

Structure of equilibria in a prototypical elastocapillary system

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A liquid drop connected to thin pliable object produces a deflection. If the volume of the liquid is sufficiently large, the deflection is modest and its shape is unique. Decreasing the volume from this state, however, leads to one of two possible minimal-volume outcomes, depending on the balance of rigidity and surface tension: The structure either returns to its original unstrained configuration or fully collapses to create a stable, highly stressed shape. Using a simple three-dimensional setup arising from self-assembly applications where the deflection is restricted to a single hinge, we characterize all the possible equilibrium shapes and show that the two possibilities arise as a consequence of a codimension-2 cusp bifurcation.

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I. INTRODUCTION

In microscale systems, a liquid drop can induce large forces from surface tension and pressure that deform elastic bodies [1,2]. When the bodies are sufficiently pliable, the deformations become significant, and one- or two-dimensional structures, such as rods and sheets, may undergo substantial out-of-plane deflections that transform them into fully three-dimensional objects [3,4]. Such transformations appear throughout the natural world—facilitating, for instance, arthropod motility [5,6], fungal proliferation [7,8], and avian feeding [9]—and their efficient and indirect approach to creating the complex configurations has motivated numerous applications [10–19]. In these wide-ranging areas, most research focuses on understanding properties of specific systems; yet, no matter the context, the results of these studies consistently contain common features, such as saddle-node bifurcations, hysteresis curves, and catastrophic collapses. Here, we explore the underlying structure of deformations exhibited by a simple physical setup (Fig. 1) to explain these recurrent behaviors. The setup is intrinsically three dimensional, unlike ones used in many studies, meaning the nonlinear equation for the liquid’s interface does not contain symmetries that reduce it to a single algebraic equation (cf., e.g., Refs. [20,21]). Instead, the interface is a nonspherical and noncylindrical constant mean curvature surface, which in addition to being more realistic makes finding the shapes of configurations much more difficult.

A scaling analysis reveals that elastocapillary deformations depend on two-dimensionless groups:

$$\lambda = \gamma l^2 / B, \quad V = V_0 / l^3. \quad (1)$$

In these groups, γ and V_0 are the surface tension and volume of the liquid, while B and l give the rigidity and intrinsic length of the mechanical component(s). The parameter λ , which is a squared ratio of l and the elastocapillary length $\sqrt{B/\gamma}$ [3,22], characterizes the overall pliability of the configuration [23], and V , which is a ratio of the lengths V_0/l^2 and l , quantifies the breadth of liquid in terms of the fundamental size of the unstrained elastic structure. Large and small values of λ , respectively, indicate flexible and rigid mechanical components, suggesting that big deformations necessary for collapsed configurations occur only when $\lambda \gg 1$ [4,24,25]. However, mismatches between the drop’s size and the elastic body’s span blur this ostensibly clear qualitative threshold. Large volumes of liquid can make it impossible for a compact, flexible body to encapsulate a drop [26], while small volumes of liquid can create large internal pressures that pull seemingly inflexible solids together [27,28].

Both λ and V play important roles in determining all possible configurations of an elastocapillary system. Most studies fix λ and investigate the structure of the deformations for varying V . This choice is appropriate when the liquid evaporates on a much quicker timescale than the elastic degradation, but it limits the view of possible behaviors. In some systems, like those built from hydrogels whose elasticity changes with water content [29], the parameter λ varies in concert with V . In others, like biological ones containing soft tissues that can rapidly relax or contract [30,31], the parameter λ varies, while V stays fixed.

