoles acting as an objective lens, through an 18×36-in. bending magnet which provided momentum resolution, through an 8-in. quadrupole magnet used as a field lens, then through another 12-in. quadrupole pair and a bending target onto a polyethylene target. This beam design, as is the case for many beam designs, has the objective of transferring a beam through distance so that adequate shielding may be provided. In this experiment the shielding of the bubble chamber from the neutrons coming from the proton target was a main concern. Angular acceptance of the beam was $\pm1\frac{1}{2}°$ vertically, $\pm9°$ horizontally, and the momentum acceptance was nominally $\Delta P/P\approx0.07$. Average external proton beam intensities were about $7\cdot10^9$ protons/pulse throughout the experiment, providing an intensity of about $2.5\cdot10^5$ 1.6-Bev/$c$ $\pi^-$ mesons on an area of about 50 cm$^2$. At times, intensities about three times as great were achieved.

Interactions of the $\pi^-$ mesons with the nucleons in the polyethylene produce $K$ mesons through reactions such as $\pi^-+(p \text{ or } n)\rightarrow(\Sigma \text{ or } \Lambda)+K^0+(0,1, \text{ or } 2)\pi$. Since the density of the polyethylene target was about 40 g/cm$^2$, or one-half of a mean free path, secondary interactions involving the production, and loss, of $K^0$ mesons were also important. One-half of the $K^0$ mesons decay very quickly to two $\pi$ mesons through the $K_1^0$ decay mode, leaving the long lived $K_2^0$ mesons. These $K_2^0$ mesons, produced at about 6° from the forward direction, passed

* A. P. Sloan Foundation Fellow.

[1] The beam design was primarily due to W. Chinowsky. The experiment, excepting the analysis, was conducted by Chinowsky, L. B. Leipuner, F. T. Shively, and R. K. Adair. H. J. Martin, R. Crittenden, and B. Musgrave contributed to the analysis and interpretation of the data; we are also indebted to Mrs. Anne L. Baker and and Miss Eileen O'Donnel for their work on the data analysis.