Drag in Cavitating Flow*

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The free streamline theory has been used for evaluation of the cavity drag of symmetrical wedges of arbitrary angle. The required conformal transformation is derived explicitly. This calculation is an extension of Riabouchinsky's theory of the cavity drag of a flat plate. As an approximation, the pressure distribution for a two-dimensional wedge is used to calculate the cavity drag of the corresponding cone of revolution. A comparison of the result of this approximation with experimental measurements made by Reichardt shows good agreement.

INTRODUCTION

THE class of two-dimensional motions of a perfect fluid in which the fluid boundaries consist in part of fixed walls and in part of surfaces of constant pressure was first considered by Helmholtz; general methods of attack on problems of this type have been given by Kirckhoff, Rayleigh, and others.[1] Of classical interest is the problem of a flat plate normal to the direction of flow in an infinite stream. The drag coefficient is found to be

$$C_D=\frac{2\pi}{\pi+4}=0.880.$$

This drag coefficient is known to be much lower than the experimental value so that this free streamline theory of the wake cannot be accepted. A similar discrepancy is exhibited in the solution for a flat plate inclined at an angle α with the direction of flow of the infinite stream. In this case, the free streamline theory gives, for the force per unit area in the direction normal to the stream, the coefficient[1]

$$C_L=2\pi \sin\alpha \cos\alpha/(4+\pi \sin\alpha).$$

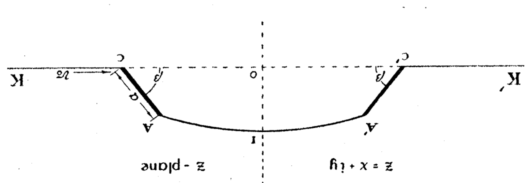

FIG. 1.

The circulation theory, which is found to be in agreement with experiment, at least for small α, gives on the other hand

$$C_L=2\pi \sin\alpha.$$

The physical reason for the failure of the free streamline theory of the wake is clear: the free streamline theory considers the wake as consisting of "dead water," or fluid at rest, up to the bounding free streamlines; actually, the wake has vortex rows which produce a suction on the rear surface of the plate comparable to the maximum pressures on the forward surface.

While the free streamline theory of the wake is only of mathematical interest in applications to fluid flow of one phase, the theory should have some physical significance for flow of liquids. As the relative velocity of flow of a liquid over an immersed body is increased, the local pressure at some points on the surface of the body will fall to the vapor pressure of the liquid. The flow may then go over into a cavity wake. While the vapor cavity has its vortex system, the inertia of the vapor is negligible compared with that of the liquid, and the kinetic energy of the vortex system has only a trivial influence on the pressure within the wake. One would then expect that the free streamline theory of the wake would apply quantitatively to this two-phase situation.

The parameter which determines the general character of cavity flow is the cavitation param-

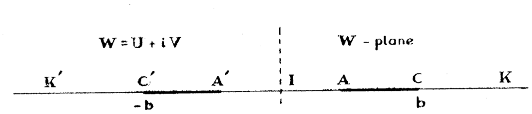

FIG. 2.

* This study was supported by the Office of Naval Research.

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[1] Cf. H. Lamb. *Hydrodynamics* (Dover Publications, New York, 1945), 6th ed. Chap. IV, §§73–78; U. Cisotti, *Idromeccanica Piana* (Tamburini, Milan, 1921–22), Part 2.

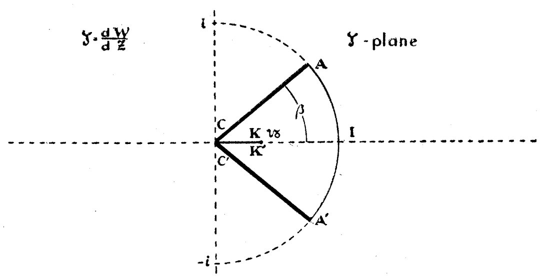

FIG. 3.