To explore how both parameters affect the equilibrium deformations, we consider a setup, shown in Fig. 1, that commonly appears in engineered, self-assembling systems [32,33]. It consists of an incompressible liquid drop (of volume V_0) completely covering two rigid $2l \times l$ rectangular

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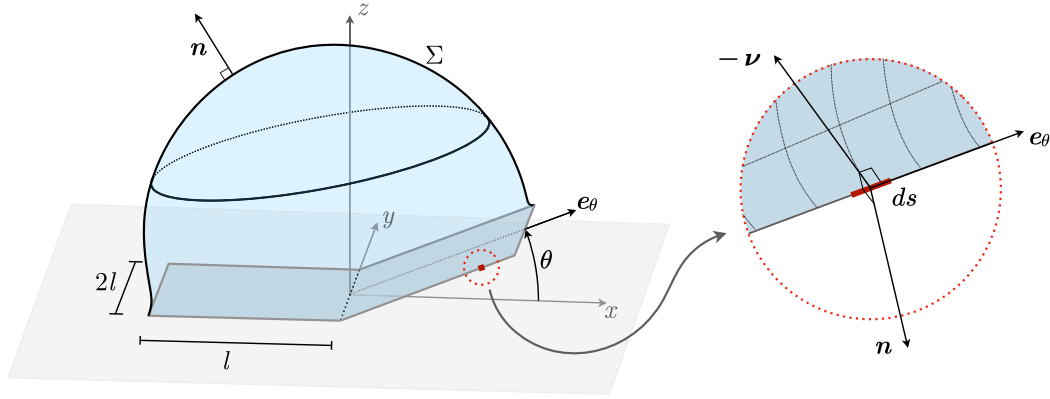


FIG. 1. A drop (of volume V_0) wetting the top of two $2l \times l$ rectangular plates that are connected by a linear torsion spring of rigidity B . The base substrate keeps the left plate stationary, while the right plate can rotate like a flap. The rotation, measured via angle θ (rad), is induced by the drop's liquid-air interface Σ , which has a surface tension γ that pulls on the perimeter of the flap. The magnification shows an arc-length increment ds on the perimeter curve with a local orthonormal frame: the boundary tangent $\mathbf{e}_\theta$, the boundary normal $-\mathbf{v}$, and the surface normal $\mathbf{n}$ of Σ . The surface tension acts along $-\mathbf{v}$.

plates that are affixed along their long side by a linear torsion spring of strength B . The base platform anchors one plate in place, while the other plate can rotate, like a flap, around the y axis. Absent other factors, the array remains flat, but in the presence of the drop, the flap twists to an angle θ due to the surface tension γ of the free liquid-air interface Σ .

II. MODEL

The vector-valued function

$$\mathbf{p}_\theta(u, v) = \begin{cases} u\mathbf{e}_x + v\mathbf{e}_y, & u \leq 0, \\ u\mathbf{e}_\theta + v\mathbf{e}_y, & u > 0 \end{cases}$$

of two variables $(u, v) \in \Omega = [-l, l] \times [-l, l]$ defines a piecewise smooth parametrization of the twisted two-plate array. Here, $\mathbf{e}_x = (1, 0, 0)$ and $\mathbf{e}_y = (0, 1, 0)$ are the standard basis vectors of the horizontal xy plane, and $\mathbf{e}_\theta = (\cos \theta, 0, \sin \theta)$ is a tangent vector on the bent plate that is perpendicular to $\mathbf{e}_y$.

Using the same material coordinates (u, v) over the same domain Ω , let $\mathbf{r} : \Omega \rightarrow \mathbb{R}^3$ be a smooth vector-valued function that parametrizes the free liquid-air interface Σ . Since the liquid completely covers the two-plate array, the contact line (i.e., boundary) $\partial\Sigma$ of Σ must remain on the perimeter curve of the plate array, regardless of the angle θ . Accordingly, the spatial coordinates must satisfy

$$\mathbf{r} = \mathbf{p}_\theta \quad \text{on} \quad \partial\Omega, \quad (2)$$

where $\partial\Omega$ is the boundary of Ω .

