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Cosmogonical Problems and Stellar Energy

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INTRODUCTION

DURING the last decade the influence of nuclear physics on many branches of science has been very large. Because of the war and postwar circumstances, communications between scientists in different countries have been rather bad so that it seems not superfluous to try to give a survey of the work which has been done in those branches of astrophysics which have mostly profited from nuclear physics during the last ten years by workers in this field in Europe and in America. It can hardly be expected that all important developments will be mentioned even in the restricted field which is reviewed here. However, it is hoped that some work which has not received due consideration because of bad communications will find a wider audience.

The present survey is restricted as far as possible to two related subjects, indicated in the title of the paper. The first subject is cosmogony in its widest sense. We shall first review recent developments in general cosmogony. After that we wish to discuss three fields which are part of a general cosmogony. The first is the discussion of the origin of planetary systems, the second the discussion of the origin of the chemical elements, and the last one a discussion of the origin of cosmic rays. The second subject especially has suffered from lack of communication. One smaller detail, the influence of the statistical weight of nuclear levels, was, e.g., discussed independently by authors in Belgium, Germany and Sweden, who all arrived at the same conclusion (Section 4, α).*

The last section of the present paper will be devoted to a brief discussion of problems connected with the production of energy in stars. Although this subject is mainly cosmology, it is also closely related to cosmogony since it puts immediately the question why there is such a great variety of stars, and the origin and development of these stars is certainly part of a general cosmogony.

Before starting the discussion of these various subjects, it must be remarked that throughout it will be attempted to draw the reader's attention to the unsolved problems in these fields and to the difficulties which have to be overcome, rather than to give a closed survey. The present-day tools of physics are becoming so powerful that it is very well conceivable that many of the as-yet unsolved problems can be solved within the next few years, or perhaps are already solved when the present paper appears in print.

The present paper covers in many respects the same subjects as the survey articles of von der Pahlen (45P1), Couderc (47C), and Unsöld (48U).** However, in many respects the present paper covers more ground, and also it may be useful to repeat some considerations because these survey papers may not be widely available.

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* For a more extensive discussion, we refer the reader to a forthcoming monograph (50H2).

** References are placed at the end of the paper. Each reference number consists of the year of publication and the first letter of the first author's surname; e.g., P. Couderc, Rev. Scientifique 85, 289 (1947) is referred to as 47C.

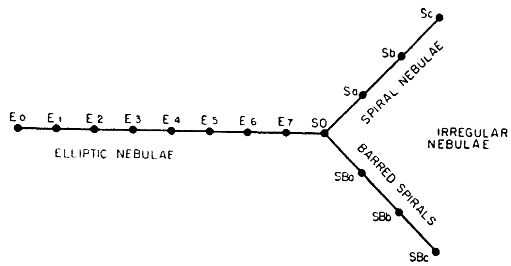


FIG. 1. Classification of extragalactic nebulae.

I should like to express my sincere thanks to Professors W. Baade, S. Chandrasekhar, G. Gamow, G. P. Kuiper, and E. Teller for putting at my disposal material which is as yet unpublished. My special thanks are due Professors S. Chandrasekhar and G. P. Kuiper for their willingness to discuss the subject matter of this paper and for their helpful criticism and suggestions.

1. SURVEY OF OBSERVATIONAL DATA†

Before starting the main discussion of the present paper, it may be well to review briefly the observational data which are relevant to this discussion. The survey of observational data which we wish to give here concerns mainly the various forms of galaxies and the various types of stars.

(α) Galaxies; Clusters of Galaxies

Our Milky Way is one of the many concentrations of stars in the universe known as galaxies. Hubble has shown that it is possible to classify these galaxies, which are sometimes referred to as extragalactic nebulae, according to their form. (See Figs. 1 and 2.) This classification starts with the elliptic nebulae, ranging from E0 to E7, where the E0 nebulae are the spherical nebulae while the E7 nebulae have a ratio 1:3 for their projected axes. After the elliptic nebulae, we have the spiral nebulae which fall into two classes: normal spirals and barred spirals. Normal spirals have spiral arms starting from the center of the nebula, while the spiral arms of the barred spirals start at the end of a straight "bar" which extends across the nucleus of the nebula. The normal spirals range from Sa to Sc, and the barred spirals from SBa to SBc. The opening of the spiral arms increases from a to c in both series. The last class of nebulae contains the irregular nebulae. Our Milky Way is a spiral nebula; Baade (44B2) gives Sb as its classification. It may be mentioned here en passant that our sun is situated far outside the center of the Milky Way (at a distance of about 30,000 light years) which is an extremely happy circumstance: If we were situated in the center of the Milky Way, the whole sky would be between 10 and 100 times as bright as the sky at full moon, which would seriously interfere with astronomical observations in the photographic region.

† We have for this section used extensively the textbook of Russell, Dugan, and Stewart (27R), and the survey papers of von der Pahlen (45P1, 47P) apart from other material quoted in the text.

Sometimes the elliptic nebulae are called early type galaxies and the spirals and irregular nebulae late type galaxies, since it was assumed that galaxies would develop from E0 via E7 to Sc or SBc types. Recent considerations (see e.g., Section 2b) have, however, made it probable that the age of galaxies measured in a suitably defined way is less for the late type than for the early type galaxies.

It is at present known that galaxies can appear in clusters, just like stars (see β). For instance, our Milky Way is a member of the so-called local group of which Baade (44B2) listed 13 members. This local group contains three spirals (the Milky Way, the Andromeda nebula (M31), and Messier 33), six elliptic nebulae, and four irregular nebulae, among which are the two Magellanic clouds.

(β) Stars, Star Clusters, Galactic Nebulae

It is usual to classify stars in the so-called Hertzsprung-Russell diagram (Fig. 3) which plots spectral class *versus* luminosity. The spectral classes extend from stars with high surface temperatures to stars with low surface temperatures. The spectral classes are denoted by letters: O, B, A, F, G, K, M, $\cdots$ the order of which has grown historically. In each class there is a subdivision, indicated by a number, ranging from 0 to 9, a B5 star having a higher surface temperature than a B7 star and so on.

Another classification, introduced by Hertzsprung, is according to luminosity. Stars of high luminosity are called giants and denoted by a g in front of the spectral class, e.g., gK0. Low luminosity stars are called dwarfs and denoted by a d, e.g., the sun: dG2. Stars of extremely high luminosity, the so-called c-stars, are also referred to as supergiants, and denoted by a c, e.g., β Orionis cB8.

If we now put all the stars in the H-R diagram, it turns out that not all regions are equally densely populated. The following regions are densely populated (see Fig. 3):

Region I, the main sequence stars, ranging from dA0 stars of visual magnitude 1.0 to dM8 stars of visual magnitude 16.3 (48K).‡ The sun is situated in the main sequence.

Kuiper has recently given improved statistical data as to the frequency of the various stars. He finds that 95 percent of all the stars are main sequence stars. Of these stars 81 percent are dM, 12 percent dK, 4 percent dG, $2\frac{1}{2}$ percent dF, and $\frac{1}{2}$ percent dA.

It must be remarked here that the conventional H-R diagrams give a very wrong impression of the relative frequencies of the various types of stars. Stars of high luminosity can be seen from very great distances and there are thus more giants than dwarfs which can be

‡ I am very much indebted to Dr. G. P. Kuiper for putting at my disposal the complete text of his talk, only an abstract of which was published.

observed. In order to get the real relative frequencies one has to consider a volume of which one knows practically all the stars. This was first done by Kapteyn at the beginning of this century. The most recent discussion is that of Kuiper (48K): It turns out that the main sequence stars are by far the more frequent.

O-, B-, and A-stars are called early type stars, and G-, K-, and M-stars late type stars. These designations reflect a now abandoned theory of stellar evolution which assumed that stars started out as O- and B-stars and ended up as M-dwarfs.** It has been observed (31S, 33W1, 33W2, 34W) that the rotational velocities of early type stars are far larger on the average than the rotational velocities of late type stars.

Region II contains about one percent of all stars. These stars are from one to four magnitudes fainter than the main sequence stars and are called subdwarfs. Region III contains the so-called white dwarfs which make up about three percent of all stars brighter than absolute magnitude 16. Region IV contains the subgiants and the red giants which together form about one percent of all stars. At the top of region IV we have c-stars or supergiants which are extremely rare (Region VII). Region V contains the variable stars, the best known of which are the Cepheid variables. The period of these variables increases if we go from the left to the right (decreasing surface temperature). It is possible to discern three groups with the characteristics given in Table I (43H).

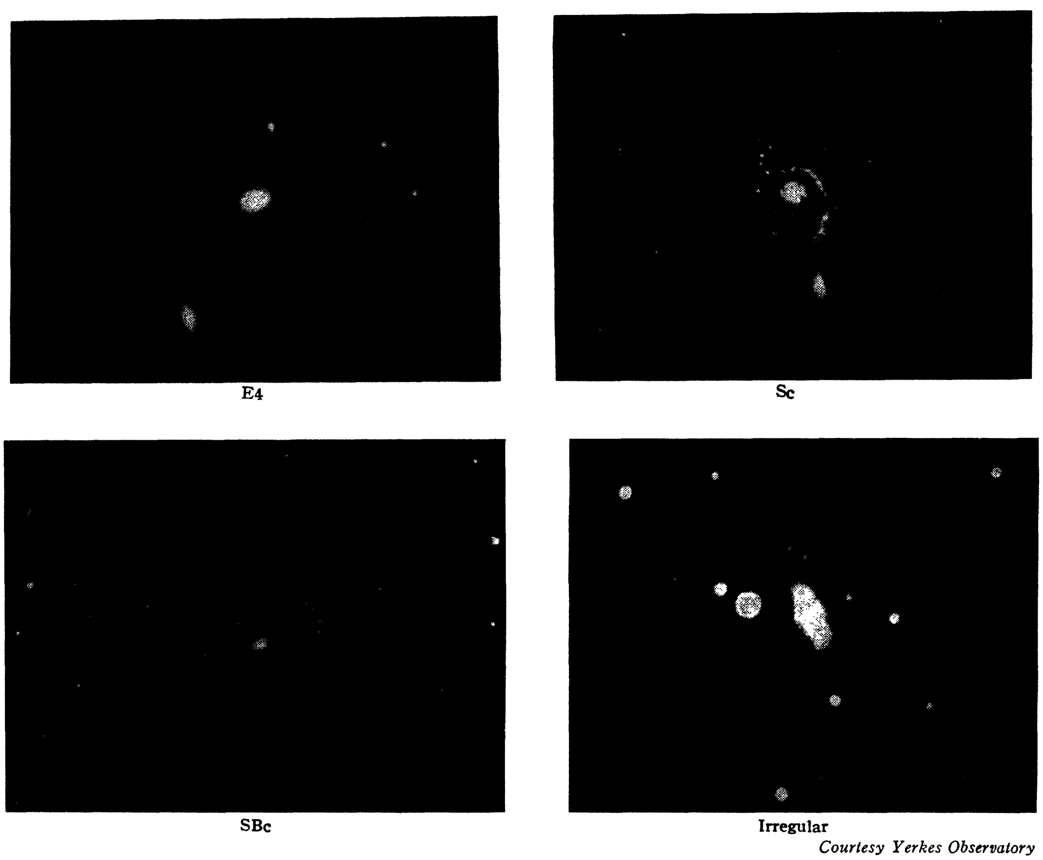
The next property is the period-density relation:

$$P\rho^{\frac{1}{2}}=\text{constant}, \tag{1.1}$$

which is valid over a large range of density values. Then there is the fact that both the luminosity curve and the radial velocity curve are skew; there also is a time lag between the two curves. Also there exists the famous Leavitt relation (08L, 12P) between luminosity and period which has been the basis of distance determination of extragalactic nebulae (13H). This property, together with (1.1), corresponds to the fact that the variables are concentrated in a narrow region in the H-R diagram. Also the form of the light curve is uniquely determined by the period as was stressed by Hertzsprung (26H).

Finally there is the so-called Guthnick effect (38G3, 39G6, 40G2, 42G1, 42G2). This is the fact that variables, after the regular light curve has been interrupted because of some unknown phenomenon, show again exactly the same period, *and the same phase retardation.*

Region VI contains the class of stars which are pre-



Courtesy Yerkes Observatory

FIG. 2. Four examples of extragalactic nebulae.

** See, however, Table V, giving more modern ideas about stellar evolution.

or postnovae. Bertaud (48B1) has given a survey of data about novae, and the pre- and postnova stage of novae. Of the 34 novae listed, at least five are recurrent. It is perhaps not irrelevant that the recurrent novae show all a small change of magnitude at the outburst.

Bertaud arrives at three correlations from his data. The data used are scarce and it seems to us that the first two relations are incorrect; the third one may be correct, but the available data seem insufficient to decide. Bertaud, first of all, finds a correlation between the change in magnitude at the outburst and the rate of decline after the outburst, but this relation does not seem to follow from the data used. For recurrent novae, Bertaud finds a relation between change of magnitude at the outburst and the interval between successive outbursts. Again the material used seems to be insufficient. For instance, T Pyxidis shows four outbursts with the following changes in magnitude: 7.9, 7.2, 6.5, and 6.5 while the interval between the outbursts is steadily declining: outbursts in 1890, 1902, 1920, and 1944. On the other hand, the maxima of U Scorpii are practically constant: 1863, $\Delta M = 9.1$; 1906, $\Delta M = 8.8$; 1936, $\Delta M = 8.8$, but the first interval is much larger than the second one. Finally, Bertaud finds a relation between the absolute magnitude of the nova at maximum and the change of magnitude.

For a theoretical explanation, certainly one of the most important observational data is the place of the pre- and postnovae in the H-R diagram. These stars fall into region VI: Practically all postnovae show an O-type spectrum, at most an early B-type (38H), but the absolute magnitudes are much fainter than for the main sequence stars of the same spectral class. If this is true for postnovae, it should also be true for prenovae (41M, 47C) since only about 0.0002 of the heat content and about 0.00001 of the mass of the star is lost in the outburst.

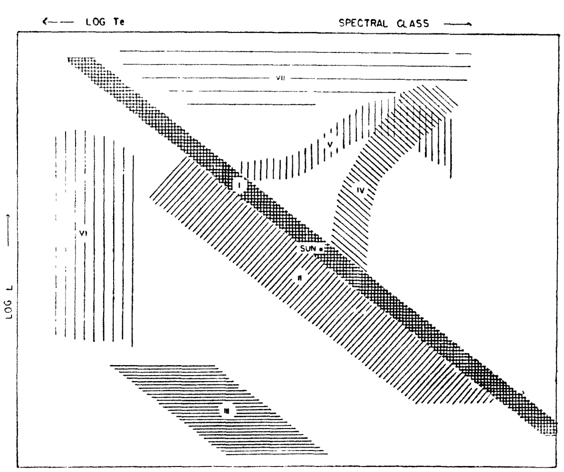


FIG. 3. The Hertzsprung-Russell diagram. Region I: main sequence; region II: subdwarfs; region III: white dwarfs; region IV: subgiants and giants; region V: variables; region VI pre- and postnovae; region VII: supergiants. The tracing is darker for regions which are more densely populated.

Before discussing the observational data about the supernovae, we wish to discuss the division of stars into populations I and II.††

Baade (44B1) showed that all stars fall into two distinct groups which he called type I and type II of stellar populations. Their characterization is given in Table II. In the first row, we have indicated which stars make up the two populations. In the second row where these populations dominate, while in the third row the difference between the mean velocities, first shown by Oort (26O), is indicated. For a more detailed discussion, we refer to Baade's paper (44B1).

It is as yet an open question whether there exists a difference in chemical composition between the two populations. Kuiper (48K) favors a difference while Unsöld (49U) believes that there is as yet no observational evidence for this conclusion.

If we now look at the different groups of stars considered up to this point, we see that region I mainly belongs to population I. Subdwarfs (region II) are high velocity stars and belong to population II, while white dwarfs (region III) occur in both populations: Sirius B is a low velocity star and o_2 Eridani a high velocity star. Region IV belongs partly to population I and partly to population II, while region V is mainly population II. Region VI also belongs to population II. The novae occur with high frequency in the central regions of the Andromeda nebula (pure population II) but are practically absent in the Magellanic clouds (almost pure population I).

Lastly, we shall look briefly into the phenomenon of supernovae. At the outburst between 10^{48} and 10^{49} erg is emitted, and a mass much larger than the solar mass is lost. The frequency of supernova outbursts is per galaxy about one each three or four centuries (42Z2). In our Milky Way there is evidence for the following supernovae: the Chinese nova of 1054 (42B1, 42M2), Tycho Brahe's nova of 1572 (45B), and Kepler's nova of 1604 (43B, 43M).

Finally, we have the division of the supernovae into two groups (44M) as given in Table III.‡‡

The frequency of supernovae II is about six times as great as that of supernovae I (42M2).

From the last row in Table III, it follows that supernovae I are probably connected with population II, and that supernovae II are probably connected with population I. This means also that a connection between supernovae and novae can only be found for novae and supernovae I, but *not* for novae and supernovae II.

We wish to conclude this section about observational data with a short survey of the various larger concentrations of mass in our Milky Way or in other galaxies.

†† I am very much indebted to Dr. W. Baade for putting at my disposal new data about populations I and II and the supernovae. The following paragraphs owe very much to letters from Dr. Baade for which I should like to express here my sincerest thanks.

‡‡ I am indebted to Dr. Baade for Table III.

TABLE I. Characteristics of variable stars.

	Short-period variables	Cepheid variables	Long-period variables
Spectral class	A, B	F, G	M
Period	Less than one day	One week	One year
Luminosity	Moderate	High	Moderate
Concentration toward the galactic plane	No	Yes	No
Close companions	Less than normal	(Less than normal?)	Normal

TABLE II. Characteristics of populations I and II.

	Population I	Population II
Stars	Main sequence stars, giants of not too high luminosity, O- and B-supergiants	Cluster variables (region V), giants up to K-giants of magnitude $M \sim -1.1$, underluminous O- and B-stars
Occurrence	Not in early type nebulae, in arms of late type nebulae; open clusters	Early type nebulae, in center of late type nebulae; globular clusters
Mean velocity of the stars	Low velocity stars	High velocity stars

First of all, there are the galactic or open clusters of stars which contain a few hundred or at most a few thousand stars. The stars in such an open cluster are rather far apart. The globular clusters, on the other hand, contain usually many thousands of stars (at least 50,000) which are very near together. As was noted before (Table II), the globular clusters are supposed to contain population II stars, and the open clusters mostly population I.

Finally, there are the galactic nebulae which can be classified in three groups: planetary nebulae, diffuse nebulae, and dark nebulae.

The planetary nebulae are roundish or oval patches of faint light often showing a good deal of light and almost always there is found a central star in the middle of the planetary nebula. It has often been remarked that there might exist a relation between planetary nebulae and novae (38G2, 47H1) or supernovae (48M3). The planetary nebulae are part of population II, which means that if they are connected with supernovae, they must probably be connected with supernovae of type I.

