Flow in Direct-Current Electromagnetic Pumps*

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A NUMBER of theoretical analyses have been carried out on the steady one-dimensional channel flow of an electrically conducting fluid in the presence of electric and magnetic fields. It is generally assumed that the electric and magnetic fields are uniform over the entire flow field or vary slowly along the length of the channel and that effects of the nonuniformities near the ends are negligible. These conditions represent an ideal limit which cannot be attained in the laboratory, so it becomes of interest to consider how the departure of the driving fields from a one-dimensional character affects the flow. A simple model that permits investigation of these effects is the direct-current electromagnetic pump (shown schematically in Fig. 1) that has an electric and a magnetic field impressed across the channel to produce a body force on the fluid in a direction along the channel. The effect of the ends of the electric and magnetic fields on the flow is studied herein theoretically and experimentally. Various modifications to the geometry of the magnet and electrodes shown in Fig. 1 are tried in order to restore a one-dimensional form to the flow.

Before proceeding to the details, consider briefly the possible variations and limitations which exist for the geometry of the magnetic and electric fields. Since they both satisfy Laplace's equation in the region of interest, under the same circumstances both have the same nonuniformities near their edges. It appears then that it is immaterial in what proportion the dimensions of the channel, magnet, and electrodes are chosen and that the roles played by each in a given setup can be interchanged. Considerations of practical importance, however, require that the air gap of the iron core electromagnet be small in order to produce a strong field, and that the distance between the electrodes not be too small in order to sustain a moderate potential difference between the electrodes. Inasmuch as the air gap between the magnet pole pieces is kept small, and therefore conducive to containing the magnetic field in this region, it can sometimes be assumed that the magnetic field is constant between the pole pieces and zero elsewhere. The area covered by the field is then governed by the cross section or plan form of the iron core.

DC ELECTROMAGNETIC PUMP

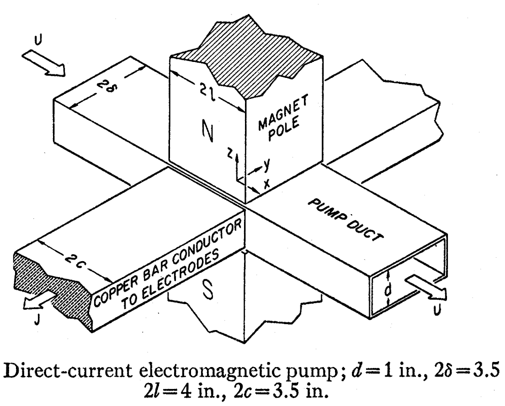


FIG. 1. Direct-current electromagnetic pump; $d=1$ in., $2\delta=3.5$ in., $2l=4$ in., $2c=3.5$ in.

Even though the separation between the electrodes is not small, a region of nearly uniform current density (see Fig. 2), that has a one-dimensional form, still exists in the center of the field. Nonuniformities in the electric-current distribution appear as infinitely large current densities at the ends and as current loops that extend upstream and downstream of the electrodes. Since the conductivity and permittivity of the fluid are constant, only small changes in this electric-current distribution can be brought about by various techniques, such as varying the potential along the side walls of the channel in a stepwise fashion with multiple electrodes. In special cases, a desired electric field can be acquired by choosing a part of the field produced by a set of electrodes. It often becomes necessary, however, to accept the electric field generated by a given set of electrodes with the result that it deviates appreciably from the desired current distribution and is not one-dimensional over a sizable portion of the flow field.

In order to realize a simple configuration for the present experiment, the electrodes are assumed to be at a constant potential and to have a length equal to the channel width. A copper sulphate salt solution ($\sigma=0.04$ mhos/in.) is used as the working fluid that flows through the clear plastic channel under the action of the electric and magnetic fields. The flow velocity ranges from 1 to 4 in./sec under the influence of 0.4- to 1.6-amp current

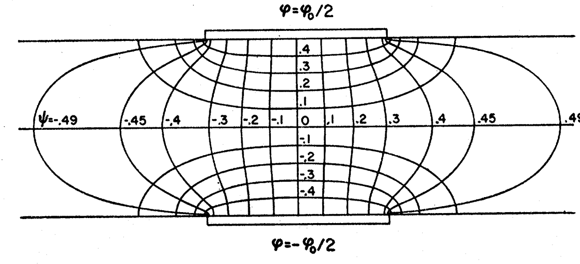


FIG. 2. Equipotential and current lines in a two-dimensional channel when electrode length equals channel width.