At the liquid-air interface Σ , the capillary and hydrostatic pressure forces balance. Since both align with the exterior normal vector field $\mathbf{n}$ of Σ , the balance provides one scalar field equation—i.e., the Young-Laplace equation [34]—that comes from equating their induced pressures, $2\gamma H$ and $-\Delta p$, over an infinitesimal surface element $d\Sigma$. In these expressions, γ is the surface tension, Δp is the Laplace pressure, and H is the mean curvature of Σ measured with respect to $\mathbf{n}$. Hence,

$$2\gamma H = -\Delta p \quad \text{on} \quad \Sigma, \quad (3)$$

where $H = (\mathbf{n}_v \times \mathbf{r}_u + \mathbf{r}_v \times \mathbf{n}_u) \cdot \mathbf{n} / 2|\mathbf{r}_u \times \mathbf{r}_v|$ (see, e.g., Ref. [35]).

On the plate, there are spring, pressure, and interfacial forces, each of which generate torques ($\boldsymbol{\tau}_s$, $\boldsymbol{\tau}_p$, and $\boldsymbol{\tau}_i$, respectively) that twist the flexible hinge and must balance in equilibrium. By definition, the torque from the torsion spring is $\boldsymbol{\tau}_s = B\theta\mathbf{e}_y$. To calculate $\boldsymbol{\tau}_p$, observe that the hydrostatic pressure pushes with magnitude Δp in the direction $\mathbf{n}_\theta = -\mathbf{e}_\theta \times \mathbf{e}_y$ at each point on the flexible plate. The hydrostatic pressure's total force is then equivalent to an effective one, $\Delta p(2l^2)\mathbf{n}_\theta$, positioned at the flexed plate's centroid $l/2\mathbf{e}_\theta$. Its associated torque $\boldsymbol{\tau}_p = (l/2\mathbf{e}_\theta) \times (\Delta p(2l^2)\mathbf{n}_\theta) = l^3\Delta p\mathbf{e}_y$. Determining $\boldsymbol{\tau}_i$ is less straightforward. The surface tension of the interface affects only the movable portion Γ_θ of plate's perimeter, which consists of the three distinct lines parametrized by $\mathbf{p}_\theta(u, l)$ and $\mathbf{p}_\theta(u, -l)$, for $u \in [0, l]$, and $\mathbf{p}_\theta(1, v)$, for $v \in [-l, l]$. For an arbitrary infinitesimal arc-length section of size ds on, say, the line $\mathbf{p}_\theta(u, -l)$, the interfacial force pulls with magnitude γds in the direction toward that liquid that is perpendicular to both $\mathbf{n}$ and Γ_θ (see the magnification in Fig. 1). Writing this direction as $-\mathbf{v}$, where $\mathbf{v} = \mathbf{e}_\theta \times \mathbf{n}$, we deduce that the interfacial force equals $(-\gamma ds)\mathbf{v}$ at the infinitesimal section. Since the position vector of the section relative to the axis of rotation is $u\mathbf{e}_\theta$, then the torque vector induced by the force becomes $(-\gamma u ds)(\mathbf{e}_\theta \times \mathbf{v})$ or, equivalently, $-\gamma u(\mathbf{n}_\theta \cdot \mathbf{v})\mathbf{e}_y ds$. (Equality follows from plugging the identity $\mathbf{e}_\theta = \mathbf{n}_\theta \times \mathbf{e}_y$ into the first expression and then simplifying the result using Lagrange's triple product expansion.) Similar steps on the other two boundaries show that this exact expression for the torque vector holds on every arbitrary infinitesimal piece of Γ_θ . Integrating implies that the total interfacial torque $\boldsymbol{\tau}_i = -\gamma \int_{\Gamma_\theta} u(\mathbf{n}_\theta \cdot \mathbf{v}) ds \mathbf{e}_y$. Because all the torques act along $\mathbf{e}_y$, the net torque equation, written as $\boldsymbol{\tau}_s = -\boldsymbol{\tau}_p - \boldsymbol{\tau}_i$, reduces to the scalar equation

$$B\theta = -l^3\Delta p + \gamma \int_{\Gamma_\theta} u(\mathbf{n}_\theta \cdot \mathbf{v}) ds, \quad (4)$$

which sets the twist angle θ . Barring a massive spillover of the liquid, the line integral along Γ_θ will be positive and cause the flap to rotate counterclockwise. The Laplace pressure Δp ,