The diffuse nebulae are irregular light patches. They are illuminated by some neighboring star which has not necessarily any connection with the gaseous system. If the illuminating star is earlier than a B2-type star, the result is an emission nebula. For later type stars, we get a reflexion nebula. Recently it has been found that the T Tauri nebula, though illuminated by a late type star, shows an emission spectrum, probably because gravitational energy is released by falling into the star. An example of a diffuse nebula is the Orion nebula. The irregular Crab nebula belongs probably rather to the planetary nebulae. It is probably the remnant of the gas blown off by the supernova of 1054 (42B1, 42M2). The star illuminating the nebula should be the stellar remnant of the supernova, a faint star with much smaller dimensions than the sun but with a much higher surface temperature (many hundred thousands of degrees).

The dark nebulae, finally, are clouds of obscuring material. They are probably composed to a large extent of smoke particles which obscure the light from far distant stars. Those clouds vary in size but contain usually several solar masses. Their dimensions also vary but are of the order of magnitude of hundreds of light years. For a more complete survey of the state of interstellar gas and smoke clouds, we may refer the reader to the literature (47O1, 48C, 48S6).

2. GENERAL COSMOGONIES

There are two different ways of attacking the problem of general cosmogony. The first one is by assuming that during a certain period τ, which is referred to as the "present epoch" or the "age of the universe," all physical laws were the same as at present and that all "universal" constants such as Planck's constant, the gravitational constant and the velocity of light have not changed during that period. One is then only interested in the developments during the present epoch and supposes all that happened before that period to lie outside the scope of these theories. The second method of approach exists when arguments are given for a systematic change of the "universal constants" and where conclusions are drawn from this assumption. In the following we shall discuss cosmogonies of both kinds after having first discussed various determinations of τ.

(a) The Age of the Universe

In this section we wish to obtain a reliable estimate of the period τ mentioned in the preceding paragraph which will be called here the age of the universe. Not so very long ago there seemed to be two possibilities: the long- and the short-time scale. If the long-time scale is accepted, one assumes that the universe as we observe it today has presented practically the same aspects for a period as long as between five and ten million million years (5 to 10×10^{12} yr.). The short-time scale, however, means that at a moment between 10^9 and 10^{10} yr. ago a fundamental "catastrophe" has taken place. At present, however, every indication seems to be in favor of the short-time scale. We shall briefly discuss the different arguments. For extensive discussions and more complete references we may refer to papers by Chandrasekhar (44C2), Bok (46B), and Unsöld (48U).

The long-time scale was generally accepted up to about 1930 when the idea of the expanding universe forced astronomers to reconsider this matter. Especially Bok's analysis (34B) of the dissolution of galactic clusters showed that Jeans' argument in favor of the long-time scale had to be considered incorrect. The vari-

TABLE III. Characteristics of supernovae.

	Type I	Type II
Mean absolute magnitude at maximum	-14.2	~ -12
Spectrum	None of the broad features identifiable up to now	Spectrum similar to that of ordinary novae, but emission bands are wider (ejection velocities of the order of 5000 km/sec.) and the emission lines indicate a higher state of excitation than that in ordinary novae
Light curves	All following with remarkable fidelity the same pattern (see 43B, 45B)	Great variety; no definite pattern at all
Occurrence	Through the whole sequence of nebular types (E to Sc)	Apparently only in systems in which very luminous stars, particularly B- and O-stars, occur. No SN II has so far been found in an E or Sa nebula or the central regions of a Sb nebula

ous arguments which at present all seem to point to the short-time scale can be divided into several groups: (α) expansion of the universe; dynamics of clusters of galaxies; (β) dynamics of star clusters; (γ) statistics of wide binaries, equipartition of kinetic energy in the Milky Way; (δ) stellar energy; and (ϵ) composition of meteoritic and terrestrial material. We shall now briefly discuss these arguments.

(α) *Expansion of the Universe; Dynamics of Clusters of Galaxies*

If one connects the red shifts of the light from distant galaxies with recessional velocities, and if one assumes that these velocities have been constant since the beginning of the expansion, one arrives at the following estimate of the age of our universe:

$$\tau = 2\times 10^9 \text{ yr.} \tag{2.1}$$

However, this procedure is too much simplified as has been pointed out by Hubble (37H) (compare also 42H1), and it is as yet impossible to arrive at a completely trustworthy estimate of τ by this method. As long as one assumes a homogeneous universe, it seems to be impossible to escape the conclusion that our universe had its smallest extension at a time not much larger than the value given by Eq. (2.1). However, Omer (49O) has shown recently that a non-homogeneous model gives rise to larger values of τ. At present there seems therefore not to be any danger that this method of determining the age of the universe would give results very different from those obtained by other methods which shall be discussed presently. Schatzman (47S1) draws attention to the fact that it is very dangerous not to take into account the variation of the luminosity of far-distant nebulae due to change of luminosity of stars with age.

A second way to arrive at an estimate of τ is by an analysis of the movement of the various galaxies in a cluster of galaxies, such as, e.g., the Virgo cluster. This analysis is similar to that of the movement of stars in star clusters, discussed under β. The evidence from this analysis is far from conclusive. Tuberg (43T) arrives at a value of τ of between 10^9 and 10^{10} yr., but Zwicky (37Z, 42Z1) arrives at an intermediate or even the long time scale from a similar analysis.

(β) *Dynamics of Star Clusters*

A more conclusive way to arrive at an estimate of the lifetime of our galaxy consists of an analysis of loose and dense star clusters.

The loose star clusters are disintegrated by galactic tidal forces caused by the differential rotation of our galaxy. Box (34B) calculated this effect. He showed that the Hyades nucleus of the Taurus cluster will be broken up in about 2×10^9 to 3×10^9 yr. The Ursa Major cluster is already practically disintegrated and will certainly disintegrate within 10^9 yr. Altogether, the lifetime of a loose cluster lies between 10^9 and 10^{10} yr.

The dynamics of dense clusters have been treated extensively by Chandrasekhar (42C1). These clusters disintegrate because the tail of the Maxwellian distribution will evaporate away. Because of mutual interactions, the Maxwellian distribution will be set up again, the high velocity stars will escape, and so on.

Chandrasekhar calculated the lifetime of these clusters, and arrived at the conclusion that a cluster like the Pleiades cluster has a mean life of the order of 3×10^9 yr., whereas globular clusters are much more stable. Their lifetime will be about 10^{12} yr.

Combining these results, we see that most galactic clusters are relatively short-lived organisms. Since, however, there are at least several hundred galactic clusters in our Milky Way, we have to come to the conclusion that our galaxy cannot be much older than a few times 10^9 yr.

(γ) *Statistics of Wide Binaries; Equipartition of Kinetic Energy in the Milky Way*

Chandrasekhar (44C1) has studied the stability of binary systems. Owing to encounters with other stars, they may be dissolved. Chandrasekhar arrives at the following formula for the lifetime of a binary with semimajor axis a, expressed in astronomical units:*

$$T_{\text{bin}} = 2.2\times 10^{15} a^{-\frac{3}{2}} \text{ yr.} \tag{2.2}$$

For $a = 100$ A.U., Eq. (2.2) gives 2×10^{12} yr., for $a = 10{,}000$ A.U.: 2×10^9 yr. We should therefore expect that if our galaxy had existed for more than, say, 10^{11} yr., the distribution of binaries over values of the semimajor axis larger than about 100 A.U. should correspond to a Maxwell-Boltzmann distribution of the total energies.

According to Ambarzumian (36A, 37A2), the fre-

* 1 A.U. $= 1.5\times 10^{13}$ cm = earth's mean distance from the sun.

quency function of the semi-major axes should be in that case:

$$f(a)da = Ca^{\frac{1}{2}}da. \tag{2.3}$$

The observed frequency curve, however, is given by (24Ö):†

$$f(a)da = Ca^{-1}da, \tag{2.4}$$

and we see that the number of wide binaries is much smaller than the number corresponding to an equilibrium situation. As Bok (46B) expresses it: "the dissolution of binary systems has only just started."

The most important argument in favor of the long-time scale was the apparent approach to equipartition of kinetic energy for the stars in the neighborhood of the sun. This argument was advanced by Halm (11H) and Seares (22S). Bok (46B) has, however, pointed out that the approach to equipartition is far from general. He also stresses that there seems to exist a correlation between the physical properties of the stars (mass, spectrum, period of variability) and their dynamical or orbital characteristics in the galaxy. It seems hardly necessary to emphasize how important these correlations may be for cosmogonic studies. The final conclusion at which Bok arrives is that the equipartition argument can no longer be quoted in support of the long time scale.

Recent work of Gondolatsch (48G4) indicates that the distribution of kinetic energy in our galaxy points even to the short-time scale, if analyzed properly. If we plot the mass of the stars against the square of their velocities, the following relation should exist, if there existed equipartition of kinetic energy:‡

$$\log M + \log\langle v^2\rangle = \text{constant}. \tag{2.5}$$

The observed relation, however, is

$$\log M + 0.98 \log\langle v^2\rangle = 2.72, \tag{2.6}$$

and Gondolatsch concludes from this relation to an age of our galaxy of the order of 10^9 yr.

(δ) *Stellar Energy*

It is well known that von Weizsäcker (37W, 38W) and Bethe (39B1) provided us with a solution to the question how main sequence stars could emit energy. We shall return to this question in Section 6. If we accept the carbon-nitrogen cycle as the source of energy, we can estimate an upper limit for the age of a star from its luminosity and its helium content. This has recently been done by Unsöld (48U) in the following way. First of all, Unsöld uses the fact that as far as is known all main sequence stars have about the same helium content (38U, 41U, 44U, 47U). If numbers of atoms are considered, the ratio of hydrogen to helium to other atoms [indicated by R (Russell mixture)] is given by

$$\text{H}:\text{He}:\text{R} = 85:15:0.24. \tag{2.7}$$

If weight percentages are considered, we have

$$\text{H}:\text{He}:\text{R} = 57:40:3. \tag{2.8}$$

If now all helium in the sun is supposed to be produced by the carbon-nitrogen cycle, the maximum age of the sun can be calculated from the sun's present luminosity:

$$T_0 = 0.4M_0c^2\Delta/L_0 = 4\times10^{10} \text{ yr.}, \tag{2.9}$$

where Δ is the part of the mass lost in the production of one helium atom from four hydrogen atoms ($\Delta = 0.007$), M_0 the solar mass ($M_0 = 2\times10^{33}$ g), and L_0 the solar luminosity ($L_0 = 4\times10^{33}$ erg sec.$^{-1}$). The factor 0.4 is due to the fact that the sun contains at present 40 percent helium.

The maximum age T of an arbitrary star follows from equations similar to Eq. (2.9) which can be written in the form:

$$T/T_0 = (M/L)/(M_0/L_0)$$

or

$$\text{Log}T = 10.63 + \text{Log}M - \text{Log}L, \tag{2.10}$$

where M and L are expressed in units M_0 and L_0.

From Eq. (2.10) Table IV has been computed.

In Table IV the luminosity has been calculated from the observed magnitude and the mass from the luminosity by using Eddington's mass luminosity relation in the form given by Kuiper (38K).

At first sight it seems that we cannot arrive at any conclusion as to the age of the universe from Table I. We see, however, that we cannot escape arriving at the conclusion that stars are at present still being created; otherwise we should never observe O- or B-stars. They use up their hydrogen so fast that even if they were born 10^9 yr. ago (which is certainly a lower limit for the age of the universe), they should have burnt out completely.

If we accept, however, the steady creation of stars, then the values of T in Table IV are of no importance.

However, Russell (42R) has shown that it is nevertheless possible to arrive at some definite conclusions against the long time scale by following the evolution of a star like our sun under the assumption that the carbon-nitrogen cycle is at every moment responsible for the energy production. He arrives then at Table V. The last line in Table V gives the period during which the sun would have stayed in that particular situation.

If the Milky Way had existed in its present state for as long a period as 10^{12} yr., we should expect that the numbers of stars with masses equal to the solar mass should be distributed among the spectral types in the second line of Table V in the ratios given by the last line of Table V. But stars of spectral types F4 to B9 with masses nearly equal to the solar mass are by far less

† Compare also (42K).

‡ Logarithms to the basis e are denoted by log; those to the basis 10 by Log.

numerous than according to the above expectations. We are again lead to the short time scale.**

As Bok (46B) has pointed out, the situation will be far more complicated if giants and dwarfs are also taken into consideration.

(ϵ) *Composition of Terrestrial and Meteoritic Material*

As often remarked, the fact that the minimum lifetime of natural isotopes found on the earth is of the order of magnitude of 10^{10} yr. points to a maximum age of the earth of the same order of magnitude. A second way of obtaining an estimate of the age of the earth consists of considering the relative abundances of the various uranium isotopes. Assuming that at the moment that the earth was formed U^{235} and U^{238} were present in about equal amounts, one can easily determine the age of the earth from the known half-lives. One then again arrives at a few times 10^9 yr. Another possibility is to use the fact that the lead isotope produced by radioactive decay of uranium is different from the preponderant normal isotope. In this way it is possible to determine the age of rocks. One arrives then at an upper limit of about 5×10^9 yr. (43K).

The same result has been obtained by an analysis of meteoritic material. Suess (38S2, 39S2) uses the K^{40} isotope and Paneth (45P2) the helium content. Both authors arrive at an age of roughly 6×10^9 yr. In view of the fact that according to Brown's recent investigations (49B), meteorites may possibly be the remnants of a planet, this should give about 6×10^9 as the upper limit of the age of our solar system.

Summarizing, we can say that practically all evidence seems to point to the short time scale of between 10^9 and 10^{10} yr.

Bok (46B) also mentioned the size of the interstellar smoke particles as evidence in favor of the short-time scale. In view of recent work of Kramers and ter Haar (46K2) and Oort and van de Hulst (47O1, 47O2), however, it seems to be impossible to arrive from this material at any definite conclusion.

TABLE IV. Lifetime of stars of various spectral classes.

Spectral class	O7.5	B0	B1	B2	B3	B5	B8	A0	A5	F0	F5	G0
Log$L$	4.9	4.2	3.9	3.6	3.3	2.8	2.2	1.7	1.0	0.7	0.4	0.0
Log$M$	1.4	1.2	1.1	1.0	1.0	0.8	0.7	0.5	0.3	0.2	0.1	0.0
Log$T$	7.1	7.7	7.8	8.0	8.3	8.6	9.0	9.4	9.9	10.2	10.4	10.6

** Dr. Chandrasekhar has kindly informed me that Dr. P. Ledoux has recently given another argument in favor of the short-time scale (49L). Ledoux assumed that the constitution of the interior parts of the sun might be different from that of the exterior parts. This should be due to the transmutation of hydrogen to helium in the interior. Integrating a suitable model for the sun and assuming that other elements take up four percent of the weight throughout the sun [compare Eq. (2.8)], Ledoux arrives at 40 percent helium in the exterior parts and 70 percent helium in the interior. If one now assumes that this difference is completely due to the carbon-nitrogen cycle and that the original composition of the sun was homogeneous and consisted of 40 percent of helium, the age of the sun can be estimated. The result at which Ledoux arrives is: 5×10^9 yr. in agreement with other arguments. For this argument it is necessary that the mixing of the different layers in the sun is bad.

(b) Von Weizsäcker's General Cosmogony

The most recent of von Weizsäcker's (47W1) many contributions to theoretical astrophysics is his general cosmogony which may well prove to be the stepping stone for many investigations in this field. In the course of the present paper, we shall, for instance, see how this cosmogony can be connected with the problem of the origin of the solar system and with the problem of the origin of the chemical elements.

Von Weizsäcker tries in this paper to extend to a general cosmogony the ideas of his paper on the origin of the solar system (44W, 46W1). He tries to take into account two basic facts. First, that all cosmic gas masses will be turbulent (compare also 41Z), and second, that most celestial bodies show rotation. He tries to give a qualitative picture of the formation of stars and systems of stars from the gas which is supposed to have filled the universe τ years ago.

His principal hypotheses are: (a) that stars and galaxies have been and are formed during the present epoch; (b) that during the present epoch all laws of physics were the same as at present; (c) that the gas filling the universe was composed of the chemical elements in the same ratios as we find them at present;†† and (d) that different parts of the gas had large relative velocities. Von Weizsäcker assumes that the origin of these relative velocities, which may be connected with the expansion of the universe, has to be looked for in periods before the present epoch started.‡‡ The origin of the chemical elements he also puts back to those early days. In all the discussions only classical physics is applied.

That it is possible to show with a qualitative theory like this one—von Weizsäcker promises more quantitative papers later on, a few of which are in the course of publication—in what direction possible solutions can be found is to a large extent due to the fact that it is possible to divide all celestial bodies or conglomerations of bodies into three groups according to their degree of rotational symmetry. In Table VI this classification is given.*

TABLE V. Development of the sun.

Percent hydrogen	91	81	71	61	51[a]	41	31	21	11
Spectral type	dK8	dK5	dK3	dG8	dG2[a]	F4	A8	A3	B9
Abs. magnitude	8.2	7.1	6.2	5.4	4.7[a]	3.9	3.2	2.6	2.1
Time interval (in 10^{10} yr.)	9	5	3	2	1.0[a]	0.5	0.25	0.11	0.04

[a] The present state of the sun.

†† This hypothesis seems unnecessary as we shall discuss in Section 4; compare also 49H1.

‡‡ Gamow (46G2) remarks that it should not be excluded *a priori* that the universe as a whole is rotating. This rotation may be the reason for the initial turbulence.

* Table VI has been taken over from von Weizsäcker's paper with one minor change. The supergiants have been omitted from

One could argue that the spheres should be grouped with the figures of rotation. If the distribution of the eccentricities of the objects in the second column is such that the corresponding group in the first column fits exactly in into that distribution at the place of zero eccentricity, those not to be treated under the "real" spheres. This is the reason for the brackets around the spherical nebulae in Table VI.

The division of the main sequence stars into spheres and figures of rotation follows the discovery of Struve, Elvey, and Westgate (31S, 33W1, 33W2, 34W) that many early type main sequence stars rotate rapidly but none of the later types.

As a final property of the subjects in Table VI it must be mentioned that the structures of clouds are again cloudy; the systems in the second column can have a cloudy structure or a regular one; the systems in the first column never have a cloudy structure.

The next point discussed in detail by von Weizsäcker is turbulence. First, he points out that cloudy structures necessarily indicate turbulence. Cloudy structure means that there are fluctuations in density and hence in pressure. These fluctuations will, however, be counteracted by currents and it is only possible to retain cloudy structure if the currents correspond to a turbulent state.

Second, the fact that at present many stars and stellar systems show rotation and that, as far as can be decided from observational data, the directions of rotation and the rotational velocities are arbitrarily distributed points, because of the conservation of angular momentum, to an earlier turbulent state of the universe.

After this, von Weizsäcker considers the development of a universe filled with a turbulent gas. Let us consider a turbulence element which is at the same time a region of slightly higher density. Such an element will act as a potential hole due to the slightly larger gravitational field strength. Particles entering this potential hole will gain energy because of this lower gravitational potential. They will lose this energy again due to the viscous interaction in the system. Hence, such a turbulence element will grow in mass, and in this way the universe will be divided up into many rotating subsystems. These subsystems may be considered to be the proto-galaxies.

Inside such a proto-galaxy there are again turbulence elements, and a similar process will happen there, giving rise to proto-clusters, and so on. The next steps will be the formation of proto-stars, proto-planets, and finally proto-satellites.