* Grateful acknowledgement is given to William Prichard Jones and Robert H. Huerta of Ames Research Center for assistance in carrying out the work reported here. Further details will appear in an NASA publication.

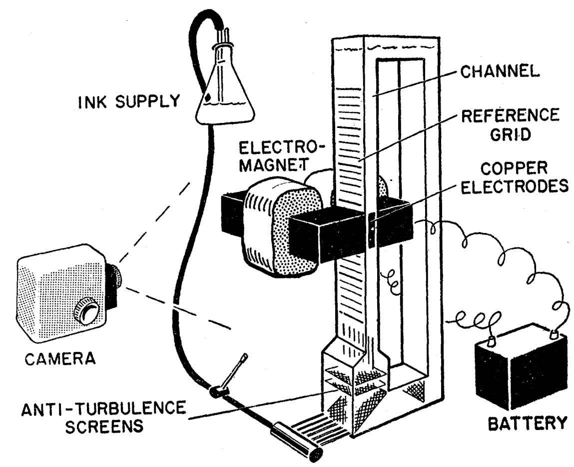


FIG. 3. Schematic drawing of test setup.

and about 4000 gauss. The reason for choosing these operating conditions is that the conductivity and flow rate are so low that the secondary induced magnetic and electric fields are each about 10^{-3} times smaller than the externally imposed fields thereby facilitating an analysis. Numerical values for the characteristic parameters are given by

$$\text{Hartmann parameter}=(\sigma/\eta)^{\frac{1}{2}}Bl\approx 10^{-2}$$

$$\text{counter emf}=UB\approx 10^{-3}\ \text{v/in.}$$

$$\text{magnetic Reynolds number}=\sigma\mu Ul\approx 10^{-8}$$

$$\text{viscous Reynolds number}=Ul/\nu=300\ \text{to}\ 1000,$$

where U, σ, η, μ, and ν are the fluid velocity, conductivity, viscosity, permeability, and kinematic viscosity, respectively. The quantities B and l are the magnetic field strength and characteristic length of the flow field.

Since the induced fields are each about 10^{-3} times smaller than the imposed fields, the analysis is greatly simplified because, under these circumstances, the effect of the fluid motion on the imposed electromagnetic fields is negligible. Classical methods can then be used to find the electric-current distribution in the flow field. These results taken together with the corresponding results for the magnetic field, which can be found theoretically or experimentally, permit the direct calculation of the body forces in the fluid.

A solution for the electric field is found by conformal mapping of the two-dimensional flow field (σ=const). The complex potential Φ is then

$$\Phi=\varphi+i\psi=(\varphi_0/2K)F(i\zeta\pi/2\delta,k), \qquad (1)$$

where φ and ψ are the potential and stream function for the electric current with $\zeta=x+iy$, $i=(-1)^{\frac{1}{2}}$, and $k=1/[\cosh(c\pi/2\delta)]$. The symbols K and $F(\ )$ represent the complete and incomplete elliptic integrals of the first kind and φ_0 is the potential difference applied across the electrodes. Typical contours for the equipotential and current lines computed by Eq. (1) are shown in Fig. 2 for the arrangement being considered in the experiment; that is, electrode length equals channel width. Over the center portion of the field, the current lines are nearly straight across the channel as assumed in the simple one-dimensional theories. Near the ends, however, the current no longer flows straight across and local values for the current density depart appreciably from the uniform value in the center region.

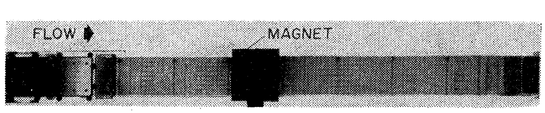


FIG. 4. Flow through an electromagnetic pump.

