The Continuous Spectrum of the Sun

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INTRODUCTION

UNDER spectroscopic examination, sunlight is found to comprise a continuous spectrum which is crossed by thousands of absorption lines. The continuous spectrum has long been attributed to the thermal radiation of the relatively dense regions of the solar atmosphere, and the Fraunhofer lines to the characteristic absorptions of the atoms comprising the less dense and less hot upper atmosphere. The layer of the solar atmosphere in which the observed continuum originates is called the "photosphere," and the higher layer in which the Fraunhofer lines originate is called the "reversing layer."

Except for irregularities produced by sunspots and certain other transitory features of relatively small dimensions, the radiation which emerges from the photosphere is found to be the same in all respects at points equidistant from the center of the solar disk. It is found, however, that radiation from different zones of the disk does not have the same characteristics. In monochromatic light of every wave-length the brightness of the disk decreases from the center to the limb, the decrease being greater in violet light than in red. This darkening at the limb is qualitatively explained by the consideration that we view an incandescent atmosphere of finite depth rather than a luminous surface. The atmosphere is not perfectly transparent, so that radiation which originates in the deeper layers is largely absorbed in passing through the superincumbent layers. Thus, we may consider that at the center of the disk radiation of any given wave-length originates, on the average, at a certain geometrical depth in the solar atmosphere, and is therefore characteristic of the temperature at that depth. But radiation originating at that depth traverses the superincumbent layers along the shortest possible path at the center of the disk, while radiation originating at the same depth but emerging near the limb follows a much longer, oblique path through the superincumbent layers. It is easy to see, therefore, that light from a point near the limb must originate, on the average, at higher and cooler levels of the solar atmosphere, and, consequently, must be less intense than light which emerges at the center of the disk.

Suppose that the sun has unit radius, and consider the light which reaches the earth from a point on the zone of radius $\sin\theta$. It will be seen in Fig. 1 that such a ray emerges from the photosphere at an angle θ with the direction of the radius vector to the point of emergence. In order, therefore, to provide a complete description of the continuous spectrum of the sun, we must be able to specify, in absolute units, for all angles of emergence and for all wave-lengths, the monochromatic intensity $I_\lambda(\theta)$ of radiation[1] which emerges from the photosphere in the direction θ. Such a description of the continuous spectrum of the sun, for the region 3400A to 21000A, is provided by the very extensive series of observations begun about 1890 by S. P. Langley, of the Smithsonian Astrophysical Observatory, and continued by C. G. Abbot and his collaborators to the present day.[2] But although the general form of the function $I_\lambda(\theta)$ has been established by these and other[3] investigators for 20 years or more, the physical interpretation of the observations has remained obscure until the past year. Today, for the first time, the physical theory is able to account in detail for the continuous spectrum of the sun.

THE CONTINUOUS SPECTRUM OF THE SUN

In the precise determination of the function $I_\lambda(\theta)$, the first requisite is a sufficiently sensitive

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[1] The emergent light is completely unpolarized.

[2] C. G. Abbot *et al.*, Ann. Astrophys. Obs. Smithsonian Inst. 1 (1900)—6 (1942). See also numerous papers in Smithsonian Miscellaneous Collections.

[3] An extensive bibliography will be found in A. Unsöld, *Physik der Sternatmosphären* (Julius Springer Verlag Berlin, 1938), p. 470.

non-selective receiver of radiation. The Smithsonian observers have employed the bolometer, as invented by Langley and developed by Abbot. In this instrument, the sensitive element is a strip of blackened platinum about 16 mm long, 0.12 mm wide, and 0.02 mm thick, which forms one arm of a Wheatstone bridge. The bridge is balanced in darkness; then, when radiation is allowed to fall upon the sensitive element, its temperature increases slightly and the bridge is unbalanced. Extensive tests have shown that, when the circuit elements are properly chosen, the galvanometer current is strictly proportional to the intensity of the light striking the receiver.

In order that observations may be made in nearly monochromatic light, the receiving strip of the bolometer is placed in the focal plane of a prism spectrograph, as shown at A in Fig. 2. B is a silvered mirror of about 1.2-m focal length, which produces a spectrum with dispersion equal to approximately 20 A/mm. The receiving strip acts as its own monochromator and intercepts a strip of the spectrum about 2A in width. C is the dispersing prism, which can be rotated to reflect any wave-length at minimum deviation to A. D is a collimating mirror, and E the slit of the spectrograph. The combination of bolometer plus spectrograph is known as a spectrobolometer.

The spectrobolometer is not suited to absolute measurements. It is well adapted, however, to the measurement of the darkening function $\phi_\lambda(\theta)$, which is defined in the relation

$$\phi_\lambda(\theta)=I_\lambda(\theta)/I_\lambda(0), \qquad (1)$$

where $I_\lambda(0)$ is the monochromatic intensity distribution at the center of the disk. In practice, $I_\lambda(\theta)$ is found, through this relation, from the measured values of $\phi_\lambda(\theta)$ and $I_\lambda(0)$. In order to determine $\phi_\lambda(\theta)$ for a given wave-length, the prism is set to transmit that wave-length to the bolometer strip. An image of the sun 21 cm in diameter is then formed by the concave mirror F at the slit E; the mirror F is stationary and is illuminated by a beam of sunlight reflected from two properly oriented plane mirrors G which are mounted at the top of a 75-foot tower. As the sun describes its diurnal arc, its image moves uniformly across the slit, so that the bolometer strip is exposed successively to monochromatic rays originating at all points along a diameter of