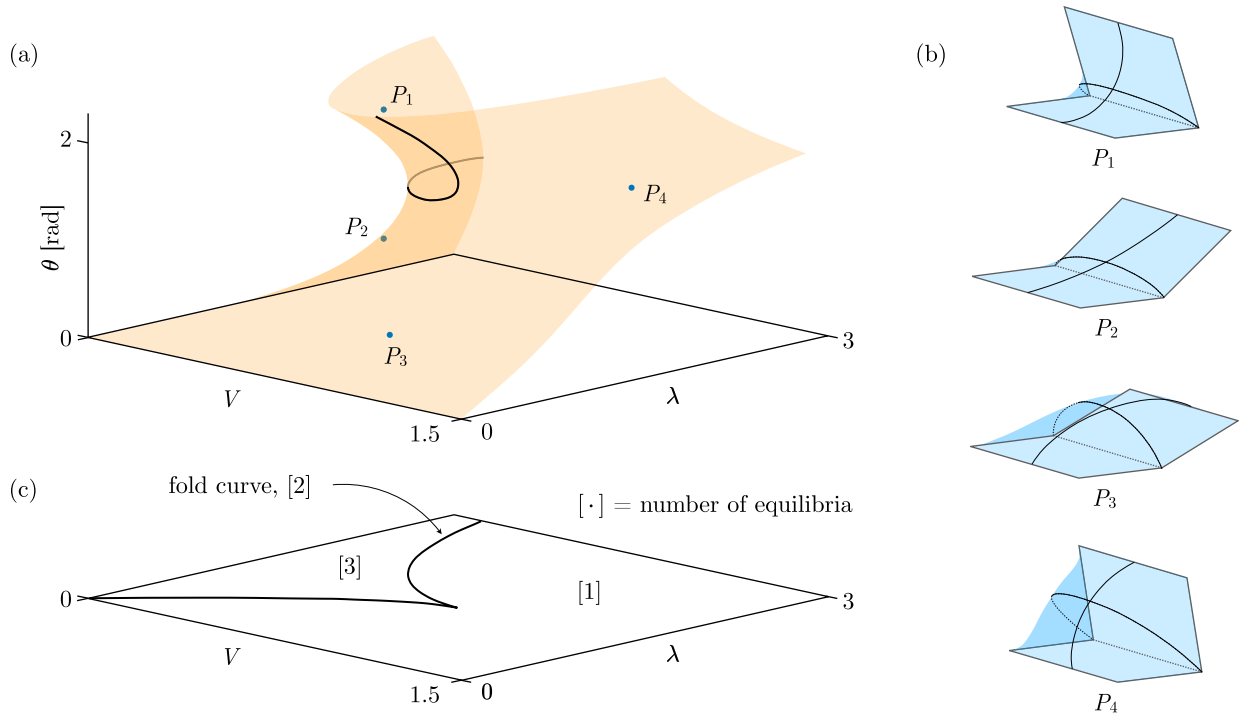


FIG. 2. (a) Solution set (i.e., bifurcation diagram) of the dimensionless model (6) plotting the rotation angle θ (rad) vs the (dimensionless) volume V and the bendability parameter λ . The solid curve gives the locations of the fold bifurcations. (b) Example equilibria at the points P_1 – P_4 , organized from top to bottom by increasing volume: $P_1 : (V, \lambda) = (0.26, 1.88)$, $\theta \approx 1.89$; $P_2 : (V, \lambda) = (0.26, 1.88)$, $\theta \approx 0.65$; $P_3 : (V, \lambda) = (0.88, 0.62)$, $\theta \approx 0.39$; and $P_4 : (V, \lambda) = (1.28, 1.82)$, $\theta \approx 1.75$. (c) Cusp catastrophe, plotted by orthogonally projecting the locations of the fold bifurcations into the parameter space. The cusp point is located at $(V^*, \lambda^*) \approx (0.849, 1.305)$.

which may be positive or negative depending on the convexity of the interface, can twist the plate in both directions.