It must be remarked here that it may well be that this picture is slightly too simplified and that in reality there are many more steps. The results of these steps can perhaps not immediately be identified with certain structures in our universe at present, but one of these structures might for instance be a cluster of galaxies like the one in Virgo. Also, the question, whether a certain gas structure can further condense, or will dissipate as soon as it is left alone, is not discussed.

the first column, last row. Because of their large extension it is very difficult to decide whether they belong in the first or the second column, as was pointed out to me at Yerkes Observatory.

TABLE VI. Classification of cosmic bodies.

	Spheres	Figures of rotation	Clouds
Galaxies	(Spherical nebulae)	Elliptic and spiral nebulae	Irregular nebulae
Clusters	Spherical clusters		Star clouds, open clusters, gas- and smoke clouds
Stars	Main sequence F5-M, giants, planetary nebulae, planets	Main sequence O-F0 binary systems, satellite systems, Saturn's rings	Meteor showers

It is now necessary to discuss the further development of such a rotating system. The rotation is the remnant of the angular momentum of the original turbulence element. The only rotational motion which does not consume energy by viscous interaction is the rotation of a rigid body, where all points have the same angular velocity. The irreversible loss of energy will therefore only cease when such a rigid rotation has been attained. However, a free gas mass will not show a rigid rotation. Apart from the very center of such a rotating mass, there will be approximate equilibrium between gravitational and centrifugal force, leading to velocities corresponding to Kepler's third law. This means that the outer regions will try to slow down the inner regions. This slowing-down process will only end when there is no rotation left. It can be shown that the following two effects take place at the same time: Deceleration of the rotation of the central parts and a separation of the mass into two, one part falling into the central mass and gaining sufficient energy to allow the second part to disappear from the system with the largest part of the angular momentum.

Von Weizsäcker has estimated the time necessary for the system to be reduced to its central mass only in the following way. The system may originally have a radius R and an average turbulent velocity v. The mixing length λ, which is the average distance traveled by turbulence elements before they lose their identity completely, is determined by the value of R and will be one order of magnitude smaller than R. The energy lost by viscous interaction will be transported by turbulence elements. They need a time R/v to travel from the center of the system to the outskirts. However, turbulence elements will mix with other turbulence elements, thus delaying this transport of energy. This delay is taken into account by inserting a factor R/λ. In this way we get an estimate of the time necessary to transport an amount of energy of the order of magnitude of the total gravitational energy of the system. If r is the radius of the final mass, the total energy to be dissipated will be larger than the original energy content, roughly by a factor R/r. The time necessary for the dissipating

TABLE VII. Approximate lifetimes of nebulae.

Spiral nebulae	$T = 7\times10^9$ yr.
Elliptic nebulae	1.5×10^9 yr.
Spherical nebulae	10^8 yr.

process will therefore be proportionally longer. The final estimate for this time will now be:

$$T \sim R^3/rv\lambda \tag{2.11}$$

which gives, of course, only an order of magnitude estimate.

Systems for which T is large as compared to τ may be called *young* systems: They have not yet lost a large part of their material and of their rotation. If T is small as compared to τ, the system will be called old. We have now found our first criterium for the age of a system: the faster the rotation, the younger the system. The age of the systems in Table III will therefore decrease with going from the first to the third column.

It is possible to check to some degree the age of the various galaxies by estimating T, using Eq. (2.11). For the spiral nebulae, elliptic, and spherical nebulae, von Weizsäcker finds the results of Table VII. We see from Table VII that, indeed, the age criterium using T coincides with the age criterium of rotation. A strange feature is that "late" type nebulae are young, and "early" type nebulae old! The development of nebulae is in the opposite direction as that assumed when the names "early" and "late" types were introduced.

It might be asked whether it is not extremely dangerous to use hydrodynamical considerations for all systems, i.e., to treat all systems as gaseous systems. The gas might be more or less used up by the formation of the stars and then the interaction might be much smaller between the different parts of the system.† The situation is, however, not quite as simple as all that. The stars are still exchanging mass with their surroundings during their development, and it is an open question whether the gas or the rotation of the system will be the first completely to disappear. We are therefore led to a third criterium for the age of a system: the presence of interstellar gas. This criterium again gives the same classification: spherical nebulae old, spiral arms young.

Finally, there exists a connection between old stars and old systems on the one hand and young stars and young systems on the other hand. If we look at the division of stars into the populations I and II (see Section 1) and remember the distribution of these populations among the various clusters and nebulae, it is immediately seen that population I which contains "young" stars (stars in the second column of Table VI) can be found in "young" regions, and that population II containing "old" stars is observed in "old" regions.

† The "stellar gas" will be a Knudsen gas, i.e., the mean free path of the gas particles, which in this case are the stars, will be large as compared to the dimensions of the system, in this case the dimensions of the galaxy.

A final difference between "old" and "young" stars or between the two populations I and II is their concentration to the galactic plane (26O). Summarizing, we can say that "young" stars show the following properties: (a) They belong to the early type stars; (b) they show rotation; (c) they belong to population I; and (d) they are concentrated toward the galactic plane. "Old" stars, on the other hand, (a) belong to later type stars; (b) do not show appreciable rotation; (c) belong to population II; (d) are not concentrated toward the galactic plane. This division into "old" and "young" stars is also in agreement with the lifetime of stars found in Table IV.

It is difficult to estimate the age of a star by using formula (2.11), mainly because of the difficulty in estimating R. Von Weizsäcker arrives at the estimate of about 10^6 yr. for the period during which the star loses most of its rotation (compare also 44W). This period is of the same order of magnitude as the time in which the O- and B-stars use up their hydrogen. It seems, therefore, possible that either the star loses its hydrogen while it is still rotating, or that the star loses its rotation before the hydrogen is exhausted.

In the first case the star may either become a giant, or give rise to an explosion like the ones which we shall discuss in Section 4. In the second case, the star remains on the main sequence and may go to later types.

Von Weizsäcker's discussion of the supernova phenomenon will be met later. His discussion of binary systems may well be insufficient because he does not take into account any interaction with other stars, and we have seen earlier (Section 2a, γ) that this interaction plays an important role in the development of binaries.

(c) The Bok-Spitzer-Whipple Dust-Cloud Hypothesis

The fact that the early type stars are found in the same regions as the interstellar gas formed an important factor in von Weizsäcker's considerations. It also led Bok and Whipple to propose a different theory for the formation of stars. The second factor leading to this theory was the fact that Spitzer (41S) had shown that radiation pressure from high luminosity, late type stars will blow smoke clouds together into conglomerations of higher density.

Bok interprets the small globular dark clouds which can be observed in our galaxy as the first stages in the birth of a star. Whipple (46W2) has given a first, mainly qualitative, analysis of this condensation process, and he arrives at the conclusion that this process may well be important. It seems to the present author that this condensation may well play a role in the processes described in von Weizsäcker's cosmogony but that it by itself is not sufficient to account for the births of stars. This follows, in our point of view, from Whipple's own conclusions.

Whipple shows that if conditions in space are ideal and exhaustion of the interstellar gas may be neglected, a star with a mass about equal to the solar mass may be formed in a period of the order of 10^9 yr. The first part of the condensation, due to the sweeping effect of the radiation pressure, develops exponentially. After the matter has become opaque, radiation pressure lessens and the growth is linear until gravitational accretion gives rise again to an exponential growth.

However, it is necessary that the relative velocities in the cloud are very small. Otherwise the time necessary to reach the final mass would be far larger, and in fact too large. Quoting Whipple: "This stipulation of extreme quiescence for the concentration with respect to the interstellar medium and the concomitant quiescence of the medium itself presents the most serious handicap to the process of stellar evolution from interstellar material. Whether the stipulations can be met in an actual obscuring cloud, is difficult to ascertain."

The second difficulty, mentioned by Whipple, is that there is a limit for the mass of the final star, corresponding to the state where radiation pressure from the new-born star equals the gravitational attraction. The luminosity of the star is then, however, much lower than corresponding to the young O- or B-stars which presumably should have been formed in this way.

The theory is able to explain the fact that the stars, born in this way, possess a high angular momentum. This angular momentum is the remnant of the angular momentum of the original cloud.

It must be remarked that no account has been taken of viscosity effects which, as Whipple also points out, will be important. Also the unimportant objection can be voiced that this theory can possibly account for the young stars but certainly not for the older ones. It is, however, certain that a final cosmogony, based perhaps on von Weizsäcker's ideas, which will give a quantitative discussion of the various processes leading to galaxies and stars, has to take into account the dynamics of smoke clouds and the influence of radiation pressure, studied by Whipple and Spitzer.

(d) Gamow's Theory of the Formation of Galaxies

In connection with the recent work of Gamow and Alpher on the origin of the chemical elements which will be discussed in detail in Section 4, Gamow also discusses the formation of galaxies during the evolution of the universe (48G1, 48G2, 49A2). At the very beginning of the expansion of our universe, i.e., at the beginning of the present epoch, the temperature and the density of the universe were much higher than at present. Their decrease with time is governed by the equations from general relativity. Gamow now uses Jeans' considerations (28J) about gravitational instability to determine the masses of the condensations forming in a gas of given temperature and density. The condition which has to be fulfilled is essentially that the total heat content of the condensation is smaller than the total gravitational energy, as long as we neglect the tearing effect from the expanding universe. The mass of such a condensation is given by

$$M=K\rho^{\frac{1}{2}}T^{\frac{3}{2}}, \tag{2.12}$$

where K is a numerical constant. Inserting numerical values into Eq. (2.12), Gamow finds for the mass of the condensations in his expanding universe:

$$M=6\times10^{40}\text{ g}=3\times10^{7}\text{ sun-masses}, \tag{2.13}$$

a value which turns out to be independent of the time at which ρ and T are taken. Since the value given by Eq. (2.13) is of the order of magnitude of the mass of a galaxy, Gamow proposes these original condensations to be the proto-galaxies. If one tries to take the expansion of the universe into account one runs, however, into serious difficulties as Dr. Gamow kindly told me (49G). One might describe the effect of the expansion as increasing the heat content—or the total kinetic energy—of the mass considered. It turns out that using for the present mean density the value 10^{-30} g cm^{-3}, used by Alpher and Gamow in all their calculations, condensation could never have taken place. It is necessary to increase the value of the mean density considerably to a value of about 10^{-27} g cm^{-3}. Then, however, the following difficulty arises. The radius of curvature of the universe can be determined from the observed red shift of the far distant nebulae and the value of the gravitational constant in the following way (34T). Introducing Hubble's constant, α, by:

$$\Delta\lambda/\lambda=\alpha r/c, \tag{2.14}$$

where $\Delta\lambda/\lambda$ is the percentual redshift of a nebula and r the distance of the nebula, and introducing the gravitational constant κ by

$$\kappa=8\pi\gamma/c^2, \tag{2.15}$$

the rate of change of any length l is given by

$$(1/l)(dl/dt)=[\tfrac{1}{3}\kappa\rho c^2-(c^2/R^2)]^{\frac{1}{2}}. \tag{2.16}$$

Since the left-hand side should be equal to α, we see that R is real or purely imaginary if the density ρ satisfies the first or the second of the two inequalities:

$$\left.\begin{array}{l}\tfrac{1}{3}\kappa\rho c^2>\alpha^2R\text{: real}\\ \tfrac{1}{3}\kappa\rho c^2<\alpha^2R\text{: imaginary}\end{array}\right\}. \tag{2.17}$$

With $\alpha=2\times10^{-17}$ sec.$^{-1}$, $\kappa=2\times10^{-27}$ g^{-1} cm, we see that the critical density is of the order of magnitude of 5×10^{-28} g cm^{-3}. Therefore, if the mean density is increased sufficiently to allow for condensations to take place, the universe has to be considered closed instead of open. It seems at present to be impossible to determine accurately the value of the mean density. De Sitter's original estimate was between 10^{-25} and 10^{-26} g cm^{-3}. Gamow and Alpher assume 10^{-30} g cm^{-3}, Omer (49O) chooses 10^{-29} g cm^{-3}, while Jordan (44J) takes 10^{-28} g cm^{-3} as do Becker (42B2) and Zwicky (39Z).

If, however, the universe is closed, its total mass is defined by its mean density, and for a density of about

10^{-9} g cm^{-3} which Alpher and Gamow need for their theory about the origin of the chemical elements, the total mass of the universe is of the order of 10^{44} g, and it is impossible to account for all the chemical elements by one process taking place at the beginning of the expansion of the universe, and the bottom has dropped out of the whole Alpher-Gamow theory.

(e) Jordan's Cosmogony

While von Weizsäcker and Whipple build their theories on the solid ground of present days' physics, Jordan (44J, 45J1, 45J2, 49J) starts from the assumption that the various constants of nature might well be varying with time instead of really being constants. To a large degree, the starting point of Jordan's cosmogony is the same as a conclusion reached previously by Dirac (37D, 38D). These two authors arrive in different ways at the conclusion that the gravitational constant varies with time and we shall give both methods of attack here. Since Dirac's final paper comes to conclusions different from those of the first note,‡ we shall only refer to his final paper.

Dirac starts from the fact that there are a number of atomic constants: charge and mass of an electron: e and m, Planck's constant $h(=2\pi\hbar)$, velocity of light c, mass of a proton M_p. These constants should really be constant, and should not depend on time. Indeed, from spectroscopic data about nebular millions of light years away it follows that the fine structure constant $e^2/\hbar c$ should be constant (39J). On the other hand, geological evidence from old rocks indicate that nuclear forces, and presumable therefore also M_p/m are constant.

From these atomic constants it is possible to form an elementary unit of length and an elementary unit of time:

$$\lambda_0 = e^2/mc^2 \tag{2.18}$$

and

$$\tau_0 = e^2/mc^3. \tag{2.19}$$

If we now express the age of the universe of, say, 3×10^9 yr. in this unit of time, we find:

$$N_1 = \tau/\tau_0 = 10^{39}. \tag{2.20}$$

On the other hand, it is possible to form a dimensionless quantity by taking the ratio of the electric Coulomb force and the gravitational force between an electron and a proton. For this ratio, one finds:

$$N_2 = (e^2/r^2)/(\gamma m M_p/r^2) = e^2/\gamma m M_p = 2\times10^{39}. \tag{2.21}$$

‡ Teller (48T) seems to have overlooked this change. He points out the dangers attached to assuming the constants of nature to change in time. Teller shows that about 3×10^8 yr. ago the surface temperature of the earth should have been about 20 percent higher than at present. However, it is quite possible that due to the large amounts of water on the earth, cloud formation might take place, thus effectively lowering the average temperature on the earth. Teller himself points out that the opacity of the sun also might be influenced by the change in the constants of nature. Finally, taking a change of the solar mass into account would invalidate Teller's argument.

From the fact that the two dimensionless quantities in Eqs. (2.20) and (2.21) are of the same order of magnitude, Dirac arrives at the principle: "*Any two of the very large dimensionless numbers occurring in Nature are connected by a simple mathematical relation in which the coefficients are of the order of magnitude unity.*" Dirac gives a few more instances of this principle. If that principle is, however, fundamental, it should be true for any time, that is, for any value of τ/τ_0. Therefore, N_2 should be proportional to the age of the universe. From this it follows immediately that γ should vary as $1/\tau$.

Dirac did not draw any further cosmogonical conclusions from this behavior; especially not since he restricts himself to a constant mass of the universe. Jordan, however, shows that this last assumption is not consistent with the other ones, as will be seen presently.

Jordan starts from six fundamental physical constants which are divided into two groups of three. The first group consists of the velocity of light, c, the gravitational constant, κ, and the age of the universe, τ. The second group consists of the Hubble constant, α, which measures the red shift of distant nebulae, the average density in the universe, μ, and the radius of curvature of the universe, R.

The radius of curvature and the mean density in the universe may be introduced by the formula, giving the total mass between distances r and $r+dr$:

$$dM = \mu\cdot 4\pi r^2 dr[1-(r^2/R^2)]. \tag{2.22}$$

For the average density, Jordan takes with Becker (42B2) and Zwicky (39Z) 10^{-28} g cm^{-3}. Jordan assumes a real value for R, i.e., a closed universe. This is not quite in agreement with the value of the average density of 10^{-28} g cm^{-3}. However, since the value of the average density is very uncertain, the assumption of a closed universe may be correct.

From dimensional considerations, it follows that α, μ, and R should be expressable in c, κ, and τ, and indeed the relations, apart from numerical factors of order of magnitude unity, are:

$$\alpha = \tau^{-1}; \tag{2.23}$$

$$R = c\tau; \tag{2.24}$$

$$\mu^{-1} = \kappa c^2 \tau^2. \tag{2.25}$$

Equation (2.14) for the Hubble constant, and Eqs. (2.22) and (2.24) can readily be explained by the assumption that the universe is a closed Riemannian space with radius of curvature R which increases with the velocity of light and which was small at a period τ ago. The finite mass of the universe, apart from a numerical factor of order of magnitude unity, is given by

$$M = \mu R^3. \tag{2.26}$$

However, Eqs. (2.25) and (2.26) are inconsistent with the point of view that both κ and M should be constants. In that case, μ^{-1} should be proportional to τ^3 and not to τ^2.

That Eq. (2.25) is a real relation and not a fortuitous relation, valid at this special moment, follows from general relativity where the relation (2.20) is known in the form (apart from a numerical factor $4\pi^2$):

$$\kappa M = R. \tag{2.27}$$

Equation (2.27) can also be written in the form:

$$\gamma M^2/R = Mc^2, \tag{2.28}$$

expressing the fact that the sum of all energies of the separate bodies in the universe is of the same order of magnitude as the total gravitational energy.** It is tempting to assume that the *total* energy of the universe is exactly equal to zero, and therefore to accept Eq. (2.28) as valid.

Jordan's next step is identical with Dirac's. Introducing λ_0 and τ_0 for the units of length and time, he also introduces a unit of mass $\mu_0 = h/\lambda_0 c$, and a unit of temperature, using Boltzmann's constant k, $T_0 = hc/\lambda_0 k = \mu_0 c^2/k$. In these units, we have for κ, R, and M, again apart from numerical factors of the order of magnitude unity:

$$\kappa = \beta^{-1}\lambda_0/\mu_0; \tag{2.29a}$$

$$R = \beta\lambda_0; \tag{2.29b}$$

$$M = \beta^2\mu_0; \tag{2.29c}$$

where β is the dimensionless constant, already met in Eqs. (2.20) and (2.21):

$$\beta \cong 10^{39}. \tag{2.30}$$

Now, one can either follow Eddington and try to explain the number 10^{39}, or follow Dirac and believe in the principle, quoted above. It then follows immediately that κ and M must vary proportional to $1/\tau$ and τ^2, respectively [as in Dirac's first note, where this conclusion was derived from Eq. (2.29c)].

It is now possible to conceive how stars can be born. Jordan lets this happen through a spontaneous birth (*Spontanentstehung*) of a mass M_0. This will not be in contradiction with the energy principle if the radius R_0 of this mass, which for the sake of simplicity is assumed to be spherical, is given by

$$R_0 = (3/40\pi)\kappa M_0, \tag{2.31}$$

which is Eq. (2.28) with the correct numerical factor.

