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Electron and Positron Polarization

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I. INTRODUCTION

WHEN Goudsmit and Uhlenbeck proposed the spinning electron the possibility arose of having spin-polarization effects for a free electron. On the basis of Dirac's theory of the electron Mott proposed in 1929 a double scattering experiment designed to demonstrate such (transverse) polarization: the first scattering in the Coulomb field of a high-Z nucleus would serve partially to polarize an incident beam of unpolarized electrons because of the spin-orbit interaction, while the second such scattering would serve as the polarization analyzer. For a quarter century thereafter this quite difficult experiment was pursued in a number of laboratories. Approximately midway through this period it became evident that the experiment could, in fact, demonstrate the sought-for polarization, and at present there remains no doubt whatever that it can be made to work.* It is perhaps fortunate that the double scattering experiment was not crucial for Dirac theory, and that the other verifications came as early as they did.

An unfortunate situation prevailed during the 1930's and 1940's in that the whole matter of electron, positron, and gamma† polarization in general was viewed rather apathetically by many physicists. Such attention as there was, was focused almost entirely on the double scattering experiment itself. Thus the polarization effects already existing in the Klein-Nishina matrix element for Compton scattering, in the Møller matrix element for electron-electron or positron-electron scattering, in the Bethe-Heitler matrix element for bremsstrahlung, to name a few of the common electron-photon processes, continued to lie dormant‡ until quite recently.

Prior to about 1956 there was no supposition that "prepolarized" beams of electrons, positrons, or photons would be available from beta decay of nuclei. Therefore any experiments then proposed, just as with the Mott proposal, would have had to be of "second order," involving a polarizing unit to be followed by an analyzing unit. Such experiments seem to have had no strong appeal to experimenters in general.

When it turned out that parity violation in beta decay was in a sense complete, as demonstrated by the beautiful cobalt-60 experiment of Wu, Ambler, Hayward, Hoppes, and Hudson, it became clear that prepolarized beams of particles (and at times prepolarized photons) must exist. This initiated a greatly accelerated development of practical techniques for handling such polarizations. This paper discusses the various methods for analyzing electron and positron polarization, most of which were introduced and proved during the short period of about half a year following the announcement of parity violation. Photon (circular) polarization§ must be brought into the discussion

* Tolhoek [reference (T)] covers the double scattering experiment thoroughly.

† Plane polarization was discussed early, but there is no *direct* connection between such photon polarization and electron polarization.

‡ That is, spin polarization was rigorously summed or averaged at each opportunity in proceeding from matrix element to transition probability. A rare exception is the paper of Franz (F38). The spin direction having been generally taken out, circular polarization information was automatically suppressed. The justification for such averaging was considered sound: "The spin direction is not measured." We have not far to look for the reason it was not measured, and the circle is thus closed.

§ Such polarization is dealt with only lightly here. In contrast to the situation regarding particle polarization, there appears to have been introduced no new approach to this problem since the time of the Wu experiment. Reference (T) discusses the Compton effect as applied to circular polarization measurement. See also a recent review (S58) by Schopper.

because it connects with the problem of measuring free particle polarization.

The large and decisive effect found in the Wu experiment was primarily responsible for the apparent ease with which the new polarization methods succeeded. Knowledge that certain polarizations are quite probably present, and in abundance, has made for quick development of suitable experimental techniques. Also this knowledge has made for ready acceptance of most experimental conclusions relating to beta-decay polarizations, however hastily drawn a few may have been.

In the beginning of this new polarization era, one might have expected that only "first-order" experiments would appear. However, with the luxury of generally large inherent polarizations to begin with, a considerable number of second-order methods have already been successfully proved. Here the polarization one wishes to know is transferred for strategic reasons from initial particle to photon, or from particle to particle, in the first stage, and then the polarization analysis itself takes place in the second stage. Thus one requires two unlikely events in series, which makes the experiment second order.

The methods and the physics on which they are based are emphasized here, so the historical order is not followed in every case. For a given topic only a sufficient number of works is reviewed to illustrate the physics involved.‖ Closely related ideas (although perhaps not proved experimentally) are sometimes mentioned to round out the discussion. No pretense is made of reviewing all "parity results" obtained by a given method. No grand averages are made of the numbers obtained with a given radioactive source, and no pronouncements are attempted on such matters as a universal beta interaction or *PC* invariance.

The discussion cannot be divided rigidly into experimental method *versus* "theory of the method." Measurement of polarization by counter techniques (the only technique described here) is a field where it seems particularly safe to predict that a hypothetical ensemble of technicians continually trying different arrangements of scattering foils, magnets, absorbers, and so forth—with no belief in any theory and therefore no real plan—would take a long time indeed to happen upon a useful method. Each technique had to be planned in advance, hopefully at times because polarization measurement is not easy.

II. GENERAL CONSIDERATIONS

A. Polarization of Free Electrons

For an electron under no forces, solutions of the Dirac equation exist which are simultaneous eigenstates of momentum $\mathbf{p}$, of energy [eigenvalues $\pm\gamma$, where $\gamma=(1+p^2)^{\frac{1}{2}}$], and, if we wish, also of helicity. That is, letting $\mathbf{p}$ define the z direction, we may have a solution which is simultaneously as eigenstate of $H\equiv\boldsymbol{\alpha}\cdot\mathbf{p}+\beta$, of $\mathbf{p}$, and of σ_z (with eigenvalues ± 1). This is possible because the operator σ_z commutes with all of the Hamiltonian *that matters* for this special case, namely, $p\alpha_z+\beta$.

But (still letting eigenvalue $\mathbf{p}$ define the z direction) we are not able to construct an eigenstate of σ_x, for σ_x anticommutes with α_z. However if the coefficient of α_z, namely p, is small, then the effect of this lack of commutation becomes correspondingly small. Thus at very low energy any component of $\boldsymbol{\sigma}$ is nearly completely measurable. For finite laboratory energy the closest we can come to having a state of precise σ_x is to take the "best" linear combination of the two eigenstates of σ_z. Mixing equal amplitudes of the latter two states with a plus (minus) sign gives $\langle\sigma_x\rangle=\pm 1/\gamma$, which tends to ± 1 as p tends to zero. Another real operator, $\beta\sigma_x$, does commute with α_z, and with β. Therefore, simply because the maximum expectation value of σ_x becomes so nearly zero at high energy, it does not follow that transverse polarization is somehow "unmeasurable" at high energies. This means only that techniques are required that aim at measuring $\beta\sigma_\perp$ rather than measuring $\sigma_\perp$ itself. Otherwise one might follow the course of first precessing the spin and then measuring σ_z.

In speaking of longitudinal polarization of an electron or positron we use $P_p=\langle\sigma_z\rangle$, where z is parallel to the velocity. P_p is thus a number which may range from -1 to $+1$. As is well known, a Lorentz transformation may be made along the z axis, to the rest system of the particle in particular, and P_p retains the same value. In the rest system we could for the moment suppose full polarization is some direction, say at angle θ_0 (polar angle) and ϕ_0 (azimuthal angle), referred to z and x directions. The spinor part of the state can then be given by $|+1\rangle\cos(\theta_0/2)+|-1\rangle e^{i\phi_0}\sin(\theta_0/2)$ in the rest system of the particle, or in the laboratory. For this situation the value of P_p is just $\cos\theta_0$.

Transverse polarization is denoted by P_s, and naturally a certain azimuth has to be implied. We shall mean that $P_s(\phi)=\langle\sigma_x\cos\phi+\sigma_y\sin\phi\rangle$ computed *in the rest system* of the particle. For the oblique situation given in the foregoing, $P_s(\phi)$ amounts to $\sin\theta_0\cos(\phi-\phi_0)$. Often in speaking of how much transverse polarization exists in a beam, one tacitly maximizes $|P_s|$ against ϕ before quoting the number, since the geometry of the experiment usually involves a natural choice for the reference azimuth ϕ_0 (or $\phi_0+\pi$).

Suppose we have a partially polarized beam (longitudinally polarized, for example), with $0<P<1$, say. We may consider that the fraction $1-|P|$ of the particles are unpolarized while the fraction $|P|$ are completely polarized, with the sign of P. Alternatively the same beam may be regarded as a mixture of $+$ and

‖ The bibliography makes no attempt to list many of the excellent experiments which have corroborated, extended or improved upon those which are listed. The total literature on the subject (experimental and computational) has become quite extensive. Inasmuch as experimental technique in general seems now to be in an improvement stage rather than in a stage of new development, it seems too early to attempt complete listing.

$-$ polarized components in the ratio of $(1+P)/(1-P)$, which holds for either sign of P. Certain mathematical approaches favor the first view. On the other hand, most calculations proceed just as easily using the second view. Since no experimental detector can be devised to differentiate between the two situations, there is no point in debating which of these two views is the correct one.

To specify the total polarization in a beam it suffices to know three components of polarization in mutually orthogonal directions.¶ Polarization is averaged as follows: If beam number one of intensity m_1 has an amount of polarization $\mathbf{P}(1)$ and a second beam *incoherent* with the first, of intensity m_2, has $\mathbf{P}(2)$, then as a whole the polarization is

$$\mathbf{P}=[m_1\mathbf{P}(1)+m_2\mathbf{P}(2)]/(m_1+m_2).$$

B. Polarization Efficiencies

One is interested in the relative response of a polarization detector for two orthogonal spin orientations (that is, mutually orthogonal states). For example, for detecting a nonzero P_p one requires an apparatus that responds differently to a particle with $P_p=+1$ than to a particle with $P_p=-1$. All the detectors reviewed here are reversible.** Suppose for $+$ setting of the detector that the probability, given a $+$ particle, that it count is C_{++}, and the corresponding probability for a $-$ particle to count is C_{+-}. Let the $-$ setting of the detector have respective probabilities C_{-+} and C_{--}. The order of the subscripts is important. Then, if $C_{--}=C_{++}$ and also $C_{-+}=C_{+-}$, the detector is called *reversible*. We write $C_{++}/C_{+-}=r=C_{--}/C_{-+}$. (This definition of r is more useful than using C_{++}/C_{-+} or C_{--}/C_{+-} as appears in a moment.) The counting-rate ratio, $+$ setting divided by $-$ setting, will be $R=(1+\epsilon P)/(1-\epsilon P)$. Here the efficiency ϵ of the detector is $\epsilon=(r-1)/(r+1)$. This efficiency applies of course to one component of polarization, that associated with the $\pm$ setting of the detector.††

In practical circumstances, due to fringing fields, for example, a counting efficiency (not a polarization efficiency) may change from $+$ setting to $-$ setting, violating the condition for reversibility. But since the physics upon which the polarization sensitivity is based is independent of such systematic effects, it may still be that the ratio r as stated above is unambiguous. We then keep the same value of ϵ independent of the systematic effect. In such a case the counting-rate ratio has to be written more generally as $R=(C_{++}/C_{--})(1+\epsilon P)/(1-\epsilon P)$. If it should be possible exactly to reverse P, or to render $P=0$, say (and neither of these things may be easy to do), then the fact that $C_{++}/C_{--}=C_{+-}/C_{-+}$ is not unity need not hurt the final result, since this ratio can be determined empirically.

To keep the discussion simple, reversible detectors are assumed from now on. A good reason for generally thinking in terms of an efficiency ϵ instead of the corresponding ratio r is that the law of propagation for a number of "in series" effects is most clearly expressed in terms of ϵ's and not r's. For example (if we may adhere to the nonpositivistic approach), consider a particle or a beam which is polarized by an amount P_0. Suppose the polarization were transferred in the first stage of operations to some intermediate particle or beam with transfer efficiency ϵ_1, the intermediate polarization, perhaps of a different kind, being now ϵ_1P_0. Let this beam be now analyzed with efficiency ϵ_2. The over-all efficiency $\epsilon_1\epsilon_2$ is the quantity that defines the change in counting rate on reversing either P_0, ϵ_1, or ϵ_2. The rate ratio on such reversal is $R=(1+\epsilon_1\epsilon_2P_0)/(1-\epsilon_1\epsilon_2P_0)$. Depolarization could be taken into account by using a series of "efficiencies for preservation of polarization," numbers ϵ_d's less than $+1$ which would then multiply the $\epsilon_1\epsilon_2$ in this example. All of the ϵ's mentioned, including that of the detector, can as well be regarded as transfer efficiencies, since the over-all product-ϵ represents numerically the efficiency for transferring the original polarization information into a counting-rate asymmetry.

Here we have chosen to carry through a pair of *probabilities* rather than a pair of *amplitudes*. It has been tacitly assumed that physical judgement has been used throughout in choosing at each juncture a pair of orthogonal states for which any coherence between their amplitudes is of no consequence. The point is that the relative phase of their amplitudes is thrown away each time we force the calculation to tell us probability throughout the chain of events. Any blunder in such choice of states leads to grief. Often, there is considerable latitude here and we are free to choose a reasonably simple pair of states for algebraic convenience. If there is any doubt as to which pair of states to choose, which is tantamount to having doubt as to which kind of polarization it will suffice to talk about, then the amplitudes themselves could be carried through.

To return to the counting-rate asymmetry, experimenters usually prefer to discuss results in terms of the fractional change in rate between the $+$ and $-$ settings of the detector. This fractional change is often denoted by δ. One has

$$\delta=2\frac{R-1}{R+1}=2\frac{R_+-R_-}{R_++R_-}=2\epsilon P. \tag{1}$$

¶ We thus have a polarization space of three dimensions for spin-$\frac{1}{2}$ particles. For photons (which are *not* spin-1 particles) a similar space can be used: The momentum direction $\mathbf{k}$ is special and pertains to circular polarization; normal to this, plane polarization may be represented, providing that azimuthal angle in this polarization space is twice the corresponding azimuthal angle in the laboratory.

** Actually, experimental situations exist and may play a role in the future where in practice the detector is not actually reversed. For example, it may have one state where it responds preferentially to $P>0$, and the other state where it is insensitive to P.

†† We could represent any polarization detector as a vector $\boldsymbol{\varepsilon}$ in a three-dimensional efficiency space. The efficiency for detecting arbitrary polarization $\mathbf{P}$ in a beam is then $\boldsymbol{\varepsilon}\cdot\mathbf{P}/|P|$. The same applies for photons.

Concerning the factor 2 that appears in expression (1): given a pair of numbers, 100 and 101, one physicist may recognize and comment that they differ (to first order) by one percent. Another may prefer to state that these two numbers exhibit an asymmetry about their arithmetic mean of 1/201, thus a nominal one-half percent effect. We prefer the first method in discussing experimental results, even though δ will exceed 100% if experiments improve to the extent that ϵP becomes larger than 0.5.

It may be felt that density matrices and Stokes parameters are not accorded their fair place in the discussion. A number of review articles have treated these matters [T, (F57)]. The basic ideas of the experiments to be described can be put in quite simple language. This reviewer can readily believe (but would find it difficult to prove) that the majority of these experiments were conceived, made to work, and interpreted correctly, without conscious recourse to the density matrix.