Given that the liquid is incompressible, the value of Δp is set by the volume V_0 of the drop. After applying the divergence theorem to the standard volume integral, this constraint is equivalent to

$$\frac{1}{3} \iint_{\Sigma} \mathbf{r} \cdot \mathbf{n} d\Sigma = V_0, \quad (5)$$

where, again, $\mathbf{r}$ is the parametrization of the interface Σ . There are no explicit terms in Eq. (5) involving the plates; however, the shape of the interface implicitly contains information about the rotation angle via its boundary, which must remain attached to the boundary of the plate array.

For convenience, we now scale all the lengths by l to nondimensionalize the governing equations. Equations (3)–(5) transform to

$$2H = -\mu \quad \text{on } \Sigma, \quad (6a)$$

$$\theta = \lambda \left(-\mu + \int_{\Gamma_\theta} u(\mathbf{n}_\theta \cdot \mathbf{v}) ds \right), \quad (6b)$$

$$\frac{1}{3} \iint_{\Sigma} \mathbf{r} \cdot \mathbf{n} d\Sigma = V, \quad (6c)$$

where λ and V are the dimensionless groups given in Eq. (1) and $\mu = l\Delta p/\gamma$ defines an unknown dimensionless ratio of pressures. Equation (2) remains the same, except that the underlying domain Ω becomes $[-1, 1]^2$. These nondimensional equations govern the shape of the scaled interface Σ , the angle of rotation of the plate θ , and the pressure ratio μ .

III. RESULTS

A straightforward family of solutions to Eqs. (2) and (6) exists when $V = 0$. No volume implies no drop, and the plate remains undeflected in the energetically preferred flat configuration, where $\theta = 0$. Physically, there is no interface; however, mathematically, there is one. It blankets the flat plate array and has zero mean curvature, which implies that μ must vanish. These choices completely satisfy Eqs. (2) and (6), leaving the value of λ free.

We calculate all the other solutions numerically. Observe that V and λ only appear in Eqs. (6b) and (6c), respectively. If we consider them as unknowns instead of ratios of known physical constants, then the triple (Σ, θ, μ) sets their values. Determining this triple requires solving the isolated subsystem consisting of Eq. (6a) with boundary condition (2) for $\Omega = [-1, 1]^2$, where the angle θ and the multiplier μ are now parameters. Setting θ as a value in $[0, \pi]$ fixes the boundary of the constant mean surface Σ and puts the subsystem into a form that can be solved by the predictor-corrector continuation algorithm outlined in Ref. [36]. Running the algorithm for different θ produces multiple distinct one-parameter families of solutions that combine together to give the whole solution set of Eqs. (2) and (6a) over the (θ, μ) parameter space. After computing V_0 and λ from (6b) and (6c) at each known triple (Σ, θ, μ) , this solution set naturally extrapolates to the solution set of the full model.

Figure 2(a) plots the solution set as a surface in the lower-dimensional (V, λ, θ) space. Although the plot does not directly represent Σ , the value of V conveys crucial qualitative

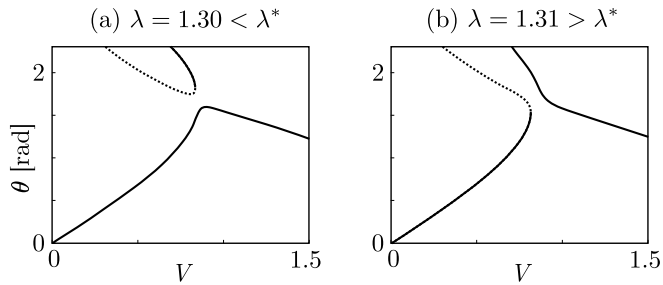


FIG. 3. Representative bifurcation diagrams, plotting the rotation angle θ (rad) vs V , for values of λ below and above the cusp point $\lambda^* \approx 1.305$. Solid and dotted curves indicate stable and unstable equilibria, respectively. The stability is inferred from a local comparison of the dimensionless energy, $|\Sigma| + \theta^2/2\lambda$, near the fold points.