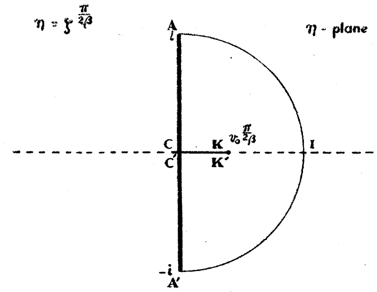

FIG. 4.

eter, σ, which is defined by the relation

$$\sigma = \frac{p_0 - p_1}{\frac{1}{2}\rho v_0^2}, \tag{1}$$

where ρ is the liquid density, p_0 is the static pressure in the undisturbed free stream, v_0 is the liquid velocity in the undisturbed free stream, and p_1 is the static gas, or vapor, pressure in the cavity. The Kirckhoff solution for the flat plate discussed earlier in this paper corresponds to $\sigma = 0$, and therefore also corresponds to a cavity which extends behind the plate to infinity. A calculation of the cavity drag for a finite cavity, that is, for $\sigma > 0$, was first made by Riabouchinsky.[2] This calculation was limited to a flat plate in two-dimensional flow, and the finite cavity was obtained by introducing an image plate downstream of the "true" plate. The free streamlines run from the true plate to the image plate. The total force on the pair of plates vanishes, but Riabouchinsky calculates the force on the forward plate only. Physical cavities do not show the perfect fore and aft symmetry of the model of Riabouchinsky; the forward portion of the cavity, however, may be fairly closely represented by this theory, in which case the drag coefficients may be accurate.

A different representation of a finite cavity behind a two-dimensional flat plate has been ascribed to Prandtl and Wagner. This theory is based on flow patterns in which the free streamlines bounding the cavity turn forward at the rear of the cavity to form a re-entrant jet. This re-entrant jet is indicated by observations on cavities for values of σ which are not too small. Calculations based on this type of solution have recently been made by Gilbarg and Rock.[3] Mathematically, this re-entrant jet extends through the plate upstream to infinity, and the area of impingement of the jet on the plate must be omitted in the force calculation to obtain a non-zero drag. This omission is physically reasonable since the re-entrant jet is ordinarily dissipated through turbulent mixing. It is of great interest that the calculations of Gilbarg and Rock show very close agreement between the re-entrant jet model and the Riabouchinsky image model.

In this paper the extension of the Riabouchinsky formulation to two-dimensional symmetrical wedges with arbitrary wedge angle will be outlined. A detailed analysis with an extensive tabulation of numerical results will be presented elsewhere.

Recently, some experimental measurements of the cavity drag of circular cones carried out by Reichardt at the Kaiser Wilhelm Institute for Flow Research have been published.[4] These measurements were performed with a free jet and appear to be very precise. There is no theory for the cavity drag for a three-dimensional body, and in view of this lack it was thought to be of value to derive a drag coefficient for cones of revolution by rotations of the corresponding two-dimensional wedge pressure distribution.

EXTENSION OF RIABOUCHINSKY'S THEORY TO A TWO-DIMENSIONAL WEDGE

The two-dimensional cavity flow past a symmetric wedge with its image is depicted in Fig. 1

[2] D. Riabouchinsky, Proc. Lond. Math. Soc. (ser. 2) **19**, 206–215 (1920).

[3] D. Gilbarg and N. Rock, Naval Ordnance Laboratory Memorandum No. 8718.

[4] H. Reichardt, "*The laws of cavitation bubbles at axially symmetrical bodies in a flow*," Reports and Translations No. 766, Ministry of Aircraft Production, August 15, 1946 (distributed by Office of Naval Research, Navy Department, Washington, D. C.).

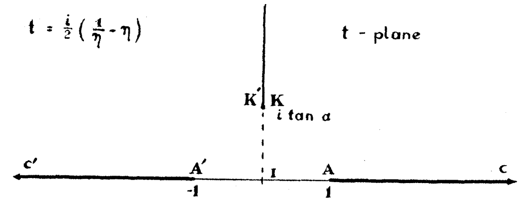

FIG. 5.

which represents the "physical" or z-plane ($z = x + iy$). Only the flow above the line of symmetry $KCC'K'$ is considered; the complete flow pattern is obtained merely by reflection in this line. Since all the results of the theory are independent of the magnitude of the velocity, there is no loss in generality in assigning the value unity to the velocity on the free streamline AA'; the velocity at infinity is v_0. The complex potential is $W = U + iV$ where U is the velocity potential and V is the stream function. The W-plane is shown in Fig. 2; the arbitrary constants in W have been fixed so that $V = 0$ on the stream line $KCAIA'C'K'$ and $U = 0$ at the mid-