With the electric field known, the body forces on the fluid can be computed if the magnetic field is determined from experimental measurements. An expression for the increase in the momentum or pressure head, $H=p+\frac{1}{2}\rho u^2$, along each streamline brought about by these body forces can be derived by integrating the momentum equations along a streamline. In the expression for H, the quantities p and ρ are the ambient fluid

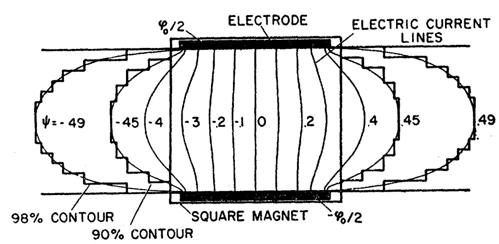


FIG. 5. Magnet plan forms compared with electric-current paths; electrode length equals channel width.

pressure, and density and u is the velocity component along the channel. Although the total integrated value actually changes over the cross section of the channel, it is nearly two-dimensional so that changes in the z direction can be assumed to be negligible. Also, the individual streamlines do not deviate appreciably from a given y=constant plane, permitting the increase in pressure head to be written as

$$\Delta H(y)=-\sigma\int_{-\infty}^{+\infty}B_zE_ydx, \qquad (2)$$

where B_z is the magnetic field strength in the z direction and E_y is the electric field intensity in the y direction.

Equation (2) takes on a particularly simple form if the further assumption is made that the magnetic field is confined to the space between the pole faces so that the function B_z may be approximated by a rectangular function. Under these circumstances, and with $E_y=-\partial\psi/\partial x$, Eq. (2) becomes

$$\Delta H(y)\approx B_z[\psi(l)-\psi(-l)], \qquad (3)$$

where $\psi(l)$ and $\psi(-l)$ are the values of the current function at $x=+l$ and $-l$, which are the values of ψ at the extremities of the magnetic field.

A schematic diagram of the test setup, with appropriate legends for the various pieces of equipment, is shown in Fig. 3. Motion of the copper sulphate solution (specific gravity$=1.040$) through the clear plastic channel (1 by $3\frac{1}{2}$ in. in cross section) is studied by observing seven ink filaments that are injected at the lower end of the test portion of the channel. Electric current, 0.4 to 1.6 amp, from two wet-cell batteries passes through the copper electrodes ($3\frac{1}{2}$ in. long) into the electrolyte where it interacts with the magnetic field to impart a body force to the liquid. In the position shown for the pump, the body force moves the fluid through the channel from the bottom to the top. For convenience of presentation and in keeping with Fig. 1, the photographs of the ink filaments to be presented in the succeeding figures show the channel in a horizontal position with the fluid flowing from left to right. The velocity of the ink filaments for several magnetic and

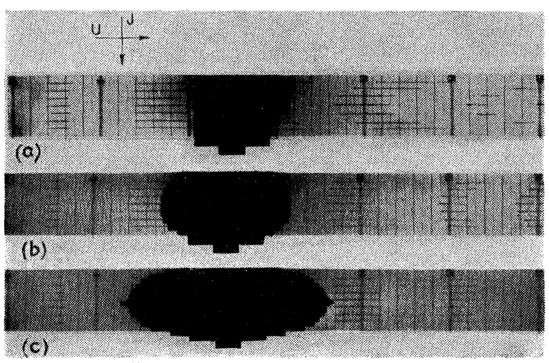


FIG. 6. Ink filament patterns depicting the velocity distribution produced by pumps with special magnet contours. (a) Square, (b) 90% contour, (c) 98% contour.

electric field configurations are determined from test results recorded by a 70-mm camera set to take pictures at 1-sec intervals. Measurement of the distance traveled along the channel by the ink filaments is aided by a reference grid (graduated in inches) placed on the back side of the channel. A current of 10 amp is maintained through the windings of the 4- by 4-in. iron core electromagnet to produce a field of about 4000 gauss in the $1\frac{1}{2}$-in. air gap.