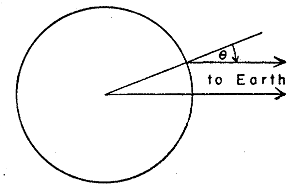

FIG. 1. The angle of emergence of a ray from a zone of radius $\sin\theta$ on the solar disk.

the solar disk. The galvanometer deflections are recorded on a moving plate, giving rise to a ∩-shaped trace. The ratio of the ordinate at any point to the ordinate at the center then gives the value of the darkening function for the corresponding values of θ and λ. Once the solar image has drifted across the slit, the prism can then be set to transmit another wave-length to the bolometer strip, the mirrors G can be reoriented, and another tracing can be made in the new wave-length. Since the object of the measurement is the determination of a monochromatic intensity ratio, there can enter no question of units or of differential extinction in the terrestrial atmosphere or in the optical train.

The result of a great many such observations of the sun is that the darkening function can be represented within the limits of experimental error by the empirical law[4]

$$\phi_\lambda(\theta)=A_\lambda+B_\lambda\cos\theta+C_\lambda\cos^2\theta, \qquad (2)$$

where A_λ, B_λ, and C_λ are quadratic functions of λ. Table I gives numerical values of $\phi_\lambda(\theta)$ for selected values of the arguments. Incidentally, it will be observed that Eq. (2) predicts finite intensity at the limb of the sun. Actually, Abbot's observations do not give the darkening function at the extreme limb, but other investigations have confirmed the fact that the limb is extremely sharp.[5]

In order that $I_\lambda(\theta)$ can be found in Eq. (1), there now remains to be determined the intensity distribution $I_\lambda(0)$ at the center of the disk. The Smithsonian observers have used the spectrobolometer again for this purpose. A plane mirror

[4] D. Chalonge and V. Kourganoff, Ann. d'Astrophys. **9**, 69 (1946).

[5] R. Wildt, Astrophys. J. **104**, 81–83 (1946) cites evidence that the brightness falls by a factor of 10 over an angular distance of half a second of arc. The sharpness of the limb indicates a thin photospheric layer, and hence a relatively steep temperature gradient.

at H in Fig. 2 is mounted on an axis parallel to that of the earth and is rotated at such a rate that the beam of sunlight reflected to the fixed plane mirror J leaves J in a direction which remains constant as the sun describes its diurnal arc. The prism is caused to rotate at uniform speed, and the galvanometer deflection is again recorded on a moving plate as the bolometer strip is exposed successively to light of different wave-lengths. The ordinate of the resulting record is therefore proportional at every wave-length to the strength of the sunlight entering the spectrograph slit.[6]

In this case, it is to be noted that no image of the sun is formed, and light from all parts of the solar disk therefore enters the slit simultaneously. It follows that the monochromatic fluxes F_λ are measured, rather than the monochromatic intensities $I_\lambda(0)$. However, it is a direct consequence of the definition of F_λ that

$$F_\lambda = 2\pi \int_0^{\pi/2} I_\lambda(\theta) \cos\theta \sin\theta d\theta.$$

Dividing by $I_\lambda(0)$ and adducing Eq. (1), we

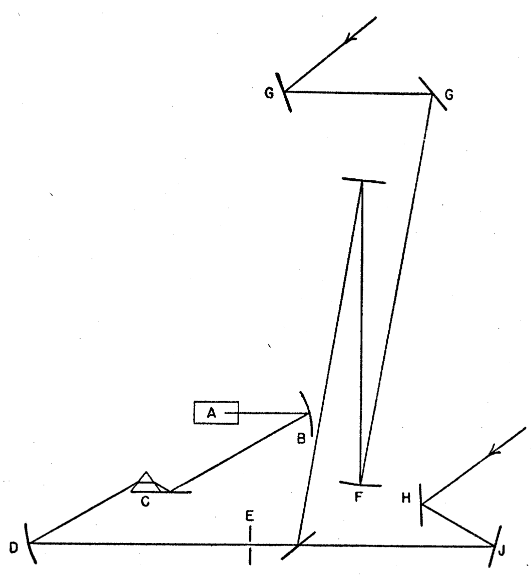

FIG. 2. The spectrobolometer of Abbot and auxiliary optical parts (schematic).

[6] The relative transmission of the optical train at various wave-lengths must be determined and allowed for. This correction is readily accomplished through the use of auxiliary mirrors, etc.

therefore find that

$$I_\lambda(0) = F_\lambda \Big/ \left[2\pi \int_0^{\pi/2} \phi_\lambda(\theta) \cos\theta \sin\theta d\theta \right]. \quad (3)$$

Since $\phi_\lambda(\theta)$ is already known, the measurement of the fluxes F_λ suffices to determine $I_\lambda(0)$ through this relation.

Really it is not even the fluxes F_λ that are measured directly. Three difficulties intervene; they are (1), the spectrobolometer cannot be accurately calibrated in absolute units; (2) after the measurements have been converted to absolute units, they will represent the energy distribution only as it is modified by the absorption in the terrestrial atmosphere; and (3) after the measurements have been corrected to the top of the terrestrial atmosphere, they will represent the fluxes at the earth's distance from the sun, and not at the photosphere.