information about the interface: as V grows, Σ protrudes farther and farther away from the flexed plates. Figure 2(b) illustrates this trend by displaying the equilibria at points P_1 through P_4 ordered by increasing volume. The curve on the solution surface in Fig. 2(a), marked by the solid black line, gives the collection of bifurcation points known as the bifurcation locus. All the points in the locus correspond to folds, except for the point with a vertical tangent, which corresponds to a cusp bifurcation. Projecting the bifurcation locus onto the parameter space produces two curves that emanate from the cusp point at $(V^*, \lambda^*) \approx (0.849, 1.305)$. These curves partition the parameter space into multiple distinct regions [see Fig. 2(c)]. At points inside the curves, there are three equilibrium configurations; at points on the curves (barring the cusp), there are two equilibrium configurations; and at points outside the curves and at the cusp, there is one equilibrium configuration.

When λ is a known parameter [4,37], the system has two regimes of behavior for varying V . The regimes, defined for $\lambda \leq \lambda^*$ and $\lambda > \lambda^*$, both have bifurcation diagrams with two distinct solution branches; however, their arrangements are obverse unfoldings of the supercritical pitchfork bifurcation, which comes from slicing the cusp point along the line $\lambda = \lambda^*$. Figure 3 juxtaposes example sketches for both. Their differences are embodied by the branch of solutions that exists for large values of V . If $\lambda \leq \lambda^*$, then decreasing V along this branch eventually produces a completely folded configuration, where the drop occupies the infinitesimally thin volume between the plates. However, if $\lambda > \lambda^*$, then the branch ends in an unfolded configuration with no liquid. In this regime, a stable folded state does exist, although it is on the other branch and is connected to stable solutions of large potential energy, $\mathcal{U} = |\Sigma| + \theta^2/2\lambda$, which makes it less accessible experimentally.

When V is a known parameter of the system [32,38], there are again two regimes of behavior. They are defined for $V < V^*$ and $V > V^*$, and each contains only one branch of solutions (see Fig. 4). When $V > V^*$, the branch is a monotonic curve of stable solutions; however, when $V < V^*$, the branch bends back on itself twice, creating two saddle-node bifurcations and an interval of three equilibrium configurations, the middle of which has the largest energy and is physically

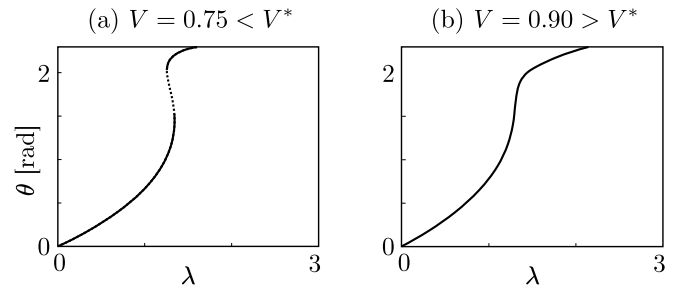


FIG. 4. Representative bifurcation diagrams, plotting the rotation angle θ (rad) vs λ , for values of V below and above the cusp point $V^* \approx 0.849$. Again, solid and dotted curves indicate stable and unstable equilibria.

unstable. This structure permits jumps and hysteresis in the equilibrium angle as λ is varied.