Consider first the flow through an electromagnetic pump that is built according to the one-dimensional model which assumes that the end effects of the electric field are negligible. Such a design, shown schematically in Fig. 1, employs a magnet that has a rectangular plan form and electrodes of about the same length that are mounted flush with the insides of a rectangular channel. Flow through a pump of this kind is illustrated in Fig. 4, where a typical photograph of the locations of several sets of ink filaments is presented. Since each set of filaments is released simultaneously, a displacement between them indicates a velocity difference in the fluid

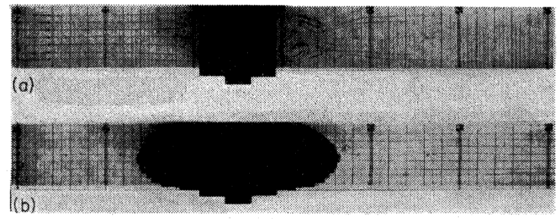


FIG. 7. Flow patterns at zero flow rate. (a) Square magnet (b) 98% contour.

near the filaments. These results show that a 4- by 4-in. magnet in combination with the $3\frac{1}{2}$-in. electrodes produces a stream that has a velocity deficiency at the center of the channel. Inspection of Fig. 2 or Fig. 5 reveals that a 4- by 4-in. magnet covers varying amounts of the electric current along the various streamlines (i.e., along $y=$constant lines). Equation (3) then predicts that the fringe current, or the current in the loops which extend beyond the magnet, does not contribute to the momentum or pressure head of the fluid, with the result that the center streamlines have a deficiency in ΔH.

A means, suggested by Eq. (3), for recovering the one-dimensional character of the flow is obtained by fitting the magnet plan form to the electric current pattern. Two magnets that have shapes which approximate the 90% ($\psi=\pm 0.45$) and the 98% ($\psi=\pm 0.49$) current lines are indicated in Fig. 5 by the heavy lines. Also shown is the plan form of the 4- by 4-in. magnet. Irregularities arise in these shapes from the edges of the iron plate stock used in constructing the pole-piece additions. No effort was made to round off the corners because they did not significantly affect the strength of the magnetic field near the centerplane of the channel. A progressive return to one-dimensional flow or a uniform stream is demonstrated in the shape of the velocity profiles shown in Fig. 6 for the square, 90 and 98% magnet contours. In order to permit an enlargement of the flow field that is of interest, the reservoirs at the two ends of the channel (see Fig. 3 or 4) are not shown. It can be seen that end effects are virtually eliminated by the 98% contour and the flow is practically uniform both upstream and downstream of the magnet. Nonuniformities in the

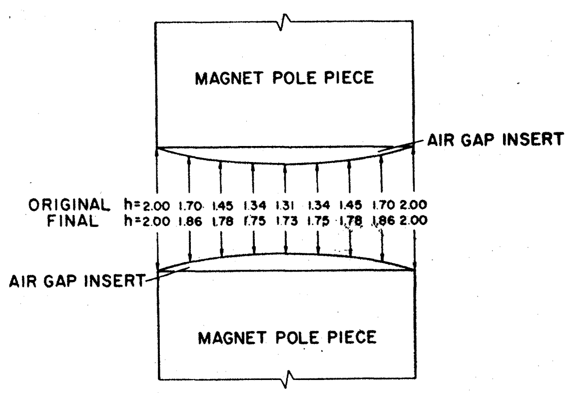


FIG. 8. Air gap inserts for square magnet.

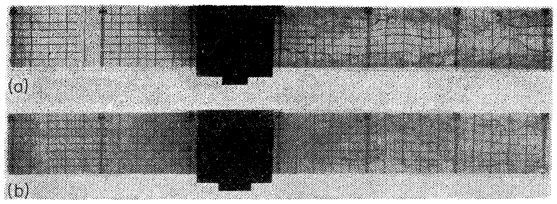


FIG. 9. Flow produced by pump with air gap inserts. (a) Original, (b) final.

electric-current distribution do, however, cause small departures from a uniform stream to occur inside the electromagnetic field region.

The variation in the velocity out of the pump that employs a square magnet (see Fig. 4) introduces vorticity into the stream. Such a vortex system is unstable and can be observed to roll up at low flow rates as illustrated in Fig. 7(a). The pump constructed with the magnet contour covering 98% of the current imparts a negligible amount of vorticity to the stream, so that no distortion of the streamlines occurs at low flow rates as shown in Fig. 7(b).