In order to circumvent the first of these difficulties, Abbot observes the integrated sunlight with an absolute instrument at the same time as the spectrobolometric tracing is made. The instrument used is a kind of radiation calorimeter, called a pyrheliometer. Through an aperture of accurately known cross section, a direct beam of sunlight is allowed to enter a cylindrical chamber having blackened walls. Water circulates through the walls at a known rate. By means of a resistance thermometer, the observer measures the rate at which the water is heated by the sunlight absorbed on the walls. Provision is made for electrical heating of the walls at an accurately known rate, so that the precision of the instrument can be adequately checked.

Let us then consider that, with the sun at a given angle z from the zenith, it is observed simultaneously with the spectrobolometer and with the pyrheliometer. The spectrobolometric tracing yields a function $\text{const} \times f_\lambda(z)$ which gives the monochromatic fluxes, at zenith angle z, in arbitrary units; the pyrheliometric observations give the integrated flux $f(z)$, at the same zenith angle, in absolute units. By integration of the spectrobolometric tracing, we may therefore determine the scale constant, for we must have

$$f(z) = \int_0^\infty f_\lambda(z) d\lambda$$
$$= \left[\int_0^\infty \text{const} \times f_\lambda(z) d\lambda \right] \Big/ \text{const}.$$

The combination of pyrheliometric and spectrobolometric observations therefore yields the monochromatic fluxes $f_\lambda(z)$, in absolute units, at the base of the terrestrial atmosphere.

The atmospheric extinction is next evaluated separately for light of various wave-lengths. It suffices to suppose that the terrestrial atmosphere comprises plane parallel layers. Let the x axis be taken in the direction of the downward normal to these layers, and let the origin be at the observer. Consider a ray of sunlight which traverses the atmosphere along an oblique line[7] through the observer and at a zenith angle z. In traversing the atmospheric layer of thickness dx, the light will travel the differential path length $dx \sec z$. According to Beer's law of absorption, the extinction will then be given by the equation

$$dI_\lambda/I_\lambda = -\kappa_\lambda dx \sec z, \qquad (4)$$

where κ_λ is the monochromatic coefficient of absorption at height $-x$. Since the angular diameter of the sun is only 32 minutes of arc, rays from all points of the disk traverse the atmosphere at essentially the same zenith angle at any given time, so that we may write Eq. (4) with sufficient precision in terms of the fluxes:

$$df_\lambda(z)/f_\lambda(z) = -\kappa_\lambda dx \sec z.$$

Integrating this equation from the top of the atmosphere to the bottom, we obtain

$$f_\lambda(z) = f_\lambda \exp[-\tau_\lambda \sec z], \qquad (5)$$

where $f_\lambda(z)$ is the flux at the base of the atmosphere, f_λ is the flux at the top, and

$$\tau_\lambda = \int_{-\infty}^{0} \kappa_\lambda dx$$

is the optical thickness of the atmosphere.

From Eq. (5) it follows that, for any given wave-length, $\log f_\lambda(z)$ is a linear function of $\sec z$. A series of determinations of $f_\lambda(z)$ made at different values of z can therefore be accurately extrapolated to $\sec z = 0$, thus yielding the desired quantity f_λ. It should be noted that Eq. (4) does not imply an absorption law of the same form for the integrated intensity. The observations of the monochromatic fluxes at various zenith angles amply confirm the predicted linear relation between $\log f_\lambda(z)$ and $\sec z$, but other observations of the integrated flux exhibit definite deviations from a linear relation. The fact that the extrapolation to the top of the atmosphere can be made accurately only for the monochromatic fluxes has been of fundamental importance in the development of the methods of the Smithsonian observers.

[7] The curvature of the ray produced by the varying index of refraction can be neglected.

TABLE I. The darkening function $\phi(\chi\theta)$, after Abbot.

$\lambda(A)$ \ $\sin\theta$	0.00	0.20	0.75	0.92	0.95
3860	1.0000	0.9797	0.7097	0.4826	0.4177
4560	1.0000	0.9857	0.7564	0.5377	0.4706
6040	1.0000	0.9894	0.8160	0.6485	0.5935
10310	1.0000	0.9978	0.8886	0.7724	0.7298
20970	1.0000	0.9965	0.9361	0.8664	0.8374

At this point, notice should be taken of two important byproducts of the measurements. The first is the determination of the optical thickness of the terrestrial atmosphere in light of various wave-lengths; these are found, of course, from the slopes of the $\log f_\lambda(z)$-*vs.*-$\sec z$ relations. The second result is the total integrated flux of solar radiation at the earth's distance from the sun; this is[8]

$$f = \int_0^\infty f_\lambda d\lambda = 1.94 \text{ cal./cm}^2 \text{ min.}$$
$$= 1.35 \times 10^6 \text{ erg/cm}^2 \text{ sec.}$$

The quantity f is the so-called "solar constant"; its exact value is of prime importance in meteorology as well as in astrophysics. As a matter of fact, the accurate determination of the solar constant has been the primary object of the Smithsonian observers.[9]

One further correction now remains to be made in the derived values of the monochromatic fluxes f_λ at the top of the terrestrial atmosphere before these are reduced to the photosphere. The quantity which is to be found is the distribution of energy in the continuous spectrum, i.e., in the

[8] In integrating over λ, an estimate must be made of the fluxes in the ultraviolet and far infra-red which do not penetrate to the ground. The estimate is based upon the blackbody distribution which most closely approximates the observed fluxes in the wave-lengths accessible to observation.

[9] For discussions relating to the small variations in f, and their effects upon the earth, see reference 2.

