Some Results on Heat Transport by Convection in the Presence of a Magnetic Field*

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1. INTRODUCTION AND THEORETICAL RESULTS

THIS paper reviews studies on the effect of an impressed magnetic field on heat transport by convection. The general situation which is investigated is the following. Consider a horizontal layer of an electrically conducting fluid (mercury) of depth d bound between two constant temperature surfaces, the bottom surface being maintained at higher temperature T_1 than the top surface T_2, and subject to an impressed magnetic field $\mathbf{H}$.

For small temperature difference (T_1-T_2), the layer of fluid must be in a state of conductive equilibrium, and under these circumstances the amount of heat transferred through the layer is

$$S=\lambda(T_1-T_2)/d, \tag{1}$$

where S is the amount of heat transferred across unit cross-section area of boundary surface in unit time and λ is the thermal conductivity of the fluid.

When the temperature difference is gradually increased, the layer of fluid becomes unstable. Beyond a certain critical temperature difference convection sets in. The exact manner of the onset of convection has been studied by Chandrasekhar[1] and others[2-5]; it is known that an impressed magnetic field inhibits the onset of convection, and the convective circulation is elongated in the direction of the magnetic field.[3,5]

For maintaining constant temperature difference, after the onset of convection, the layer of fluid eventually reaches a state of convective equilibrium. Then, by using the relation which defines the Nusselt number[6] Nu, we may write

$$S=\lambda \mathrm{Nu}(T_1-T_2)/d. \tag{2}$$

It is convenient to introduce the nondimensional parameters of the problem[1] before proceeding further; these are the Rayleigh number

$$\mathrm{Ra}=\alpha(T_1-T_2)gd^3/\kappa\nu$$

and

$$Q=\sigma\mu^2\cos^2\vartheta H^2d^2/\rho\nu, \tag{3}$$

where g is the acceleration of gravity, ρ the density, H the strength of the magnetic field, ϑ the inclination of the direction of $\mathbf{H}$ to the vertical, and α, σ, κ, μ, and ν are the coefficients of volume expansion, electrical conductivity, thermometric conductivity, magnetic permeability, and kinematic viscosity, respectively.

In terms of the Rayleigh number and the Nusselt number, Eqs. (1) and (2) can be combined and rewritten into the following form of a single nondimensional equation:

$$S=\mathrm{Nu}\,\mathrm{Ra}, \tag{4}$$

where S now denotes the nondimensional amount of heat transferred through the layer of fluid. In Eq. (4) the value of the Nusselt number is

$$\mathrm{Nu}=1, \tag{5}$$

by definition in a state of conductive equilibrium.

The value of the Nusselt number, after the onset of convection, may depend on the material properties of the fluid and the characteristics of the motion of convection, such as the pattern of motion and the velocity of convection. By choosing appropriate units which curtail no entire loss of generality, we may write the Nusselt number in a state of convective equilibrium in the form[7,8]

$$\mathrm{Nu}=1+(\mathrm{Pr}\,\mathrm{Ra})^{-\frac{1}{2}}\langle w\theta\rangle_{\mathrm{av}}, \tag{6}$$

where $\mathrm{Pr}=\nu/\kappa$ is the Prandtl number, w is the vertical velocity of convection, θ is the temperature fluctuation associated with convection, and the angular parentheses with subscript "av" denotes the average over space for

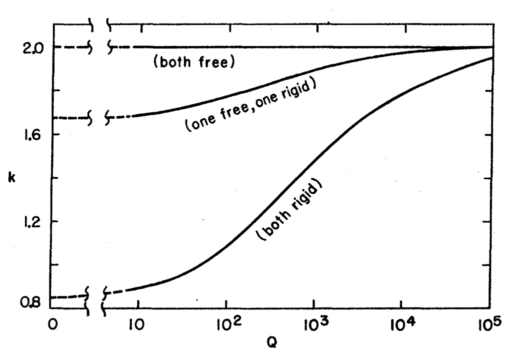

FIG. 1. Variation of value of k for various values of Q and boundary conditions.

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steady convection and the average over space and time for oscillatory convection.

