Astronomical Time

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INTRODUCTION

IT is well known by this time that the operational approach to science has been exceedingly fruitful, the special theory of relativity being perhaps the outstanding example of a contribution to knowledge that resulted from consideration of the actual operations performed by scientists. It is my purpose here to try to describe the determination of time in terms of the operations involved, and to do it in language intelligible to physicists. The attempt has not been made before, so far as I am aware. The novelty of the approach makes it somewhat difficult to select the topics to be discussed, and to decide what degree of refinement is desirable. Readers who find the going difficult may be helped by reference to two earlier articles on the same subject (Clemence, 1952, 1956),* in which a somewhat different point of view is taken.

I hope that the present attempt may conduce to a more general understanding of the special theory of relativity, which appears to hold some mysteries still for about half the persons who write about it. That this statement is true is demonstrated by the frequent appearance in scientific journals of articles in pairs, where the two authors take opposite sides. In many of these articles the issue involves something that is supposed to be done with a clock, and it is evident that at least one of the authors has no clear idea of what a clock is.

The operational approach is, in practice if not in definition, to some extent a matter of degree. We may describe operations that actually have been performed, or operations that could easily be performed but have not been, or operations that could be performed only with very great effort, or operations that cannot be performed. Among the last we may further distinguish operations that may sometime become possible and those that will not. At one end of the scale we are always sure of our ground but find it difficult to extend our horizon; at the other the horizon becomes boundless but there is nowhere to stand. It is at the latter end that many of the silly questions of students, as well as most of the famous paradoxes of science, originate. The examples of the earth that is supposed to stop rotating and of Maxwell's demon come readily to mind. In my opinion it is better, when introducing a novel subject, to stick pretty closely to the facts; and that is what I shall do here. In the interest of clarity I shall, however, take the liberty of simplifying the actual operations greatly. For example, no mention will be made of the moon as a clock, although it is an indispensable means of subdividing and refining the measurements of the revolution of the earth around the sun. The various noninertial frames of reference that are in use merely to serve the convenience of observers, will not be described. The rotation of the earth, although it served as the fundamental standard of time until a few years ago, and is still extensively used as an adjunct to the new fundamental standard, will be passed over with slight mention. My aim is to describe only the basic concepts in the simplest possible operational terms.

* See the list of references at the end of the paper.

TIME AND TIME INTERVALS

The measurement of time is, basically, the counting of the repetitions of a recurring phenomenon. No measuring rod is required. The measurement of lengths always involves the comparison of two lengths: the length to be measured and the unit of length, whereas the measurement of time requires no such comparison, but merely the counting of the units themselves as they recur. The length of any object is obtained by counting the number of times that the unit of length is contained in it, whereas the time of any event is given by stating the actual count of units of time that has been arrived at when the event is observed to occur. Thus, a determination of time always involves a judgment of simultaneity.

While in principle the measurement of time involves only counting, in practice measurements of lengths are introduced in connection with determinations of time for two different purposes. One is as an aid in the subdivision of the unit of time into parts too small to be immediately perceptible to the senses. The other is as an aid in fixing an epoch with the utmost precision. In neither case is the fundamental principle affected. Probably measurements of lengths are introduced for convenience rather than necessity; microscopes have reached a higher state of development than magnifiers of time intervals. The latter are, however, used to some extent. One method of timing solar eclipses is by taking motion pictures at high speed simultaneously of the sun and a clock, the judgment of simultaneity being made by identifying the first frame on which no portion of the sun appears.

The origin of time, that is, the instant when the count of units is supposed to have begun, is called the fundamental epoch. It is completely at our choice, and is determined by custom and consent. It is common practice to count years from the beginning of the Christian era, months from the (arbitrary) beginning of the year, days from the beginning of the month, and hours, minutes, and seconds similarly. The vagaries of the calendar are due chiefly to the incommensurability between the day and the year.

Just as the meter sticks in common use are more or less accurate copies of the standard meter, so our ordinary clocks may be regarded as more or less accurate copies of the standard of time.