Before ending the general discussion of polarization measurement we might inquire a little into measurability of more than one component of polarization. Consider some state α which means definite spin direction for the particle (in its rest system). Only one orthogonal state exists, for a spin-$\frac{1}{2}$ particle; call it β. Let us have α and β normalized in the same way, with the phase between them fixed. For any state of affairs, the polarization of the kind α is computed from the probability of finding that the particle is in state α compared to the probability it is in state β. Thus

$$P_\alpha = \frac{\text{prob}(\alpha) - \text{prob}(\beta)}{\text{prob}(\alpha) + \text{prob}(\beta)}, \tag{2}$$

and P_β is $-P_\alpha$.

Instead of states α and β we might use a state called A, which is $\alpha + e^{i\phi}\beta$ and its orthogonal state B which is $\alpha + e^{i(\phi+\pi)}\beta$. Then the polarization of kind A, denoted P_A, is given in terms of prob(A) and prob(B) by an expression such as (2). So long as ϕ is a real number, polarization P_A is complementary to polarization P_α, since if $P_\alpha^2 = 1$, then $P_A^2 = 0$, and conversely. We may generate a third polarization $P_{A'}$ which is complementary to P_A and to $P_\alpha (A' = \alpha + ie^{i\phi}\beta, B' = \alpha + ie^{i(\phi+\pi)}\beta)$. For a particle polarized by an amount $\mathbf{P}$, we have

$$P_\alpha^2 + P_A^2 + P_{A'}^2 = P^2 \leqq 1. \tag{3}$$

Of course, expression (3) is a statement of probabilities, as with expression (2). If one measures, by utterly precise means, P_A for a particle, one finds either $+1$ or -1, and expression (3) is then amended to read $P_A^2 = 1$, $P_\alpha^2 = P_{A'}^2 = 0$.

A longitudinal polarization detector gathers *all* the longitudinal polarization information from a beam, whereas a transverse detector leaves unexplored the complementary component of transverse polarization. It is possible however to arrange to collect concurrently all the transverse polarization information in a beam of particles or photons, in which case the detector is not characterized by a single angle ϕ (supposing α to correspond to the beam direction) but strictly consists of two or more detectors operating in parallel.

C. Right *versus* Left Helicity

A right-handed particle or photon means one that "rotates" as it moves in the same manner as a standard right-hand screw. This can also be stated: When angular momentum and velocity are parallel the particle is right-handed, when antiparallel, left-handed.

In classical optics the opposite definition is in use: Right circularly polarized light is that in which the electromagnetic vectors show clockwise rotation as viewed by an observer into whose eyes the light enters, that is, an observer facing *opposite* to the direction of propagation of the light.

III. EXPERIMENTAL METHODS

A. Classification of Methods

Three different ways of analyzing spin polarization have been experimentally proved. One method makes use of the spin-orbit interaction when the particle is scattered by the Coulomb field of a nucleus, the method proposed originally by Mott (M29). Here any transverse polarization generally leads to a right-left asymmetry in the scattered intensity. This amounts to talking about a "force" $-\nabla(\boldsymbol{\sigma}\cdot\mathbf{H}')$ for e^-, where $\mathbf{H}'$ is the magnetic field experienced by the particle in moving through the Coulomb field $\mathbf{E}$, namely $-(\mathbf{v}/c)\times\mathbf{E}$. In the second method one makes use of the $\boldsymbol{\sigma}\cdot\mathbf{H}$ energy, where $\mathbf{H}$ now stands for an applied magnetic field. This method finds application where positronium is formed from polarized positrons. The third method is where one *compares* the polarization of the particle to be analyzed with a *reference* polarization residing in atomic electrons which are already aligned. In this category belongs use of electron-electron scattering (and positron-electron scattering), annihilation of very low energy positrons in a ferromagnetic material, or use of annihilation-in-flight of more energetic positrons in a magnetized foil. Also there belong here the second-order techniques where the particle polarization information is transferred to helicity of a photon, this helicity being then analyzed by Compton scattering against the polarized reference electrons.

B. Mott Scattering (and Spin Precessors)

Properly, the Mott scattering method applies to transverse polarization analysis only. For reasons of Coulomb repulsion it is not well suited for positron analysis. As one suspects from contemplating the fingers, an "up" spinning electron approaching "straight ahead" a nucleus of (positive) charge Ze should prefer to scatter to the right rather than to the left because of the force $\nabla\boldsymbol{\sigma}\cdot(\mathbf{v}\times\mathbf{E})$ implied by its magnetic moment moving in the steeply graded field $\mathbf{B}'=\mathbf{E}\times\mathbf{v}$. The quantum mechanical results, first estimated by Mott and more recently computed by Sherman (S56), predict generally large right-left asymmetry in the scattered intensity. The azimuthal variation of the scattered intensity at angle ϕ is $\sin(\phi-\phi_0)$ for an incident electron polarized at azimuth ϕ_0. Conversely, the asymmetry in intensity between azimuth $\phi+90°$ and the conjugate azimuth $\phi-90°$, defined as

$$\delta_\theta(\phi)=\frac{I(\phi+90°)-I(\phi-90°)}{I(\phi+90°)+I(\phi-90°)} \tag{4}$$

is a measure of the transverse polarization at azimuth ϕ. A coefficient of proportionality relates the thing we presume to be measuring, the transverse polarization $P_s(\phi)$, and the observed δ. We call this coefficient S_θ following Sherman, and write $\delta_\theta(\phi)=S_\theta P_s(\phi)$.

For given energy, there is a best region in polar scattering angle θ at which to work, from the point of view of size of the asymmetry S_θ. Some of Sherman's results for the right-left asymmetry for an unscreened point scatterer ($Z=80$) are given in Table I. This shows at each of several speeds v/c, the maximum S_θ, the angle $\theta_{\max}$ at which it occurs, and the two adjacent angles called $\theta_{\frac{1}{2}}$ at which the asymmetry is down to one-half maximum value. At a certain angle θ ($\sim 60°$ at $v/c=0.4$ and shifting to $\sim 30°$ at $v/c=0.9$) the asymmetry S_θ becomes zero and changes sign. To avoid ambiguous factors of two, note that $S=+0.4$, for example, means that the intensity to the right (see beginning of this subsection for orientation) exceeds that to the left for an up-spinning electron incident, the ratio of intensities being 1.4/0.6.

What is not shown in Table I, but which could be very important when considering possibilities of systematic error in an actual experiment, is the variation with θ of the ordinary differential cross section. This gradient turns out to be quite steep at angles of experimental interest. A representative value, for the speeds shown in Table I, is 150% change in differential cross section per radian at $\theta=90°$. The gradient at $\theta_{\max}$ ranges from roughly 70% per radian for $v/c=0.4$ to about 150% per radian for $v/c=0.9$.

Also important is the fractional change in ordinary differential cross section for given fractional change in energy. At $\theta=90°$ this amounts to about 1.6, and would be somewhat larger if put in terms of fractional change in momentum.

TABLE I.

v/c	$S_{\max}$	$\theta_{\max}$	$\theta_{\frac{1}{2}}$	
0.9	0.51	150°	101°	170°
0.8	0.48	137°	90°	167°
0.7	0.45	130°	85°	165°
0.6	0.43	126°	83°	162°
0.5	0.40	123°	82°	161°
0.4	0.37	121°	85°	159°

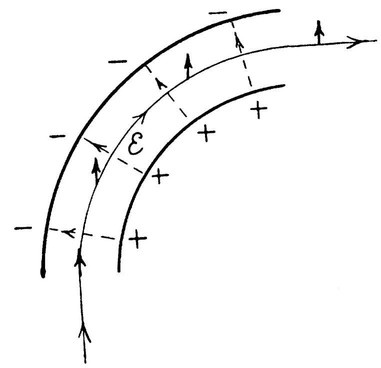

FIG. 1. Electrostatic spin-precessor due to Tolhoek and DeGroot (T51).

In the first successful detection of longitudinal polarization of beta particles from unpolarized nuclei by Frauenfelder and co-workers (Fa57), a means of first precessing the spin from longitudinal to transverse had to be employed in order to be in a position to use Mott scattering as the detector. A device originally proposed by Tolhoek and DeGroot (T51) was employed in order to "erect" the spins. Such a spin-precessor is shown in Fig. 1. It is essentially a nonrelativistic device and so the energies at which it works best are well matched to a Mott-scattering analyzer. Neglecting relativistic considerations, its action is simple and direct: The spin has no way to know that the electric field is continually changing the particle momentum $\mathbf{p}$, and thus the spin tends to retain its original direction in space, independent of sign or magnitude of the g value. (The device could more accurately be described as a momentum precessor.)

In the Frauenfelder experiment the Coulomb scattering act, (taking place in a thin gold foil) in analyzing the transverse polarization as the beam emerged from the spin erector, was measuring the longitudinal polarization that had entered the spin erector. Inasmuch as this spin precessor is a reversible device, that is, no information is lost through its use, we could classify this experiment as first order.

The angular aperture used was 95°–140°. A pair of end-window Geiger counters recorded the right- and left-scattered betas. The scattering foil was either 0.05 or 0.15 mg/cm^2 gold on a low-Z backing. The results of the first measurement, using a Co^{60} source, are well known: These negative betas were predominantly left-handed, the magnitude of the asymmetry obtained being taken to imply longitudinal polarization at emission of magnitude v/c, at least at sufficiently high beam energy that all the "dirt" effects that had at times plagued certain of the double-scattering experiments were probably not serious. There was a drop in apparent polarization of over a factor two on changing the mean v/c from 0.49 to 0.47, the decrease amounting to essentially two standard deviations.

We continue to introduce Coulomb scattering experiments in terms of the kind of spin precessor used, because the Mott scattering approach to polarization is

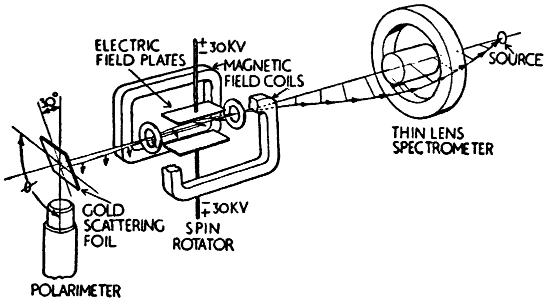

FIG. 2. Use of crossed-field spin-precessor. In the apparatus of Cavanagh *et al.* (CR57) original longitudinal spins entering at the right of the spin-precessor are precessed to emerge pointing downward. Polarimeter unit consists of scattering foil plus scintillation counter which can rotate about the beam axis.

scarcely new in the laboratory, and has been reviewed (T). Actual use of spin precessors is however quite new.

Another spin precessor is the crossed-field device. The beam is arranged to travel at right angles to an electric field **E** and a magnetic field **B** which are themselves at right angles. In the type to be described the fields are homogeneous over a certain region (it may be clearer to say that the geometry is rectangular). The crossed fields define a speed $\beta_0 = E/B$. If one arranges $0<\beta_0<1$, then in the moving system characterized by speed β_0 in the laboratory, there is no electric field, and the magnetic field is $B'=B(1-\beta_0^2)^{\frac{1}{2}}$. A Dirac particle at rest (for example) in this moving system has its spin precessed at angular speed $\omega'=eB'/m_0c$, the sense depending on the sign of the g value. In the laboratory system this means that a particle that has traveled a distance $z=2\pi\rho_0(1-\beta_0^2)^{-\frac{1}{2}}$ has its spin precessed through an angle $\theta=2\pi$. Here, $B\rho_0$ stands for laboratory momentum of the central ray. For distances in the laboratory Δz, differing from integral multiples of the distance just cited, the precession angle of a general ray may deviate somewhat from that of the central ray, which remains $\Delta\theta=(1-\beta_0^2)^{\frac{1}{2}}\Delta z/\rho_0$. But such deviations are readily computed.

A crossed-field device with suitable slit system can be a mass spectrometer,‡‡ and not simply a "velocity-filter." It might be possible to make practical use of this fact in dealing with polarized particle beams.

Cavanagh and co-workers (CR57) have used a crossed-field device as a spin-erector. The beta energy was preselected by means of a magnetic lens (Fig. 2). This lens does nothing to alter P_p since $\boldsymbol{\sigma}\cdot\mathbf{v}$ in a region where $\mathbf{E}=0$ is a constant of the motion. Next the crossed-field precessor interchanges longitudinal and transverse polarization components in the beam. Finally, Mott scattering at nominal $\theta=90°$ is used to analyze for transverse polarization. The scintillation detector (of size characterized by 0.3 radian) plus the thin gold scattering foil form a unit, the polarimeter unit, which may be rotated in azimuth ϕ. The results show the expected right-left asymmetry [the observed δ as in Eq. (1) being as large as 20%], but in addition a curious and hard to explain up-down asymmetry presumably of instrumental origin. It was possible by reversing fields to split off this latter (orthogonal) asymmetry from the desired one. In the first experiments Co^{60} and Au^{198} negative betas were shown to have longitudinal polarization consistent with $-v/c$. Alikhanov *et al.* have also used a crossed-field spin precessor (A58).

Regarding attenuation of the sought-for asymmetry due to the scattering foil having finite thickness, Fig. 3 shows an extrapolation against foil thickness. This gives a graphic demonstration of how finite foil thickness, presumably through plural scattering and possibly depolarization, reduces the observed asymmetry.

A third device for transferring polarization information from longitudinal to transverse (or vice versa) has been used specifically to prepare a beta beam for Mott scattering analysis. This we call a "spin reprojector." It consists in scattering the original beam by atoms, whether singly or multiply does not matter to a first approximation. The transverse polarization of the electrons after scattering at $\theta=90°$ represents the input longitudinal polarization, to the extent that the original spin direction is unchanged on scattering.§§ The action is similar to that of the macroscopic electrostatic spin precessor.

The resulting transverse polarization lies in the plane of the scattering. Even taking into account the fact that the spin-orbit effect may not be entirely negligible, this cannot lead to any polarization in this plane. The spin-orbit effect and any helicity memory act only in competition to dilute and dull somewhat the transfer efficiency of interest, since the sum of the squares of

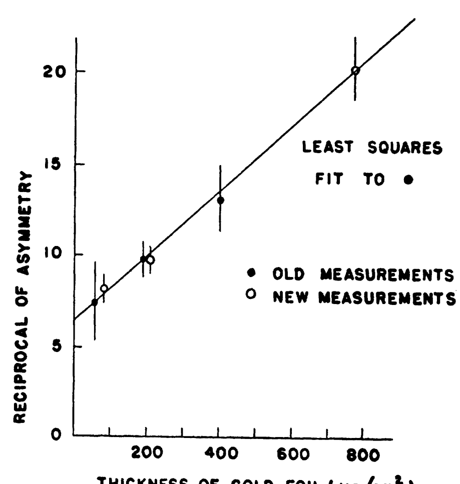

FIG. 3. Effect of scattering foil thickness on the measured asymmetry, from Cavanagh *et al.* (CR57).