For the maximum density of this body, we may assume at most nuclear densities or:

$$M_0/R_0^3 \lesssim \mu_0/\lambda_0^3, \tag{2.32}$$

and find from Eqs. (2.31) and (2.32) a minimum value of the mass created in this way:

$$M_0 \gtrsim \kappa^{-\frac{3}{2}}\lambda_0^{\frac{3}{2}}\mu_0^{-\frac{1}{2}}, \tag{2.33}$$

where numerical factors of order of magnitude unity have been neglected. By use of Eq. (2.29a), Eq. (2.33) can also be written in the form:

$$M_0 \gtrsim \beta^{\frac{3}{2}}\mu_0 \approx 10^{34}\ \mathrm{g}. \tag{2.34}$$

These masses are therefore indeed of the order of stellar masses.†† According to Eq. (2.34) there should be a close relation between age and mass of a star, a relation which really exists. This fact, that the young O- and B-stars are heavy, might be considered to be in favor of Jordan's considerations.

Jordan's next considerations of the development of this mass M_0 into a star of normal density weakens his case, however, greatly. First of all, Jordan points out that the energy necessary to create the star does not necessarily come from the gravitational energy of the star itself, but according to the interpretation of Eq. (2.28) from the gravitational field of all the matter which is already present. In that case, the argument leading to Eq. (2.34) loses much of its rigor. Second, it is difficult to see how the star will obtain the hydrogen, necessary to start the carbon-nitrogen cycle. At the initial high densities, heavy elements will be favored, and also later on (see also the considerations in Section 4).

The final difficulty is to understand that the O- and B-stars have such large angular momenta. Altogether there seems to be more against than in favor of this theory, although the argument for a variation of κ with time seems to be rather strong. Jordan himself objects to a possible way out consisting of the assumption that the creation of mass takes place by the production of neutron- or proton-pairs. The difficulty there, however, seems that in this case there are just as many antiparticles (negative protons, and antineutrons, decaying by positron emission into negative protons) created as normal particles. Since it seems feasible to expect that mass will be created in the neighborhood of existing masses, it seems that the antiparticles will be annihilated and the net result would be that the new mass should be present in the universe in the form of radiation. However, it seems hardly possible that really the mass of the universe is mainly made up of radiation.

We refer to Jordan's first paper for a discussion of the early stages of the universe in this theory. In Section 6, we shall return to his ideas about the supernova phenomenon, which is supposed to show the birthcries of a new star, and the variable stars. Jordan (45J2, 47J) has also developed the theory of general relativity with a gravitational constant which is now a field quantity. A

** Gamow and Teller (39G4) calculated the total kinetic energy of nebulae, which is about equal to Mc^2, and they arrived at the conclusion that it is much larger than the total gravitational energy. Jordan points out that the data available to compare accurately these two energies are by far too scanty to decide the point. Here again the value of the average density is of great importance. Jordan's conclusion that Gamow and Teller's conclusion is in contradiction to relation (2.27) from general relativity is incorrect since their result refers to an open and Eq. (2.27) to a closed universe.

†† It has already been pointed out previously by Chandrasekhar (37C) in connection with Dirac's considerations that the ratio of stellar masses to the mass of a proton was of the order of magnitude $\beta^{\frac{3}{2}}$.

discussion of this matter lies, however, far outside the scope of the present paper, and the reader is referred to Jordan and Müller's paper.‡‡

We have not discussed here the discussions in general relativity pertaining to cosmogonical theories. For a recent account of these discussions, we refer to a paper by Bondi (48B2) where an extensive bibliography can be found.

3. MICROCOSMOGONY

In this section we wish to discuss briefly the latest developments in the theories about the origin of the solar system. For a more detailed discussion we may refer the reader to earlier papers (48H1, 48H2). In this section we shall first discuss von Weizsäcker's theory with the extensions proposed by the present author, which in our opinion is still the most promising way of attacking this problem. After that the recent theories of Alfvén, Hoyle, and Whipple will be sketched.*

A related problem which is still unsolved is that of the origin of the comets. An extensive analysis of this problem was recently given by van Woerkom (48W2) [see also Lyttleton (48L) and Oort (50O)].

(a) An Improved Version of Kant's Theory

Until recently, Kant's theory about the origin of the solar system was considered to be insufficient to explain the many regularities of the planetary system. However, von Weizsäcker (44W, 46W1) has drawn attention to the consequences of taking into account hydrodynamical considerations and the present author (48H2) has extended this analysis by also considering carefully the condensation processes in a more rigorous way than was done by Hoyle (47H4). Various authors have given more or less extensive accounts of von Weizsäcker's theory (45G1, 46C, 48H1, 48H2). We shall, therefore, only sketch the theory here as it stands at present and point out which points still need explanation.

This new version of the old Kant theory follows immediately from von Weizsäcker's general cosmogony which was discussed in Section 2, although the general cosmogony was published a few years after the microcosmogony. It was seen in Section 2 that during the evolution of our universe galaxies and stars were born from turbulent vortices. At a certain stage of this development we may therefore assume to be left with a fast rotating sun, surrounded by an extended envelope, which may have contained up to one-tenth of a solar mass. Because of the rotation, the shape of this envelope will have been like that of a lens. The temperature in this lens will have decreased inversely proportionally to the square root of the distance from the sun as follows from a careful analysis of the physical conditions in this envelope (48H2):†

$$T = ar^{-\frac{1}{2}} \tag{3.1}$$

where r is measured in astronomical units, and $a=400$ degrees A.U.$^{\frac{1}{2}}$.

In the disk there will be turbulence. It is possible to give reasons (44W, 47T, 48H2) why it is plausible that some regular system of vortices may have been set up, although this point is still far from definitely settled. If this is accepted, von Weizsäcker shows that one can easily understand the Titius-Bode law which states that the mean distance of the nth planet from the sun is given by‡

$$r_n = r_0\epsilon^n, \tag{3.2}$$

where $\epsilon=1.9$, and where the asteroids are counted as a planet.

The difference in properties of the group of the four inner planets (Mercury, Venus, Earth, and Mars) on the one hand, and the group of the four major planets (Jupiter, Saturn, Uranus, and Neptune) on the other hand, can be explained by the condensation process in the disk. Because of the fact that the temperature is decreasing with increasing distance from the sun, more compounds are supersaturated and able to condense in the outer parts of the disk than in the inner parts. Also it turns out that in the regions of the inner planets only inorganic compounds will take part in the condensation process while in the outer regions the condensation products contain mainly organic compounds. Owing to this difference in amount of condensation the condensation products in the outer regions will grow faster than those in the inner regions. The result will be that in the outer regions gravitational capture will have played an important role but not in the inner regions, since the disk was dissipated before gravitational capture in the inner regions started. We can therefore understand that the inner planets are small and dense while the outer planets are large but have a small specific density. A more quantitative analysis gives good agreement between the calculated and observed masses of the planets, and also good agreement for their densities (compare also 49B2 and 50H1).

Even if the explanation of the Titius-Bode law by von Weizsäcker is accepted there remains one crucial point which has to be explained. That is the fact that the sun is at present rotating at such a low rotational velocity. If we could accept for a moment that the sun's rotation was already slow when the condensation in the disk started, there should be no difficulty as has

‡‡ Hoyle (48H3) also discusses the possibility of continuous creation of matter and in connection with this idea a new cosmological model of the universe.

* Jeffreys (48J) has also recently discussed some of the more recent theories, notably those of Lyttleton and Hoyle.

† This result can easily be obtained, if one assumes that the energy emitted by any volume in the disk follows Stefan-Boltzmann's law ($\sim T^4$) while the energy received from the sun is inversely proportional to r^2 (no absorption). Energy balance then gives Eq. (3.1).

‡ This expression is slightly different from the usual form of the Titius-Bode law but describes the observational data just as accurately, in fact slightly better.

been pointed out elsewhere (46H1), since the condensation process will not appreciably alter the relative angular momentum per unit mass. However, we saw that from general ideas it seems proper to start from a situation where the sun is still rotating at high velocity. Von Weizsäcker suggested that material from the disk should fall onto the sun with zero angular momentum. An easy calculation shows, however, that this process is not sufficient to slow down the sun to its present slow rotation. As another possibility, one might hope that the viscous drag from the disk on the sun might have been the agent by which the sun was slowed down; the outer parts which rotate much slower than the inner parts will try to slow down the center (compare Section 2, b). This process is also insufficient, the discrepancy between the necessary and the possible effect being a factor of the same order of magnitude as for the process proposed by von Weizsäcker, *viz.*, about 100,000.

Recapitulating, we may say that Kant's theory has been brought to life again, but that there remain two big questions to be solved. The first and most important one is the question of the angular momentum of the sun.** It seems that the solution of this problem, if one does not put the solution of this problem into the initial conditions so that nothing has to be proved, may well be connected with earlier stages in the development of our galaxy. The second question is whether it is possible to prove rigorously from hydrodynamical considerations that a regular system of vortices is possible in a gaseous disk rotating around the sun.††

(b) Recent Dualistic Theories

If von Weizsäcker's theory is not deemed to be satisfactory, it may be worth while to consider the theories proposed by Alfvén, Hoyle, and Whipple and discuss whether these theories can give more satisfaction.

The Swedish physicist Alfvén has developed a theory (42A2, 43A2, 46A2) taking into account the magnetic forces in ionized matter. He follows here the attempts of Birkeland (12B) and Berlage (30B). Alfvén shows first of all that if a proton is moving with the earth's velocity in the earth's orbit the gravitational force is smaller than the force exerted by the sun's magnetic moment by a factor 60,000. Second, Alfvén (42A1) shows that due to currents in an ion cloud evoked by the sun's magnetic field, the sun's rotation can be slowed down sufficiently in a period as short as 10^6 years.‡‡

Alfvén now proposes that the sun on its journey through space met an interstellar gas cloud and became surrounded by it. The atoms in this cloud should start to fall toward the sun and become ionized by collisions once their kinetic energy has increased sufficiently. These ions will move along the magnetic lines of force and reach equilibrium positions in the equatorial plane of the sun. On the assumption that the atoms are moving uniformly toward the sun and that the ionization takes place at a fixed distance from the sun, the subsequent mass distribution in the equatorial plane can be calculated. This distribution agrees roughly with the mass distribution in the series of the outer planets.

Alfvén is not able to give a process which might account for the inner planets. But the main objection against this theory lies in the fact that a careful analysis of the physical characteristics of the encounter of the sun with an instellar gas cloud shows that the process proposed by Alfvén is impossible (see 48H2).

A different attempt was made by Hoyle (44H, 45H) who introduced into Lyttleton's idea (41L) to start from a binary system the new feature of a supernova outburst. The binary companion of the sun should have become a supernova. This supernova outburst should have removed the second star and sufficient material should have been left behind to form the planets. However, it seems that Spitzer's objections (39S1) against Lyttleton's theories are again valid and should be sufficient to discard this theory. Spitzer points out that the temperature of the gaseous filament in which the planets should be formed will be so high that the filament will expand before any condensation can start. (Nölke had earlier pointed out that tidal action will have the same effect.) The only possibilities are then either a total dissipation of the filament or the formation of a gaseous envelope in which case the development would be the same as in von Weizsäcker's theory.

The third recent dualistic attempt is that of Whipple which is connected with the Bok-Spitzer-Whipple dust-cloud cosmogony. Whipple (47W2) has as yet not published any detailed calculations about this theory so that it is rather difficult to discuss it quantitatively at this moment but a few objections can already be raised.

Whipple starts from a cloud of radius of about 30,000 A.U. in which solid particles and gas are mixed. Contraction of this cloud should produce both the sun and the planets. In order to account for the present distribution of the angular momentum, Whipple assumes that the cloud itself possesses negligible angular momentum while the planets are assumed to be formed from condensations in a stream in the cloud. In this way, the sun is understood to possess a low angular momentum while the orbital angular momentum of the planets has its beginning in the stream in the cloud. The proto-planets will sweep up matter with zero angular momentum and consequently spiral inward.

Apart from the objection that the solution of the important question of the origin of the present distribution of the angular momentum in the solar system is

** See, however, 49H5.

†† Nölke (48N) has criticised von Weizsäcker's theory severely. His criticism centers around the formation of a regular system of vortices and the formation of one planet along each one of the circles separating the circles of vortices. As was remarked by von Weizsäcker (48W1) in his reply to this criticism, Nölke's criticism is concentrated on minor points and does not touch essential ones.

‡‡ In Alfvén's paper there exists a numerical error in the final formula; he gives 10^5 yr. instead of 10^6 yr.

put into the theory more or less *ad hoc*, one other objection can be voiced already now. From the process proposed by Whipple one should expect that the planets which are nearer the sun would be larger than the outer planets. They have spiraled inward more than the outer ones and should therefore have acquired more mass. However, observation shows an opposite trend: The outer planets are by far larger than the inner ones.

4. THE ORIGIN OF THE CHEMICAL ELEMENTS*

In the last ten years, there has been an extensive discussion whether it is possible to account for the present abundances of the chemical elements and their isotopes. There are two schools of thought. The first one tries to find a statistical equilibrium in which all the elements are formed. The second one tries to picture a non-equilibrium process by which long ago the elements were formed. The second kind of theory has mainly been developed by a group of theoretical physicists around Gamow. The first kind of theory has been discussed by numerous astronomers and physicists from all over the world, and we shall give here a more or less historical account of this equilibrium theory, and after that briefly discuss the α-β-γ-theory.

(a) Equilibrium Theories

The first author to suggest that the chemical elements might have been formed by nuclear reactions in stars was Sterne (33S). Von Weizsäcker (37W, 38W) investigated whether the elements might have been formed with their present abundances in a state of statistical equilibrium which was subsequently frozen down. He came to the conclusion that the light and the heavy elements could not have been formed in the same equilibrium situation. These considerations were, however, mainly qualitative. Chandrasekhar and Henrich (42C2) followed up this suggestion and tried to determine what temperature and density corresponded to the relative abundances of the isotopes of five chosen elements, if those abundances had once been equilibrium abundances at that temperature and density. Using the temperature and density found in this way, they calculated the equilibrium abundances of all the chemical elements. They found agreement with the observed values of the abundances for elements of atomic number up to $A=40$. Since the paper of Klein, Beskow, and Treffenberg (46K1) arrives by a slightly different way at practically the same conclusion, we shall discuss their method rather than the method used by Chandrasekhar and Henrich since Klein, Beskow, and Treffenberg's method will be used also in later discussions.

A Gibbs grand ensemble (48G3) will be used with neutrons and protons as the constituents. The concentration of a nucleus with N neutrons, and Z protons in a given energy state E will be given by:

$$C(N,Z)=g(2\pi M\theta/h^2)^{\frac{3}{2}} \exp([\mu N+\lambda Z-E(N,Z)]/\theta), \quad (4.1)$$

where θ is the modulus of the ensemble: $\theta=kT$; μ, λ are the chemical potentials of the neutrons and protons, respectively; M is the mass of the nucleus, $M=NM_n+ZM_p$; E is the energy of the nucleus from which we have subtracted the mass energies of the constituent protons and neutrons since inclusion of these energies would only change the values of λ and μ; and g is the multiplicity of the state in question.

If we neglect for a moment excited states of the nuclei, the energy E will be the mass defect of the nucleus under consideration. We have now three parameters which can be chosen freely. Klein, Beskow, and Treffenberg chose θ and $\mu+\lambda$ so as to get agreement between the calculated values of $C(N,Z)$ for nuclei with $N=Z$ and the observed values for which they take, as all other authors, the data of Goldschmidt (37G). The difference $\mu-\lambda$ was chosen so as to get agreement for elements where $N\neq Z$. In this way, they found:

$$\begin{aligned} &\theta=1\ \text{Mev}, \quad T\sim10^{10}\ \text{deg.},\\ &\mu=-7.6\theta,\\ &\lambda=-11.6\theta. \end{aligned} \quad (4.2)$$

Using these values, they find good agreement between the calculated and observed values for $C(N,Z)$ up to $A=60$ to 70.

The total density of the matter is now given by:

$$\rho=\sum C(N,Z)M=4\times10^8\ \text{g cm}^{-3}. \quad (4.3)$$

For heavier elements, the observed abundances are by far higher than the calculated ones as also was the case in the calculations of Chandrasekhar and Henrich.

From Eq. (4.2) it is immediately clear that the proton density is only a fraction $e^{-4}\sim0.02$ of the neutron density. At first sight this seems strange. However, from the reaction

$$\text{neutron}=\text{proton}+\text{electron}+\text{neutrino}+\text{energy}, \quad (4.4)$$

it follows immediately that this reaction will proceed in the neutron direction at high temperatures and high densities.

One might be afraid that the introduction of electrons and neutrinos necessarily introduces two new chemical potentials. Klein, Beskow, and Treffenberg, however, show (a) that positrons are practically absent; (b) that the electron density and therefore the electron chemical potential is now determined by the condition for electrical neutrality, i.e., by the chemical potentials of the nuclei, or in last instance by λ and μ; (c) that the neutrino density will be equal to the electron density, both particles being produced by reaction (4.4).†

* Compare also 49H2.

† This will be true as long as gravitational effects may be neglected; see under δ.

The high neutron density has an important consequence. There will be a tendency to form nuclei with a N/Z ratio larger than that of nuclei observed in nature. This high N/Z ratio will give rise to β-decay of some of the nuclei. The calculated abundances show indeed a too high value for nuclei with large N/Z values and a relatively low value for small N/Z values.

If one is inclined to accept the idea of the formation of the chemical elements in a statistical equilibrium, one has to put the question in what way the theory sketched here can be improved, since it cannot account for the heavy elements in the form presented. It is then clear that among the problems to be investigated, there are the following ones: (α) The existence of excited nuclear states; (β) electrostatic effects due to the attraction between nuclei and electrons in very dense matter; (γ) the question as to where the processes considered here can have taken place; (δ) gravitational effects; (ϵ) the question of the transition from the high densities and temperature to normal conditions; (ζ) the transition from neutron rich nuclei to normal nuclei; and (η) the possibility to connect these processes with a general cosmogony.

At present all these effects have been studied more or less extensively and, luckily, it is possible to consider them separately, although, of course, a final theory has to consider them all at the same time. The order in which the various effects are discussed is rather arbitrary but follows more or less the historical development.

(α) *The Existence of Excited Nuclear States*

Independently, scientists in Sweden, Belgium, and Germany arrived at the same conclusion as far as the influence of excited states is concerned. It is possible to get agreement between calculated and observed values up to higher values of A, but even so the heaviest nuclei cannot have been formed in the same process. Géhéniau, Prigogine, and Demeur (47G) show this in a rather rough way.‡ Unsöld (48U) shows it by taking the rotational states into account. The most complete treatment is probably given by Beskow and Treffenberg (47B1) whose arguments we shall give here.

If we are only interested in the total number of nuclei with N neutrons and Z protons, irrespective of their stationary states, we should write:

$$C(N,Z)=(2\pi M\theta/h^2)^{\frac{3}{2}} \times\exp[(\mu N+\lambda Z-E_0)/\theta]\textstyle\sum_\omega\sum_\nu \quad (4.5)$$

instead of Eq. (4.1) where the total energy E is equal to $E_0+E_\omega+E_\nu$: E_0: binding energy (or mass defect); E_ω: rotational energy; E_ν: vibrational energy, and where the rotational and vibrational partition functions are given by

$$\textstyle\sum_\omega=\sum g_\omega \exp(-E_\omega/\theta) =\sum_{j=0}^{\infty}(2j+1)^2\exp(-j(j+1)\hbar^2/2I\theta), \quad (4.6)^{**}$$

$$\textstyle\sum_\nu=\sum g_\nu\exp(-E_\nu/\theta)\approx\int g_\nu\exp(-E_\nu/\theta)dE_\nu, \quad (4.7)$$

where we have put $E_\omega=j(j+1)\hbar^2/2I$ (I: moment of inertia). The rotational partition function can easily be computed (see, e.g., 48U).