To the first approximation, Eq. (6) can be reduced to[7,8]

$$\mathrm{Nu}=1+[k(\mathrm{Ra}-\mathrm{Ra}_c)/\mathrm{Ra}], \tag{7}$$

where k is a constant and Ra_c denotes the critical Rayleigh number at which convection sets in. It may be worthwhile to point out that to the first approximation the Nusselt number can be expressed in the form similar to Eq. (7) in the absence of an impressed magnetic field,[7,8] as well as for the cases when a layer of fluid is subject to Coriolis forces[9] or the simultaneous action of a magnetic field and rotation.

In the present problem, the effect of an impressed magnetic field enters Eq. (7) through the value of k and the critical Rayleigh number. Both the value of k and the critical Rayleigh number are functions of the parameter Q and the boundary conditions of the problems, such as whether the bounding surfaces of the layer are both free, are both rigid, or one free and the other rigid.

Theoretical study developed on the basis of Chandrasekhar's earlier work has resulted in quantitative predic-

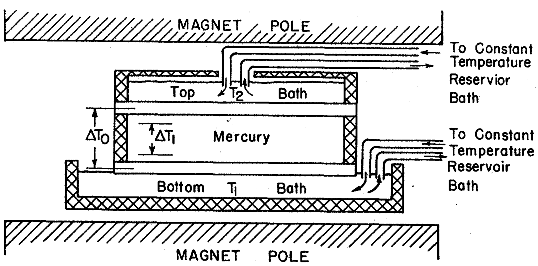

FIG. 2. Schematic diagram of experimental arrangement.

tions of the value of k for various values of Q and boundary conditions.[7] In such theoretical study it was assumed that steady cellular convection occurs beyond the marginal state of stability in a pattern of motion which can be represented by the solutions of the linear theory. The results of computations are summarized in Fig. 1 in terms of k and the parameter Q for all three cases of boundary conditions.

It can be noticed that for large values of Q the value of k gradually approaches to the same constant value of 2.00 for the case when both bounding surfaces are free. The change can be attributed to the increasing effect of Ohmic dissipation which becomes dominant and overtakes the effect of viscous dissipation for large Q. This is in agreement with what Chandrasekhar has pointed out,[10] namely, for large values of Q, the critical Rayleigh number follows the asymptotic behavior

$$\mathrm{Ra}_c \sim \pi^2 Q, \tag{8}$$

and the critical temperature gradient becomes independent of viscosity.

[9] G. Veronis, J. Fluid Mech. **5**, 401 (1959).
[10] S. Chandrasekhar, Daedalus **86**, 323 (1957).

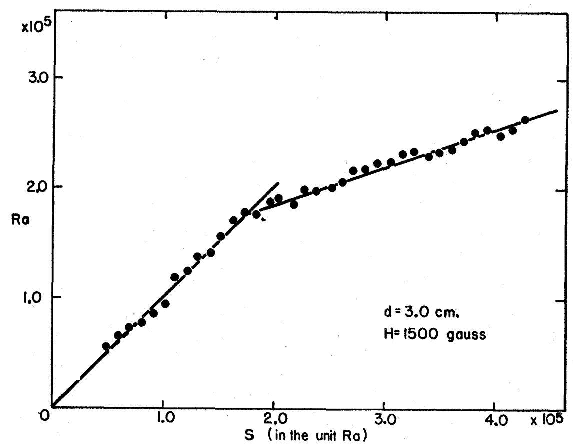

FIG. 3. Observed amount of heat transfer at various values of the Rayleigh number.

By rewriting Eq. (4) we may summarize the present discussion:

$$S=\mathrm{Ra} \quad \text{for} \quad \mathrm{Ra}<\mathrm{Ra}_c \tag{9}$$

and

$$S=(1+k)\mathrm{Ra}-k\mathrm{Ra}_c \quad \text{for} \quad \mathrm{Ra}>\mathrm{Ra}_c. \tag{10}$$

The essential features of Eqs. (9) and (10) are that linear functions of the temperature difference characterize the amount of heat transferred through the layer both in the cases of conductive and convective equilibrium. For the latter case, however, the statement must be reserved only to the first approximation.