‡‡ Compare the nonrelativistic applications by Bleakney and Hipple (B38).

§§ Bernardini, *et al.* (B58) treat this matter quantitatively. They show that there can be considerable helicity memory in a Coulomb scattering event.

three mutually complementary polarizations cannot exceed unity. A fairly low-Z scattering material is preferable from this point of view.

Spin erection by means of this reprojection method has been used by Pond (Po57) on P^{32} electrons. It has been used in a very nice geometry by Deshalit and coworkers (De57). Their apparatus is shown in Fig 4. The spin reprojector is in the form of a sheet of aluminum (denoted by symbol σ_1) forming an arc of a circle. From the geometry of the circle one may then have (for point source and point second-scatterer) constant "total" scattering angle θ, in this case 90°, over a large range of emission angle at the source. This affords a gain in intensity which is needed in a second-order experiment. The resulting asymmetry for P^{32} electrons of energy exceeding 900 kev, at nominal second-scattering angle $\theta=75°$, was 5%. This was interpreted as compatible with full v/c polarization.

Spin erection by means of Coulomb scattering was suggested independently also by Tassie (Ta57). Another possibility for spin erection is based on Møller scattering [see Sec. III D (1)].

A high-energy precession effect relating to the anomalous magnetic moment has been computed by Mendlowitz and Case (M55). A spin precessor based on such an effect, used in conjunction with a Mott scattering analyzer might suffer from a limitation due to energy mismatch. To illustrate with a simple situation where a Dirac particle travels normal to a magnetic field, if the g value were simply two, this would cause the preservation of spin polarization projected onto the velocity vector. In terms of "time dilation," the zero-energy value of the cyclotron frequency (the frequency of rotation of the velocity vector) and the spin precession frequency are each multiplied by the same factor, $(1-\beta^2)^{\frac{1}{2}}$. The two quantities, velocity and spin direction, then stay in phase for a hypothetical Dirac particle. Otherwise said, $\boldsymbol{\sigma}\cdot\mathbf{v}=\boldsymbol{\sigma}\cdot(\mathbf{p}-e\mathbf{A})$ is a constant of the motion because it commutes with the Dirac Hamiltonian. The interesting result of the Mendlowitz and Case calculation is that the additive "Schwinger part" of the magnetic moment (or of the g value) is predicted to be immune to this time-dilation factor.

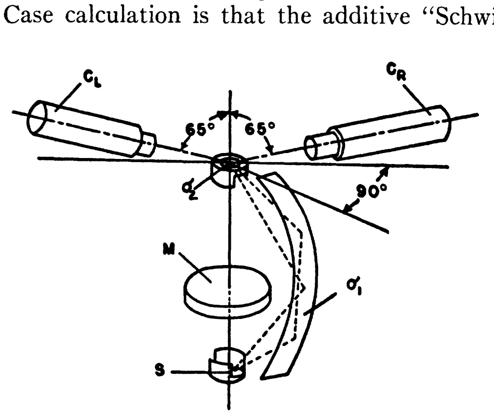

FIG. 4. Apparatus of DeShalit *et al.* (De57), illustrating "spin reprojection" by a first scattering of the betas from source S in the curved aluminum sheet, σ_1. Analysis of transverse polarization lying in the plane of $S-\sigma_1-\sigma_2$ is effected by Mott scattering at σ_2.

If the velocity of the particle is effectively along the magnetic field (so that we might transform along the field, find the particle moving only normal to the field, but with practically nonrelativistic energy) then the spin ought to precess at the rate of $\omega_{cyc}(1+\alpha/2\pi)$. Higher terms are not mentioned because free-particle experiments are still concerned with the first‖‖ non-Dirac term. It would then take the order of 137 cyclotron revolutions for the spin vector relative to the velocity vector (in the moving system) to change by one radian. However, for the case of primary interest here, where the particle with substantial energy moves normal to the magnetic field, it would require *only* about $137/E$ cyclotron revolutions to precess the spin one radian further than the velocity vector. Here E is the total energy of the particle in units of its rest energy.

This computation has recently been done again by Carrassi (C58) using a somewhat different approach, with the same result obtained by Mendlowitz and Case. Such an effect would be of interest for itself, independent of any "practical" use. For de-erecting transverse polarization of highly relativistic particles it could possibly be useful, since P_s becomes increasingly difficult to measure directly as kinetic energy increases above the rest energy.

To return to the Mott scattering method itself. It would seem that the effect of the screening of the nuclear charge by the atomic electrons will have to be understood quantitatively, as will plural and multiple scattering effects in finite scattering foils, before precision polarization numbers can be obtained with confidence. These matters are under active scrutiny in a number of laboratories, and therefore after a time a good understanding of this useful method must emerge.¶¶

C. Energy Term $\boldsymbol{\sigma}\cdot\mathbf{H}$

The Pauli energy term $\boldsymbol{\sigma}\cdot\mathbf{H}(e\hbar/2mc)$ affords a means of determining whether the spin $\boldsymbol{\sigma}$ is parallel or antiparallel to an applied magnetic field $\mathbf{H}$. For measuring σ_z of free beta particles it has so far not been employed.*** Niels Bohr showed that a Stern-Gerlach experiment on bare electrons is not a good way to go about such a measurement. This has been discussed thoroughly by Mott and Massey (MM).

One may however capture the particle of interest into a bound system (see again the discussion by Mott and Massey) and then measure $\boldsymbol{\sigma}\cdot\mathbf{H}$. Concerning the preservation of the spin direction during capture, one notices that it is the charge e of the particle and not its

‖‖ *Note added in proof.*—However, see recent precision results for g factor of a free electron obtained by the University of Michigan group, Bull. Am. Phys. Soc. Ser. II, 4, 250 (1959).

¶¶ It will then (by definition) be possible to perform a double-scattering experiment to high precision, with no anomalies.

*** A resonance experiment on free electrons has been performed by Dehmelt (D58).

magnetic moment that makes the process go. Hence, so long as the energy transfers involved (usually tens of volts only) are small with respect to m_0c^2 of the particle, there is no spin-flipping *mechanism* during the act of capture.

Let us specialize to formation of ground-state positronium since this has been employed for polarization measurement. Here an "up" positron allowed to form positronium (say by radiative capture with an effectively free electron) will not leave electron holes more "up" than "down." The fine structure of the resulting positronium is too small to cause selective capture to singlet rather than to triplet states (the former being bound more tightly by ΔE of only about 10^{-3} ev). It is necessary to examine a little more closely to find out where the original positron polarization information goes.

Consider still a free "up" polarized positron (that is with its spin along $+z$) which eventually is captured by one of a group of electrons which as a whole had no spin preference with respect to the z direction. Suppose there are no applied fields. On a time scale, the triplet-singlet energy difference ΔE is by no means inconsequential: ΔE corresponds to $\sim 10^{-12}$ sec. Thus at a time later than capture of the order of 10^{-12} sec, a (resonably adiabatic) inquiry whether the positronium state is singlet or triplet makes physical sense. This arises spontaneously in the process of decay by annihilation into two or three photons, respectively. At this stage, having allowed singlet and triplet state amplitudes to have lost their relative phase, the original spin information is largely lost: To be sure it could be partially retrieved if one looked only at certain kinematically special triplet (three-quantum) annihilation events, measuring photon helicity—rather difficult experimentally. For singlet annihilation (into two photons) there is no information that can be traced back to the original positron polarization, either in the angular correlation with respect to the $z=0$ plane, or in the helicities of the photons, the parent state being $J=0$ [see, for example, Yang's paper on dematerialization, (Y50)]. In the case of the singlet state, the original polarization has not actually been lost, but resides in the sign of the hole where the captured electron was, with complete preservation of information. For the three triplet substates as a whole the information resides 67% in the helicity of the residual photons, and 33% in the sign of the hole.††† The important point is that by examining only the two-photon residue, no information remains. This is in contrast to higher energy direct annihilation where quite large retention of polarization information is possible [see Sec. D (5)].

If the positronium is formed in the presence of an applied magnetic field, along z, say, then in effect some phase coherence between singlet and triplet states (of same J_z, thus only $M=0$ states are affected) can be "frozen in" until the annihilation takes place, and not only for the 10^{-12} sec permitted by the triplet-singlet energy difference. Under these circumstances some polarization information will now be retained. We should recall how the Zeeman splitting goes for ground-state positronium [see the positronium review articles (D53) and (D54)]. Here, J is no longer a good quantum number in the presence of the field $\mathbf{H}$, but only its projection along the field. Let us use an arrow for up or down positron spin, and likewise for the electron spin. Let the positron's spin state stand first. The state $\uparrow\uparrow$ and the state $\downarrow\downarrow$ are each unperturbed by the applied field. The $M=0$ part of the Hamiltonian, normally diagonal, now has, in virtue of the $\boldsymbol{\sigma}\cdot\mathbf{H}$ energy, part of which belongs to the positron and part to the electron, matrix elements connecting triplet and singlet. On rediagonalizing the Hamiltonian, the *nominal* triplet state is $(\uparrow\downarrow+\downarrow\uparrow)-\epsilon(\uparrow\downarrow-\downarrow\uparrow)$, and the nominal singlet state is $\epsilon(\uparrow\downarrow+\downarrow\uparrow)+(\uparrow\downarrow-\downarrow\uparrow)$. The mixing amplitude ϵ is practically linear with H, below 20 kilogauss, and becomes unity at extremely high field. The signs are such that positive H_z means positive ϵ.

In the high field limit, the two states of interest become (for positive H_z) $\downarrow\uparrow$ for triplet and $\uparrow\downarrow$ for singlet, indicating complete spin memory. Even at modest field values, say about 15 kilogauss, the disparity in $\langle\sigma_z\rangle$ for the positron (or the electron) between triplet and singlet is about a factor two. We can think of this as a capture efficiency $\epsilon_{\text{cap}}=-0.33$. This means that initial positron polarization (called P_z) would be 33% preserved in the capture act, residing finally in the asymmetry between the population of triplet and singlet states. As explained earlier, capture to $M=1$ states becomes generally a three-quantum annihilation problem and is not considered.

The discussion now divides according to whether the positronium is formed in a gas or in condensed material.

Positronium in gases.—At a field strength of 15 kilogauss about 0.97 of the triplet ($M=0$) states annihilate via two photons, and for practical purposes, *all* the singlet states do. Except for a peculiarity relating to the kinetics of the positronium atom just formed, the polarization information (all but some 3% of it) would now be lost, since it was agreed earlier that we should not attempt to measure helicity of the three quantum events. The triplet state has at this field value a lifetime $\sim 3\times 10^{-9}$ sec, the singlet state its normal value $\sim 10^{-10}$ sec. Nascent positronium in argon is known to have recoil energies of 1 or 2 ev. The magnetically quenched triplet states yield two photons observed to have such strong angular correlation at 180°, when suitable slowing (thermalizing) agents are present in the gas, that undoubtedly the positronium has lost essentially all its original recoil energy in 3×10^{-9} sec.

††† Thus two-thirds of the time, J, $M_J=1$, $+1$, and one-third of the time 1, 0, restricting to final $J=1$. The transfer efficiency from initial positron polarization to hole polarization is thus one-third.

This behavior was known some time before polarization became of interest (H56, He57).

One may judge from the observed mutual angular correlation for the two-photon events whether the parent state was nominal triplet or singlet, the connection between an observable and the original positron spin direction being thereby completed. With reasonably narrow slits (about 1×10^{-3} radian of projected angle) the former states can be counted about five times as efficiently as the latter, at first glance implying the large efficiency for this stage of the operations of 0.67. However, inasmuch as there are many competing annihilations recorded (about five times as many positrons as we have just discussed are at the same time making competing direct annihilations in the gas sample) the effective efficiency because of the diluting action of such events drops to about 0.20.

To summarize: Let a hemisphere of positive betas emitted in the general z direction come to rest in a gaseous annihilator without depolarization. The geometrical efficiency (for projecting any longitudinal polarization P_p onto the z direction, calling the latter polarization P_z) is then 0.5. The capture efficiency ϵ_{cap} to ground-state positronium as discussed earlier would be 0.3 (at 15 kilogauss), the detector efficiency as discussed immediately above could be typically 0.2. The expected δ (on reversing $\mathbf{H}$) is then $2\times0.5\times0.3\times0.2\times P_p = 0.06P_p$. In the initial experiment of Page and Heinberg (PH57) it was found that Na^{22} positrons characterized by v/c at emission of 0.75 were polarized with $P_p > +0.30$, the lower limit being considered conservative in that no backscattering corrections from the gold backing nor depolarization estimates were made (Fig. 5). The detector efficiency was purposely overestimated.

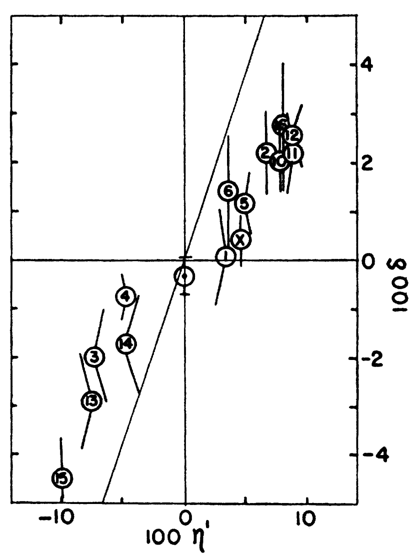

FIG. 5. Use of positronium in a gas as polarization detector (PH57). For longitudinal positron polarization at emission of +0.75, measured δ would follow oblique line whose slope is proportional to polarization.

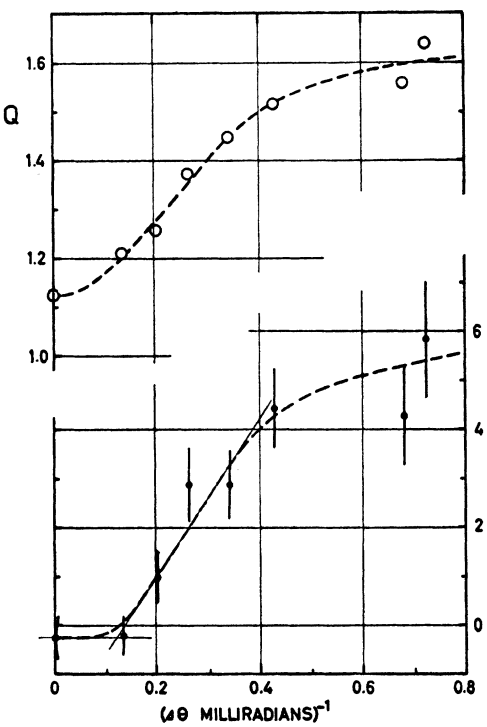

FIG. 6. Positronium in a gas (PO58). Lower curve: extrapolation of measured δ to zero effectively yields a measure of width of singlet contribution to the two-photon angular correlation. Upper curve: quenching ratio Q at infinite slit width yields fractional positronium formation in gas sample. Variation of Q with slit width yields width of triplet events.