For the density of the vibrational levels, Beskow and Treffenberg use a formula given by Bohr and Kalckar (37B) and independently by van Lier and Uhlenbeck (37L).†† In this way, g_ν is given by

$$g_\nu dE_\nu=(48)^{-\frac{1}{2}}\exp[\pi(2AE_\nu/3\epsilon_0)^{\frac{1}{2}}]dE_\nu/E_\nu, \quad (4.8)$$

where ϵ_0/A is the energy difference between the lowest states. Using expression (4.8), one can calculate $\sum_\nu$. Then the calculated abundance curve can be corrected for the factor $\sum_\nu\sum_\omega$ and it turns out that there now exists agreement between the calculated and observed curve up to atomic weight around 100.‡‡

(β) *Electrostatic Effects*

In two papers, van Albada (46A1, 47A) has discussed the effect of electron degeneracy and electrostatic effects at high densities. He points out that the energy of a nucleus with Z protons is not only given by the binding energy and vibrational and rotational energies. There is also a contribution from the electrons which are present, not only as far as their presence introduces a new chemical potential as was already mentioned, but also because due to the electrostatic field of the electrons neutron-rich elements will become more probable. The best way to introduce this effect is by introducing an electrical potential into the expression for the density in the grand ensemble, as is done by Beskow and Treffenberg in their final paper (47B2) (see under δ). This then automatically takes care of all effects. However, following van Albada's considerations, we shall show here how, indeed, the equilibrium is shifted toward heavier and neutron-rich nuclei. This is not so easily seen in Beskow and Treffenberg's discussion since gravitational effects are mixed up with the electrical effects.

‡ These authors actually overestimate this factor so much that their calculated curve agrees with the observed one for values of A right up to 250.

** This equation is the same as the one used by Unsöld.

†† It would be of interest to repeat the calculations by using for g_ν the expression given by Wergeland (45W) (compare 49H3).

‡‡ The quantity Σ_ω behaves as $A^{5/2}$ (see 48U) while $\log\Sigma_\nu$ is roughly proportional to A. For the total factor $\Sigma_\omega\,\Sigma_\nu$, we find from Beskow and Treffenberg's figures roughly:

A	50	100	150	200	250	300
$\log\Sigma_\omega\,\Sigma_\nu$	5	9	13	17	21	25
$\log g$	100	300	500	800	1000	1200

In the last row, we have inserted the values of the correction factor introduced by Géhéniau, Prigogine, and Demeur. It is immediately clear that these authors have grossly overestimated the effect of the excited states.

Van Albada considers the total energy of a nucleus with N neutrons and Z protons ($N+Z=A$) and calculates which atomic weight will be the most probable for a given density at zero temperature. This value of A will be given by that value for which the total energy per unit weight or the packing constant is minimum. This packing constant, F, consists of four different parts. The first one is the normal packing constant for which we may use the formula given by Bohr and Wheeler (39B3):

$$F_1=\alpha[(\tfrac{1}{2}A-Z)/A]^2+\beta[(\tfrac{1}{2}A-Z)/A] -\gamma+kZ^2A^{-4/3}+sA^{-\frac{1}{3}}, \quad (4.9)$$

where we have used van Albada's notation. In millimass units, we have:

$$\left.\begin{array}{c}\alpha=83 \text{ mMU},\ \ \beta=0.81 \text{ mMU},\ \ \gamma=6.65 \text{ mMU},\\ k=0.63 \text{ mMU},\ \ s=15 \text{ mMU}.\end{array}\right\} \quad (4.10)$$

In Eq. (4.9) the first term represents the well-known parabola of mass defects for a series of isotopes. The second term stems from the difference in mass between proton and neutron. The fourth term is the Coulomb energy contribution, and the last term the surface energy contribution.

The second part of F arises from the electrostatic energy of the electrons. The density of the electrons is connected with the total density, if only nuclei of one kind are present, by

$$\rho_{\text{el}}=(Z/A)\cdot(\rho/m_H). \quad (4.11)$$

The total electrostatic energy is due now to the attraction between the nucleus and the electron cloud on the one hand, and to the repulsions in the electron cloud on the other hand. The radius of the sphere in which all these charges are present is determined by the relation:

$$(4\pi/3)\rho_{\text{el}}r^3=Z, \quad (4.12)$$

and the total electrostatic energy becomes

$$Am_HE_{\text{es}}=-(3/2)Z^2e^2/r+(3/5)Z^2e^2/r =-(9/10)Z^2e^2(4\pi\rho/Am_H)^{\frac{1}{3}}$$

or

$$F_2=E_{\text{es}}=-p\rho^{\frac{1}{3}}Z^2A^{-4/3} \quad (4.13)$$

with

$$p=1.9\times10^{-5} \text{ mMU}. \quad (4.14)$$

The third part consists of the energy of the degenerate electron gas:

$$F_3=q\rho^{\frac{1}{3}}(Z/A)^{4/3} \quad (4.15)$$

with

$$q=4.2\times10^{-3} \text{ mMU}. \quad (4.16)$$

The final contribution comes from exclusion effects. If a nucleus has a large excess number of neutrons, these neutrons cannot move in the same volume since the lower quantum levels are already occupied by other neutrons. The volume of the nucleus has therefore to increase, and it is to be expected that the volume will be about twice the normal volume if all protons are transformed into neutrons. For this correction we may therefore try:

$$F_4=-2\alpha[(\tfrac{1}{2}A-Z)/A]^4-\tfrac{2}{3}kZ^2A^{-4/3}[(\tfrac{1}{2}A-Z)/A]^2 +(4/3)sA^{-\frac{1}{3}}[(\tfrac{1}{2}A-Z)/A]^2. \quad (4.17)$$

Together, we now have

$$F=F_1+F_2+F_3+F_4 \quad (4.18)$$

or

$$F=\alpha y^2-2\alpha y^4+\beta y-\gamma+kZ^2A^{-4/3}-\tfrac{2}{3}kZ^2A^{-4/3}y^2 +sA^{-\frac{1}{3}}+(4/3)sA^{-\frac{1}{3}}y^2-p\rho^{\frac{1}{3}}Z^2A^{-4/3}+q\rho^{\frac{1}{3}}(Z/A)^{4/3}, \quad (4.19)$$

where

$$y=(A-\ Z)/2A. \quad (4.20)$$

It is seen immediately (a) that the term F_2 will shift the equilibrium toward heavier nuclei, and toward high Z/A values for high densities; (b) that the term F_3 will shift the equilibrium toward smaller Z/A values for high densities; (c) that the term F_4 will also shift the equilibrium toward higher atomic weights. Also, for not too high densities, F_1 alone will determine the equilibrium.

From Eq. (4.19) we can calculate for a given density which atomic weight is the most probable at zero temperatures. From the equations $\partial F/\partial Z=0$, $\partial F/\partial A=0$, we can determine both Z/A and A for the most probable nucleus, i.e., the nucleus for which F is minimum. Van Albada works the other way round and calculates the density for which a given atomic weight is the most probable. After that he calculates the charge of these nuclei. In this way he gets Table VIII.

In the second row of Table VIII, we have inserted the charge of a nucleus of atomic weight A, following from F_1 only. We see that for high densities Z becomes much lower than Z_A: neutron-rich nuclei are favored. Van Albada shows that for densities higher than those In Table VIII, instead of heavier nuclei free neutrons will be formed; the electrons are pressed into the protons, so to say.

Van Albada finally considers the possibility of the inverse process of fission. If we assume that the density slowly was raised from 10^{10} g cm^{-3} to 10^{11} g cm^{-3}, the nuclei with $A=80$ would have to be rebuilt into nuclei with $A=130$, but the process where two nuclei with $A=80$ will form one nucleus with $A=160$ will also liberate energy. Studying these processes, van Albada arrives at the conclusion that any atomic weight between $0.70A_0$ and $1.40A_0$ may be found at a density corresponding to a most probable atomic weight A_0. In the last row of Table VIII, we have inserted the value of the upper limit for the atomic weight.

The drawback of van Albada's discussion is that he does not attempt to calculate the relative abundances of the different nuclei. However, he shows beyond doubt that high densities will result in *heavier* nuclei and nuclei for which the ratio Z/A is *lower* than normal, a fact which is important for the further discussion as we shall see.

(γ) *Where Are the Elements Formed?*

Many people would undoubtedly welcome a solution in which all nuclei were formed in one process, in one large mass, containing the total mass of the universe. However, as was pointed out by Gamow (46G1) and Klein (47K), the lifetime of a mass M, much larger than a stellar mass, will be very short, if its average density is large.

According to general relativity (34T) a closed universe uniformly filled with matter of density ρ will have a radius R and mass M given by

$$R=[3/(\kappa c^2\rho)]^{\frac{1}{2}}, \quad M=[108\pi^4/(\kappa^3c^6\rho)]^{\frac{1}{2}}, \quad (4.21)$$

where

κ = Einstein's gravitational constant

$$=2\times10^{-48}\ \text{g}^{-1}\ \text{cm}^{-1}\ \text{sec.}^2. \quad (4.22)$$

The lifetime of such a universe will be of the order of magnitude

$$\tau=(4\kappa c^4\rho)^{-\frac{1}{2}}, \quad (4.23)$$

where τ is the period during which the dimensions will have increased by a factor e, i.e., the density decreased by a factor 20. For a mass of only 1000 solar masses and a mean density of 10^{11} g cm^{-3}, τ is only 10^{-3} sec. If we therefore wish to account for the formation of the chemical elements by an equilibrium theory, we have to look for smaller masses and then we are immediately led to the interiors of stars.

This was done by van Albada, Hoyle (47H2), Beskow and Treffenberg (47B2) and indirectly by Mayer and Teller (49M). The last authors do not introduce real stars but they restrict themselves specifically to masses of stellar order of magnitude. Also in Beskow and Treffenberg's discussion no specific mentioning of a special type of stars is made but they do make use of the gravitational field of a star, as we shall see under (δ). Van Albada's speculations are qualitative. He identifies tentatively stars with degenerate cores as the sources of heavy elements; the outburst accompanying the distribution of these elements over space is identified with the supernova phenomenon and the remnant should become a white dwarf. Elements of moderate atomic weight should be formed in stars with condensed, hot cores which van Albada tentatively identifies with red giants.

Hoyle's attack seems to us to be more promising. Hoyle considers a star the hydrogen of which has been used up. This star will contract and the central temperature and density will increase. This will go on until rotational instability will occur. This instability occurs when centrifugal and gravitational forces become of the same order of magnitude. Depending on the initial rotational velocity, the temperature and density in the center may rise sufficiently to allow for the formation of heavy elements. In Table IX we give Hoyle's values for the central temperature and density, at the moment the gravitational and centrifugal forces become equal, as functions of the initial rotational velocity. The total stellar mass is assumed to be 10 solar masses, the initial central temperature 3.5×10^7 deg. and the initial central density 3 g cm^{-3}.

From Table IX it is seen immediately that unless the rotational velocity was very large, the central regions probably will finally have obtained sufficiently high temperatures and densities [compare Eqs. (4.2) and (4.3)] so that heavy elements could have been formed. The final outburst, when the star becomes rotationally unstable, accounts for the distribution of the elements over space. This outburst is identified either with a nova- or with a supernova outburst. The mass which is left behind is identified as a white dwarf.

A condition necessary if rotational instability can be reached is that the mass of the star exceeds Chandrasekhar's limit (39C2) which is about three solar masses. This means that the processes discussed here are confined to A-, B-, and O-stars. This also agrees with the condition that the hydrogen has to be used up before this collapsing process starts (compare Table IV). Altogether, we have to look to early type stars for the formation of heavy elements, as long as we restrict ourselves to main sequence stars. It is perhaps not irrelevant that there seems to exist a connection between supernovae II and population I (see Section 1).

(δ) *Gravitational Effects*

If we really accept the idea that heavy elements are formed in certain stars, it follows immediately that we also have to investigate the influence of gravitational fields. This has been done by Beskow and Treffenberg (47B2) whose arguments follow Gibbs' method (75G). If φ_g is the gravitational potential, φ_e the electrical potential, the equilibrium condition for a nucleus of mass $M=ZM_p+NM_n$, and charge Ze will be given by

$$(Z\lambda+N\mu)+(ZM_p+NM_n)\varphi_g+Ze\varphi_e=\text{constant}, \quad (4.24)$$

where λ and μ are again the chemical potentials of the protons and neutrons, respectively. This equation is obviously satisfied for any pair of values for N and Z if

$$\mu+M_n\varphi_g=\text{constant}, \quad (4.25)$$

$$\lambda+M_p\varphi_g+e\varphi_e=\text{constant}, \quad (4.26)$$

which are the equilibrium conditions for neutrons and protons.

The gravitational potential is determined by the density ρ through the Poisson equation:

$$\nabla^2\varphi_g=4\pi\gamma\rho, \quad (4.27)$$

where γ is the gravitational constant. Using Eq. (4.25), we get the following relation between μ and ρ:

$$\nabla^2\mu+4\pi M_n\gamma\rho=0. \quad (4.28)$$

We could get a similar equation for λ from the Poisson equation for φ_e. For all practical purposes, however, we can use the equation expressing electrical neutrality. This equation can be written in the form:

$$C_-=\sum C(N,Z)Z+C_+, \quad (4.29)$$

where C_- and C_+ are the concentrations of electrons and positrons. The concentrations $C(N, Z)$ are functions of λ and μ [see Eq. (4.1)], while C_- and C_+ are determined by the chemical potentials of the electrons and positrons, μ_- and μ_+. The various chemical potentials are connected in the following way:

$$\mu_n = \mu_p + \mu_- + \mu_\nu \quad (4.30a)$$

$$\mu_p = \mu_n + \mu_+ + \mu_{\nu^*} \quad (4.30b)$$

$$\mu_+ + \mu_- = 0 \quad (4.30c)$$

$$\mu_\nu + \mu_{\nu^*} = 0. \quad (4.30d)$$

Equation (4.30a) follows from reaction (4.4); $\mu_\nu(\mu_{\nu^*})$ is the chemical potential of the neutrinos (antineutrinos). Equation (4.30b) follows from the reaction

$$\text{proton} = \text{neutron} + \text{positron} + \text{antineutrino}, \quad (4.4b)$$

and Eq. (4.30c) follows from the reaction

$$\text{photon} = \text{electron} + \text{positron}, \quad (4.30)$$

where the chemical potential of the photons is equal to zero. Equation (4.30d) follows then from a combination of Eqs. (4.30a), (4.30b), and (4.30c). Since the particles are not conserved in the reactions considered, mass terms have to be included in the chemical potentials:

$$\mu_n = \mu + M_n c^2, \quad \mu_p = \lambda + M_p c^2. \quad (4.32)$$

If gravitational effects are taken into account, most of the neutrinos will be formed in pairs with antineutrinos and no longer by reaction (4.4), in any case as long as the temperature is high. In that case, $C_\nu \approx C_{\nu^*}$, and therefore $\mu_\nu \approx \mu_{\nu^*} \approx 0$. From Eqs. (4.30a), (4.30b), (4.30c), and (4.32), we have

$$\mu_- = -\mu_+ = \mu - \lambda + (M_n - M_p)c^2. \quad (4.33)$$

Using Eq. (4.33), we can express C_- and C_+ as functions of λ and μ. Equation (4.29) then gives us a relation between λ and μ. Equation (4.28) gives us a relation between μ and ρ, and finally from:

$$\rho = \sum C(N, Z)M \quad (4.34)$$

we get a third relation between ρ, λ, and μ. It is therefore possible to solve λ, μ, and ρ as functions of the position in the star [compare Eq. (4.28)]. If we know the chemical potentials λ and μ we can calculate the $C(N, Z)$ using Eq. (4.1) or better the corrected Eq. (4.5). Integrating over the whole star we get the abundances of the various elements. Beskow and Treffenberg have done this, using throughout the star $\theta = 1$ Mev. It turns out that if μ/θ becomes larger than 5.7 the density becomes of the order of nuclear densities: We can no longer talk about separate nuclei, the center is one large nuclear core. In the integration, one parameter can be chosen: The value of μ/θ at the center, or if $(\mu/\theta)_c = 5.7$, the radius of the nuclear core, r_0. The result is Table X, taken from Beskow and Treffenberg's paper.

TABLE VIII. Atomic weight and specific charge at various densities.

A	60	80	100	120	140	160	180
Z_A	27	35	43	51	59	66	73
Z	27	32	37	41	45	49	52
Logρ	6.7	10.0	10.6	10.9	11.1	11.2	11.3
$A_{\max}$	84	112	140	168	196	224	252

It is immediately seen that heavy nuclei are sufficiently abundant. However, there is one effect which has not been studied by Beskow and Treffenberg and which may be of very great importance: the fact that all the nuclei formed in the star are neutron-rich. Another difficulty is that in actual stars, the temperature will not be constant throughout the whole star. The temperature may, however, well be practically constant throughout a sufficiently large volume to give Beskow and Treffenberg's abundances as first approximations.

(ϵ) *Transition to Normal Conditions*

There is still a very important problem which has to be solved quantitatively before any of the theories discussed here can be accepted. This is the question as to whether the supernova outburst can be sufficiently fast so as to allow for a "freezing down" of an appreciable fraction of the equilibrium concentrations. There are a few factors which are in favor of the assumption that this outburst is really sufficiently fast. Hoyle (47H2) has studied equilibrium conditions in detail and he finds that for temperatures below 4×10^9 deg. equilibrium will not be attained sufficiently rapid, i.e., if the temperature drops fast from above this limit to below it, the equilibrium situation at the higher temperature will probably be frozen down to a large extent. The break-up of the star may be a matter of minutes. As Hoyle points out, the fact that it only takes a few days for a supernova to reach maximum gives an upper limit. In the first stages of a supernova outburst, most of the emitted radiation will be in the ultraviolet, and therefore unobservable. Hoyle estimates that the transition to much lower temperatures and densities will take place in a few seconds.

In favor of the freezing down is the fact, shown by Klein (47K) that the time scale is drastically changed at high temperatures. This entails that all processes will develop much slower than in normal conditions. The strong gravitational field may also act in the same direction. Van Albada also expresses the optimistic opinion that the break-up will be sufficiently fast.

(ζ) *Transitions from Neutron-Rich Nuclei*

Mayer and Teller (49M) have considered in detail the development of neutron-rich nuclei. They start with very heavy nuclei which will break up as soon as they are free, or show fission. The fragments have an excess of neutrons and also will be highly excited.

TABLE IX. Central temperatures and densities at moment of rotational instability.

Rotational velocity in km/sec.	1	5	10	20	40	100
T_c	$>4\times10^9$	$>4\times10^9$	$>4\times10^9$	$>4\times10^9$	$>4\times10^9$	1.3×10^9
ρ_c	1.7×10^{17}	10^{13}	1.7×10^{11}	2.6×10^9	4×10^7	1.7×10^6

Neutron evaporation will follow and the nuclei formed in this way (perhaps after some β-decay processes) will be the nuclei found at present.