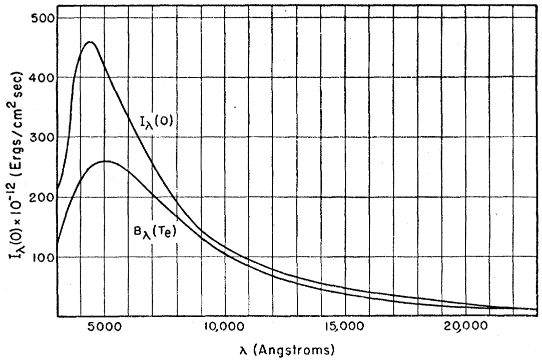

FIG. 3. The monochromatic intensity $I_\lambda(0)$, per unit solid angle and per cm wave-length interval, at the center of the solar disk. Derived by Minnaert and Mulders from the measures of Abbot, Plaskett, and Fabry. $B_\lambda(T_e)$ is the intensity in the Planck spectrum for 5750°K.

part of the spectrum which lies between the absorption lines. However, the resolution of Abbot's spectrobolometer is too small to reveal in the tracings any but the strongest Fraunhofer lines. The fluxes f_λ derived by Abbot are therefore too small in every wave-length region in just the proportion of the energy which is absorbed in the lines. By a photometric calibration of the Rowland scale of line intensities, G. F. W. Mulders has evaluated this depletion of the spectrum separately for the various spectral regions.[10] The total depletion is found to be 8.3 percent, but in the near ultraviolet, where strong absorption lines are the most numerous, the depletion amounts to as much as 40 percent.

After correction for the energy absorbed in the unresolved Fraunhofer lines, the fluxes f_λ can next be reduced to the photosphere. Since the space between the photosphere and the top of the terrestrial atmosphere is essentially transparent, the flux in every wave-length varies simply as the inverse square of the distance, and we have

$$F_\lambda = (a/R)^2 f_\lambda,$$

where a is the distance from the earth to the sun, and R is the radius of the sun. The values of F_λ determined from this relation can now, finally, be put into Eq. (3) to yield the monochromatic intensities, in absolute units, at the photosphere. Figure 3 exhibits the results of this procedure.

Several more byproducts of primary importance come out of the investigation at this point. In the first place, the net integrated flux from the photosphere can be determined; this is

$$F = \int_0^\infty F_\lambda d\lambda = (a/R)^2 f$$
$$= 6.27 \times 10^{10} \text{ erg/cm}^2 \text{ sec.}^{11}$$

The integrated flux from a blackbody is related to the temperature according to Stefan's law,

$$F = \sigma T^4.$$

It is yet to be seen how closely the continuous spectrum of the sun approximates that of a blackbody, but it is natural at this point to calculate the temperature of the blackbody which would emit the same integrated flux as the photosphere. The temperature so defined is said to be the "effective temperature" of the sun; it is

$$T_e = (F/\sigma)^{\frac{1}{4}} = 5750°\text{K}.$$

A. Unsöld[12] estimates that the probable error of this result is about 30°. The blackbody distribution $B_\lambda(T_e)$ is also shown in Fig. 3.

THE OPACITY OF THE SOLAR ATMOSPHERE

In order to interpret the continuous spectrum of the sun at the center of the disk, let us first assume that it originates at a single definite level of the solar atmosphere, and that thermodynamic equilibrium prevails at that level. The second assumption allows us to ascribe some definite temperature to the level and to calculate the corresponding Planckian spectrum. The first assumption then requires that the continuous spectrum of the sun be the same as this Planckian spectrum.

In point of fact, neither assumption is correct. In the first place, the radiation which reaches the earth from the center of the disk must originate over an appreciable interval of depth in the solar atmosphere, such that at the top of the interval the gas is effectively transparent and only weakly luminous, while at the bottom it is effectively

[10] G. F. W. Mulders, Zeits. f. Astrophys. **11**, 132 (1936).

[11] 7800 hp/ft.^{2}! The energy absorbed in the Fraunhofer lines is not counted in this figure, since it is scattered back into the photosphere. Multiplication of F by the superficial area of the photosphere yields the luminosity of the sun; it is 3.78×10^{33} ergs/sec. Since the mass of the sun is 1.98×10^{33} g, the average rate of energy generation is therefore 1.90 erg/g sec.

[12] Reference 3, p. 32.

opaque and intensely luminous. Moreover, in the second place, the enormous net flux of radiation outward from the solar atmosphere disposes of the possibility that thermodynamic equilibrium can prevail. In view of these considerations, the very appreciable deviations of $I_\lambda(0)$ from the Planckian spectrum $B_\lambda(T_e)$ can hardly occasion any surprise.

Since no direct interpretation of the continuous spectrum of the sun is possible in terms of Planckian radiation from a surface of given temperature, we must consider in some detail the actual transfer of radiation through the solar atmosphere. If, at a given level in the atmosphere, the monochromatic emission per cubic centimeter and per unit solid angle is j_λ, and if the absorption per cm from a ray of intensity I_λ is $\kappa_\lambda I_\lambda$, then the change of monochromatic intensity per unit length along a ray at this level will be

$$dI_\lambda/ds = j_\lambda - \kappa_\lambda I_\lambda,$$

where ds is the element of path length. This is the equation of transfer of radiation; in this form, it is valid under very general circumstances.