An alternative method for producing a uniform stream, and thereby recovering one-dimensional flow, with the square magnet and the $3\frac{1}{2}$-in. electrodes is acquired by satisfying Eq. (3) in another manner. It consists of an adjustment to the field strength rather than to the plan form of the magnet to fit the electric-current pattern. In order to achieve this, the air gap is constructed so that the magnetic field strength B_z compensates for the deficiencies in $[\psi(l)-\psi(-l)]$ along each streamline. Coordinates for the air gap were first estimated by simple one-dimensional considerations and then corrected experimentally by several trial and error attempts. Values for the original and the final or approximately optimum shapes are presented in Fig. 8 and the flow produced by each in Fig. 9. A stream which has a uniform velocity (i.e., is once again one-dimensional) is produced by the final model tested [Fig. 9(b)], but turbulence is imparted to the stream by extraneous forces which arise from the curvature of the magnetic lines of force.

Electric-current barriers that are placed along the center of the channel and that extend some distance upstream and downstream of the electrodes (see Fig. 10, for example) have been proposed[1] as a means for reducing the end effects or fringing of the electric field. In the ideal case, the momentum imparted to the center of the stream is the same as that imparted to the sides. It is not always possible, however, to make the barriers as long as desired so that some current can flow around the far ends. Flow out of such a pump, that is equipped with 11-in. barriers (Fig. 10), bears out the fact that barriers of finite length are partially effective. An estimate of the resistance of the several current paths shows that about one-seventh of the current goes around the ends, resulting in a loss of about 14% in the total head at the center of the channel.

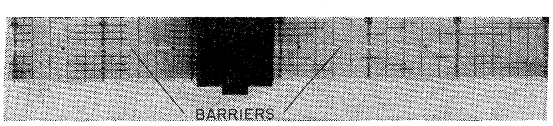


FIG. 10. Flow when channel is equipped with two 11-in. electric-current barriers.

The foregoing discussion has been devoted to the fringing or end-current loops of the electric field and one attempt has been made to reduce the spreading out of the current paths. It is possible to get an electric field which is essentially one-dimensional, by choosing the center portion of the field produced by very long electrodes for the location of a magnetic field of rectangular plan form. Figure 11 illustrates the flow from such a pump where the end effects are indeed negligible as evidenced by the uniformity of the stream. Curvature of the electric current paths near the center of the 12-in. electrodes is so small that it is not possible to measure a variation in the velocity. The disadvantage lies in the fact that only about one-third of the current interacts with the magnetic field to increase the momentum of the stream.

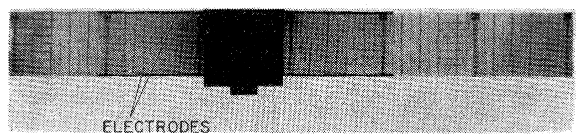


FIG. 11. Uniform flow produced by long electrodes.

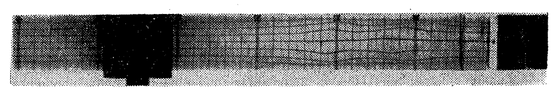


FIG. 12. Flow distortion caused by turning off magnetic field.

Availability of large quantities of alternating-current power has prompted the design of alternating-current electromagnetic pumps.[2] Large eddy currents that arise from the alternating magnetic field (i.e., $\nabla\times\mathbf{E} = -\partial\mathbf{B}/\partial t$) produce extraneous body forces that distort the streamlines and cause wasteful Joule heating. Figure 12 shows a distortion in the salt-water pump used in this study caused by turning off the magnet at the end of a test run, while the fluid coasts to a stop. The hourglass shape is caused by the nonuniform pressure fields at the ends of the magnetic field.

The results presented demonstrate that the finite length of the electromagnetic fields can cause sizable deviations from one-dimensional flow. It is possible, however, to correct for these end effects by either shaping the magnetic field or varying its intensity to fit the electric field, or by a combination of these two techniques.

[1] A. H. Barnes, *Nuclear Reactor Experiments*, J. Barton Hoag, Editor (D. van Nostrand Company, Inc., Princeton, New Jersey, 1958), pp. 272–279.

[2] J. F. Cage, Jr., Machine Design **25**, 178 (1953).

DISCUSSION

Session Reporter: N. H. Kemp

G. Jungclaus, *Institut für Angewandte Mathematik und Mechanik, Freiburg i. Br., Germany:* I think this is a very good way to produce any velocity profile which one wants. It is very hard sometimes in experimental technique to produce, for instance, the velocity profile at the end of a turbine.