Consider now such a fission product with charge Z. The number of neutrons is not fixed but may vary over a certain range of values. The total internal energy of this neutron-rich nucleus will be varying, probably according to a Gaussian curve. When the neutrons evaporate, they will also carry away some energy in the form of kinetic energy. The evaporation process will stop when there is not sufficient energy left to evaporate another neutron. The probability of the process ending at a definite neutron content N, or of finding the final nucleus with N neutrons, will be given by a Gaussian curve:

$$P(N, Z)=K_Z[E(N, Z)-E(N-1, Z)] \times\exp\{-[E(N, Z)-E_0]^2/\alpha^2\}. \quad (4.35)$$

The spread is assumed to be at least several units, in which case the probability for evaporating down to N, but not to $N-1$ neutrons will be proportional to the binding energy of the last neutron. K_Z is a normalization factor which depends on Z. Finally, E_0 is the binding energy of the nucleus for which $P(N, Z)$ is maximum. E_0 will also depend on Z.

The nuclei produced in this way will now produce the stable nuclei by β-decay. In order to calculate the abundances of the various isotopes, Mayer and Teller assumed α to be constant, and also that the difference between $E_0(Z)$ and $E(N_A, Z)$ was constant. Here, $E(N_A, Z)$ is the energy content of a nucleus with charge Z, for which the packing constant [see Eq. (4.9)] is minimum. Mayer and Teller choose:

$$\alpha=24.15\text{ mMU}, E(N_A, Z)-E_0(Z)=35.69\text{ mMU}. \quad (4.36)$$

Taking into account the known β-decay processes and using Eq. (4.35) they find good agreement between calculated and observed isotopic abundances.

We shall not discuss here their "polyneutron" model but refer to their paper. We shall assume in the following that the heavy nuclei which will show fission are produced in some stars which are becoming rotationally unstable. For the result for the distribution over the various isotopes it is immaterial how these very heavy, neutron-rich nuclei are formed.

(η) *Connection with General Cosmogony*

If we believe that, indeed, all chemical elements are formed in O-, B-, perhaps A-stars, and if we accept for the moment von Weizsäcker's general cosmogony (see Section 2), we can draw the following picture (49H1).

At the beginning of the present epoch, the universe was filled with hydrogen only.* In this hydrogen gas, mass concentrations were formed which finally developed into stars. The energy of these stars is in the beginning provided by gravitational energy. When the central densities and temperatures increase gradually, the deuterium reaction and finally the carbon-nitrogen cycle will start. If the total mass of the star is sufficiently large, the star will be an O- or B-star. It will use up its hydrogen in a period, short as compared to the present epoch. The star will break up after that due to gravitational instability. If the angular momentum at the moment that the hydrogen is used up is not too large, the central portions of the star will have attained densities and temperatures sufficiently high that heavy nuclei, even heavier than the heaviest found now, were formed. The catastrophic outburst will distribute these nuclei, or in any case a fraction of them, over space. Because of the processes discussed by Mayer and Teller, these heavy nuclei will go over into stable nuclei by fission and subsequent β-decay. The resultant isotopic abundances will be about the same as those found in nature.

In order to put this whole theory on a quantitative basis, it will be necessary to investigate (a) whether a sufficient amount of heavy nuclei can have been formed by the outbursts of only a small fraction of all the stars; (b) whether a careful study of the collapsing process of an actual star bears out the qualitative conclusions given here; (c) whether a quantitative analysis of the freezing down process gives really the results mentioned here; it is then necessary to take into account the finite period of the break-up and the processes discussed by Mayer and Teller.

It is not difficult to ascertain that the density of O- and B-stars is sufficiently large and their lifetime sufficiently small to account for heavy nuclei in their present abundances. Using Kuiper's figures (48K) (see also Section 1), it is possible to give a rough estimate of the fraction of the total mass of the universe which might have taken part in break-up processes leading to heavy nuclei. One arrives then at the estimate that about one percent of the total mass of the universe may have been involved in such processes. If one then compares the abundances calculated by Beskow and Treffenberg with the actual observed ones, one arrives at the conclusion that this fraction is sufficiently large.

The origin of the light elements is another problem which has to be looked into rather carefully although there seems to be no difficulty as yet. The abundances found by Beskow and Treffenberg are a good deal smaller than the observed ones (compare Table VII). However, many O- and B-stars will have had such a large angular momentum that the central density at

* The final result is independent of the constitution of the initial gas. Hydrogen only is, however, the simplest possible assumption.

the moment of the break-up was not sufficient to produce the heaviest nuclei, but only the light and medium ones. A final verdict as to the plausibility of this process can only be reached when we have a quantitative law for the distribution of angular momentum over the various stars. However, the angular momentum of some early type stars seems larger than the limiting value (compare Table X).

For a discussion of the effects due to the still large neutron density at the beginning of the break-up, we refer to Mayer and Teller's paper.

Before discussing Gamow's recent theory about this subject, we should like to stress that *any* theory about the origin of the chemical elements has to take into account the processes described here, since even if other processes may have played a role, the life time of O- and B-stars is so short that many of them must have come to the end of their life during the present epoch. In that case, they will have had cores of very high densities where heavy elements will have been formed. The subsequent outburst will have distributed these elements over space. Only a quantitative analysis can decide whether or not these processes are sufficient by themselves, but any other process will have been accompanied by the O- and B-star outbursts.

(b) The α-β-γ-Theory

In order to avoid the difficulties which equilibrium theories encountered before Beskow and Treffenberg's paper (47B2), Gamow (46G1) suggested that the observed abundances of the chemical elements do not correspond to an equilibrium state, but, on the contrary, represent a dynamical building-up process which was arrested by the rapid expansion of the universe. This idea has recently been developed quantitatively by Gamow, Alpher, Herman, and Smart (48A1, 48A2, 48A3, 48A4, 48A5, 48G1, 48S5, 49A2, 49G, 49S2, 49T2, 50A).

This building-up process should have taken place in the first seconds of the present epoch, when the universe had just started expanding. The idea is that in those first seconds the density in the universe had decreased so much that one can speak of a neutron gas instead of just a kind of nuclear fluid. These neutrons will decay, producing protons. After that subsequent proton (neutron) captures and β-processes will produce heavier elements. Since the neutron density will still be much higher than the proton density, neutron capture will be the preponderant process, and it is to be expected that the abundance of the various nuclei will in first approximation be inversely proportional to their neutron capture cross section. This cross section increases steeply with atomic weight up to A about 100 and then remains about constant (46H3), apart from being extremely small for the so-called "magic number" nuclei (48M1). This fits in well with the observed abundance curve showing a sharp decrease up to $A\approx 100$ and then a leveling off, and also high abundances for the "magic number" nuclei.

TABLE X. Relative abundances of elements in different stellar models.[a]

Model Mass of the core Total mass		I — 39	II 0.06 39	III 0.1 15	IV 0.29 0.31	Abundances in the solar system	
$\log C(A)$,	$A=1$	58.7	58.7	58.3	55.3	56.9	H^1
	$A=4$	56.2	51.9	56.4	50.8	55.5	He^4
	12	51.7	50.3	48.1	47.1	54.9	C^{12}
	60	52.3	49.9	49.6	49.0	53.5	Fe^{56}
	120	51.2	49.3	49.2	49.0	48.5	Sn^{120}
	240	49.3	48.8	48.8	48.8	46.9	U^{238}
	300	47.2	49.0	49.1	48.8	—	—

[a] The four models for which Beskow and Treffenberg's values are given here are: I: $(\mu/\theta)_c=3$; II: $r_0=6.10^6$ cm; III: $r_0=7.10^6$ cm; IV: $r_0=10^6$ cm. The masses of the core and of the total star are expressed in solar masses.

In order to be able to account for all the nuclei at present, the universe is assumed to be open (compare the discussion in Section 2, d). The equations governing the change of the relative abundances n_i are (48G1, 48A1, 48A4):

$$dn_i/dt=\lambda_{i-1}n_{i-1}-\lambda_i n_i;\quad i=1,2,\cdots,\tag{4.37}$$

where we have made use of the fact that only neutron capture will change the relative abundances. The coefficients λ_i depend on the neutron capture cross section and the collision frequency. As long as the temperature is so high that all neutrons are fast neutrons, the capture cross sections may be treated as being independent of the neutron velocity. The collision frequency, however, depends strongly on the temperature and on the density of the ylem.† For the sake of simplicity let us assume for a moment that the whole process took place during an interval Δt, during which ρ and T were constant. The final abundances will then depend only on $\rho\Delta t$, the quantity determining the λ_i's. Alpher's calculations then show that for a good agreement between observed and calculated data, one should have

$$\rho\Delta t\approx 10^{-6}\text{ g cm}^{-3}\text{ sec.}\tag{4.38}$$

Since the time interval should be of the order of magnitude of the half-life of the neutron:

$$\Delta t\sim 10^3-10^4\text{ sec.},\tag{4.39}$$

the density during this interval should have been of the order of magnitude

$$\rho\sim 10^{-9}\text{ g cm}^{-3}.\tag{4.40}$$

This density corresponds to a moment about a few hours after the start of the expansion of the universe: The elements were formed at the very beginning of the present epoch. In order to see this, we can consider the cosmological model of the expanding universe (34T).

From this picture it follows that the temperature will decrease with time. The building-up process can only start after the temperature has decreased to the

† Alpher has revived this obsolete noun which means "primordial substance from which the elements were formed."

neighborhood of 10^9–10^{10} deg. For such a temperature, however, the mass density of the blackbody radiation given by

$$\rho_{\mathrm{rad}} = aT^4/c^2 \tag{4.41}$$

(a is Stefan-Boltzmann's constant) will be much higher than the value given by Eq. (4.40): during the first stage of the expansion of the universe, this expansion was governed by radiation and not by matter.

For the expansion, we have the equation [compare Eq. (2.16)]:

$$(1/l)(dl/dt) = [(\kappa\rho c^2/3) - (c^2/R^2)]^{\frac{1}{2}}, \tag{4.42}$$

which can be written in the form:

$$(1/l)(dl/dt) = [\kappa a T^4/c^2]^{\frac{1}{2}}, \tag{4.43}$$

if we neglect the last term and the contribution from the density of the matter. Since T will be proportional to $1/l$ for an adiabatic expansion, Eq. (4.43) can be integrated, and gives:

$$T = (4\kappa a/3)^{-\frac{1}{4}} t^{-\frac{1}{2}}. \tag{4.44}$$

Inserting numerical values, we have

$$T = 2\times 10^{10} t^{-\frac{1}{2}}\ {}^\circ\mathrm{K}. \tag{4.45}$$

For the mass density of the radiation, we then get from Eq. (4.41):

$$\rho_{\mathrm{rad}} = 3\cdot(32\pi\gamma t^2)^{-1} = 4\times 10^5 t^{-2}\ \mathrm{g\ cm^{-3}}. \tag{4.46}$$

The density of matter cannot be calculated in this way. However, it is possible to determine this density by demanding that in the final composition of the universe, hydrogen forms about 50 percent of all matter. Considering thereto the equations governing the deuteron formation, and putting

$$\rho_{\mathrm{mat}} \sim l^{-3} \sim T^3 \sim t^{-\frac{3}{2}} \quad \text{or} \quad \rho_{\mathrm{mat}} = \rho_0 t^{-\frac{3}{2}}, \tag{4.47}$$

we can calculate ρ_0 and obtain

$$\rho_{\mathrm{mat}} = 5\times 10^{-4} t^{-\frac{3}{2}}\ \mathrm{g\ cm^{-3}}. \tag{4.48}$$

We see that the density given by Eq. (4.40) is attained for $t \sim 6000$ sec., when the temperature is still about 2×10^8 °K.

It may be asked whether the results of the neutron capture building-up process would change appreciably from the results obtained by assuming a constant density and temperature during a period Δt, if the values of ρ_{mat} and T of (4.44) and (4.48) were used. Alpher and Herman (48A5) have indicated how this can be done. They show also that inclusion of the change in neutron density due to β-decay will not alter the conclusion reached on the basis of the simplified picture. Smart (49S2) has discussed the effects of nuclear stability on the neutron capture process.

Against this theory various objections can be raised. One of those was mentioned in Section 2: In order to get at all condensation from the gas into galaxies and subsequently into stars, one needs a mean density in the universe which is so high that the universe is closed. A closed universe, however, with a density of 10^{-9} g cm^{-3} needed for the neutron capture theory, contains far less than the total present mass of the universe (compare also the discussion in Section 2, c). The fact that light nuclei like Li, Be, and B have very small abundances can easily be understood (48A2) if we take into account the fact that long after the neutron capture processes have stopped thermonuclear reactions will go on, depleting the amounts of Li, Be, and B originally found, by the nuclear reactions discussed, e.g., by Bethe (39B1).

A second difficulty which does as yet not seem to be solved satisfactorily is the presence of the so-called "shielded" isotopes, i.e., isotopes which could not have been formed by β-decay (a nucleus ${}_Z{}^AX$, where ${}_{Z-1}{}^AX$ is stable). Smart (48S5) suggested the (γ, n) reaction as the process producing these isotopes. As Dr. Turkevich kindly pointed out to me, if the (γ, n) reaction should have been important at all, it should have undone the work of the neutron capture process. This solution has therefore to be treated with the greatest care and suspicion. As a possible, not very promising solution, Dr. Turkevich suggested the (neutrino, electron) reaction.

The most serious difficulty, also mentioned by Alpher (48A4) is the fact that there are no stable nuclei of mass 5 or 8. At $A=5$ there exists a serious bottleneck, which has been studied extensively by Fermi and Turkevich (49T2).‡ First of all, they show that indeed after a few thousand seconds, the relative abundances of neutrons, H, D, T, He_3, and He_4 are about the same as those found at present. They take into consideration all possible reactions between these nuclei. For the temperature and density they use the (varying) values given by Eqs. (4.44) and (4.48). However, it seems to be impossible to go beyond $A=4$. All non-capture reactions are endothermic enough to be out of question. Fermi and Turkevich then considered capture processes like

$$\mathrm{He}^4 + \mathrm{T} \rightarrow \mathrm{Li}^7 + \gamma, \tag{4.49a}$$

$$\mathrm{He}^3 + \mathrm{He}^3 \rightarrow \mathrm{Be}^6 + \gamma, \tag{4.49b}$$

but the cross sections for these reactions are by far too small in the absence of resonance. Resonance has not been observed and does not appear likely. Under the most favorable assumptions, only a fraction of about 10^{-7} of the total mass got beyond the bottleneck while for a successful theory at least a fraction of one-tenth should have passed. Other possibilities do not seem to be any better. Dr. Turkevich mentioned a remote possibility of a chain involving the unstable C^{10} nucleus. The reaction

$$\mathrm{C}^{10} + \mathrm{T} \rightarrow \mathrm{Li}^6 + \mathrm{Be}^7 \tag{4.50}$$

should be the first step in a series of reactions leading

‡ I am greatly indebted to Dr. Turkevich for permitting me to quote their results before they have been published.

to two C^{10} nuclei. The short lifetimes of the nuclei involved do not, however, make this a very promising way out.

It may be added here that this difficulty of not being able to pass the bottleneck of $A=5$ may not be so serious in the case of the establishment of the equilibrium abundances met in the equilibrium theories since the density is in that case so much larger that three-body collisions and even more-body collisions will play a very important role and may well lead to heavy elements in a reasonably short time interval.

Summarizing we feel that at present the equilibrium theory offers a better solution for the problem how to account for the observed abundances of the chemical elements than the α-β-γ-theory.

5. ORIGIN OF COSMIC RAYS

Although in the case of cosmic rays, we are leaving the realm of astrophysics and entering the field of pure physics, most of the existing theories of the origin of cosmic rays borrow astrophysical results so that a brief account of the present situation does not seem to be out of place in the present paper. We shall briefly discuss successively the theories of Klein (44K), Hoyle (47H3), Fermi (49F), and Richtmyer, Teller, and Alfvén (49R1, 49A1, 34D, 48M2). Finally, we shall venture to suggest a fifth possibility which would fit in with von Weizsäcker's general cosmogony and with the equilibrium theory of the origin of the chemical elements (49H4).

(α) Klein's Theory

In 1942 Millikan, Neher, and Pickering (42M1) suggested that cosmic rays might be due to the spontaneous annihilation of atomic nuclei in interstellar space. As was pointed out by Klein (44K), it is extremely difficult to imagine annihilation processes in which single nuclei lose their mass energy. Klein therefore looked for a mechanism through which annihilation could take place and found it in the annihilation process by which negatively charged atomic nuclei would annihilate positively charged atomic nuclei in much the same way as a positron-electron annihilation takes place.

Let us now assume that one-half of all the material of our universe consists of normal matter and the other half of antimatter, i.e., positrons instead of electrons and nuclei built up out of negative protons and antineutrons (which should have a magnetic moment, the direction of which with respect to the spin momentum should be opposite to that of ordinary neutrons, and which should decay into a negative proton by the emission of a positron). These two kinds of matter should be separated in space since they would annihilate each other violently, if brought into contact. It should therefore, tentatively, be assumed that one-half of all the extragalactic nebulae consisted of normal matter and the other half of antimatter. It is, of course, well known that there are no means to decide by spectroscopic observations whether a star or nebula consists of normal or of antimatter.

If this situation should exist, intergalactic space would contain both normal and antinuclei which could annihilate each other. Let us consider for a moment only the case of helium atoms. The number of cosmic rays produced per unit volume of intergalactic space, per second, will be given by

$$N=n\rho_1\rho_2 v\sigma, \tag{5.1}$$

where n is the number of cosmic rays produced in one annihilation process, ρ_1 and ρ_2 the number of normal and antihelium atoms, v their average relative velocity, and σ the effective annihilation cross section.

As soon as the two atoms come close to each other, the electrons and positrons will annihilate each other and the cross section σ will be essentially the cross section for the annihilation process between an α-particle and an anti-α-particle. It can then easily be shown that this annihilation will mainly result from collisions where the two α-particles form a S-state. The cross section for that process is estimated by Klein to be of the order of magnitude

$$\sigma\sim 3\times 10^{-18}\ \mathrm{cm}^2. \tag{5.2}$$

If the annihilation takes place from a S-state, the number of cosmic rays produced will probably be two. The average relative velocity is determined by the kinetic temperature of the intergalactic gas, for which Klein assumes 1000°K, which seems to be a rather high estimate. However, the temperature is not very important in the final conclusions reached.

One might now ask how large the intergalactic densities have to be in order to produce the observed cosmic-ray intensity. This has been done by Klein, who arrives at the estimate

$$\rho_{\mathrm{He}}\sim 2\times 10^{-8}\ \mathrm{atom/cm}^3. \tag{5.3}$$

Klein also shows that the annihilation processes are sufficiently rare not to influence the intergalactic gas density appreciably.