Later data (Fig. 6), where effectively this latter efficiency is measured for the particular gas sample in use, by extrapolating measured δ against reciprocal angular resolution, gave $P_z = 0.25$. This implies a lower limit on P_p of $+0.50$. The low-Z source-backing in the latter experiment (quartz) improved matters somewhat. Detailed angular correlation measurements (PO58) showed clearly the transfer from "broad component" to "narrow component" when the H field was changed from parallel to the positron momentum to antiparallel.

In a sense this kind of experiment is a comparison one: between the sign of $\langle\sigma\rangle\cdot\mathbf{H}$ and the theoretical sign of the triplet-singlet energy difference for ground-state positronium. Here it was of practical concern for the first time which lay higher in energy, the triplet or singlet states.

Positronium in condensed materials.—Since the time when DeBenedetti and Richings (D52) found positrons to live longer in certain dense materials (specific gravity unity or more) than free positrons should, a number of diverse experimental results have been in accord with the hypothesis that positronium may exist in a certain class of substances. We mention the detailed measurement of these long lifetimes (Bell and Graham B53), the measurement of the abnormally high three-quantum yield [Pond (P54), Graham and Stewart (G54)], of the abnormally narrow component in the two-photon angular correlation [(P55) (S55)], measurement of the magnetic quenching of the triplet state showing that this state provides broad-component two-quantum events [Page and Heinberg (PH56)], detailed time-delay measurements as a function of magnetic field showing that the ↑↑ and the ↓↓ states (in Teflon) were not spontaneously mixed with others [Deutsch (D56)], the observed decrease of three-quantum yield in a strong magnetic field giving a consistency with ordinary positronium [Telegdi *et al.* (T56)]. Admittedly, we are now on less firm ground than when dealing with *bona fide* positronium as in a gas, but it is interesting that positron polarization measurements can be made utilizing the concept of positronium in condensed materials. The class of materials which exhibits positronium effects has been called the "long component class." Otherwise it might be called the narrow component class. It might suffice for the remainder of the discussion to call these materials "plastics."‡‡‡

We might assume for argument that the triple-singlet energy difference for positronium in the condensed materials is the same as for free positronium. The transfer efficiency from initial polarization of the slowed positrons to the population disparity between triplet ($M=0$) and singlet states would then be the same figure at the same magnetic field as for free positronium. From this point on, the picture is quite different.

For our purposes, each positron which comes to rest in the annihilator gives two photons. Ignoring polarization for the moment, let us discuss a typical "plastic" such as fused quartz. The original conjecture was that the narrow component occurred in parallel§§§ with the delayed component (P55). If we retain this view, and also pretend that all measurement is ideally precise, we shall have the singlet states giving only narrow component, denoted N, the triplet states in absence of applied field giving only broad component, denoted B, with nonpositronium events giving also a broad angular distribution, B'. In actual measurement, B' has never been distinguished from B. Discussion is now restricted to the two $M=0$ states, singlet and triplet, unless otherwise stated.

Application of a strong field H, strong enough to compete successfully with the rather fast (one or two by 10^{-9} sec lifetime) "pickoff annihilation," causes a detailed transfer of events from B to N [(Pa55) (PH56) (W57)]. Fields of 10 to 20 kilogauss are typically required. If an angular correlation apparatus is set to accept only 180° events, within about 10^{-3} radian, the coincidence rate increases with H, as in the so-called quenching curves of Fig. 7. Here the detector is perhaps four times as efficient for N events as for B events.

Regarding the possibility of positron polarization measurement, the situation in one respect is not as fortunate as with positronium in gases: "up" positrons in an "up" field ought to form more singlet states than triplet, the singlet states give only N events, but the triplet states instead of giving only B events (as at zero applied field) now might divide 0.6 of the time into B and 0.4 of the time into N, for example. In other words the H field which is required to furnish the initial "capture efficiency" acts unfortunately to dull the action of the detector from distinguishing triplet from singlet events.

In order to see what response to expect for plastics let us supply some rather rough numbers, and assume that the field is such that one-third of the triplet ($M=0$) decays appear as N events. For orientation, at $H=0$, one has 0.5 of the positrons of interest forming singlet states and 0.5 triplet ($M=0$) states. The fraction f_N of these yielding N events is thus 0.50. For unpolarized input positrons, at $H=H_{\frac{1}{2}}$, one has still the equal division into singlet *versus* triplet states, but f_N becomes now 0.67. For completely polarized stopped positrons, at $H=H_{\frac{1}{2}}$, the capture efficiency for keeping the polarization information might be 0.33 (depending indirectly on the lifetime of the delayed component). Thus, for field **H** parallel to the spins, a singlet/triplet population ratio of 0.67/0.33 is implied, and f_N is computed as 0.78. For the opposite field setting one gets $f_N=0.55$. In order to arrive at a final number, suppose that 0.35 of *all* incident positrons go into ($M=0$) positronium, and that the detector response $N:B$ is 4:1. Using such numbers one would "predict" a measured δ in the coincidence rate on reversing field of 0.14, given positron polarization P_z to be unity. Having field **H** and spin parallel should yield the greater coincidence

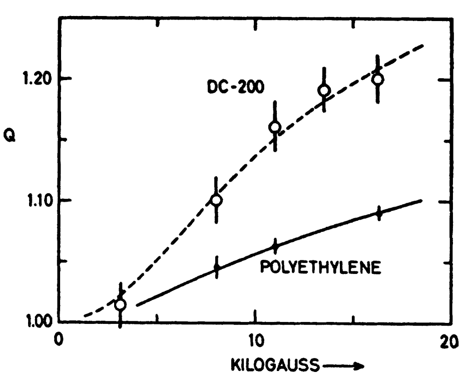

FIG. 7. Quenching curves for positronium in plastics. Fractional increase in two-photon coincidence rate within 1.5 milliradians of 180°, as a function of applied field H, corrected for focusing effects. Actual transfer from broad B component to narrow N component on applying field is less than one-fourth of $Q-1$. From Page and Obenshain (PO58).

‡‡‡ Inasmuch as whether or not the material tends to harbor positronium seems to connect with whether or not it is crystalline, some one has facetiously suggested the following nomenclature: materials not having positronium effects shall be Xtals, materials having them shall be Ytals. (This would save the term Xtal from extinction in the physical journals.)

§§§ Thus a positronium annihilation event is either narrow or it is delayed (at $H=0$). Attributing the delayed events to triplet positronium, one expects from statistical weights three times as many delayed events as narrow events.

rate, which is opposite to the response in the gas experiments just described.

The magnitude of the response of plastics to polarized positrons is next to impossible to estimate *a priori* because so many essentially unknown numerical factors enter. But the ***sign*** of the response may be traced back to one hypothesis, that the normal fate of the triplet states is broad annihilation (a half-width like 10×10^{-3} radian), and the normal fate of the singlet states is narrow annihilation (half-width less than 3×10^{-3} radian).

To come finally to an actual experiment, Page and Obenshain (PO57) used a hemisphere of Na^{22} positrons (gold-backed) which impinged on the plastic sample. The δ in the coincidence rate at 180° with angular resolution 1.5×10^{-3} radian was measured on reversing **H**. Only for the narrow-component materials was a nonzero δ found, and its sign agree always with that anticipated in the foregoing discussion. With no absorber between source and sample, and running typically at $H=13$ kilogauss, δ was measured for fused quartz, polystyrene, DC-200 high viscosity silicone oil, polyethylene, and Teflon. Expressed in percent the results in order were 1.7 ± 0.5, 1.7 ± 0.7, 1.3 ± 0.6, 1.3 ± 0.4, and 1.9 ± 0.5. With an interposed absorber of 0.0005-in. tantalum, the Teflon response increased to $4.8\pm1.1\%$, while a (low Z) Mylar foil of like density in mg/cm² increased the Teflon response to $4.5\pm0.9\%$. The very roughly estimated response quoted above would yield 7% expected δ for a hemisphere of fully (longitudinally) polarized positrons, neglecting back reflection.

Hanna and Preston (Ha58) have checked the sign of the response of polyethylene against the "iron detector." Page and Obenshain have checked the sign of the iron detector against the plastics [see Sec. III D (2)].

The fact that plastics in general give a detectible response to positron polarization must be taken as evidence that depolarization on slowing is small (even in tantalum, $Z=73$) and as supporting evidence that something very much like positronium must exist in this class of materials, and finally that the separation‖‖‖ into broad and narrow components for triplet and singlet states is rather clear-cut.

D. Comparison Methods

As discussed earlier (Sec. III A) a comparison method is one where the spin direction to be analyzed is compared somehow with the spin direction of atomic electrons already known. It is sometimes implied that comparison methods in general are rather inefficient. However, if one is discussing the particular physical act which is the heart of a given method, then it often turns out that efficiencies are closer to unity than to zero. It is best to keep separate the physics of the analysis act from the question of what degree of (reference) polarization might be achieved in practice for the average electron in a typical "iron" foil for example. In the discussion of several comparison methods, efficiency means efficiency per unit degree of polarization of the target electron, unless some statement is made to the contrary.

(1) Møller and Bhabha Scattering

The most direct and intimate comparison of spin directions is afforded by the electron-electron and positron-electron scattering processes, which we label "Møller" and "Bhabha" scattering respectively. The matrix element of Møller (M32), from which have stemmed the well-known Møller and Bhabha scattering formulas, has within it much polarization information which, prior to about 1957, was not much considered. Bincer (Bi57) seems to have been the first to publish computations on the polarization aspects of such scattering.

Speaking first about electron-electron collisions, and in the nonrelativistic limit, elementary quantum mechanics tells us that if a pair of identical spin-$\frac{1}{2}$ fermions are to undergo mutual scattering (letting all spins be random), one-half of the time the spins are antiparallel, the particles therefore distinguishable after the collision (they are always distinguishable before the collision) and the scattering cross section has its classical value at all angles in that there are now no interference (exchange) effects. For the other half of the time the spins are parallel, in which case we have indistinguishability after the collision, interference effects come into play, and, for 90° scattering in the center-of-mass system in particular, there can be no scattering. On the whole there is a progressive shift from the classical cross section at small angles to one-half of the classical value in the large angle limit, 90°.

The alternative view (which is not particularly useful for polarization discussions) is that three-fourths of the time the pair of spins form a triplet state, nullifying the scattering at 90°, while the remaining one-fourth of the time they are in a singlet state, now with constructive interference for fermions which doubles the cross section at 90°. The same numerical result is obtained at any angle as with the previous view. This result hinges on there being no spin-flips during the collision and again no spin sensitive forces in the scattering "force," which guarantees the first proviso.

Longitudinal polarization analysis.—For the relativistic discussion, let us focus attention on helicities. Writing the appropriate free particle spinors for definite initial and final helicities, then applying the Møller interaction and subtracting Feynman diagrams,

‖‖‖ Further results by Bell and co-workers (GB57) on the fraction of positrons in the delayed component for fused quartz in particular (now reported to be 0.51 instead of the earlier estimate of about 0.3) have removed the numerical disparity that existed for some time between fractional narrow component and fractional delayed component. The recent polarization applications however, being based on a balance method, might be argued to lend strong support to the contention that this separation is clear-cut.

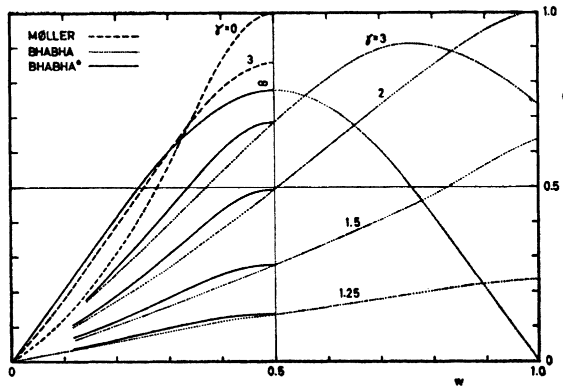

FIG. 8. Differential Møller and Bhabha scattering. For a completely polarized target electron, the efficiency ϵ for detecting longitudinal polarization of e^- or e^+ respectively, is plotted against fractional energy transfer w. Total energy of incident particle is γm_0c^2. Møller and Bhabha expressions coalesce as $\gamma \rightarrow \infty$. Solid curves, labeled Bhabha,* apply if final e^+ and e^- are *not* distinguished by the apparatus. Inasmuch as cross section generally changes rapidly with w, partial areas under curves may have but limited practical significance. Curves have been plotted from the formulas of Bincer (Bi57).

that is, writing down the Møller matrix element, the following facts emerge. We refer mostly to 90° scattering in the center-of-mass system where those polarization effects which arise entirely from the subtraction of diagrams are generally a maximum.

In the first place, single flips of helicity are taken out by the interference (at 90° scattering angle). In the second place, particle helicity tends to be better conserved as the energy increases.¶¶¶ We are not referring to the more trivial effect where at small angle the helicity naturally is well preserved. The helicity of one of the input particles would be in fact 80% conserved at only 0.6-Mev laboratory energy, in the sense that either of the output particles viewed in the center-of-mass system preserves 0.8 of the input helicity (for 90° scattering). This fact is used later. In the third place, and this is the important point upon which the polarization experiments have been based, the relative cross section (at 90°) between input spins parallel and antiparallel, for *longitudinal* polarization only, which was zero in the low energy limit climbs only to $\frac{1}{8}$ in the high-energy limit, the corresponding efficiency being therefore 7/9. The complete expression for this cross-section ratio as given by Bincer (Bi57) is

$$\frac{\sigma_p}{\sigma_a}=\frac{2x^2+\beta^2(3x^2+x^4)+\beta^4(1+x^2)}{1+x^2+\beta^2(2+3x^2-x^4)+\beta^4(5-4x^2+x^4)}, \tag{4}$$

where β is the speed of either particle in the center-of-mass system and $x=\cos\theta$ where θ is the center-of-mass angle of scattering. This result is shown in Fig. 8, now put in terms of analyzing efficiency ϵ for a longitudinally polarized target electron. The abscissa is the conventional fractional energy transfer, $w=\frac{1}{2}(1-\cos\theta)$. The two dashed curves labeled "Møller" and the limiting shape at high energy labeled ∞ apply here.

Bincer has also computed the companion expression for Bhabha scattering (Bi57). A few curves computed from these results are also shown in Fig. 8. These are the dotted curves labeled "Bhabha." At first glance, the e^+ and the e^- are distinguishable and thus one expects no polarization sensitivity in the sense of a correlation between positron and electron helicities for a given probability of scattering (of course, both initial and final e^+ helicity tend to be correlated, and the same applies for the e^-, on any simple theory). The point here is that there exists the virtual annihilation process, and this accounts for a good part of the ordinary (unpolarized) positron-electron scattering cross section (A54). This process is represented by one of the Feynman diagrams. It manifests itself now in making possible the polarization effect shown in Fig. 8. The effect dies out at quite low energy, where the annihilation amplitude has diminished.