If we define an epoch as any specified instant of time, then the duration between two distinct epochs is an interval of time. Whenever the time at which an event occurred is stated, an interval of time is implied: the interval between the fundamental epoch and the epoch of the event. In an important sense a determination of time is a determination of the interval of time since the fundamental epoch. So it may be said that we do not measure time but only intervals of time. The important distinction is that when the time of an event is stated, the other end of the interval (the fundamental epoch) is known, whereas when an interval of time is stated its place in the scale of time remains indeterminate. For example, if I say that I visited a friend for half an hour, less information is conveyed than if I say that I visited him from 8:00 until 8:30 P.M. If, in addition, I state the day, month, and year, nothing further can be said about the time of the visit.

VERIFICATION OF EXPERIMENTS

In the making of many experiments, particularly those involving the determination of the rate of change of physical magnitudes with the time, the physicist imagines that he has no need of a clock; a stop watch seems to serve him equally well or better. What he requires is a known interval of time, which need not be fitted in its place within the entire scale of time. The reason he has no interest in the whole scale of time appears to be that he is accustomed to take his intervals of time on faith from the astronomer; experience has shown that the intervals of time so obtained are much more accurate than any measurement the physicist usually makes. Very likely it is for this reason that the time is usually regarded as the independent variable whenever relations are established between it and other variables. But there is no basic reason why determinations of time intervals should be more accurate than other measurements, and in fact they may not always be so. This fact has an important bearing on the verification of experiments. Evidently an experiment in which a time interval plays an important part cannot be said to be completely verified unless the time interval itself is verified.

It is well known that the difference between a scientific experiment and a nonscientific one is that the scientific experiment may be verified, the verification nearly always being made at a different place and time. In verifying the length of a solid object it is usually considered sufficient to measure it a second time, of course with suitable controls of temperature, errors of measurement, and other sorts of variables. An important question to ask is how an interval of time may be verified. Once it is past, the same interval cannot again be measured. How then can we be sure that a second measured with a stop watch today bears any known relation with a second measured fifty years ago? A partial answer to the question is that most of our experiments today give results consistent with those of fifty years ago. But a more precise answer is possible. Our confidence in our determinations of time, and of intervals of time, is determined solely by our ability to calculate the time of past and future events, and to confirm the calculations by comparing them with actual observations or with records of observations. I refer of course to events that are not controlled by man. The events most useful for the purpose have been those the time of which can be calculated by the laws of celestial mechanics, most notably solar eclipses and occultations of stars by the moon. It should be noticed that when the observation of an event is used to verify our determinations of time (and intervals of time) we are making the time depend on the event and not the event on the time; in a sense the time becomes the dependent variable instead of the independent one. Of course the true state of affairs is that an equation expresses a relation between two or more variables; none of them is the independent variable in any absolute sense, but each is regarded sometimes as independent and sometimes as dependent, according to the nature of the operations being performed.

Returning to the verification of experiments, let us imagine a class of experiments in which a time interval plays an important part, as for example the measurement of velocities or accelerations, and in which a variable may be measured with a precision comparable to that with which the interval of time involved is known. Let us imagine further that two experiments performed at different times give inconsistent results. It is evident that the question arises whether it is the physical measurement or the time interval that is at fault. If it is possible to fit the time interval into its place in the whole scale of time, which is already known to be consistent with an important class of experiments (the observation of astronomical phenomena), that is, if we know the actual times between which the experiment was conducted, then the time interval may be said to be confirmed.

Our scale of time, then, together with the laws of celestial mechanics, is basically the means by which we know, both in advance and in retrospect, when astronomical phenomena may be observed. The comparison of a prediction with an observation is commonly called a comparison between theory and observation. Among the great number of sorts of astronomical phenomena that may be observed there are several discrepancies between theory and observation. In all such cases that I know of, the reason is simply that the calculations are so difficult as to be beyond our powers. The most notable example is the rotation of the earth, which is not uniform, but is subject to accelerations of apparently random nature, such that if two different intervals of five years are chosen at random, the average duration of a day in one of them is likely to differ from the duration in the other by a few parts in 10^9 (Brouwer, 1952). Presumably the cause is to be found in the interior of the earth; its nature is not well understood.

It was stated that the measurement of time is basically the counting of the repetitions of a recurring phenomenon. In the case of certain phenomena the statement is clear enough; for example there is no difficulty in counting the ticks of an ordinary watch or the luminous periods of a flashing electric sign, and thus arriving at a measurement of time. Such measurements are not very useful, because they are not sufficiently long continued nor sufficiently accessible to all. What is required for a fundamental measurement of time is a phenomenon that can be observed by anyone who provides himself with the necessary equipment, and that will give results consistent with all observable phenomena. The phenomena actually used for measuring time are the revolutions of the earth around the sun. It is not obvious how revolutions of the earth are to be counted, and it is best to approach the subject in easy stages, by first considering how we may count rotations of the earth.