Since the total thickness of the photospheric layers is small compared with the radius of the sun, we may regard the solar atmosphere as stratified in plane parallel layers, as shown in Fig. 4. If we take the x axis in the direction of the outward normal to these layers, the element ds of path length becomes $dx \sec\theta$, where θ is the angle between the outward normal and the ray, and the equation of transfer becomes

$$\cos\theta dI_\lambda/dx = j_\lambda - \kappa_\lambda I_\lambda.$$

It now proves convenient to define the element of optical depth in the relation

$$d\tau_\lambda = -\kappa_\lambda dx,$$

and to regard τ_λ as the independent variable. In terms of τ_λ, then, the equation of transfer is

$$\cos\theta dI_\lambda/d\tau_\lambda = I_\lambda - j_\lambda/\kappa_\lambda. \qquad (6)$$

Equation (6) is linear in I_λ, and the general solution can be written down at once. The quantity I_λ is a function of optical depth τ_λ and of direction θ; since there is symmetry about the outward normal, I_λ will not depend upon the azimuthal coordinate of the ray. For our purposes, however, we require the solution of Eq. (6) only for the emergent radiation, i.e., for the intensity at $\tau_\lambda = 0$. This is readily shown to be

$$I_\lambda(\theta) = \int_0^\infty (j_\lambda/\kappa_\lambda) \exp[-\tau_\lambda \sec\theta] \sec\theta d\tau_\lambda.$$

It will now be recalled that the ratio j_λ/κ_λ of the coefficients of emission and absorption is, in thermodynamic equilibrium, just the Planck function $B_\lambda(T)$ for the appropriate temperature. It has already been pointed out that the solar atmosphere cannot be in strict thermodynamic equilibrium, so that it is not possible to ascribe a proper temperature to any level. Nevertheless, there is reason to suppose that, except in the highest levels, it is possible to assign to each level τ_λ a temperature $T(\tau_\lambda)$ such that the Kirchoff law is a valid approximation, i.e., such that

$$j_\lambda/\kappa_\lambda = B_\lambda[T(\tau_\lambda)].$$

This assumption is said to define the state of local thermodynamic equilibrium; it is justified by the consideration that the interaction between continuous radiation and matter proceeds by transitions of atoms and ions into and out of continuous states, and is therefore governed largely by the kinetic characteristics of the matter.

Assuming, then, that the photospheric layers are in local thermodynamic equilibrium, we have

$$I_\lambda(\theta) = \int_0^\infty B_\lambda[T(\tau_\lambda)] \times \exp[-\tau_\lambda \sec\theta] \sec\theta d\tau_\lambda. \qquad (7)$$

In this form, the solution of the equation of transfer suggests an immediate physical inter-

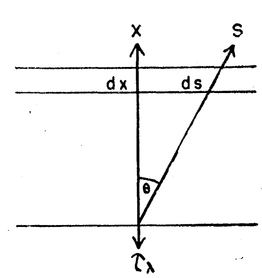

FIG. 4. The relation between geometrical height and optical depth in the solar atmosphere.

pretation: the radiation which emerges from the photosphere in any direction is the simple sum of the Planckian emissions from all the levels of the photosphere, the emission from each level being cut down, however, by an amount which depends upon the optical path length through the superincumbent layers.

Since $I_\lambda(\theta)$ is given by the observations, it should now be possible to derive from Eq. (7) the function $\tau_\lambda(T)$, which describes the absorbing properties of the solar atmosphere. A number of different procedures have been devised for this derivation; we shall employ the relatively direct method of Chalonge and Kourganoff.[4] Upon division by $I_\lambda(0)$, Eq. (7) becomes

$$\phi_\lambda(\theta)=\int_0^\infty \frac{B_\lambda[T(\tau_\lambda)]}{I_\lambda(0)}\exp[-\tau_\lambda\sec\theta]\sec\theta d\tau_\lambda,$$

and this can be regarded as an integral equation for the fraction $B_\lambda[T(\tau_\lambda)]/I_\lambda(0)$. It is found that the solution, which leads to the observed expression for $\phi_\lambda(\theta)$, as given in Eq. (2), is

$$B_\lambda[T(\tau_\lambda)]/I_\lambda(0)=A_\lambda+B_\lambda\tau_\lambda+\tfrac{1}{2}C_\lambda\tau_\lambda^2. \qquad (8)$$

Corresponding to any given values of T and λ, all the quantities which appear in Eq. (8) are now known except $\tau_\lambda(T)$. It is therefore possible to solve directly, in any prescribed wave-length, for the optical depth $\tau_\lambda(T)$ down to the level in the atmosphere where the temperature has any prescribed value T. Chalonge and Kourganoff have carried out this solution; their results are summarized by the full curves of Fig. 5. Wave-

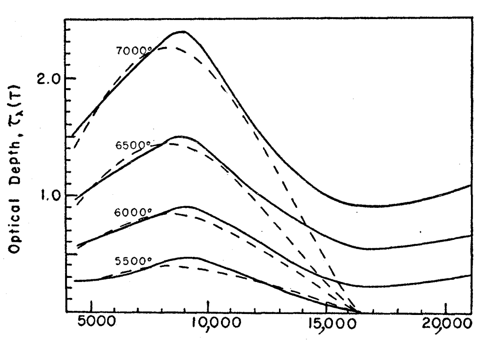

FIG. 5. Optical depths in the solar atmosphere, after Chalonge and Kourganoff. The full curves are derived from the observations of Abbot, Plaskett, and Fabry, as corrected by Mulders. The dashed curves are derived from Chandrasekhar's theoretical absorption coefficient of H^-.

lengths to the violet of 4000A are not considered because of the relative uncertainty of Abbot's data in this region of the spectrum.