Inasmuch as the Bhabha polarization efficiency ϵ as a function of w gives a rather unrealistic picture of how an experiment might be designed (because the differential cross section typically changes by a factor 30 between $w=0.25$ and $w=0.75$), we show also some modified Bhabha curves. These efficiency curves, labeled Bhabha,* would apply if the two particle detectors, operated in coincidence (or a single detector, if such an experiment were done), were completely insensitive to whether the counted particle is a final e^+ or a final e^-.

To summarize the picture presented in Fig. 8, the Møller method for longitudinal polarization measurement could be classed as a low-energy method, based on symmetry (Pauli principle)—the fact that a high efficiency persists to all energies can be regarded as good fortune. As for the Bhabha method, which again rests on symmetry, high energy is required to provide sufficient annihilation amplitude that the symmetry may come into play, so it is essentially a high-energy method. The left half of Fig. 8 clearly shows these energy trends. Another view is that at high energies the two methods are just the same (the infinity curve), but that at lower energies different details appear, simply because the forbidden zone of width $2m_0c^2$ is no longer negligible.

The Møller scattering method for the purpose of polarization detection was first employed by Frauenfelder and co-workers (Fb57) on negative betas from P^{32} and from Pr^{144} with the result that both beta groups were left-handed, and by an amount consistent with v/c at emission. Their apparatus is shown in Fig. 9. For reasons of impingement of beam, the scattering foil is inclined to the beam direction, so the efficiency has to be multiplied by the cosine of the small relevant angle. What is measured here is not an absolute differential cross section, or relative cross section against angle of

¶¶¶ This is not surprising since γ_5 does commute with the Møller interaction, but we remember that neither particle at finite energy can be in a pure eigenstate of γ_5.

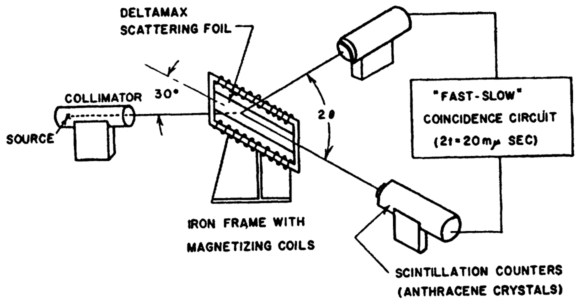

FIG. 9. Møller scattering setup of Frauenfelder and co-workers (Fb57).

collision, but only a comparison of a coincidence rate *proportional to* cross section between the two opposite states of magnetization of the scattering foil. For this reason the apparatus need not be especially elaborate.

The full definition of a true Møller event is that (i) $\mathbf{p}_1+\mathbf{p}_2=\mathbf{p}_0$. That is, momentum of particle No. 1 and particle No. 2 (labeled by which counter they enter) shall add up to the incident momentum. (ii) $K_1+K_2=K_0$. The kinetic energies likewise shall add up to the original kinetic energy. Coincidence detection of Møller events has been effected in the past by requiring for example that $(\mathbf{p}_1+\mathbf{p}_2)\cdot\mathbf{p}_0=p_0^2$ (P51), (A54), which is a compromise between complete rigidity of definition and counting rate. At energies of say 0.5 to 2.0 Mev, at the fairly high fractional energy transfers, say $0.4<w<0.5$, both $\mathbf{p}_1$ and $\mathbf{p}_2$ make sufficiently large angles with $\mathbf{p}_0$ that this simple criterion rules out multiple or plural scattering in the target foil, particularly after the Møller event, as well as against energy loss on the part of either particle No. 0, 1, or 2. Thus by setting (besides p_0) a certain value of $\mathbf{p}_1\cdot\mathbf{p}_0$, coincidences are found only at the expected value of $\mathbf{p}_2\cdot\mathbf{p}_0$ if the experiment is clean. See in this connection "coherence curves" (A54).

Returning to the experimental setup of Fig. 9, where a considerable spread was accepted in the incident energy K_0, typically a factor two or three, one would lose essentially all opportunity to check for "coherence" between K_1 and K_2. There does still remain the possibility for an "angular coherence" check, which however due to the relativistic narrowing is sharpest only for fixed K_1+K_2.

For the isotope Au^{198}, results obtained with the setup of Fig. 9 were interpreted (Fc57) as implying negligible longitudinal polarization for these particular negative betas (some seven standard deviations less than the expected v/c value). The Møller method was subsequently proven, for the case of Au^{198}, by Benczer-Koller *et al.* (BW58). The apparatus used by the latter group did not differ in essential details from that in Fig. 9. A magnetic lens system was used to bring the beam into the apparatus and focus it on the "iron" foil. It has been pointed out (W58) that precautions were taken in the latter experiment to discriminate against true time-coincident Compton electrons (due to the accompanying Au^{198} gammas) which could otherwise produce false Møller events, and that these precautions could probably account for the difference between the two results. The latter result was v/c polarization to seven standard deviations, and left-handed.

Use of the Bhabha method for longitudinal polarization measurement was first attempted by Frauenfelder and collaborators (Fd57) on the high-energy positron group from Ga^{66}. The results were interpreted as meaning that these positrons were probably not polarized (a matter of about three standard deviations below the expected v/c value). However, three different methods [see Sec. III D (2), (3), and (5)] applied to these same betas have since yielded large (right-handed) helicity. Since we know there is nothing wrong with the ordinary Bhabha differential cross section (A54) which itself is highly sensitive to whether one takes into account the interference effects, it must be presumed that the Bhabha polarization method is intrinsically sound.

Transverse polarization analysis.—It is interesting to contrast the possibilities for transverse polarization analysis with the longitudinal case just discussed. Specific discussion is for the electron-electron case. One may examine again the basic Møller matrix element, and restrict again to 90° scattering angle in the center-of-mass system where the algebra is easiest and the physics most meaningful for our purposes. For the efficiency of a transversely polarized target electron we have the nonrelativistic value unity at sufficiently low energy. With increasing energy, the efficiency falls off very rapidly compared to the longitudinal case just described: if we average over azimuth of the plane of collision the efficiency ϵ is down to 0.5 at only 0.65-Mev laboratory energy, 0.3 at 1.25 Mev, 0.1 at about 3.4 Mev and ultimately becomes zero. If, instead, the plane of the collision is restricted to include the initial "spin vectors" involved (to include the spin vector of the polarized target electron will suffice) then the efficiency falls off almost as fast with energy, but does have a nonzero residue in the high-energy limit of $+0.11$. Or, if the plane of collision is required to be at right angles to this direction, ϵ saturates at -0.11 in the high-energy limit, having passed through zero at about 3-Mev laboratory energy.

For practical purposes, the transverse Møller method does not compare favorably with the longitudinal Møller method. The details for the transverse Bhabha method can be had by inspecting the matrix element again. A paper by Stehle (St58) lists in convenient form both Møller and Bhabha matrix elements.

Spin erection.—A further comment on Møller scattering relates to its hypothetical use as a spin erector (compare the electrostatic spin-precessor and the reprojection spin erector discussed in Sec. III B). The helicity memory through the Møller event can be rather large: given an incident longitudinal particle, let it make a single collision in an unpolarized target. Then either of the offcoming particles, as viewed in the center-of-mass system will have a certain memory of the original helicity. Speaking as usual of 90° scattering, this efficiency for retention of helicity is +0.6 at 0.25 Mev, +0.8 at 0.6 Mev, and +0.9 at 2 Mev in the laboratory. The high-energy limit is 8/9. If then we transform either of the final electrons to the laboratory system we find that the over-all efficiency for transfer of longitudinal polarization P_p into transverse polarization P_s(lab) departs from zero $\sim 0.71E$, reaches the value 0.4 at 0.25 Mev and 0.5 at 1.0 Mev. It then falls with increasing laboratory energy E, approximately as $0.89(E+2\text{ Mev})^{-\frac{1}{2}}$, decreasing to 0.25 at 10 Mev. The direction of the resulting P_s is clear from the description—it points away from the line of collision for right-handed input. Longitudinal target polarization likewise could be transferred to P_s(lab) using the same numbers, given an unpolarized incident beam. Again, the transfer of polarization from P_p to P_p(lab) can be had from the transfer efficiencies just quoted by multiplying in each case by $(E+1\text{ Mev})^{\frac{1}{2}}$.

If one asks now for the transfer efficiency from P_s to P_p(lab) in a 90° Møller scattering act from an unpolarized target, which is the inverse of spin erection, one finds quite different numbers from the spin erection case. Thus in contrast to the spin-erectors already described: macroscopic electrostatic device, crossed fields device, or the use of atomic scattering for spin reprojection, this is an illustration of a nonreversible device. For $P_p \rightarrow P_s$(lab), the spin direction of the participating target electron and also of the conjugate recoil electron are (individually) about 90% certain. For $P_s \rightarrow P_p$(lab) these spin directions are about 90% uncertain, except at very low energy.

As for use of Bhabha scattering for spin erection or de-erection, the various transfer efficiencies at high energies will be nearly the same as for the Møller case. For low energies, even for 90° scattering some memory of "which one is the positron" will persist through the collision, and at very low energy the various transfer efficiencies at any angle will be apparent from the geometry of the collision. The discussion has by now become quite academic. For positrons, or indeed electrons, except at quite high energy one would generally be better off to use single, plural or multiple Coulomb scattering for de-erection of spin.

Finally, neither Møller nor Bhabha formulas show any right-left asymmetry as there is in Mott scattering, even allowing both target and beam to be (transversely) polarized.

(2) *Annihilation of Slow Positrons in a Ferromagnet*

Suppose that initially polarized positrons could be brought to rest (that is to essentially thermal energies) in a ferromagnetic material, and still retain some of the original polarization, so that finally $\langle\sigma_z\rangle \neq 0$. We consider complete polarization for simplicity. The ultimate fate of over 99% of these stopped positrons is two-quantum annihilation, and against electrons of a definite spin polarization, namely opposite to that of the positrons. The total annihilation gamma momentum represents, to the extent that the positrons themselves contribute negligible momentum, the momentum (Fourier transform of wave function) of those electrons susceptible annihilation (for example, La57).

In iron, one believes that the magnetic electrons (the "two" out of the 26 electrons per atom) surely must have different momentum components than the average electron. Thus, quite aside from positron polarization, there would have to be some correlation between amount of departure from collinearity of the two annihilation photons and whether or not a "magnetic electron" had been annihilated. Since the deeper bound electrons, which are not polarized, also are presumably not very accessible to a slow positron for annihilation, this may tend to enhance the correlation. What could be measured in the two-quantum angular correlation is the relative abundance of the higher momentum portion (thus, in the wings of the curve, say from 5 to 10×10^{-3} radians away from 180°) for the two analyzer settings: magnetization along $+z$ and along $-z$. Thus if $\langle\sigma_z\rangle$ for the positrons were positive, then + magnetization must yield more of the higher momentum events than − magnetization, *providing* the magnetic electrons tend to have higher momentum than the average electron with which a positron may annihilate.

This method was devised and used successfully by Hanna and Preston (Ha57). Their first polarization results showed (accepting that the magnetic electrons have generally higher momentum) that Cu^{64} positrons are right-handed. In later work (Hb57, Ha58) various ferromagnetic materials and a number of different positron emitters have been compared. Figure 10 shows the apparatus used in certain of the experiments of Hanna and Preston. Instead of using the more standard

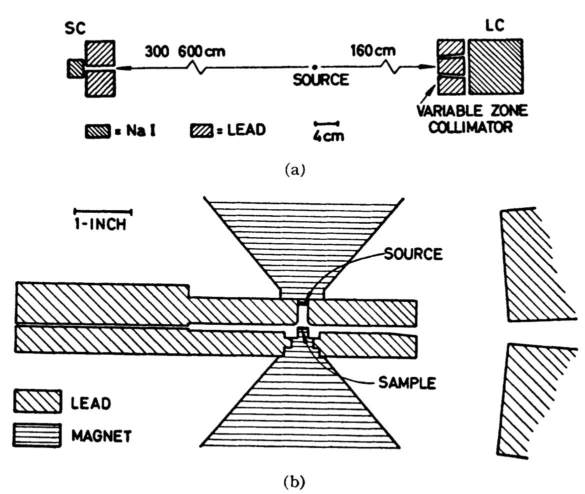

FIG. 10. Angular correlation apparatus of Hanna and Preston (Ha58) for measuring two-photon angular correlation from polarized positrons annihilating in a ferromagnet.

momentum measurer as originally introduced by DeBenedetti and collaborators (D50) which essentially defines projected angle between photon momenta $-\mathbf{k}_1$ and $\mathbf{k}_2$, they used an annular detector slit located opposite to a point detector so that the actual angle θ between $-\mathbf{k}_1$ and $\mathbf{k}_2$ is defined. The particular choice of geometry for the gamma detection is not crucial for making polarization measurements however. If this technique eventually makes contact with any anisotropy of the momentum of the polarized electrons (with respect to crystal axes) then choice of detector geometry can be quite important.

The measured δ on reversing the state of magnetization, as obtained by Hanna and Preston, range from 5% to 10% depending on controllable circumstances, such as inserting absorbers between source and iron analyzer to "clean up" the positron beam.

There is no reason to suppose that the positrons on slowing have partaken of the polarization of the ambient electrons to any extent. To consider how annihilation-in-flight competes with the ordinary energy loss processes, the fact that a positron has survived so as to finally appear within tens of milliradians of 180° in the angular correlation predisposes it to be polarized, by a fraction of a percent, and parallel to the predominant electron spins. However, such added polarization would be the same for both states of magnetization (if referred to the magnetizing field direction) and therefore could not influence the measured δ to first order (and to any order, no matter how large such added polarization might be, it could not give a false sign to the measured δ).

As with analysis of positron polarization by means of positronium in the "plastics," absolute polarization measurement by the present technique awaits further development. But relative polarization measurement is feasible now.

The sign of the response of annihilation-at-rest in iron, the sign of the response of positronium in plastics,

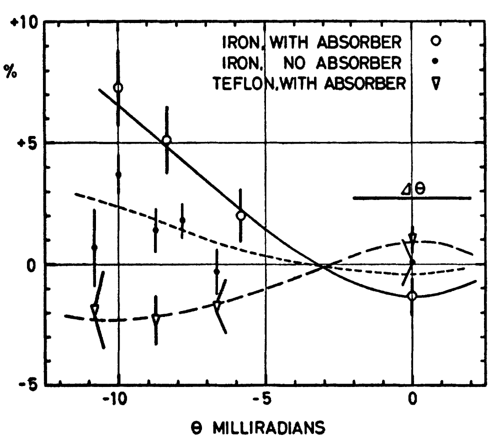

FIG. 11. Response of iron annihilator to polarized sodium-22 positrons compared to response of a plastic, both run at same resolution $\Delta\theta$ in same apparatus. Measured δ on reversing magnetic field is opposite in sign for the two materials. Positive δ means higher coincidence rate for H parallel to positron spins. (PO58).

and the sign of the response of positronium in gases, are mutually consistent (the iron method has, for entirely different reasons, the same sign as for gases; the method using plastics has the opposite sign). Figure 11 shows a comparison plot between measured δ for iron (mild steel) and for a plastic (Teflon), run in the same apparatus at the same angular resolution. The resolution chosen (4×10^{-3} radian) seemed to be suited to the iron, but is known to be too small for optimum response in Teflon (III C). In any case the opposite response to the same positrons is evident. Hanna and Preston have checked (using Cu^{64} positrons) the response of polyethylene with respect to iron and it was opposite (Ha58).