2. EXPERIMENTAL INVESTIGATIONS

A series of experiments has been performed with the arrangement shown in Fig. 2 in order to examine the predicted effect of an impressed magnetic field on the heat transport by convection. A horizontal layer of mercury was placed in a uniform vertical magnetic field. The layer of mercury was bound by two stainless

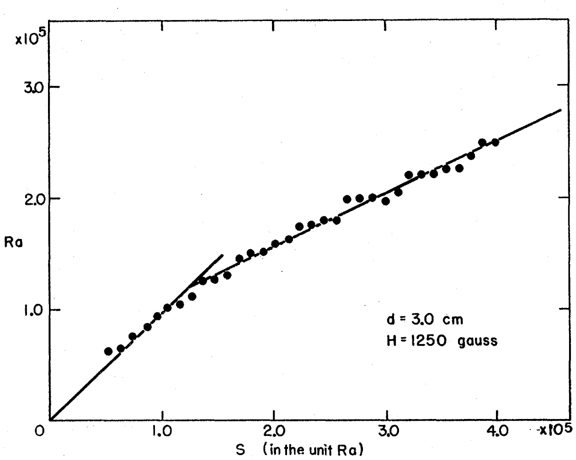

FIG. 4. Observed amount of heat transfer at various values of the Rayleigh number.

TABLE I. Summary of experimental and theoretical results.

Value of Q	Ra_c Exptl	Ra_c Theoret	k Exptl	k Theoret
9.730×10^3	1.27×10^5	1.22×10^5	1.303	1.788
1.407×10^4	1.75×10^5	1.67×10^5	1.989	1.828

steel plates which were in contact with two constant-temperature baths. By changing the temperature difference between the top and bottom baths, various states of thermal equilibrium were realized in experiments.

It was possible to determine the onset of convection and the amount of heat transferred through the layer by measuring the thermal emf of two thermopiles (indicated in Fig. 2 as ΔT_0 and ΔT_1) and the temperatures of the top and bottom baths, respectively. Typical results of experiments are shown in Figs. 3 and 4.

The results shown in Fig. 3 were obtained with a layer of mercury of depth 3 cm under impressed magnetic field of strength 1250 gauss ($Q=9.73\times10^3$). The results shown in Fig. 4 refer to those obtained with mercury of same depth under strength of magnetic field 1500 gauss ($Q=1.407\times10^4$). In both experiments the temperature difference between the top and bottom baths were varied in the range between 0° and 40°C. It may be seen in Figs. 3 and 4 that the present experimental results support the predicted linear heat-transport relation of Eq. (10) after the onset of convection.

The value of k determined by least-squares fitting of Eq. (10) to the experimental data and the critical Rayleigh number obtained in the experiments are listed in Table I with the results of theoretical predictions.[1,7] Qualitative, if not quantitative, agreement in the value of k and the critical Rayleigh number between the theoretical predictions and experiments can be seen in Table I.

The present preliminary experimental results have shown that to the first approximation, heat transport by convection can be characterized by Eq. (10). As k becomes constant and Ra_c follows the asymptotic relation (8), $Ra_c\sim\pi^2Q$, for large value of Q, we may conclude that under these circumstances the effect of an impressed magnetic field reduces the amount of heat transported by convection by a factor proportional to π^2Q.

3. CONCLUDING REMARKS

In many astrophysical and geophysical contexts we encounter situations in which oscillatory and turbulent convections are present. In this connection, it may be of interest to note the experimental results obtained when convection sets in as overstability.[11] In overstable convection, the velocity and temperature fields are both oscillatory and to the first approximation a definite phase difference exists in the fluctuations of these quantities.[9] Then it may follow from Eq. (6) that the heat transport by overstable convection is reduced due to the phase difference. Experiments[11] have confirmed such reduction in the heat transport by overstable convection, and further shown discontinuous increase in the heat transport for higher temperature differences. This discontinuous increase in the heat transport may be explained as loss of definite phase difference, and the transition towards turbulent convection.

[11] I. R. Goroff, Proc. Roy. Soc. (London) **A254**, 537 (1960).

DISCUSSION

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M. Łunc, *Polska Akademia Nauk, Warsaw, Poland:* Is it always stable even with finite amplitude perturbations regardless of electrical conductivity; when the bottom surface is cold and the top surface is kept warmer?

Y. Nakagawa: The manner of onset of instability depends on the electrical conductivity of fluid, and overstable convection can set in in a good electrically conducting fluid. In mercury one can show that it is stable even with finite amplitude perturbations.