The question now arises whether it is possible to account for the observed wave-length dependence of the opacity. The curves of Chalonge and Kourganoff are so similar in shape as to suggest at once that the same agent is responsible for the opacity at all levels. For, if this is the case, the variation with wave-length of the optical depth down to any given level is necessarily the same as the variation with wave-length of the strength of the relevant microscopic absorption process. Thus, for example, if the opacity of the solar atmosphere were due to the photoelectric ionization of some specific atom, then the coefficient κ_λ of absorption at each level would be just the product of the coefficient k_λ of absorption per atom and the number n of atoms per cm^3 in the appropriate lower state. The optical depth down to any given level would then be simply

$$\tau_\lambda(T)=\int_{x(T)}^\infty k_\lambda n dx=k_\lambda N(T),$$

where $N(T)$ is the total number of absorbing atoms above one cm^2 at the level of temperature T.

It is natural to attempt to attribute the opacity first to the photoelectric ionization of the atom most abundant in the solar atmosphere. The Fraunhofer spectrum indicates that this is hydrogen; for, while most of the strongest Fraunhofer lines are produced by transitions out of atomic states having excitation energies between zero and three volts, the Balmer lines, which are among the strongest in the spectrum, originate from the second quantum state of hydrogen, which has an excitation potential of 10.15 volts. Since only a minute fraction ($\sim 10^{-8}$) of the hydrogen atoms in the solar atmosphere can exist in the second quantum state at the temperatures and pressures which must prevail there, it follows from the strength of the Balmer lines that hydrogen must be exceedingly abundant. Recent quantitative investigations[13] have

[13] See, for example, B. Strömgren, Festschrift für Elis Strömgren (E. Munksgaard, Copenhagen, 1940), pp. 218–257.

led to the result that hydrogen atoms must outnumber the metals in the ratio of nearly 8000:1.

A hydrogen atom in the ground state can absorb continuous radiation[14] only at wavelengths shorter than the limit of the Lyman series, which falls at 912A. The absorption of a quantum of wave-length less than 912A produces a photoelectric ionization, the excess of radiant energy above the amount required for ionization going into kinetic energy of the ejected electron. The quantum-mechanical discussion of this process indicates that the atomic absorption coefficient falls off to the violet of the Lyman limit very nearly as λ^3. Similarly, the absorption coefficient for a hydrogen atom in the second quantum state falls off as λ^3 from the limit of the Balmer series, at 3646A, and is zero to the red of this limit. Photoelectric ionizations from still higher states follow the same law. The continuous absorption coefficient per cm^3 of hydrogen will therefore be a sum of the atomic absorption coefficients for the various energy states, each coefficient being weighted by the number per cm^3 of hydrogen atoms in the corresponding state. Since the atomic absorption coefficients are essentially discontinuous at the limits of the respective line series, the resulting coefficient per cm^3 is also a highly discontinuous function. However, the curves of Fig. 5 exhibit no significant discontinuities at the limits of the Balmer and Paschen series, and their general form is entirely incompatible with the discontinuous λ^3 law. It must therefore be concluded that the photoelectric ionization of hydrogen atoms is not mainly responsible for the opacity of the solar atmosphere.

After hydrogen, the elements most abundant in the solar atmosphere are probably helium, carbon, nitrogen, oxygen, magnesium, silicon, and iron. Of these elements, and the others which are abundant, only the metals, which are of relatively low ionization potential, can contribute to the opacity in the visual and infra-red; for only these elements can be appreciably excited to states from which photo-ionization will produce continuous absorption in the relevant spectral regions. No one of the metals by itself can be nearly abundant enough to produce the observed opacity, but it is conceivable that, acting together, they may suffice. The quantum-mechanical discussion of continuous absorption processes for complex atoms is much more difficult than for hydrogen, but the result indicates that the λ^3 law provides an adequate approximation. The difficulty again is that no metal absorption edges are observed in the solar spectrum. The absence of any one absorption edge can always be attributed to a scarcity of the corresponding element, but it is clearly impossible to account in this way for the absence of *all* metal absorption edges, and yet maintain that the metals are responsible for the opacity.

TABLE II. Variation functions and energy limits for H^- and He.

Variables $s=r_1+r_2$ $t=-r_1+r_2$ $u=r_{12}$ Form of wave function (atomic units)	$-W$ (ev) H^-	$-W$ (ev) He
$e^{-\alpha s}$	12.8	77.086
$e^{-\alpha s}(1+\beta u)$	13.8	78.265
$e^{-\alpha s}\cosh\beta t$	13.90	
$e^{-\alpha s}(1+\beta u+\gamma t^2)$	14.22	78.569
$e^{-\alpha s}\cosh\beta t(1+\gamma u)$	14.237	
$e^{-\alpha s}(1+\beta u+\gamma t^2+\delta s+\epsilon s^2+\zeta u^2)$	14.251	78.591
$e^{-\alpha s}\cosh\beta t(1+\gamma_2 s+\gamma_4 u+\gamma_5 us)+te^{-\alpha s}\sinh\beta t(\gamma_3+\gamma_6 u)$	14.263	
$e^{-\alpha s}(1+\beta u+\gamma t^2+\delta s+\epsilon s^2+\zeta u^2+x_6t^4+x_7t^6+x_8t^4u^2+x_9t^2u^2+x_{10}t^2u^4)$	14.281	
Experimental		78.603

In addition to the atoms and positive ions which are present in the solar atmosphere, there are known to be some diatomic molecules. From the relative weakness of their absorption bands, it follows that they must be far too rare to contribute appreciably to the opacity. But the solar atmosphere must also contain appreciable numbers of free electrons, liberated mainly from the metals of low ionization potentials. By themselves, these, again, cannot account for the observed opacity. However, it was suggested by Wildt,[15] in 1938, that some of the electrons would combine with hydrogen atoms to produce negative ions of hydrogen-H^-, and that the subsequent photoelectric dissociation of H^- might be responsible for the opacity. Wildt went on to show that, according to the theory of dissociative equilibrium, ions of H^- must actually be more abundant than H atoms in the second quantum state, at least through a large part of the solar atmosphere.