The function of the absorber, which has been applied both at Argonne Laboratory and at Pittsburgh, is simply to enhance the measured δ, by selecting the higher energy portion of the actual positron spectrum which emerges from the source material plus the source backing. Thus, over and above the obvious fact that the effective v/c is slightly higher, clearly one is selecting particles of better polarization in the z direction. Depolarization of a positron beam in flight appears not to be a very serious matter.

A further comment on the three annihilation-at-rest methods for positron polarization measurement, is that it is generally quite difficult to join on in series a spin erector or de-erector, or an energy analyzer, principally because of the large ambient magnetic field in the immediate vicinity of the active region. For transverse polarization measurement, the annihilation-at-rest in a ferromagnet would clearly be the most easily adapted, since the magnetic field strength just outside the active sample need not be objectionably large.

(3) *Annihilation-in-Flight in Polarized Electron Target*

A method which is intrinsically nonrelativistic, but which can be used at fairly high energy nonetheless, is based primarily on the fact that at rather low energies, positron and electron helicities must be the same for two-quantum annihilation in flight to take place (three-quantum annihilation-in-flight is smaller by $\sim 1/137$ and does not concern us here). There is, however, a strong departure from the nonrelativistic situation as regards efficiency for a longitudinally polarized analyzer to detect longitudinal beam polarization, at least assuming any and all two-quantum events to be recorded. This behavior is shown in Fig. 12, the solid curve. A crossover in sign of response occurs at about 4-Mev beam energy, so that at higher energies annihilation takes place more readily with spins parallel (positive efficiency ϵ).

However, as was realized by Frankel and co-workers (Fr57), the low-energy behavior (that is, $\epsilon=-1$) can be retained at any beam energy provided that the line-of-annihilation (in the center-of-mass system the two photons define a line of annihilation) lies exactly along the line of collision. In practice one chooses a sufficiently

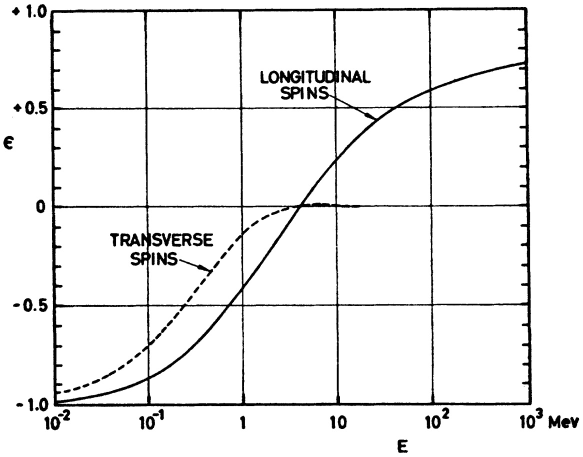

FIG. 12. Total cross section for annihilation-in-flight. Efficiency per polarized electron in target for analysis of longitudinal positron polarization (solid curve). Efficiency for transverse polarization analysis (dashed curve). Positive ϵ means parallel spins are favored to annihilate. From (P57).

narrow cone in the center-of-mass system that the high relativistic type events (those tending to make $\epsilon=+1$) are partially excluded from being counted. Thus, by a judicious choice between loss of efficiency for too large a cone and loss of counting rate for too narrow, a first-order experiment can be done on rather high-energy positrons. Frankel *et al.* used Ga^{66} (energies up to 4 Mev) with the result that these positrons are quite highly polarized, and right-handed, in line with the result of Deutsch *et al.* using another method [Sec. III D (5)]. In this type of experiment one necessarily runs into the oblique foil situation as in the Møller experiments (compare Fig. 9).

For energies much above 4 Mev, say 40 Mev, one would probably be better off to accept all annihilation events.

Transverse polarization.—The analog of the longitudinal case just discussed is mentioned briefly, for completeness sake, by examining the cross section for total annihilation-in-flight for a transversely magnetized foil. The efficiency per polarized electron is shown in Fig. 12 (as the dashed curve) as a function of positron energy in the laboratory. It is down to 0.5 by about 0.25 Mev and is essentially zero by about 3 Mev. Even restricting to the line of annihilation parallel to the line of collision, the efficiency falls off equally fast with increasing beam energy.

If the line of annihilation is restricted to be at right angles to the line of collision, the azimuthal dependence of the photon intensity becomes essentially $\cos^2\theta$ or $\sin^2\theta$ at high energy, depending on whether the input spins are parallel or antiparallel. This effect is about 50% efficient at 3 Mev and 90% efficient at 30 Mev (P57). Regarding counting rate such an approach would be expensive for two reasons: restricting the azimuth, and by restricting to crosswise annihilation events which unfortunately are fewer than the others (the ordinary Dirac differential annihilation cross section varies with beam energy so that a factor two is lost at 6 Mev, and a factor three at 20 Mev).

A second-order (hypothetical) method for transverse positron polarization analysis using annihilation-in-flight is discussed in context in Sec. III D (5).

(4) *Polarized Bremsstrahlung*

Longitudinal.—A spectacular and useful method for analyzing longitudinal polarization of energetic e^- and e^+ was devised largely intuitively by Goldhaber, Grodzins, and Sunyar. The method is to extract the longitudinal particle polarization information from the beam by means of the usual "forward" bremsstrahlung. This affords a natural, convenient, and (by polarization standards) copious source of circularly polarized photons. The secondary beam of photons is analyzed by comparison with atomic electrons (see Appendix C on the Compton method for photon helicity measurements).

Figure 13 shows the δ obtained by Goldhaber *et al.* (G57) on reversing the (transmission type) Compton analyzer through which the bremsstrahlung spectrum from Y^{90} betas was counted. These photons were found to be highly polarized. The Compton analyzer was here throwing in no spurious δ since the crossover in sign of the response is observed to come almost exactly where it should according to the Compton formulas, at 0.64 Mev.

Early calculations by McVoy (M57) showed that the transfer efficiency P_p to photon circular polarization P_k should be $\sim$0.95 at least for the truly forward photons, and near the tip of the bremsstrahlung spectrum. One of McVoy's curves (M57) implied**** that

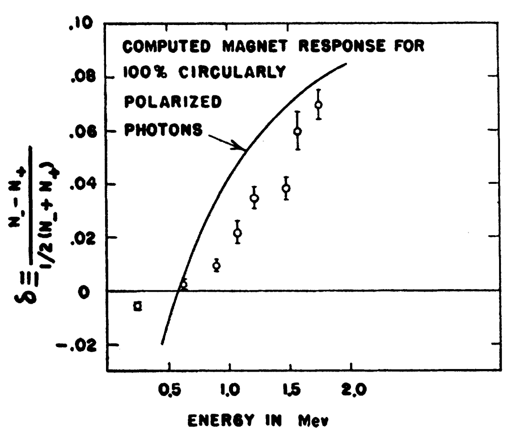

FIG. 13. Polarized bremsstrahlung from Y^{90} betas. Measured δ on reversing transmission type Compton analyzer as function of photon energy accepted. From Goldhaber *et al.* (G57).

**** See corrected curve (Mc58). The main effect in correcting the curve is that as $k/(E_1-1)$ decreases from unity the transfer efficiency begins to fall off considerably sooner than was at first believed, with the result that typically one should expect somewhat such high efficiency should persist over a rather wide range of fractional photon energy, $k/(E_1-1)$. Later, bremsstrahlung computations were made over a range of photon angle as well as fractional photon energy by Fronsdal and Überall (F58). Computation on this problem has also been done by Claesson (C57) and by Böbel (B57). The problem of particle depolarization during such slowing as takes place before the bremsstrahlung act seems not to be an all-important problem, but must ultimately be faced.

On the basis of the first curve of McVoy (M57) some precise measurement of longitudinal particle polarization has been reported. For example, P^{32} betas were measured somewhat in the nature of a by-product in a series of experiments by Boehm and Wapstra (Bo58). Here the implied longitudinal polarization was -0.97 ± 0.06 of v/c. Until such time as (i) the bremsstrahlung method has been seriously checked, experiment *versus* theory, and (ii) the number of polarized electrons/cm^3 in a given ferromagnetic material under given conditions is truly known, polarization measurement by this method to absolute precision approaching 5% should be considered as somewhat on the optimistic side. As matters stand, item (i) cannot be divorced from item (ii), since the former is currently based upon the latter.

Aside from their intrinsic interest, bremsstrahlung sources are being widely used as calibrators for photon-helicity measuring devices. The bremsstrahlung process occupies a strategic position among the various polarized sources and measuring devices which eventually must come to be well understood.

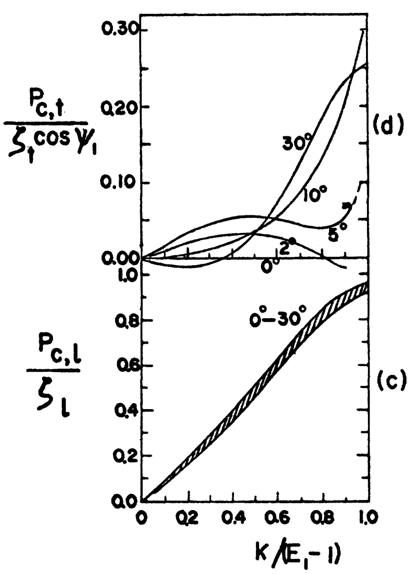

FIG. 14. Bremsstrahlung calculations. Theoretical transfer efficiency from longitudinal electron polarization to circular polarization of photon (lower curves) as function of fractional photon energy. Transfer from transverse particle polarization to photon helicity (upper curves). Photon emission angle θ is shown. Incident electron energy is 2.5 Mev. From Fronsdal and Überall (F58), with some details omitted.

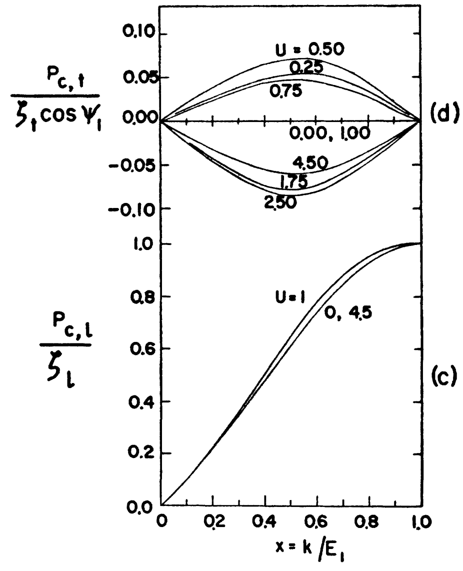

FIG. 15. Bremsstrahlung calculations. See also Fig. 14. Incident beam energy is 52 Mev. Photon emission angle in radians given by $U=103\theta$. From (F58).

Circular polarization of bremsstrahlung was the method used in the first successful helicity measurement of betas from muon decay. The experiment of Culligan *et al.* (Cu57) which yielded right-handed positrons, was the first suggestion that came from a *helicity* measurement that either lepton conservation or the tensor interaction in certain beta-decay processes would have to be given up. Since then, bremsstrahlung measurements on the helicity of both the negative and the positive betas from decay of the negative and positive muon, respectively, has been performed (Ma58). Both signs turn out to be the same as for the corresponding beta from nuclear beta decay, and the sign obtained by Culligan is thereby confirmed.††††

Typical results from the computation by Fronsdal and Überall (F58) are given in Figs. 14 and 15.‡‡‡‡ To a first approximation, all of the forward cone of bremsstrahlung has about the same transfer efficiency (from P_p to P_k) at a given fractional photon energy.

Transverse polarization.—The transfer efficiency from P_s to photon helicity P_k has been computed by Claesson (C57), by Böbel (B57), and by Fronsdal and Überall (F58). The efficiency here is considerably diminished from the longitudinal case, $P_p \rightarrow P_k$, as may be seen from the upper curves in both Figs. 14 and 15.

less circular polarization, by perhaps 10 to 15%, unless the conditions correspond to the tips of both the beta spectrum and the bremsstrahlung spectrum (which is scarcely practical).

†††† See also Culligan, Frank, and Holt, Proc. Phys. Soc. (London) **73**, 169 (1959) for further work.

‡‡‡‡ These authors are thanked for sending results prior to publication.

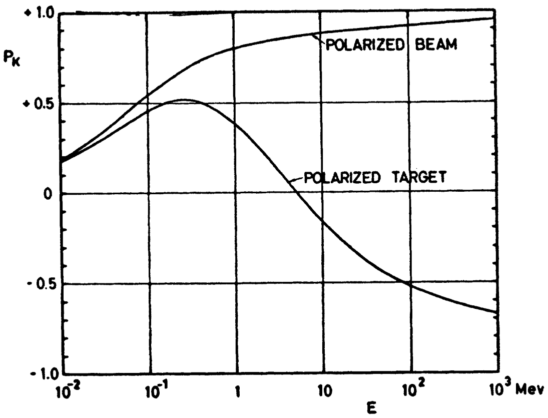

FIG. 16. Annihilation-in-flight. Computed transfer of helicity from positron to the annihilation quantum of higher energy in the laboratory system (upper curve). Right-handed positron yields right-handed photon (positive P_k). If target, instead of beam, is polarized, lower curve applies. From (P57).

(5) *Circular Polarization of Annihilation Photons*

As has been stressed before, there is no helicity information in the photons from nonrelativistic annihilation of positrons (nor is there in the photons which lie strictly along the line of collision, at any beam energy). However, for annihilation in flight at quite modest energies, the photon in the "opposite hemisphere" to whichever of the particles is longitudinally polarized takes on to some extent the same helicity as that particle had. This transfer efficiency is implied by the upper curve of Fig. 16, which specifically gives the circular polarization of the *higher* energy member of the pair of photons, per unit polarization of the beam. The target is assumed to be unpolarized. In actual circumstances, depending on geometry and energy discrimination in the photon detector, one may effectively accept only photons at center-of-mass angle $0<\theta<\theta_{\max}$, and usually with varying efficiency as a function of θ. What is shown here is simply the situation where all photons $0<\theta<90°$, and only those, are presumed to be analyzed (all counted and analyzed with equal efficiency). The process here is typically relativistic, and thus the higher the beam energy the more convenient the method becomes (with regard to energy discrimination, and geometrically). This may be compared with the experimental situation in Sec. III D (3), where a balance had to be maintained between competing effects.