ROTATION OF THE EARTH

In a first approximation all that is necessary is to mount a telescope vertically, which can be done with the aid of a plumb bob, to provide a fine wire stretched north and south in the focal plane, and to note the instant when the image of any star as it passes through the field of view is quenched by the wire. The period of the rotation of the earth may be defined in a first approximation as the time interval between two successive passages of the same star. The purpose of the telescope is principally to constrain the observer's vision along a line of sight that is fixed with respect to the earth.

This crude procedure, if the observer is within say 40 degrees of the equator of the earth, gives a precision of about a part in 10^6, most of the error being made by the observer. By providing him with mechanical aids the error is reduced to a part in 10^7. In order to obtain still higher accuracy it is necessary to take account of the changes in the direction of the axis of rotation of the earth relative to the star. In the language of Newtonian mechanics, these changes are caused by the gravitational action of the moon and sun on the equatorial bulge of the earth, the moon and sun not moving in the plane of the equator of the earth. I shall pass over the operations by which the direction of the axis of rotation is calculated, because they do not differ in principle from the operations connected with the motions of planets around the sun, which will be treated at some length later. After the motion of the axis of rotation is allowed for, it is found that the next most important source of error is the accidental errors of observation. It is practicable, by observing many stars instead of one, to reduce the effect of the accidental errors by about a factor of 10, and thus the period of one rotation of the earth becomes measurable to a few parts in 10^8. At this stage the remaining error is not predominantly due to one cause, but to a combination of several. Among them are the individual motions of the stars relative to one another, variations in the direction of the axis of rotation relative to the observer, and the remaining accidental errors of observation. By means of statistical analyses of stellar motions and special series of observations the error may be still further reduced to a part in 10^8. Beyond this it does not seem necessary to go, because the variations in the period of rotation, mentioned earlier, set a working limit on the usefulness of the applications.

Before leaving the subject of the rotation of the earth, a few remarks about the frame of reference are in order. The frame of reference implied by what has already been said is the system of stars that are observed. Since each star is moving relative to the others, there is no reason to suppose that the mean motion is zero. It is more likely, *a priori*, that there is a small residual rotation of the stars as a system, and indeed it turns out to be so. Two methods are known for determining the residual rotation: the kinematic method and the dynamical one.

The kinematic method depends on extending the frame of reference outward from the earth to the most distant visible objects, the extragalactic nebulae (Wright, 1950). If the red shifts in nebular spectra are interpreted as velocities of recession, then the more distant of the nebulae that can be well observed are receding from us at very high velocities, more than 10 percent of the velocity of light. The velocities across the line of sight are entirely unknown. But if we assume that the cross-velocities are of the same order of magnitude as the velocity of recession, then, by reason of their very great distance, the *angular* velocity across the line of sight is negligibly small for many centuries. The stellar motions are so small that several decades elapse before they can be satisfactorily measured with respect to the nebulae; the work is now in progress. The indications are that the uncertainty in the present frame of reference sets a limit on the precision of time determinations such that the duration of a day cannot be measured with a precision greater than a part in 2×10^{15} at present. It would appear that this very high precision does not constitute an effective limitation on any experiments, but such a conclusion would be false. The effective precision with which the motion of the planets around the sun, or the moon around the earth, have been observed is about 0.1 second of arc per century, which corresponds precisely to the precision with which the frame of reference is known. The observations of the present day are more accurate than those of a hundred years ago, and so an improvement in the frame of reference is desirable, at least in connection with studies of celestial motions. The dynamical method of establishing a frame of reference will be described later, in connection with the determination of time from planetary motions, a subject to which we now turn.

THE SECOND

For half a century astronomers have found that the rotation of the earth does not measure time with sufficient precision for their purposes. During much of this time there has been some confusion. There was natural reluctance to give up a measure of time that had been regarded as fundamental since time immemorial. Indeed, the day had become so firmly fixed in mind as an invariable unit of time that only a daring thinker could imagine any alternative. At last, however, the work of E. W. Brown (1926), W. de Sitter (1927), and H. Spencer Jones (1939) in succession proved beyond a doubt that it was necessary to relinquish the day in favor of a unit of time that would better correlate observable phenomena. By now the realization of the facts has become sufficiently general that the definition of the second has recently been changed by international authority. Whereas formerly the second was defined as 1/86 400 of a mean solar day, it is now 1/31 556 925.974 7 of the tropical year for 1900 January 0 at 12 h. ephemeris time.