There is one objection which can be voiced against this theory and that is the following one: If the helium density is as large as that given by Eq. (5.3), the sodium density would be of the order of magnitude

$$\rho_{\mathrm{Na}}\sim 10^{-12}\ \text{to}\ 10^{-13}\ \text{at}\ \mathrm{cm}^{-3}. \tag{5.4}$$

In interstellar space, Ti absorption lives have been observed. The Ti density in interstellar space inside our galaxy is of the order of magnitude of 10^{-8} at cm^{-3}, which is between 10,000 and 100,000 times larger than the intergalactic sodium density given by Eq. (5.4). However, the average distances between galaxies are of the order of magnitude of millions of light years, while the average distances between stars are of the order of magnitude of hundreds of light years. Hence, if Ti can be observed in interstellar absorption, Na should be observable in intergalactic absorption. The absence of intergalactic absorption is thus an argument against Klein's hypothesis.

(β) Hoyle's Theory

In connection with his paper on the origin of the chemical elements (47H2, compare Section 4a) Hoyle (47H3) has made some remarks on a possible mechanism for the production of cosmic rays. Let us consider the outburst of an early type star which in the equilibrium theory of the origin of the chemical elements should give rise to the heavy elements. As was discussed in Section 4, the nuclei thrown off by the star during the explosion will have a large atomic weight and small specific charge. Hoyle assumes

$$A \gg 10^6, \quad Z/A \gg 1. \tag{5.5}$$

Hoyle now assumes that these lumps of nuclear material might be separated from their accompanying electrons by a mechanism which he leaves unexplained. There will then occur large electric potentials, of the order of magnitude

$$\Phi = K/Ze, \tag{5.6}$$

where K is the kinetic energy of the lump. The reason for formula (5.6) is that the separation which costs energy should get this energy at the expense of the kinetic energy. Since the lumps leave the star with velocities close to the velocity of light, K will be of the order of magnitude of the rest mass, or

$$K \approx AM_pc^2 \approx A \times 10^9 \text{ ev}. \tag{5.7}$$

If now a proton is ejected by the lump *after* the separation has taken place, it will obtain an energy of the order of magnitude of Φ, i.e., combining Eqs. (5.5)–(5.7) of the order of magnitude of maybe 10^{12} to 10^{15} ev. This process would then be responsible for the cosmic-ray energies larger than, say, a few thousand million ev. The energy range of 10^9 ev is immediately provided by nuclei leaving the star with kinetic energies of the order of magnitude given by Eq. (5.7).

Hoyle's theory easily explains the occurrence of light nuclei in cosmic rays; these nuclei leave the exploding star with velocities, close to the velocity of light. According to experiments of Bradt and Peters (48B3, 49B1) about 30 percent of the primaries are nuclei of Z between 2 and 26.

Hoyle shows that the intensity of cosmic rays, observed here on earth, is compatible with the idea that cosmic rays are formed in supernova outbursts. Richtmyer and Teller (49R1) give as the energy density of cosmic rays, using Rossi's data (48R2):

$$\rho_{\text{tot}} \cong 3 \times 10^{-10} M_pc^2 \text{ erg/cm}^3, \tag{5.8}$$

from which follows for the flux

$$S_{\text{obs}} \cong \rho_{\text{tot}} c \sim 10^{-2} \text{ erg cm}^{-2} \text{ sec.}^{-1}. \tag{5.9}$$

The total energy produced in the form of cosmic rays by supernova outbursts in a spherical shell lying between the distances r and $r+dr$ from the observer is given by

$$4\pi r^2 dr\ \nu\epsilon,$$

where ν is the mean space density of extragalactic nebulae, and ϵ is the total cosmic-ray energy produced per galaxy per second. The intensity observed at the center of the shell will be given by

$$\nu\epsilon dr, \tag{5.10}$$

and the total intensity will be obtained by integrating expression (5.10) and will be given by

$$S \sim \nu\epsilon R \tag{5.11}$$

where R is the radius of the universe for which we shall assume the value 3×10^{27} cm [compare Eq. (2.24)]. Assuming a mean internebular distance of 10^6 light years, we have $\nu \sim 10^{-72}$ cm^{-3}. For ϵ Hoyle assumes that a few percent of the total mass energy of the exploding supernova will be transformed into cosmic-ray energy. Since the pre-supernovae used in the theory of the origin of the solar system are early type stars, we can assume that per supernova outburst about $0.1 M_0 c^2$ is produced as cosmic-ray energy, where M_0 is the solar mass. Assuming one supernova outburst per galaxy per 300 years, we get for ϵ about 2×10^{43} erg/sec. per galaxy. These values finally give us for S:

$$S_{\text{calc}} \sim 0.06 \text{ erg cm}^{-2} \text{ sec.}^{-1}, \tag{5.12}$$

which is of the same order of magnitude as the observed value (5.9).

Hoyle also shows that the fluctuations due to single supernova outbursts (a) are negligible if the observer and the outburst are in different galaxies; (b) in the same galaxy will die out in a period much shorter than the average period between cosmic-ray producing outbursts.

Richtmyer and Teller (49R1) object to Hoyle's theory on the ground that "it seems remarkable that this process (the supernova outburst) gives rise to no other observable energies that are as great or greater than the energies contained in cosmic rays." This objection does not seem to us to hold, since the enormous release of energy observed in the visual region during a supernova outburst certainly shows the release of enormous amounts of energy.

Our main objection is that it seems difficult to find the stripping process necessary for the production of the hardest cosmic rays, and, second, that it seems unnecessary as we shall discuss in (ϵ).

(γ) Fermi's Theory

Fermi (49F) has recently suggested that charged particles in interstellar space might gain energy by "colliding" with the interstellar magnetic fields. Let us therefore first of all consider these magnetic fields which arise from the movement of charged particles in interstellar space. According to Alfvén (43A3), waves will travel in the interstellar matter with a velocity

$$v = H/(4\pi\rho)^{\frac{1}{2}}, \tag{5.13}$$

where H is the magnetic field strength and ρ the density. In the interstellar material, kinetic energy will be converted into magnetic energy until a quasistationary situation has been reached where the kinetic velocity will be of the same order of magnitude as the velocity of propagation of the magnetoelastic waves (5.13). If we assume for the interstellar matter between the gas clouds a density of about 10^{-25} g/cc (see 48S6) and v about 10^6 cm/sec., H comes out to be about 10^{-6} gauss. This magnetic field is, of course, large enough to keep even the most energetic cosmic rays inside our galaxy.

Cosmic rays "colliding" with the magnetic lines of force will be reflected and on the average gain energy since there will be a tendency to move toward equipartition of energy, and since the magnetic lines of force presumably move with a velocity of the order of magnitude of 10^6 cm/sec., the cosmic rays will gain energy. An elementary estimate is obtained by Fermi by considering the collisions as collisions against reflecting objects of nearly infinite mass moving with random velocities v of the order of magnitude of 10^6 cm/sec. The average gain per collision is then given by

$$\delta\epsilon \sim \beta^2 \epsilon, \tag{5.14}$$

where $\beta = v/c$, and the energy ϵ includes the rest energy of the charged particle. If the particle starts with non-relativistic energy, after N collisions its energy will be given by

$$\epsilon = Mc^2 \exp(\beta^2 N). \tag{5.15}$$

Fermi shows that this gain is counterbalanced by ionization loss as long as ϵ is smaller than 200 Mev.

It is known that cosmic rays can be absorbed by nuclear collisions in which the energy is converted into a spray of mesons. The cross section for this process is about $\sigma \sim 2.5\times 10^{-26}$ cm^2, corresponding to a mean free path in interstellar space of about $\lambda \sim 4\times 10^{25}$ cm, corresponding to a density of about one atom per cm^3. The mean lifetime of a cosmic ray will then be

$$T \sim \lambda/c \sim 4\times 10^7 \text{ yr}. \tag{5.16}$$

If we assume that cosmic rays are produced continuously at a constant rate, the distribution of the rays as to age should be:

$$f(t) \sim \exp(-t/T). \tag{5.17}$$

If the average time between scattering collisions is τ, the energy of a particle of age t is given by Eq. (5.15):

$$\epsilon(t) = Mc^2 \exp(\beta^2 t/\tau). \tag{5.18}$$

Combining Eqs. (5.17) and (5.18) one finds for the energy spectrum:

$$f(\epsilon)d\epsilon = \delta(Mc^2)^{\delta} d\epsilon/\epsilon^{1+\delta}, \tag{5.19}$$

where $\delta = \tau/\beta^2 T$. The strong point here is that one, indeed, finds an inverse power law like the one observed. From observations it follows that the exponent should be about 2.9, and hence

$$\delta \sim 1.9 \quad \text{or} \quad \tau \sim 1.9\beta^2 T. \tag{5.20}$$

From Eq. (5.20) it follows that the mean free path between the scattering collisions should be of the order of magnitude $\lambda' \sim 10^{18}$ cm, which does not seem to be unreasonable.

In the last part of his paper Fermi discusses in detail the gain during scattering collisions and the injection mechanism. He points out that in his picture it seems difficult to account for the heavy nuclei in the cosmic radiation. He also points out that one should in this picture expect different energy spectra for heavy nuclei than for protons.

It seems to us that Fermi's picture only gives half the story, i.e., gives a way to account for the energy spectrum, once our knowledge of interstellar magnetic fields is more developed than at present. However, the origin of the original cosmic rays of energies up to 2×10^8 ev for protons, and up to a few times 10^9 ev for light nuclei, which are accelerated by the Fermi mechanism, is still left open. Under ϵ we shall show a possible way to account for these original rays; Hoyle's ideas, of course, suggest a different way out.

(δ) The Solar Origin of Cosmic Rays

Dauvillier (34D) suggested in 1934 that the primary cosmic rays might be high energy γ-rays arriving on the earth from the sun. Menzel and Salisbury (48M2) suggested that the varying electromagnetic field of the sun might act like a linear accelerator, thus providing us with charged particles of sufficiently high energy. Neither Dauvillier nor Menzel and Salisbury gave quantitative arguments to help their theory. Recently Teller and collaborators (49R1, 49A1) have again investigated this possibility of a solar origin of the cosmic rays. Richtmyer and Teller (49R1) have discussed extensively whether or not such a theory is feasible. Their main argument in favor of a solar origin is the high energy density of the cosmic rays. They point out that if the density of cosmic rays were the same all over the universe, about 10^{-4} of the total energy of the universe, including the rest energy of matter, would be in the form of cosmic rays. The strength of this argument as it stands seems doubtful to us. It loses all its strength since there are strong reasons to assume that interstellar magnetic fields are sufficiently strong to keep even the most energetic cosmic rays inside our galaxy. Spitzer (46S3) has estimated that the interstellar magnetic field should be of the order of magnitude of 10^{-12} gauss. Assuming a radius of 30,000 parsec. ($=10^{23}$ cm), this field can keep cosmic rays inside our galaxy of energies up to:

$$\epsilon_{\max} \sim 300Hr \sim 3\times 10^{13} \text{ ev}, \tag{5.21}$$

which is of the same order of magnitude as the maximum energies observed (cf. also 37A1, 49A1). Richtmyer and Teller argue that it is difficult to find a mechanism which would renew the cosmic-ray density in our galaxy every 40 million years—cosmic rays only having a lifetime of about 4×10^7 yr. (see (5.16))—since this mechanism would need about 10^{-4} of the total energy emitted by the stars.

After this, Richtmyer and Teller proceed to investigate the necessary conditions to be satisfied by a solar origin theory. It is necessary to have a field of about 10^{-5} gauss extending all through our planetary system. A weaker field would let the cosmic rays escape and a stronger field should have been observed. This field will limit the cosmic rays to energies up to 10^{14} ev as far as protons, and to 10^{16} ev as far as heavy nuclei are concerned.

Alfvén (49A1) in a paper accompanying Richtmyer and Teller's discussion first shows that magnetic storms on the sun will probably not produce sufficiently strong magnetic fields, but that the interstellar magnetic field itself might be sufficiently strong. If equipartition exists between kinetic and magnetic energy, the field strength should be given by the equation**

$$H^2/8\pi=\tfrac{1}{2}\rho v^2, \tag{5.22}$$

which with $\rho\sim10^{-24}$ g/cm^3, $v\sim10^6$ cm/sec. gives

$$H\sim3\times10^{-6}\text{ gauss}. \tag{5.23}$$

Alfvén points out that the decay of this field will be very slow. This, however, also means that it will take a long time to set up such a field. In view of the fact that it seems well possible that there is a large interchange of matter between the stars and the interstellar material (compare Section 2b), one might ask whether the periods involved are sufficiently small for such an interstellar magnetic field to be set up. This question certainly merits closer investigation, the more so since Spitzer (46S3) explicitly states that he has reasons to prefer a value of H many powers of 10 smaller than that given by Eq. (5.23).

Alfvén suggests as a mechanism for the production of cosmic rays the following. Let us assume that there are already high energy particles present, of energies of at least 10^8 ev—the origin of these particles, the energy of which corresponds to a temperature of the order of 10^{12}°K is left completely open. Alfvén then shows that the varying electromagnetic field of the sun will not be able to accelerate these particles sufficiently. However, the acceleration may occur in trapped orbits in the magnetic field of the sun. Alfvén shows that for particles describing such a trapped orbit in the neighborhood of the earth, the variation of the electromagnetic field due to magnetic storms will be sufficient to double the energy of the particles in about one day. Alfvén finally indicates how it might be possible in this frame work to explain the observed energy spectrum. The main difficulty of Alfvén's ideas as of Fermi's theory is to account for the first high energy particles.

** Compare Eq. (5.13).

(ε) Again: The Supernova Origin?

Since it is rather probable that interstellar magnetic fields are sufficiently strong to keep the cosmic rays inside our galaxy, it seems to be worth while to reconsider the possibility of supernova outbursts as the mechanism producing the high energy particles needed in both Fermi's and Alfvén, Richtmyer and Teller's theories. It is well known that the energy production accompanying a supernova outburst is enormous. Assuming about 10^{49} erg per outburst, which is certainly a lower limit since we can only observe the energy emitted in the visible region, this corresponds to the rest energy of about 10^{28} g, i.e., a mass of the order of magnitude of the earth's mass. Let us assume that a fraction, f, of this energy is emitted in the form of cosmic rays. What will now be the total flux density due to supernova outbursts? Assuming again one supernova outburst per 300 years per galaxy (42Z2), the total output per galaxy per sec. is $f\times10^{39}$ erg. Assuming a mean lifetime of 4×10^7 yr., and a total volume of our galaxy of about 10^{11} cubic parsec., we arrive at a total energy flux density:

$$S_{\text{calc}}'\sim f\cdot10^{-2}\text{ erg cm}^{-2}\text{ sec.}^{-1}, \tag{5.24}$$

from which it follows that, if $f\sim1$, sufficient energy is provided by supernova outbursts. A total cosmic-ray energy of about 10^{49} erg produced per supernova outburst does not seem unreasonable. It is about 10^4 times less than the energy amount suggested by Hoyle. It has in this connection to be borne in mind that a supernova blows off about one solar mass during the outburst and it does not seem unreasonable that a fraction of about 10^{-5} of this mass leaves the supernova with velocities near the velocity of light (compare Hoyle's arguments, under β).

It also must be expected that particles with very high energies will leave the supernova. For the production of these high energy particles we may perhaps offer the following suggestion, first proposed by Cermuschi (39C1). Fission of very heavy nuclei, which presumably will be produced during the preliminary stages of the supernova phenomenon (compare Section 4), will produce high energy nucleons. Assuming a specific charge of one-fourth (compare Table VIII) and using formula (4.9) for the packing constant, it is easy to calculate the energy available in these heavy nuclei. The results of these calculations (compare 49H4) are tabulated in Table XI, giving the energy content E in ev as a function of atomic weight A. From Table XI it is seen that, indeed, even the highest energies may be produced in fission processes.

As yet this is only a tentative suggestion but it might be of interest to investigate, if possible, the following points more carefully both in connection with the equi-

TABLE XI. Energy content of nuclei with $Z/A=\frac{1}{2}$.

A	100	300	500	1000	10000	100000	1000000
E in ev	3.10^8	8.10^8	2.10^9	4.10^9	2.10^{11}	10^{13}	4.10^{14}

librium theory of the origin of chemical elements and in connection with the origin of cosmic rays: (a) How large a fraction of the mass of a supernova will be converted into heavy nuclei; (b) how many high energy nucleons will be produced by fission and what is their energy spectrum; (c) how many light nuclei with velocities in the neighborhood of the velocity of light and thus with energies sufficiently high for Fermi's injection mechanism will be ejected; (d) how many light nuclei, especially α-particles will be produced by tripartition (compare, e.g., 49T1).

Summarizing, we may perhaps conclude that at present there is no reason to exclude the possibility that the original cosmic rays are produced by supernovae and that their present energy spectrum is determined by the Fermi mechanism, a solution which the present author favors at present.

6. STELLAR ENERGY

It is only with the greatest reluctance that the present author writes about this subject. However, it seemed to be worth while to give here a sketch of the present situation in this field. The problems connected with stellar structure will not be discussed. The reader can find a discussion of these problems either in Chandrasekhar's monograph on the subject (39C2) or in Vogt's monograph (43V) or in some of the papers quoted below. As was rightly remarked by Hoyle and Lyttleton (39H), this part of the problem is certainly as important as the investigation of possible reactions which can maintain the luminosity of the star. Also, the discussion of the structure of the stellar interior has to decide in the last instance which nuclear reactions can be used. At present there seems to be slightly too much qualitative work by physicists perhaps not always founded on a sufficiently quantitative basis, while, on the other hand, many quantitative astronomical papers do not stress the qualitative picture without which a real understanding of the problems is so difficult.

Following the description of the Hertzsprung-Russell diagram given in Section 1, the discussion will be divided into six parts which are devoted respectively to: (a) main sequence stars; (b) white dwarfs; (c) red giants; (d) variable stars; (e) novae; and (f) supernovae. It will be seen that apart from the first two groups, satisfactory explanations of the production of energy in these stars is still lacking.

(a) Main Sequence Stars

It is by now general knowledge that von Weizsäcker (37W, 38W) and Bethe (39B1) have indicated the nuclear reactions which can account for the energy production in main sequence stars. The cycle of nuclear reactions is:

$$\left.\begin{array}{r} H_1^1+C_6^{12}=N_7^{13}+\gamma,\\ N_7^{13}=C_6^{13}+e^+,\\ H_1^1+C_6^{13}=N_7^{14}+\gamma,\\ H_1^1+N_7^{14}=O_8^{15}+\gamma,\\ O_8^{15}=N_7^{15}+e^+,\\ H_1^1+N_7^{15}=C_6^{12}+He_2^4.\end{array}\right\} \tag{6.1}$$

Bethe has shown that this reaction, which finally forms one helium nucleus out of four protons under the release of energy, can account for the energy output of the main sequence stars. In Section 2a, δ, we have already commented on the age of stars following from this picture of energy production.

We may draw attention to the important point mentioned by Bethe that the first half of the hydrogen content is used up at a rate more than ten times slower than the rate at which the second half is used up. This entails, of course, that the hydrogen content of most stars is about constant, as was observed by Unsöld (38U, 41U, 44U, 47U). This also makes it possible to establish a mass-luminosity relation!

The energy output from reaction (6.1) in its dependence on density and temperature is given by:

$$\epsilon=a\rho^2T^{18}, \tag{6.2}$$

which is valid for temperatures of the order of the central temperature in the sun. Since the energy output depends so strongly on the temperature, it will be understood at once that most of the energy of the star is produced at the very center.