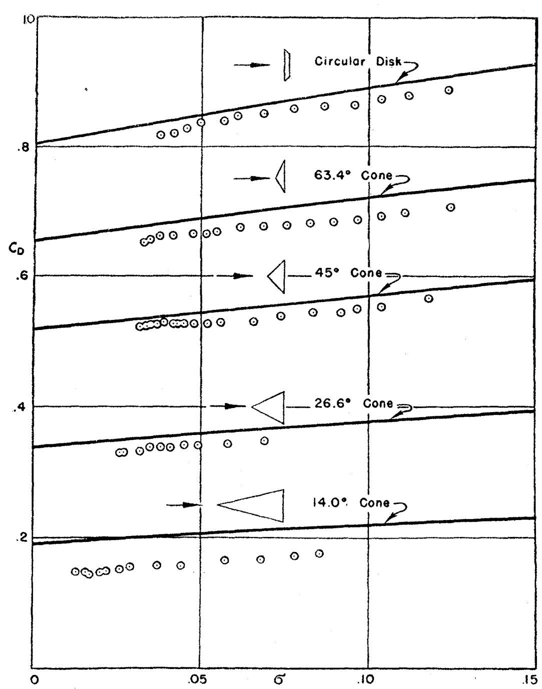

FIG. 6. The drag coefficient C_D as ordinate is plotted as a function of the cavitation parameter σ as abscissa. The lines are calculated by the approximation indicated in the text, and the circled points are the measured values of Reichardt. The angles indicated are the interior half angles of the symmetric cones of revolution.

point, I, of the free stream line. If u, v are x, y components of the fluid velocity, and if the convention is followed that

$$u = -\partial U/\partial x, \quad v = -\partial U/\partial y,$$

then

$$dW/dz = \zeta = -u + iv; \tag{2}$$

i.e., ζ is the reflection of the complex velocity in the y. The hodograph, or ζ-plane, is shown in Fig. 3.

The mathematical problem reduces to the determination of the unique one-to-one conformal transformation $W = W(\zeta)$ which maps the interior of the ζ-sector on the upper half W-plane so that corresponding boundary points coincide. The solution is completed by integration of the relation $\zeta = dW/dz$. The ζ-sector $ACC'A$ goes into a semicircle of unit radius in the η-plane (Fig. 4) by the transformation

$$\eta = \zeta^{\pi/2\beta}. \tag{3}$$

Further, the transformation

$$t = \frac{i}{2}\left(\frac{1}{\eta} - \eta\right) \tag{4}$$

takes the boundary of the semicircle in the η-plane into the real t axis; the interior of the semicircle goes into the upper half t-plane (Fig. 5). In particular, KK' for which $\eta = v_0^{\pi/2\beta}$ goes into $t = i\tan\alpha$ where the parameter α is determined by the relation

$$v_0^{\pi/2\beta} = (1 - \sin\alpha)/\cos\alpha. \tag{5}$$

Finally, the transformation

$$t = W\tan\alpha/(b^2 - W^2)^{\frac{1}{2}}; \quad W = bt/(\tan^2\alpha + t^2)^{\frac{1}{2}} \tag{6}$$

takes the upper half t-plane into the upper half W-plane with the desired correlation of the boundary points. The parameter b fixes the cavity length CC'.

The drag coefficient, C_D, is expressed as follows:

$$C_D = \int_{AC} (p - p_1)dy \Big/ \tfrac{1}{2}\rho v_0^2 \int_{AC} dy, \tag{7}$$

where p is the fluid pressure over the upstream face of the wedge and p_1 is the pressure in the cavity. Also, from Bernoulli's equation,

$$\begin{aligned} p-p_1 &= (\rho/2)(1-v^2) \\ &= (\rho/2)(1-|\zeta|^2), \end{aligned} \tag{8}$$

where ζ is to be evaluated along AC. Thus,

$$C_D=(1/v_0^2)\cdot\left(\int_{AC}(1-|\zeta|^2)dy \Big/ \int_{AC} dy\right). \tag{9}$$

APPROXIMATE CAVITY DRAG FOR CONES OF REVOLUTION

Thus far, there has not been available a theoretical evaluation of the cavity drag of bodies of revolution. The mathematical complexity of such problems is very great; yet it is for bodies of this type for which there is the most direct experimental interest. A calculation of the cavity drag for cones of revolution has been made by the authors under the assumption that the pressure distribution $(p-p_1)$ over the cone is the same as that found from the two-dimensional flow about a wedge of the same internal angle. The accuracy of such an approximation is a question which must be determined by experiment, and if the approximation is shown to be reasonably good, support is given to general application of this procedure for the determination of cavity drag for bodies of revolution.

The only experimental data available to the writers bearing on this question are those of Reichardt. A comparison of the results of the present approximation and Reichardt's data is given in Fig. 6. The agreement is quite satisfactory except for the case of the cone of half-angle $\beta=14.0°$. An explanation of this particular discrepancy was not obvious to the writers in view of the good agreement elsewhere. It would appear most desirable to have further experimentation in this field over the widest possible range of variation of cavitation parameter.