The effect of the new definition is to make the second equal to the average of the old one during the 18th and 19th centuries. Whereas time reckoned in the old seconds was called mean solar time (universal time if reckoned from midnight on the meridian of Greenwich), time reckoned in the new seconds is called ephemeris time. The difference between the two has now accumulated to an amount such that a clock keeping ephemeris time would be about 30 seconds fast on a clock keeping universal time. The difference will not cause any confusion, because our ordinary clocks will continue to keep universal time as before, and when ephemeris time is desired, the necessary corrections will be applied in the calculations. Frequency standards, on the other hand, will be operated on the new time; inasmuch as the time of day cannot be obtained from a measurement of frequency, no confusion can possibly arise. An alternative possibility would have been actually to have altered our clocks to ephemeris time, but this procedure would have worked a hardship on navigators and surveyors, who must determine time in field operations in order to find their longitude; they have no means of determining ephemeris time, and it is better to throw the hardship on persons who have the means of coping with it.

There are two important differences between the new and old definitions of the second. The specification of a particular instant of time in the definition requires us to know at all times the relation between the present tropical year and the one for 1900.0. The other difference is that the tropical year, while it is in a sense the average duration of a revolution of the earth around the sun in a specified frame of reference, is not equal to the duration of any particular revolution; in fact one particular revolution may differ from another by as much as 20 minutes, or a part in 3×10^4. The mean solar day, on the other hand, is precisely equal to any rotation of the earth in its specified frame of reference.

REVOLUTION OF THE EARTH

Before considering what is actually counted in determining ephemeris time, let us pause to examine how a particular revolution of the earth may be observed without recourse to any theory. The annual revolution of the earth around the sun produces an apparent motion of the sun against the background of stars. I shall ask the reader to admit without proof that the angular position of the sun with respect to the stars in the nearby background can be ascertained with high precision at any instant; the observational techniques by which it is done are too complex to describe here, but they do not involve any new principle. If the apparent path of the sun among the stars is traced out with care for some years, it will be found that the path is very nearly a great circle on the apparent celestial sphere. The great circle is slowly rotating about one of its diameters, at the rate of about 47 seconds of arc in a century, and the maximum departure of the sun from the great circle is less than a second of arc. Both the rotation of the great circle and the departures of the sun from it are caused, again in the language of Newtonian mechanics, by the gravitational action of the other planets on the earth, Venus and Jupiter predominating. It is easy to choose a star not far from the axis of rotation of the great circle, and to count successive passages of the sun by this star. The frame of reference is the same as was used for counting rotations of the earth, and it may be refined and made more precise in the same way.

EQUATIONS OF MOTION

In the actual measurement of ephemeris time it is necessary to have recourse to the equations of planetary motion. Stated as concisely as possible, the method is as follows. The solution of the equations gives the angular position of the sun among the stars as a function of the ephemeris time. The solution is then inverted, so as to give the time as a function of the position of the sun. An observation of the sun is then used to establish the time at the instant of observation.

The equations of planetary motion employed are the equations of general relativity, but not in the form that they take most naturally. For numerical applications it is easier to begin with the Newtonian form of the equations, and to express the difference between the two as a power series, the successive terms of which contain increasing powers of the velocity of light in the denominator. It is found in practice that only the first term of the power series is necessary in order to obtain a solution comparable in precision with the observations (Eddington, 1924; Chazy, 1928, 1930).