The bifurcation surface in Fig. 2(a) also captures the equilibrium shapes set by the competition between bending and surface tension only. Changing the liquid drop in the setup to a soap film and replacing the plates with two rectangular wire frames removes the hydrostatic pressure difference at the interface of the two media. Accordingly, $\mu = 0$, and there is no volume constraint in the model. The parameter V simply gives a measure of the film, instead of being a physical restriction. Solutions of Eq. (6) for $\mu = 0$ then define two curves on the surface in Fig. 2(a). One curve (i.e., $(V, \lambda, \theta) = (0, c, 0)$ for all $c \geq 0$) comes from the flat-film-undeflected-plate state, and the other, corresponding to a deformed interface and deflected plate, arises from flat state at $\lambda = 2$. Projecting these curves onto the (λ, θ) plane reveals the upper part of a subcritical pitchfork bifurcation (see Fig. 5). Subcriticality implies that the configuration is sufficiently bendable when $\lambda > 2$ and immediately snaps to a fully folded structure when soap film and wire frame come into contact. This snapping is different than the smooth buckling observed in other setups with soap films spanning deformable wire frames (see, e.g., Ref. [39]. Preliminary work suggests that if the long side of the rectangular frame varies, then the pitchfork bifurcation shifts from $\lambda = 2$ and becomes supercritical when the side length reaches a set threshold.

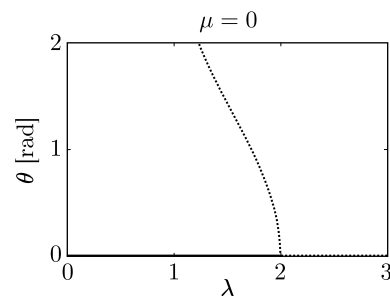


FIG. 5. Bifurcation diagram obtained from projecting the $\mu = 0$ curves on the catastrophe surface in Fig. 2 onto the (λ, θ) plane. Setting $\mu = 0$ (i.e., no pressure) physically represents replacing the drop with a soap film and the plates with rigid wire frames. The resulting equilibria form by balancing elasticity with surface tension. At $\lambda = 2$, there is a subcritical pitchfork bifurcation (the diagram shows only the upper half), where the $\theta = 0$ state becomes unstable.

IV. DISCUSSION

The cusp catastrophe appears to be an intrinsic feature of all elastocapillary systems. For instance, consider a general setup composed of two elastic components that are separated by a gap that is bridged by a liquid with at least one free interface. The liquid interface creates a stress via hydrostatic pressure and surface tension that induces a mechanical deformation, resulting in a strained shape that is unique when the liquid's volume is sufficiently large. Decreasing the volume, say via evaporation, will lead to one of two different final configurations depending on the relative rigidity of the structure, as measured by the system's bendability parameter λ . A large rigidity implies that the system will return to its unstressed arrangement with no liquid bridge. A small rigidity implies that the system will completely collapse to a configuration where the two elastic components are bonded together by a very thin film. However, both of these final states exist regardless of physical parameters—the elastic rigidity just dictates which is accessible under evaporation—and there are two families of solutions branching from them, along with a bifurcation at a critical value of the rigidity that dictates which branch connects to the unique large volume state. Since this bifurcation involves two parameters, it is of codimension 2 and, hence, a cusp catastrophe.

Altering the simple setup in various ways, such as allowing for dewetting [40,41] or stretching [42,43], further complicates the governing equations and adds more parameters to the

model, yet the structure of the solution space deduced from the preceding general argument appears robust. Studies on these modified systems present features, including hysteresis [44] and unfolding bifurcations [4,27,28], that appear from slices of a cusp catastrophe.

A striking physical consequence of the cusp is the jump in θ brought about by varying the parameters across the bifurcation curves. In engineering applications, this jump-inducing cross typically happens when V is varied since temporal scales of evaporation are often smaller than those characterizing the degradation of the elastic components. In biological systems though, we can easily argue that crossings will happen much more frequently in the other direction, with varying λ due to the rapid contraction or relaxation of soft tissues. This mechanism provides a promising biomimetic method for loading structures and dissipating energy.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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