The case where the target and not the beam is polarized is also indicated in Fig. 16, but is of no particular concern to us here. One sees however how the crossover in sign at 4 Mev relates to the similar crossover in the solid curve of Fig. 12.

Deutsch and collaborators (DS57) have used annihilation-in-flight in an unpolarized converter (Lucite) to measure the helicity of the positrons from Ga^{66} (the high-energy group has 4-Mev end point). Their apparatus is shown in Fig. 17. A selected portion of the beta spectrum is focused onto the converter, the high-energy (forward) photons being then analyzed for circular polarization in the transmission Compton analyzer shown. The computed δ, based on fully polarized (right-handed) positrons compared with the measured δ as a function of photon laboratory energy (for given input beta energy) implied essentially 100% longitudinal polarization. The measured values of δ ranged to 8%.

This second-order method has been used without preselecting the energy of the positrons by Boehm *et al.* (Bo57).

Transverse polarization.—For the process just considered (annihilation-in-flight against an unpolarized target) there is generally a nonzero transfer from any P_s in the particle beam to P_k of the annihilation radiation. This is a maximum at center-of-mass angle 90°, that is when the line of annihilation lies crosswise to the line of collision (P58). The magnitude of this efficiency generally exceeds the analogous transfer term for the bremsstrahlung process. At 3 Mev the transfer efficiency is a maximum of 0.55, and it is rather symmetric about 3 Mev on logarithmic energy scale, falling to half of this value at 0.3 Mev or at 60 Mev. This hypothetical method suffers from intensity limitations for the same two reasons mentioned in Sec. III D (3).

IV. CONCLUDING REMARKS

A. On Calculations

So far, all helicity calculations cited have proceeded from theory that is already very well believed, and reasonably well checked experimentally. For example (to include Compton scattering for completeness), the "spin effects" were always there, both theoretically and experimentally, in the nonhelical details of the differential cross section. This includes loss of definition of plane polarization in Compton scattering. The same is true of the existence of ground-state positronium and the details of its fine structure. The spin terms were known to be present in Møller and Bhabha differential scattering, in the annihilation-in-flight cross section, and in the Bethe-Heitler approximation for the bremsstrahlung process. No really new physics is being used here in making use of standard processes for polarization measurement.

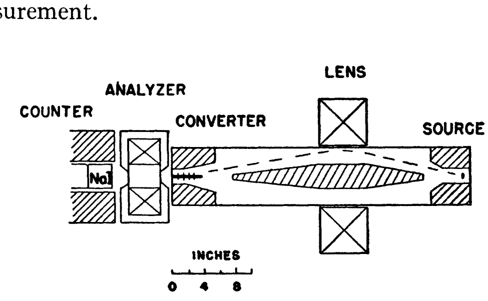

FIG. 17. Experiment to measure positron helicity by transferring polarization information to annihilation (in flight) photon, which is analyzed for helicity in Compton analyzer. From Deutsch and colloborators (DS57).

In order to make practical use of these spin effects, lists of matrix elements (transition amplitudes) from fully specified state to fully specified state are good things to have accessible. Then none of the polarization implications in the original theory are averaged out until the people devising a particular experiment are ready to do so.

For fast particles, transverse polarization is generally less amenable to measurement than is longitudinal polarization. Perhaps it is not too early to begin looking beyond those polarization effects which are already in the extant theory. Under this heading would come an enquiry into the possibilities of right-left asymmetry in bremsstrahlung intensity from transversely polarized particles. Here one would have to go beyond the Bethe-Heitler approximation, reminiscent of how, in Coulomb scattering without radiation, one has to go beyond the first Born approximation in order to find any polarization effects other than "reprojection" of spin.

B. On Experiment

Of great assistance in succeeding with the experiments discussed here has been the fact that all the methods are really *null* methods. In these experiments, a large number of parameters such as solid angle, energy straggle, backgrounds, are arranged to be *the same* for both "states" of the analyzer. By and large it is nowadays as easy to demonstrate spin-helicity effects, using the large input polarizations now known to be available from beta decay, as it is to demonstrate in a nonpolarization experiment some relativistic feature of a differential cross section, or to measure an absolute cross section, to compare with Dirac theory. The reason is that none of these latter experiments can be made into a null method. In exploring details of a differential cross section there is always some important parameter that changes: the energy of a photon, an angle of obliquity in a scattering foil, signal-to-noise ratio. These things that matter will change always in a systematic way with the independent variable of the particular experiment. Hopefully, these systematic changes are computable, measurable or otherwise understood. As for absolute cross-section determinations (except perhaps for total Compton cross section), these are notorious for having many factors which must be controlled.

With the labor and care necessary for say a 10% measurement of differential cross section, one now gets typically a 1% measurement on a change in counting rate which relates linearly to the polarization being measured. The intrinsic reliablity and power of the balancing or null approach makes this possible. Suppose the polarization effect causes a 10% change in counting rate, then one would have a measurement of polarization good to 10%. For smaller output asymmetries, one must try to balance more thoroughly. In any case, it appears that the potential power of the null methods needs to be stressed, especially in telling of polarization measurement to persons outside the immediate field.

C. On Calibration

We look forward to the time when a number of standard (that is, reproducible) beta and bremsstrahlung sources and analyzers are available. Numbers will have to be attached, these numbers will have to be cross-checked with each other somehow, and of course "theory" and the numbers will have to be compatible.

Aside from precision "numerical" calibration which is the course presently being followed, there is a different approach which might be mentioned. Just as an unpolarized beam has no polarized part, a fully polarized beam has no unpolarized part. This is not trivial: It should be recalled that if the problem were to demonstrate that a certain beam is unpolarized to within an infinitesimal, perhaps 0.5%, then a polarization analyzer whose efficiency is known only crudely (say within a factor two) might suffice—one performs a null experiment. In dealing with degrees of polarization P which are much closer to unity than to zero, it might be worthwhile to consider possibilities for measuring $1-|P|$, instead of P itself. Thus, if $|P|$ were $\sim$0.97, a measurement on $1-|P|$ good to only 30% is equivalent to a measurement on $|P|$ good to one percent. The sign of P would of course be known before one embarked on this hypothetical precision experiment. Admittedly, such experiments would have to be of second order and therefore quite difficult. Yet, it would be aesthetically pleasing to have proof of, say, 100% polarization presented in the form of a null experiment, to complement the more traditional approach which would hinge upon a chain of calibration numbers.

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APPENDIX A. BETA DECAY AND POLARIZATION

Allowed Fermi Decay

The connection between neutrino polarization and beta polarization can be illustrated in simple sketches (Fig. 18). We assume a pure V (vector) interaction, and let a positron be emitted for illustration. The other

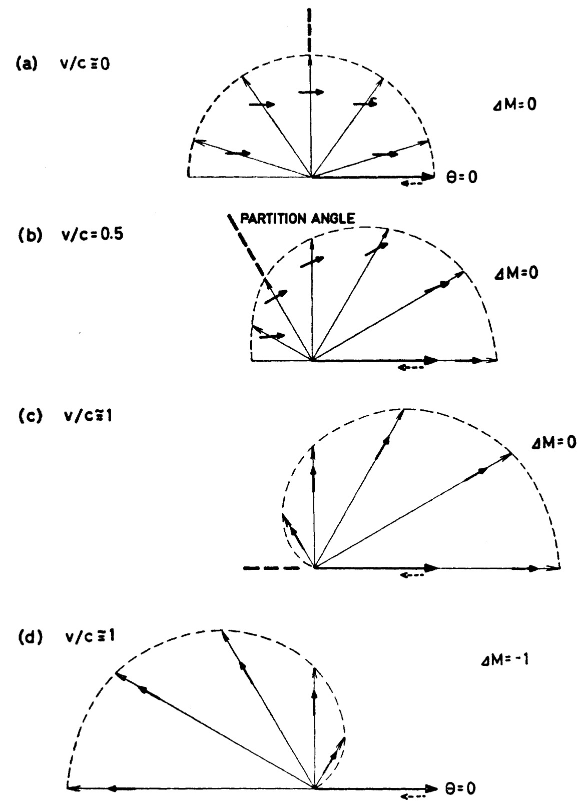

FIG. 18. Relations between polarization vectors and momentum vectors. The emitted neutrino momentum $\mathbf{q}$ defines direction $\theta=0$. Length of a given radius vector to the dotted envelope is proportional to the probability that the beta momentum $\mathbf{p}$ shall be in that direction. Spin-polarization vectors are shown by short arrows. Beta-polarization arrows (each representing degree of polarization unity) generally have meaning only in the rest system of the beta particle. Neutrino helicity -1 is used for illustration. Sketches apply for pure V or pure A interaction.

particle emitted is called a neutrino and not an antineutrino. In sketches (a), (b), and (c) the neutrino proceeds along the line $\theta=0$, to our right. Suppose it is left-handed. The sketches show (i) the angular correlation of the beta momentum $\mathbf{p}$, with respect to neutrino momentum $\mathbf{q}$ (the envelope drawn is the relative probability of beta emission for given angle of emission, to be multiplied by d^3p); (ii) the polarization vector in the *rest system* of the beta particle, where the magnitude of the polarization is always unity since in this situation no degree of freedom remains.

These sketches rest on the fact that the *amplitude* for the beta to be at angle θ and to be right-handed is $(1+b)\cos(\theta/2)$, while the (coherent amplitude for it to be left-handed is $(1-b)\sin(\theta/2)$, where $b=p/(\gamma+1)$ and γ means total energy of the beta. It follows that the beta-neutrino correlation (unpolarized) is

$$W(\theta)=1+\frac{2b}{1+b^2}\cos\theta=1+\frac{v}{c}\cos\theta. \qquad \text{(A1)}$$

Alternatively from these amplitudes, the longitudinal polarization is

$$P_p(\theta)=\frac{(v/c)+\cos\theta}{1+(v/c)\cos\theta}. \qquad \text{(A2)}$$

Beginning again with these amplitudes, the longitudinal polarization, *independent of* θ is $+v/c$. We could have reversed the roles of beta and neutrino by requiring say a right-handed beta, then the angular correlation of the neutrino with this beta is just $(1+\cos\theta)$. A neutrino of zero rest mass is taken here.

Expression (A2) shows there is a "partition angle" at which the beta polarization is exactly transverse. This angle, defined by $(v/c)+\cos\theta=0$, moves closer to 180° as v/c increases. The drawings show how P_p tends to unity in the high-energy limit.

To pass from pure V interaction to pure S (scalar) interaction, one may reverse $\mathbf{q}$ and the neutrino helicity and otherwise keep the same patterns as sketched in (a), (b), and (c) of Fig. 18.

Nowhere in this discussion was anything said about nonconservation of parity: We could argue that what was plotted or given in the foregoing expressions was intended to represent only *half* of the Einstein-Podolsky situation. Then, by supplying the recipe for the *other* helicity of the neutrino, which says to reverse all polarization vectors, the picture would be completed. [Expression (A1) would be unchanged.]

It seems that only one-half of the picture occurs in nature, but this need have nothing to do with our trying to illustrate the connection between the momenta and helicities. It does supply the motive and explains our choice of *longitudinally* polarized neutrino in these figures.

Allowed Gamow-Teller Decay

In the foregoing section, although we pretended to be discussing only the pure Fermi case, drawings (a), (b), and (c) apply equally well to pure GT—to the $\Delta M=0$ part at least. For the A (axial-vector) interaction we may keep the drawings as they stand, for pure T (tensor) interaction we can reverse $\mathbf{q}$ and the neutrino helicity. To depict the situation for an offgoing right-handed neutrino, reverse all polarization vectors. To complete the picture for GT, a $\Delta M=-1$ transition is sketched as part (d) of Fig. 18, only the high-energy limit being shown for illustration. A $\Delta M=+1$ transition would be represented by reflecting picture (d) in the plane $\theta=90°$ (including the polarization arrows). Throughout we have been taking the axis of quantization for J to be along $\theta=0$.

As regards the unpolarized angular correlation, we now have competing parts: the angular correlation for a picture like (b) is $1+v/c\cos\theta$, and for a picture like (d) is $1-v/c\cos\theta$. However, transition (d) occurs twice as often as (b), at least for unpolarized initial and final nucleus. By not forcing a definite helicity on the neutrino, $|\Delta M|=1$ transitions occur twice as often as

$\Delta M=0$ transitions, as required by spectroscopic stability. Or, requiring definite helicity for the neutrino, picture (b) represents "half" of a triplet $M=0$ state (of neutrino plus beta). Thus, either way we look at it, the angular correlation is the same,

$$W(\theta)=1-\frac{v}{c}\cos\theta. \tag{A3}$$

Picture (a) and a nonrelativistic version of (d) are what apply in a pure *GT* *K*-capture event. With respect to $\mathbf{q}$, the ΔM imparted to the nucleus is either -1 or zero (for a left-handed neutrino emitted), the former transition occuring with twice the probability of the latter.

Now if we require, for the first time, a given "ΔM" in the laboratory, the neutrino will seldom proceed along the axis of quantization, and simple sketches such as these are no longer feasible. So many important experiments are related to J_z in the laboratory that the discussion should be pursued a little further. Inspection of equation (1) of Jackson *et al.* (J57) shows, for pure *A* interaction, say, that the $\Delta M=0$ transition has angular correlation (z axis fixed in the laboratory)

$$1-\frac{\mathbf{p}\cdot\mathbf{q}}{E_eE_\nu}+2\frac{p_zq_z}{E_eE_\nu}. \tag{A4}$$

Of course, $\mathbf{p}$ and $\mathbf{q}$ *separately* are isotropic in the laboratory since one cannot "align" a spin-$\frac{1}{2}$ particle but can only polarize it. From (A4) the correlation between $\mathbf{p}$ and $\mathbf{q}$ can be readily worked out to be $1-\mathbf{p}\cdot\mathbf{q}/3E_eE_\nu$, which is the same as (A3).

For comparison the $\Delta M=-1$ correlation [again from (J57)] is

$$\left(1-\frac{p_z}{E_e}\right)\left(1+\frac{q_z}{E_\nu}\right) \tag{A5}$$

and, although $\mathbf{p}$ and $\mathbf{q}$ are now each tied closely to the z axis, the correlation between them is the same as for the $\Delta M=0$ case just treated.

If we had the simple case J, $M=1$, -1, making a transition to $J=0$, the neutrino (taken to be left-handed) would be emitted as $1+\cos\theta$, the beta (independently) as $1-v/c\cos\theta$, the factor $\frac{1}{3}$ being now absent since only one possibility for ΔM has been allowed. On the other hand, if we ask for the polarization of daughter nucleus (let us now go from $J=0$ to $J=1$ for illustration) having found a beta along z, then the $\Delta M=0$ contribution is again in competition with $|\Delta M|=1$, and the factor $\frac{1}{3}$ often associated with *GT* transitions reappears. The beta—circularly polarized gamma correlation is treated in detail by Alder *et al.* (A57) for example.