The equations of motion take many different forms according to the system of coordinates employed. Let us fix our attention on the equation for the orbital longitude, which is the angular distance of the sun, reckoned in the direction of motion from an origin to be described later, in the slowly rotating great circle before mentioned. Denoting the orbital longitude by L, we may put the solution of the equation in the form

$$L=C+Nt+St^2+E\sin[C-P+(N-R)t] + \text{periodic terms.}$$

In this equation C, N, E, P are four constants of integration, which must be determined by observations of the sun. The constant C is the value of L, diminished by all periodic terms, at the fundamental epoch, say the beginning of the year 1900, from which the ephemeris time t is reckoned positively and negatively. The constant angle N is called the mean motion; evidently its numerical value depends on the unit in which t is reckoned. The unit of t might be chosen so as to make N unity, but it is preferable for practical reasons to connect t with the rotation of the earth, so that ephemeris time will not differ greatly from mean solar time. In the 19th century, when the speed of rotation of the earth was supposed to be constant, the unit of t was taken to be precisely 365.25 rotations, and N was determined by comparing the equation with observations during the 18th and 19th centuries. We still retain the same numerical value for N, whence it follows that the unit of t is 365.25 *average* rotations of the earth during the centuries mentioned, and N is nearly but not quite 2π radians. The remaining two constants of integration, E and P, do not require discussion. The constant S as it comes out in the calculations is about 10^{-11}; it is important to notice that although it has the form of an acceleration, it is not to be physically interpreted as such. Its presence is due merely to the form of the solution; other forms are known in which S vanishes, but those forms are associated with very difficult calculations, and they have not been used in practice. The periodic terms, a few hundred in number, are all of the form

$$k\sin(nt+K),$$

where k, n, and K are constants obtained during the course of the solution, k ranging up to about 10^{-4} and the periods varying between about a month and 1800 years. Finally, the constant R, according to Newtonian theory, is about 3×10^{-4}, which is increased a part in 1500 under general relativity. The small increase in R is the only difference between the solutions as given by Newtonian mechanics and by general relativity.

Having obtained the solution it is easy to make a table, or ephemeris, showing values of t and of L in parallel columns. An observation of L may then be used to determine t by interpolating in the table. It can now be seen that what is actually being counted, with the aid of the table, are units of t, which can be translated into ephemeris seconds using the new definition of the second already given.

So far nothing has been said to justify use of general relativity in preference to Newtonian mechanics. If observations of the sun were the only means of determining ephemeris time we should have no operational reason (so far as the measurement of time is concerned) for preferring one to the other. In fact, however, equations similar to the one already given may be calculated for the other planets, the moon, and the principal satellites of Jupiter, and the ephemeris time determined from observations of any of these objects. When this is done it is found that the observations of Mercury and Mars cannot possibly be reconciled with each other without introducing the modified R required by general relativity, or some modification of very nearly the same amount. The same conclusion is indicated, though with less certainty, by observations of Venus and the sun. The other objects are less affected by changes in R, or are observed with less accuracy, and no conclusions can be drawn from them.

FRAME OF REFERENCE

The origin from which L is reckoned may be established kinematically, as was described in connection with the rotation of the earth. But it may also be established dynamically, by comparisons of ephemeris time determined by different planets (Clemence, 1951). To understand how it is done let us imagine that our adopted frame of reference, including the origin from which L is reckoned, is slowly rotating about some axis. In general the rotation will have a component along the great circle in which L is measured, and this component will be absorbed in the constant of integration N, and also in $N-R$. A small change in $N-R$ produces in L a periodic oscillation with period $N-R$, phase a quarter-period different from that of the term with coefficient E, and amplitude proportional to t. Since the oscillation is present in L it is also present in the ephemeris time, which is determined from observations of L, but must remain undetected as long as the ephemeris time is determined from observations of the sun alone. Let it now be supposed that the ephemeris time is also determined by observing the L of another planet, which is reckoned in nearly the same plane as the L of the sun. The values of C, P, and N being different for the sun and the planet, the oscillation in the ephemeris time has a different phase and period, and if the ephemeris times from the sun and planet are compared, there will be found to be an oscillating discrepancy between them. It will be found possible to remove the discrepancy by adjusting the two values of N by the same small amount, which is equivalent to altering the frame of reference previously adopted. Summing up the whole matter in a few words, the dynamical frame of reference is specified by the condition that observations of different celestial bodies shall yield the same ephemeris time.

Since the equations of motion give the values of the second derivatives of the coordinates with respect to the time, the periodic oscillations just referred to are the doubly integrated effects of periodic accelerations. So the dynamical frame of reference may also be defined operationally as that frame in which the planets exhibit no periodic accelerations not given by the equations of motion.