[14] Light of all wave-lengths except those of the Lyman lines is here considered continuous.

[15] Wildt, Astrophys. J. **89**, 295 (1939).

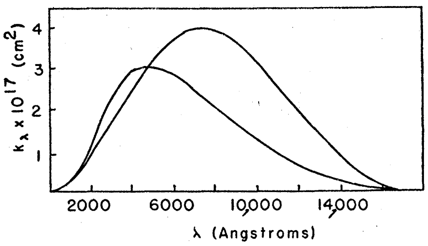

FIG. 6. The absorption coefficient of H^-, after Chandrasekhar. Two different variation functions have been used with the matrix element of the dipole moment.

The negative ion of hydrogen had been detected in certain experiments[16] with canal rays, but nothing whatever was known of its properties until Bethe[17] and Hylleraas[18] discussed it from a theoretical point of view in 1930. The fact of the existence of H^- was established by an application of the Ritz principle, which can be formulated as follows. Let Ω be any function of the coordinates of a dynamical system, which is normalized to unity, and let

$$W = \int \Omega^* H \Omega d\tau,$$

where H is the Hamiltonian operator of the system. Then the Ritz principle asserts that if E is the lowest energy of the system, and ψ the corresponding wave function, $W \geq E$, and, furthermore, that $\Omega \rightarrow \psi$ as $W \rightarrow E$. The application of the Ritz principle to the discussion of H^- is then as follows. We take as the system one comprising a proton and two electrons, and we let

$$\Omega = (1+\beta r_{12}) \exp[-\alpha(r_1+r_2)],$$

where r_1 and r_2 are the nuclear distances of the respective electrons, r_{12} is their separation, and α and β are parameters to be determined by the conditions which minimize W, *viz.*, $\partial W/\partial\alpha = \partial W/\partial\beta = 0$. The result of the calculation is that $W = -13.8$ volts. However, the energy of a system comprising an H atom in the ground state and an electron at rest at ∞ is already -13.54 volts, which is larger than W. It follows from the Ritz principle that this state of a proton and two electrons is not the state of lowest energy, and that there must exist a state of energy less than -13.8 volts, in which both electrons are bound to the nucleus.

Having once established the stability of H^-, it became a matter of great importance to approximate the energy of its ground state as accurately as possible. In order to accomplish this, a large number of variation functions have been tried, having more and more disposable parameters. This work has been done mainly by S. Chandrasekhar and his collaborators at the Yerkes Observatory.[19] Table II lists the variation functions which have been used and the energy limits to which they have led. The last column, incidentally, gives the energies predicted by these variation functions for helium, for which the best results agree to one part in 5000 with the experimental value. It is clear that the energy integrals seem to converge to a value very near -14.28 volts; the chances are that this is an accurate value for the energy of the ground state of H^-.

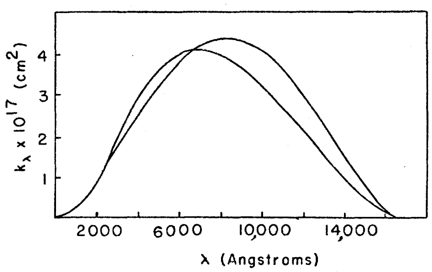

FIG. 7. The absorption coefficient of H^-, after Chandrasekhar. The same two variation functions as in Fig. 6 have been used with the matrix element of the momentum operator.

The atomic absorption coefficient depends essentially only on the square of the matrix element of the dipole moment; apart from a constant factor, the matrix element is[20]

$$\int \psi_d^*(z_1+z_2)\psi_c d\tau.$$

In this expression, ψ_d^* is the (complex conjugate) wave function of the initial (bound) state, which is a discrete state—hence the subscript d—and

[16] See, for example, O. Tüxen, Zeits. f. Physik **103**, 463 (1936).

[17] H. A. Bethe, Zeits. f. Physik **57**, 815 (1930).

[18] Hylleraas, Zeits. f. Physik **60**, 624 (1930).

[19] S. Chandrasekhar summarizes the work in Rev. Mod. Phys. **16**, 301 (1944).

[20] The x and y components of the matrix element vanish if the z axis is taken in the direction of motion of the photoelectron at ∞.

ψ_c is the wave function of the final (dissociated) state, which is a continuous— c —state. For ψ_c, one uses the product of two wave functions: first, the wave function of a hydrogen atom in the ground state, and second, the wave function of a free electron.[21] For ψ_d, one can only use one of the variation functions of Table II. Unhappily, however, this approximation of the true wave function of H^- by a variation function proves troublesome for the following reason. In minimizing the energy integral according to the Ritz principle, one determines well only those parts of the wave function which contribute appreciably to the energy, while the other parts of the wave function are only poorly determined. In particular, the more remote parts of ψ_d are quite uncertain. However, the matrix element of the dipole moment weights most heavily just these parts of ψ_d. The extent of the uncertainty is illustrated by the curves of Fig. 6, which gives the atomic absorption coefficient of H^- as computed with each of two variation functions which lead to the same energy within 0.03 volt. According to one variation function, k_λ has its maximum at about 4000A, while according to the other the maximum lies near 8000A.