APPENDIX B. SHORT COMPENDIUM OF HELICITY SIGNS

The experiment of Wu *et al.* (Wu57) showed that the negative beta particles from polarized Co^{60} nuclei come off predominantly opposed to the field H. Thus these betas as predominantly opposed to the direction of the original nuclear spin, because ground-state Co^{60} has a positive g value (W55). This decay has a decreasing spin change ($5\rightarrow 4$) and therefore the beta and the emitted antineutrino jointly carry off angular momentum parallel to the original nuclear spin, unless we suppose that conservation of angular momentum is in question. Since none of this angular momentum carried off is orbital (this is an allowed transition) it has to be carried entirely by the spins of the two particles. Suppose, for argument, that the original nuclear polarization were complete. Then the two particles must carry off one unit of angular momentum referred to the z axis, the field axis. They are both spin-$\frac{1}{2}$ particles and so must "aid each other." The Wu result implies then that the betas from Co^{60} have to be predominantly left-handed. To summarize, granting the sign of g, the sense of the spin sequence here, and granting the conservation of angular momentum, the "sign" that emerges from the Wu experiment is just the helicity of those betas.

Soon after the Wu experiment, the left-handed helicity of the Co^{60} betas was checked by Frauenfelder and collaborators (Fa57) by means of Mott scattering. Page and Heinberg found that positrons from Na^{22} were predominantly right-handed (PH57). Simultaneously Schopper (S57) showed that Na^{22} and Co^{60} (both having the same sense in the spin sequence) emitted photons of opposite handedness, when measured at nominal 180° to the line of the emitted beta, the sign for Co^{60} being in accord with the Wu experiment. Ga^{66} believed to decay via a pure Fermi transition was shown by Deutsch and co-workers (DS57) to give highly polarized right-handed positrons. [No attempt is made to record many of the early polarization experiments which quickly extended the list to include, for example, mixed (*F* and *GT*) transitions, forbidden transitions, etc.]

Strictly, none of these experiments shows anything about neutrino helicity (that is, whether it is left or right), since the neutrino direction is not measured. What is measured in the three kinds of experiment just mentioned, Wu experiment, polarized beta experiment, and beta—circularly polarized gamma experiment, is a a sense of circulation and coincidentally a *beta* direction, nothing more.

The helicity of the neutrino emitted in *K* capture (presumably the same particle as the one emitted together with a positive beta) was measured *directly* by Goldhaber *et al.* (G58) and found to be left-handed.§§§§ One concedes again conservation of angular momentum and certain j values of the levels involved.

§§§§ The sign and order of magnitude of the neutrino helicity for the decay used by Goldhaber ($Eu^{152m}\rightarrow Sm^{152*}$) has now been checked at Uppsala (M58).

To review further would lead into the status of beta-decay theory. The algebraic signs relating to a small and specific set of decays are given, signs which have not been in jeopardy and presumably will stand. This is done for the benefit of any person who may have had the notion that all had been confusion with regard to helicity measurement.

APPENDIX C. COMPTON ANALYSIS OF PHOTON HELICITY

The use of a Compton analyzer for circularly polarized radiation rests on a spin-helicity correlation present in the matrix element of Klein and Nishina. Details|||||||| of the spin-helicity part of differential or total cross section are given in (G53), or see the review article (T). The Compton analyzers may be divided into two types as indicated in the following.

Transmission Type

Here the spin-polarization sensitivity of the *total* Compton cross section is utilized. The analyzing efficiency, per polarized "iron" electron, has a faint unexplored maximum of 0.06 at about 0.2 Mev, and is zero at 0.64 Mev. From this energy to 14 Mev, where the efficiency is 0.4, it increases almost as the logarithm of $E/0.64$ Mev, and at very high energy becomes unity. Before helicity measurement became of general interest, this type was used in second order (that is, the two sets of spins in an iron analyzer acted as the polarizer-analyzer combination to verify that the spin-helicity term could be seen experimentally). A transmission analyzer was first used in first order, to measure degree (and sense) of polarization in a beam, by Trumpy (T57). This was a "nonparity" experiment, as it used capture gamma rays produced by prepolarized slow neutrons. The first application of this analyzer in a "parity" experiment was by Goldhaber *et al.* (G57), to demonstrate and measure the polarization of bremsstrahlung by Y^{90} beta rays. Lundby and co-workers have used it in beta—circularly polarized gamma correlation work (L57).

Owing to the inescapable null in the efficiency curve at 0.64 Mev, the transmission analyzer is best suited for rather high energy work, but has been used successfully on a gamma line as low as 0.9 Mev (G58).¶¶¶¶ Normally, a gamma ray is not seriously degraded in energy in passing through the iron absorber. Finally, this type of analyzer has zero sensitivity to any competing *plane* polarization in the photon beam.

Differential Type

In this type one counts, at a certain angle, photons scattered by polarized electrons. Some years before parity work, the differential analyzer was proven at Leyden (W55) on the Ni^{60} gamma rays following the beta decay of polarized Co^{60}. The method has since been revived by Schopper (S57) and by Boehm and Wapstra (BW57) for use in beta—circularly polarized gamma correlation measurements. Scattering at very large angles has recently been employed by Bernardini *et al.* (Be58) to measure the helicity of inner-bremsstrahlung. Unless one is prepared to make absolute scattering measurements (which are quite out of the question, compare Sec. IV B) a differential analyzer is quite sensitive to *plane* polarization,***** this sensitivity being independent of spin direction. Therefore, for a given geometry of source, scatterer and photon detector, a flat statement of "circular polarization efficiency" in terms of *fractional* change in intensity of scattered photons on reversing magnetization cannot strictly be given, unless some proviso is made regarding ellipticity (or planeness). This feature was at once recognized by the Leyden group, but appears to have largely escaped mention since.

Owing to counting rate considerations the scattering geometry is often such that any lines in the incident gamma spectrum are quite spread in energy as seen by the detector.

|||||||| For reasons which are not clear, expressions applicable for experimental design are occasionally attributed to an older publication (F38).

¶¶¶¶ Also compare Fig. 13.

***** In a beta-gamma cascade exhibiting nontrivial angular correlation, or where for any reason the gammas to be analyzed may have nonzero plane polarization referred to the plane of the Compton scattering act, this effect could conceivably affect numerical reduction of the data.

Bibliography

(H) W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, 1944), second edition, or later.
(JR) J. M. Jauch and F. Rohrlich, *Theory of Photons and Electrons* (Addison-Wesley Press, Cambridge, 1955).
(KS) *Beta- and Gamma-ray Spectroscopy*, Kai Siegbahn, editor, (North-Holland Publishing Company, Amsterdam, 1955).
(MM) N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, New York, 1949).
(S) L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, New York, 1949), or later editions.
(SBD) Schweber, Bethe, and DeHoffmann (Row, Peterson and Company, Evanston, Illinois, 1955), Vol. I.
(T) H. A. Tolhoek, Revs. Modern Phys. **28**, 277 (1956).

A54 Ashkin, Page, and Woodward, Phys. Rev. **94**, 357 (1954).
A57 Alder, Stech, and Winther, Phys. Rev. **107**, 728 (1957).
A58 Alikhanov, Eliseiev, Lubimov, and Ershler, Nuclear Phys. **5**, 588 (1958).
B38 W. Bleakney and J. A. Hipple, Phys. Rev. **53**, 521 (1938).
B53 R. E. Bell and R. L. Graham, Phys. Rev. **90**, 644 (1953).
B57 G. Böbel, Nuovo cimento **6**, 1241 (1957).
Bi57 A. Bincer, Phys. Rev. **107**, 1431 (1957).
Bo57 Boehm, Novey, Barnes, and Stech, Phys. Rev. **108**, 1497 (1957).
BW57 F. Boehm and A. H. Wapstra, Phys. Rev. **106**, 1364 (1957).
B58 Bernardini, Brovetto, and Ferroni, Nuclear Phys. **8**, 294 (1958).
Be58 Bernardini, Brovetto, DeBenedetti, and Ferroni, Nuovo cimento **7**, 416 (1958).
Bo58 F. Boehm and A. H. Wapstra, Phys. Rev. **109**, 456 (1958).
BW58 Benczer-Koller, Schwarzschild, Vise, and Wu, Phys. Rev. **109**, 85 (1958).
C57 A. Claesson, Arkiv Fysik **12**, 569 (1957).
Cu57 Culligan, Frank, Holt, Kluyver, and Massam, Nature **180**, 751 (1957).

CR57 Cavanagh, Turner, Coleman, Gard, and Ridley, Phil. Mag. **2**, 1105 (1957).
C58 M. Carrassi, Nuovo cimento **7**, 524 (1958).
D50 DeBenedetti, Cowan, Konneker, and Primakoff, Phys. Rev. **77**, 205 (1950).
D52 S. DeBenedetti and H. J. Richings, Phys. Rev. **85**, 377 (1952).
D53 M. Deutsch, *Progress in Nuclear Physics* (Butterworths-Springer, London, 1953), Vol. 3, p. 131.
D54 S. DeBenedetti and H. C. Corben, Ann. Revs. Nuclear Sci. **4**, 191 (1954).
D56 M. Deutsch, International Conference on Quantum Interactions of the Free Electron, University of Maryland (1956).
De57 DeShalit, Kupermann, Lipkin, and Rotham, Phys. Rev. **107**, 1459 (1957).
DP57 H. deWaard and O. J. Poppema, Physica **23**, 597 (1957).
DS57 Deutsch, Gittleman, Bauer, Grodzins, and Sunyar, Phys. Rev. **107**, 1733 (1957).
D58 H. G. Dehmelt, Phys. Rev. **109**, 381 (1958).
F38 W. Franz, Ann. Physik **33**, 689 (1938).
F57 U. Fano, Revs. Modern Phys. **29**, 74 (1957).
Fa57 Frauenfelder, Bobone, VonGoeler, Levine, Lewis, Peacock, Rossi, and DePasquali, Phys. Rev. **106**, 386 (1957).
Fb57 Frauenfelder, Hanson, Levine, Rossi, and DePasquali, Phys. Rev. **107**, 643 (1957).
Fc57 Frauenfelder, Bobone, VonGoeler, Levine, Lewis, Peacock, Rossi, and DePasquali, Phys. Rev. **107**, 909 (1957).
Fd57 Frauenfelder, Hanson, Levine, Rossi, and DePasquali, Phys. Rev. **107**, 910 (1957).
Fr57 Frankel, Hansen, Nathan, and Temmer, Phys. Rev. **108**, 1099 (1957).
F58 C. Fronsdal and H. Überall, Phys. Rev. **111**, 580 (1958).
G53 S. B. Gunst and L. A. Page, Phys. Rev. **92**, 970 (1953).
G54 R. L. Graham and A. T. Stewart, Can. J. Phys. **32**, 678 (1954).
G57 Goldhaber, Grodzins, and Sunyar, Phys. Rev. **106**, 826 (1957).
GB57 R. E. Green and R. E. Bell, Can. J. Phys. **35**, 398 (1957).
G58 Goldhaber, Grodzins, and Sunyar, Phys. Rev. **109**, 1015 (1958).
H56 M. Heinberg and L. A. Page, Bull. Am. Phys. Soc. Ser. II, **I**, 168 (1956).
Ha57 S. S. Hanna and R. S. Preston, Phys. Rev. **106**, 1363 (1957).
Hb57 S. S. Hanna and R. S. Preston, Phys. Rev. **108**, 160 (1957).
He57 M. Heinberg and L. A. Page, Phys. Rev. **107**, 1589 (1957).
Ha58 S. S. Hanna and R. S. Preston, Phys. Rev. **109**, 716 (1958).
J57 Jackson, Treiman, and Wyld, Phys. Rev. **106**, 517 (1957).
L57 Lundby, Patro, and Stroot, Nuovo cimento **6**, 745 (1957).
La57 G. Lang and S. DeBenedetti, Phys. Rev. **108**, 914 (1957).
M29 N. F. Mott, Proc. Roy. Soc. (London) **A124**, 425 (1929).
M32 C. Møller, Ann. Physik **14**, 568 (1932).
M55 H. Mendlowitz and K. M. Case, Phys. Rev. **97**, 33 (1955).
M57 K. McVoy, Phys. Rev. **106**, 828 (1957).
Ma58 Macq, Crowe, and Haddock, Conference on Weak Interactions, Gatlinburg (1958).
M58 I. Marklund and L. A. Page, Nuclear Phys. **9**, 88 (1958).
Mc58 K. McVoy, Phys. Rev. **110**, 1484 (1958).
P51 L. A. Page, Phys. Rev. **81**, 1062 (1951).
P54 T. A. Pond, Phys. Rev. **93**, 478 (1954).
P55 Page, Heinberg, Wallace, and Trout, Phys. Rev. **98**, 206 (1955).
Pa55 Page, Heinberg, Wallace, and Trout, Phys. Rev. **99**, 665(A) (1955).
PH56 L. A. Page and M. Heinberg, Phys. Rev. **102**, 1545 (1956).
P57 L. A. Page, Phys. Rev. **106**, 394 (1957).
PH57 L. A. Page and M. Heinberg, Phys. Rev. **106**, 1220 (1957).
Po57 T. A. Pond, postdeadline paper, Washington American Physical Society Meeting (1957).
PO57 L. A. Page and F. E. Obenshain, Bull. Am. Phys. Soc. Ser. II, **2**, 260 (1957).
P58 L. A. Page, Phys. Rev. **109**, 2215 (1958).
PO58 L. A. Page and F. E. Obenshain, final O. O. R. report, University of Pittsburgh (November, 1958) (unpublished).
S55 A. T. Stewart, Phys. Rev. **99**, 594 (1955).
S56 N. Sherman, Phys. Rev. **103**, 1601 (1956).
S57 H. Schopper, Phil. Mag. **2**, 710 (1957).
S58 H. Schopper, Nuclear Instr. **3**, 158 (1958).
St58 P. Stehle, Phys. Rev. **110**, 1458 (1958).
T51 H. A. Tolhoek and S. R. DeGroot, Physica **17**, 1 (1951).
T56 Telegdi, Sens, Yovanovitch, and Warshaw, Phys. Rev. **104**, 867 (1956).
T57 G. Trumpy, Nuclear Phys. **2**, 664 (1957).
Ta57 L. J. Tassie, Phys. Rev. **107**, 1452 (1957).
W55 Wheatley, Huiskamp, Diddens, Steenland, and Tolhoek, Physica **21**, 841 (1955).
W57 S. D. Warshaw, Phys. Rev. **108**, 713 (1957).
Wu57 Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).
W58 C. S. Wu, *Proceedings of the Rehovoth Conference on Nuclear Structure*, H. J. Lipkin, editor (North-Holland Publishing Company, Amsterdam, 1958), p. 380.
Y50 C. N. Yang, Phys. Rev. **77**, 242 (1950).