It may be instructive to inquire how the Newtonian frame of reference differs from the frame of general relativity, since it is not immediately evident how the modification of the N for different planets can be distinguished from the modification of the Newtonian value of R required by general relativity. The answer is that different modifications of R are required for the different planets, these modifications being given by general relativity itself, whereas one and the same modification of N is required for all the planets, this modification being given by a comparison between observation and theory. Thus, if it is attempted to establish a dynamical frame of reference using the Newtonian equations of motion, it will be found that a different frame is required for every planet.

There is nothing in the operations that obviously requires the dynamical frame of general relativity to be identical with the kinematical frame discussed earlier. It turns out experimentally that if there is any rotation of one with respect to the other, the integrated effect during a century is less than 0.1 second of arc. The precision of observations and the difficulty of the calculations do not permit us to say more at present, but it is expected within another decade or two to reach another decimal of a second.

Throughout the discussion of the determination of ephemeris time I have been careful to avoid any mention of measurement of lengths, in order to show that it need not be introduced. Indeed it need not. Rectangular coordinates are introduced only as a computational convenience. Ephemeris time could in fact be determined by observations of the sun and moon alone, completely neglecting the circumstance that they are moving during the interval while light is coming to us from them, the effects of the motions being absorbed in the constants of integration for the most part. Residual errors would appear as periodic discrepancies between observation and theory, which would be too small to be detected with certainty. For the general correlation of astronomical phenomena it is, however, necessary to make allowance for the finite velocity of light, and for convenience it is done in all of our theory. Since the procedure is not germane to the determination of time I shall not describe it here. It is pertinent to remark only that the distances are not measured directly but inferred from the angular velocities, that the unit of distance is completely arbitrary, and that the interval of time required for light to traverse the unit of distance may be determined with sufficient precision (three or four significant figures) from observations of the phenomena themselves.

CLOCKS

Let us turn our attention to the definition of a clock. We leave aside the subject of atomic clocks, which although important, is outside the scope of this article, and confine attention to astronomical clocks. Evidently in common practice a clock is a mechanism for measuring time that is approximately synchronized with mean solar time, that is, with the rotation of the earth. But this definition is not suitable for precise applications, because as we have seen, the rotation of the earth is subject to unpredictable variations. It seems to me that a clock must be defined as a mechanism for measuring time that is continually synchronized as nearly as may be with ephemeris time. This definition implies an observer who initiates the synchronization, and the observer must be located on the earth, since ephemeris time is defined only for an observer on the earth. We may imagine the clock to be transported anywhere we like, either on or off the earth, but we must always imagine as well that there is an observer who remains on the earth and synchronizes the clock with ephemeris time, either by means of light signals, or in some other specified manner. Perhaps it may be possible to extend the definition of ephemeris time, so as to render it determinable by an observer not on the earth; the definition would then lose something of its operational quality, because the operations described would not be those actually performed, but those that are imagined to be performed, which is not quite the same thing.

PRECISION OF TIME INTERVALS

The relative precision with which an interval of ephemeris time can be determined depends partly on the precision of the man-made clocks that are used to interpolate between astronomical observations and to smooth out to some extent the accidental errors of the observations, partly on the precision of the astronomical observations, and partly on the time itself, since the precision of clocks and of observations have been continually increasing for several centuries. Also it is necessary to distinguish between the precision attained in ordinary practice, and that attained when an extraordinary effort is made. Furthermore, a time interval can always be stated more precisely some time after it is past than it can be immediately after the measurement is made, because interpolations of astronomical observations are more precise than extrapolations; for this reason the principal time services of the world issue bulletins, which are always some weeks or months in arrear, giving revised estimates of the times at which their radio signals were propagated, and the Bureau International de l'Heure issues, a year or so later, still more accurate estimates. In view of all these considerations no statement about the precision of time intervals can be very simple and at the same time very meaningful. It is nevertheless possible to give some understanding of the subject in a few words.

Intervals of time varying from very short ones, say 10^{-9} second, to those of moderate length, say 10^6 seconds, can be determined with moderate effort to a precision of a part in 10^8; longer intervals, of about 10^7 seconds, to a precision of a part in 10^9; and still longer ones, of 10^8 seconds or a little longer, to a precision of a part in 10^{10}. The longest interval that can be said to have been actually measured is the interval from some ancient eclipses of the sun to the present time, about 10^{11} seconds, which is probably known with a precision of a part in 10^8; the uncertainty is mainly in the interpretation of the ancient records.

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