If the central temperature of the star is much lower than that of the sun, reaction (6.1) no longer supplies the energy and the energy output depends less strongly on the temperature:

$$\epsilon'=a'\rho^2T^{3.5}; \tag{6.3}$$

the reaction cycle is now (38B):

$$\left.\begin{array}{l} H_1^1+H_1^1=D_1^2+e^++\gamma,\\ H_1^1+D_1^2=He_2^3+\gamma,\\ He_2^3+He_2^4=Be_4^7+\gamma,\\ Be_4^7 \qquad =Li_3^7+e^++\gamma,\\ H_1^1+Li_3^7=2He_2^4.\end{array}\right\} \tag{6.4}$$

This reaction cycle accounts for the energy output of the late type main sequence stars.

(b) White Dwarfs

Although the carbon-nitrogen cycle (6.1) or the deuterium cycle (6.4) can be used to explain the luminosity of the main sequence stars, it seems at first difficult to explain the low luminosity of white dwarfs such as, for instance, Sirius B. This is easily seen. The radius of Sirius B is about 0.01 of the solar radius, but the mass is equal to that of the sun. If we assume the same central temperatures, their luminosities would show the ratio:

$$L_{\mathrm{Sir}}/L_0=(R_{\mathrm{Sir}}/R_0)^3(\rho_{\mathrm{Sir}}/\rho_0)^2\sim 10^6, \tag{6.5}$$

but the observed ratio is only 0.01. Eddington suggested as a possible solution the absence of carbon, the catalyst of the cycle (6.1).

There exists, however, a more probable solution, as was shown by Schatzman (46S1). Schatzman uses a simplified stellar model where the stellar material is supposed to consist of hydrogen atoms, helium atoms, and "Russell atoms," representing the Russell mixture. Because of the strong gravitational field of the stars, "Russell" and He atoms will be concentrated toward the center and the hydrogen will float on the surface. For the energy production the extension of the transition layer where hydrogen and "Russell" atoms are mixed is important since we need the hydrogen for the energy production and the "Russell atoms" as the catalysts for the carbon-nitrogen cycle. The extension of this layer can be calculated by usual statistical methods. The thickness, Δr, of this layer depends on the central temperature T_c and the gravitational field of the star in the following way:

$$\Delta r=1.5\times 10^8 T_c/g \text{ cm}, \tag{6.6}$$

where g is the gravitational acceleration at the surface of the star. For the sun, we have

$$T_c=2\times 10^7,\quad g=3\times 10^4,$$
$$\Delta r=10^{11}\text{ cm}(r_0=7\times 10^{10}\text{ cm}), \tag{6.7}$$

so that throughout the sun all constituents are mixed: Δr is large as compared to the solar radius, r_0.

For Sirius B, however, we have

$$T_c\sim 10^7,\quad g\sim 10^{18},\quad \Delta r\sim 10^7\text{ cm}(r_0\sim 10^9\text{ cm}): \tag{6.8}$$

here exists, indeed, a mixed layer which has a height small as compared to the dimensions of the star; the separation is nearly complete. It is now possible to calculate the total energy output of such a star, and the calculated values agree very closely with the observed ones. For low luminosity white dwarfs, the deuterium cycle takes over again from the carbon-nitrogen cycle. Although Schatzman originally thought that the observed luminosity agreed only with the calculated one if the original Fermi selection rule (34F) for β-radioactivity, which states that the $H_1^1+H_1^1$ reaction is forbidden, was taken, he later (48S1) arrived at the conclusion that the Gamow-Teller (36G) selection rule had to be preferred.

In Schatzman's picture, it is also possible to understand the absence of metallic lines in the spectra of white dwarfs (48S2). This absence is again due to the gravitational separation which brings all metal atoms toward the center of the star.

It must be remarked here that Fowler (26F) was the first to draw attention to the fact that the interior of a white dwarf must be a degenerate gas. This study of degenerate stars has been the subject of many papers, notably those of Chandrasekhar (31C, 35C, 39C2).

Problems connected with the origin of white dwarfs have been discussed by Hoyle (47H5) and Schatzman (47S2).

(c) Red Giants

Although it seems possible to understand the energy output of the main sequence stars or white dwarfs, the theories proposed up to now for the energy generation of red giants, variable stars or novae are still inadequate. It may well be that these problems can be tackled satisfactorily in the near future, although it is quite possible that more observational data are needed before this can be done.

Gamow and Teller (39G5) originally suggested that red giants should be young stars which used up their light elements. This suggestion was also discussed by Greenfield (41G2). However, van Albada (43A1, 45A) has shown among other things that the distribution in the Hertzsprung-Russell diagram does not agree with this assumption which was also admitted by Gamow himself. Gamow and Keller (45G2) have recently given a new model. However, Mrs. Harrison (46H2) has shown that the model used by Gamow and Keller does not lead to a star of the red giant type.

Most authors seem, however, to agree that an important point is the fact that the hydrogen content of the envelope which is part of the giant, must be different from that of the interior of the star (42H2, 45A, 45G2, 46M, 47W3). Also there seems now to exist general agreement that red giants are old stars which agrees with Baade's or von Weizsäcker's classification.

Recently Richardson and Schwarzschild (48R1) have considered a completely different model. They do not assume any change in chemical composition between the interior and the exterior. However, they assume that between the core where the carbon cycle takes place and the outer atmosphere there exists an isothermal zone which carries most of the energy in acoustic waves. Richardson and Schwarzschild point out that as yet their considerations are only speculative since the turbulent velocity in the core has to be rather larger than is usually assumed. At present, however, their mode of attacking the problem of energy generation in giants seems to be the most promising.††

(d) Variable Stars

The problems of variable stars are still far from satisfactorily solved. Of all the variables, the Cepheids have received the most extensive investigations. In 1879, Ritter (79R) suggested that the variation of the light curve of the Cepheids might be due to a pulsation.

†† Dr. Schwarzschild has kindly informed me that further computations on this model are under way. Dr. Li is studying the influence of a change of composition in connection with this model, and Mr. Epstein is using this model in connection with pulsating stars.

Eddington (17E, 18E1, 18E2) developed this idea in a quantitative way. Such a theory is able to explain immediately the relation between the period and the density (1.1) although the periods predicted by this theory are somewhat shorter than the observed ones. This is rather serious since Rosseland (43R) in his George Darwin lecture pointed out that one should expect longer periods for anharmonic pulsations than for the harmonic pulsations of Eddington's theory.

Another difficulty is that the pulsation theory is not able to explain the phase retardation, the period-luminosity relation or the skewness of the light curve. As we shall see, recent work, however, has shown considerable progress along these lines.

Milne (33M) has stressed in this connection the importance of relaxation phenomena which also show a skewness of the curves. The same suggestion was independently made by Wesselink (39W1). The difficulty is then to understand why such a relaxation process would take place. Eddington (41E) suggested the following solution. Let us suppose that the retardation of the outflow of energy is connected with a hydrogen layer which is alternately ionized and neutralized during the pulsation of the star. The corresponding change in the ratio of the specific heats makes it then possible that energy is stored in that layer during a part of the period. The calculations as they are at present seem to indicate that in this way the necessary phase retardation can be obtained. It is then also possible to understand the period-luminosity relation and the fact that there seems to exist an upper limit for the magnitude range of the Cepheids. It would be of extreme importance if this work of Eddington's could be extended so as to take second-order effects into consideration since these second-order effects are very important as soon as the amplitude of the oscillation is finite. It may be remarked here that Schwarzschild (38S1) has drawn attention to the fact that the pulsation should have the character of a running wave at the outside of the star but of a standing wave inside.

Rosseland (43R) has shown the importance of the second-order terms on the form of the light curve. His results, and also later work by Bhatnagar and Kothari (44B3), seemed to indicate a discrepancy between the observed and the calculated amplitudes of the light curves. Sen (48S4), however, has shown that it is possible in this way to get good agreement between theory and observational data as far as period, skewness of the velocity curve, *and amplitude* of the light curve of δ Cephei is concerned, using a non-homogeneous model of the star (see also 49S1).‡‡

If Sen's model should give a solution for the Cepheids, there still remains the question of the other variables. Also there are the problems of the origin of these variable stars which are rather difficult as was, e.g., remarked by Hoyle (47H2).

‡‡ Rosseland (49R2) has given an extensive account of the pulsation theory in a recently published monograph.

There are two other, different, ways of attacking this problem which we should like to mention here, *viz.*, Hoyle and Lyttleton's theory (43H) and Jordan's theory (45J1).

Hoyle and Lyttleton follow up Kuiper's observations of β-Lyrae (41K). They propose that every Cepheid should be a very close binary with a common atmosphere. If one accepts that this atmosphere does not share the rotation of the system, an oscillating gravitational field will act on the atmosphere, resulting in a *forced* oscillation of the atmosphere, as against a *free* oscillation in the case of the pulsation theory.

Hoyle and Lyttleton seem to be able to explain the observed velocity curve, and the observed period density relation (1.1). However, the region occupied by their models in the H-R diagram differs greatly from region V.

Jordan leaves completely the realm of present-day physics. He introduces a *fourth law of thermodynamics* which we shall not discuss here. This law is connected with the assumption of an upper limit of the temperature scale at

$$T_0 = hc/k\lambda_0 \sim 10^{12} \text{ deg.}, \tag{6.9}$$

where λ_0 is the elementary length introduced by Eq. (2.18).

All equations of state have to be changed so as to take into account that a temperature always has to be smaller than T_0. If one then plots the adiabates in a temperature *versus* volume diagram, it turns out that under circumstances, for dense matter, *an increase in temperature will accompany an adiabatic expansion.*

Jordan now assumes that the star is regulated by a core of nuclear density, where alternately the neutrons go over into protons and electrons, and the other way round. Owing to the increase in temperature accompanying the expansion, there will indeed be a time lag between the velocity curve and the light curve, as is observed. A second effect which can be explained in this theory is the fact that there is a lower limit for the periods of Cepheids. One should, indeed, expect that the period should always be larger than the half-life of the neutron. The next effect is that the amplitudes should show a very marked maximum since we have now a forced vibration. Also, it is possible to understand why only the first mode contributes to the oscillation. There are two mechanisms regulating the oscillation of the outer regions, *viz.*, the mechanical vibrations and the changing radiation pressure. In the first mode, the two are in phase, but in the other modes, they are partly in phase and in other regions out of phase. Finally, in this theory, the Guthnick effect can be understood. This effect needs a rather sensitive central mechanism like the one proposed by Jordan or the one proposed by Eddington. It seems, however, that the Guthnick effect is an effect which cannot easily be understood in Hoyle and Lyttleton's theory. Jordan's theory is at present still in its first, qualitative, stages which makes it difficult to judge it adequately, but it seems to us that as long as there seems to be a good possibility to arrive at more reasonable explanations, his theory is slightly too fantastic.

(e) Novae

The situation in the case of the novae is very unsatisfactory. There are authors who believe that novae are the next step in the series of variables. In favor of this idea is the fact that there exist recurrent novae. The amplitude of the oscillation should then be so large that the normal oscillation makes place for a nova outburst. However, the position of the prenovae in the H-R diagram, at the short period end of region V and not at the long period end, is certainly in contradiction to this idea. This is, e.g., probably sufficient to discard Hoyle and Lyttleton's theory (43H). Their idea is that the binary which in their theory (see d) accounts for the Cepheid variability finally becomes unstable through accretion, the separation decreases and the final configuration is rotationally unstable. At that moment matter will be ejected.* This matter will sometimes leave the system, or in other instances partly fall back onto the star, and then give rise to a second outburst. Against this theory there are the following objections: First of all, the prenova should in this theory be expected to lie to the right of the main sequence and not to the left of it (compare d). Second, it seems to us that the period between two outbursts would be very much smaller in this theory than the observed intervals. Hoyle and Lyttleton show that this theory can account for the observed change in magnitude at the outburst.

Jordan's explanation of the nova phenomenon has again the disadvantage of being extremely sketchy and of being connected with a departure from normal physics. He assumes that a post-supernova (see f) has a nuclear core, i.e., a central region of nuclear density, which might in this case be explosive because of the β-radioactivity of the neutron. The planetary nebulae should now not be the results of novae but of supernovae outbursts. Minkowski (48M3) also stresses the possibility that the relation between planetary nebulae and supernovae is much closer than between planetary nebulae and novae.

McLaughlin (41M) remarked already that the fact that the pre- and postnovae are practically identical stars, excludes the Gamow-Schoenberg neutrino-collapse theory (41G1) of novae outbursts as a possibility.

Finally, Biermann (39B2) has given a theory which is able to explain the emitted energy, observed frequency of novae, and the observed expansion velocities of the gases. The frequency of nova outbursts is about once in every 10^7 to 10^8 yr. per prenova. Biermann's idea now is that a prenova is an early type star with very little hydrogen, and with average magnitude, i.e., with a lower magnitude than of the same type main sequence star. The energy source of these stars should then be contraction. According to Biermann, the final stage of this star would be a white dwarf, although this seems to be in contradiction to our present knowledge that there is every reason to believe that white dwarfs may well possess an appreciable amount of hydrogen (46S1). It would be important to check in how far a higher hydrogen content would change Biermann's considerations, especially since it seems that the low hydrogen content is essential for his theory.

Biermann shows that stars with little hydrogen will show an instability zone, the ionization energy of which will be released at an outburst. It is shown that this ionization energy is sufficient to explain the change in magnitude of the novae: a total energy release of about 10^{44} erg. The instability zone goes to a depth where the temperature is about one-tenth of the central temperature, and about one-tenth of the total mass is involved. The thermal velocities at these temperatures are of the same order of magnitude as the observed expansion temperatures. Finally, Biermann is able to explain the observed frequency of nova outbursts.

(f) Supernovae

The energy production in supernovae and the cosmogonical implications, though still far from finally solved, seem to become slowly understandable. Whipple (39W2) suggested that a collision between two stars would produce the desired effect. It seems, however, difficult to understand, in that case, why there is no smooth transition between novae and supernovae (40Z).

The next possibility was that proposed by Baade and Zwicky of a sudden collapse of a star into a neutron core. This theory has been criticized by von Weizsäcker (47W1) since in this case the relation between novae and the fast rotating O- and B-stars cannot be understood. The star can only collapse as fast as it can lose its angular momentum.

Von Weizsäcker (47W1) himself suggests that the rotational instability of an O- or B-star which has used up its hydrogen might be the solution for the supernova phenomenon. The same process has also, independently, been suggested by Hoyle (47H3). Hoyle's theory is slightly further developed than von Weizsäcker's, but there still remains a large amount of quantitative analysis to be made. It would especially be of interest to try to give a theoretical synthesis of the spectrum which could be expected if such an outburst took place. The work of Payne-Gaposchkin and Whipple (40G1, 41W) should have to be extended to higher temperatures corresponding to the temperatures of the inner parts of the star involved in the outburst.

Chandrasekhar (see, e.g., 42S) suggested that the supernova phenomenon was connected with the fact that there exists an upper limit for the mass of a de-

* It must be remarked here that this process is not a slow development through equilibrium configurations but a rapid dynamical process.

generate star. If the stellar mass were larger than this limiting mass, a supernova outburst to get rid of the excess mass might result.

Another very promising theory is that proposed by Schatzman (46S2, 48S3). In his theory of white dwarfs (see b), Schatzman has shown that white dwarfs of large mass will increase in luminosity and central temperature when time goes on, that is, with decreasing hydrogen content. This will finally lead to an instability. His theory now proceeds in the following way. The transition layer becomes unstable and this instability will be propagated as a shockwave.† Matter is expelled and the exclusion energy of the electrons in the Fermi sea in the star will be converted into kinetic energy.

Schatzman shows that the energy emitted in this process is of the right order of magnitude. Because only under special circumstances the shockwave will reach the surface, only part of the white dwarfs will give rise to supernovae: hence the rarity of the phenomenon. Here again, it might be worth while to calculate a theoretical spectrum.

We may perhaps express the optimistic point of view that Schatzman's supernovae can be identified with the supernovae of type I while the supernovae of type II are due to the process proposed by Hoyle and von Weizsäcker.

We have not discussed in the present paper the evolution of stars although this is certainly an extremely important cosmogonical problem. We may refer here to the many papers which have appeared on this subject (e.g., 38G1, 39G1, 39G2, 39G3, 42S, 47C).

† Rosseland (46R) has also stressed the importance of shockwaves for a theoretical explanation of nova or supernova outbursts.

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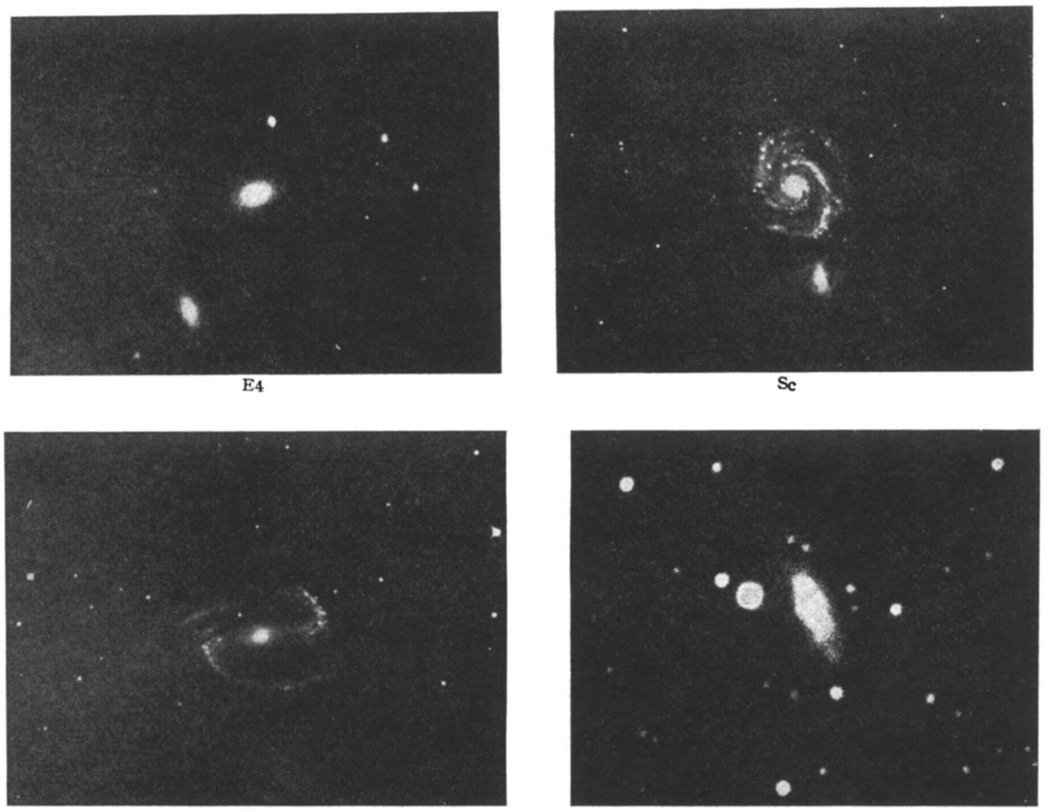

E4 Sc

SBc Irregular

Courtesy Yerkes Observatory

FIG. 2. Four examples of extragalactic nebulae.