J. Ackeret, *Eidgenössische Technische Hochschule, Zürich, Switzerland:* Did you have difficulties with electrolytic effects, bubble formation, etc.?

V. J. Rossow: It was because of these effects that we tried a number of solutions and ended up using copper sulphate solution at a specific gravity of 1.040 because that mixture approaches a nearly 100% plating solution. The slightest amount of sulphuric acid or other impurities may cause gas to be given off at the electrodes.

A. R. Kantrowitz, *Avco-Everett Research Laboratory, Everett, Massachusetts:* Have you ever tried any experiments decelerating fluids with a magnetic field?

V. J. Rossow: We have tried to decelerate the fluid in this pump, but the conductivity is so low that it does not slow up any faster with the magnetic field on or off.

A. R. Kantrowitz: Even if you reverse the electric current?

V. J. Rossow: It stops in a hurry then.

A. R. Kantrowitz: This would be quite interesting, you see, because the nonuniformity that you have in acceleration, contrasted to deceleration, is like a diffuser versus a nozzle. You will get separation. You saw interesting things when you used a paddle to stop the flow, and I thought you might see the same things in a generator.

V. J. Rossow: Yes, it is quite possible.

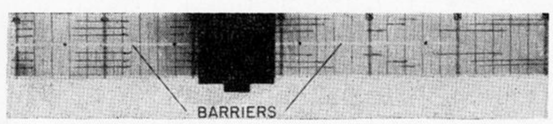


FIG. 10. Flow when channel is equipped with two 11-in. electric-current barriers.

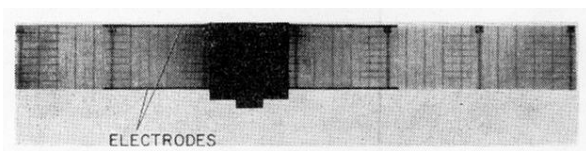


FIG. 11. Uniform flow produced by long electrodes.

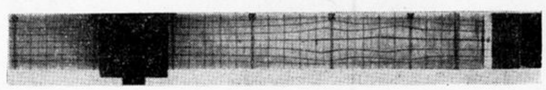


FIG. 12. Flow distortion caused by turning off magnetic field.

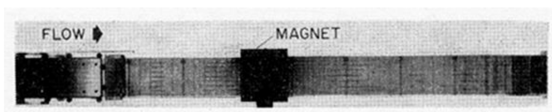


FIG. 4. Flow through an electromagnetic pump.

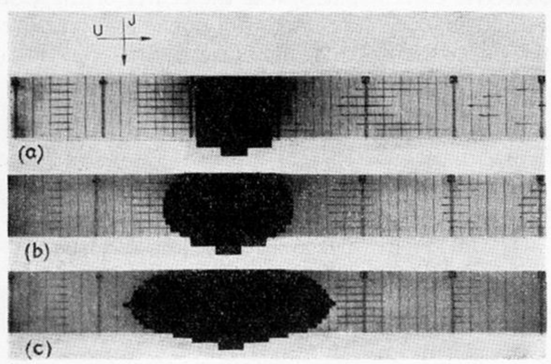


FIG. 6. Ink filament patterns depicting the velocity distribution produced by pumps with special magnet contours. (a) Square, (b) 90% contour, (c) 98% contour.

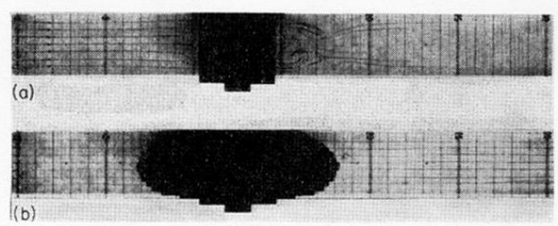
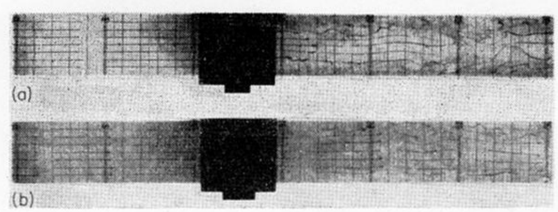


FIG. 7. Flow patterns at zero flow rate. (a) Square magnet (b) 98% contour.



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