It will be recalled that in the classical theory the radiation of a dipole can be expressed in terms of its momentum or its acceleration, as well as in terms of its dipole moment. Analogously, in quantum mechanics the matrix elements of the dipole moment, the momentum operator, and the acceleration operator are all related. In fact, it can be shown that, if the energies of the initial and final states are E_d and E_c, respectively, then

$$\int \psi_d^*(z_1+z_2)\psi_c d\tau = -\frac{1}{E_d-E_c}\int \psi_d^*\left(\frac{\partial}{\partial z_1}+\frac{\partial}{\partial z_2}\right)\psi_c d\tau = \frac{1}{(E_d-E_c)^2}\int \psi_d^*\left(\frac{z_1}{r_1^3}+\frac{z_2}{r_2^3}\right)\psi_c d\tau.$$

Chandrasekhar has found[22] that the matrix element of the acceleration operator is again extremely sensitive to the form of ψ_d; however, the matrix element of the momentum operator is not. The improvement effected by the use of the momentum operator is illustrated in Fig. 7; the two curves have been computed with the same two variation functions employed in the derivation of the curves in Fig. 6, but the divergence between the curves is appreciably diminished.

Chalonge and Kourganoff[4] have adopted the higher curve of Fig. 7 and have compared the wave-length variation with that which they have deduced for the opacity of the solar atmosphere. In order to effect the comparison, for example, with that curve of Fig. 5 which gives $\tau_\lambda(7000°)$, they have simply multiplied Chandrasekhar's k_λ at all wave-lengths by the number $N(7000°)$ of H^- ions above the 7000° level. This number they have chosen as 5.12×10^{16}, in order to achieve the best general agreement between the observed and predicted curves. The dashed curve depicts the result of this procedure; the other dashed curves have been similarly derived by multiplication of Chandrasekhar's values for k_λ by the appropriate values of $N(T)$. The close agreement of the predicted and observed curves between 4000A and 13,000A conclusively establishes H^- as the principal source of opacity in the solar atmosphere.

Only in the extreme infra-red can the photodissociation of H^- still not account for the observations. However, Chandrasekhar and F.

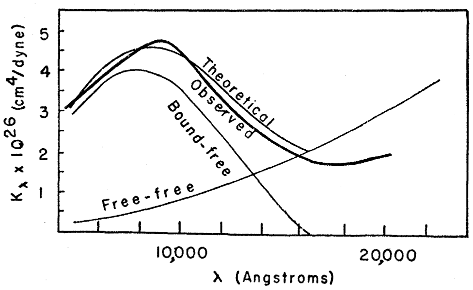

FIG. 8. The absorption coefficient of H^-, after Chandrasekhar. The coefficient is here given per neutral H atom and per unit electron pressure at $T=6300°$K. The heavy curve is the observed coefficient, after Chalonge and Kourganoff.

[21] Actually, the ejected electron is not quite free even after dissociation. In Astrophys. J. **103**, 41 (1946), Chandrasekhar and F. H. Breen have evaluated the wave functions of an electron moving in the Hartree field of a hydrogen atom, and in Astrophys. J. **102**, 395 (1946), Chandrasekhar has used these wave functions in the evaluation of the matrix elements.

[22] S. Chandrasekhar, Astrophys. J. **102**, 223 (1945).

H. Breen have shown[23] that the free-free transitions of H^- suffice to remove this last discrepancy. In the earlier studies of the continuous transitions of an electron in the field of a hydrogen atom, it was assumed that an *s*-electron could be adequately represented by a plane wave. However, the field of the hydrogen atom proves to modify this representation considerably for the relatively slow electrons which occur in the solar atmosphere, and the matrix elements calculated with the modified wave functions lead to an absorption coefficient which exceeds the older one by a factor of order ten. Figure 8 depicts the new absorption coefficient for free-free transitions, together with the absorption coefficient for photo-dissociation. The agreement of the resulting total absorption coefficient with the observations can now be considered satisfactory over the whole observed range of the solar spectrum.

Finally, there remains to be interpreted the structure of the photospheric layers that is implied by the function $N(T)$, as derived by Chalonge and Kourganoff. Since hydrogen is so very abundant, it will be readily seen that with the ionization equation $N(T)$ gives, essentially, the density as a function of temperature; the pressure then becomes determinate through the gas law. The derivation of these physical parameters in the photospheric layers involves an application of the theory of radiative equilibrium, and is beyond the scope of this paper. Rapid progress toward a completely satisfactory theory of the photospheric structure is now being made. But even in the absence of a definitive atmospheric model, the results already achieved may well be said to represent a real triumph of theoretical physics and observational astrophysics: in the one case, for the prediction of the existence and properties of an ion never studied in the laboratory; and in the other case, for the verification of these predictions by the discovery, through its absorption spectrum, of the ion in the atmosphere of the sun.

I wish to express my thanks to Dr. C. G. Abbot and to Dr. S. Chandrasekhar, both of whom have read the manuscript and offered helpful comments.

[23] S. Chandrasekhar and F. H. Breen, Astrophys. J. **104**, 430 (1946).