

Dynamical vortex phase diagram of two-dimensional superconductivity in gated MoS₂Yu Saito^{1,2,*}, Yuki M. Itahashi¹, Tsutomu Nojima³, and Yoshihiro Iwasa^{1,4,†}¹Quantum-Phase Electronics Center (QPEC) and Department of Applied Physics, The University of Tokyo, Tokyo 113-8656, Japan²California NanoSystems Institute, University of California at Santa Barbara, Santa Barbara, California 93106, USA³Institute for Materials Research, Tohoku University, Sendai 980-8577, Japan⁴RIKEN Center for Emergent Matter Science (CEMS), Wako 351-0198, Japan

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Recent discoveries of two-dimensional (2D) superconductors have uncovered various aspects of physical properties including vortex matter. In this paper, we report transport properties and a dynamical phase diagram at zero magnetic field in ion-gated MoS₂. In addition to the universal jump in the current-voltage characteristic showing unambiguous evidence of the Berezinskii-Kosterlitz-Thouless transition, we observed multiple peaks in the temperature and current derivative of the electrical resistance, based on which a dynamical phase diagram in the current-temperature plane was constructed. We found current-induced dynamical states of vortex-antivortex pairs, containing that with the phase slip line. Also, we present a global phase diagram of vortices in gated MoS₂ which captures the nature of vortex matter of clean 2D superconductors.

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I. INTRODUCTION

Superconductivity in two-dimensional (2D) materials [1] not only expands a new materials paradigm [2–9], but also offers opportunities to explore various novel phenomena of 2D superconductors, which were masked by disorder or randomness in conventional metallic thin film superconductors with amorphous or granular structures. Examples include the quantum metallic state (a vortex liquid state due to quantum fluctuation) as a ground state [10–12], the quantum Griffiths state [13–16], enhanced in-plane upper critical field [17–19], nonreciprocal superconductivity [20–22], and unusual behavior of superconducting gap against magnetic field B [23,24]. Such series of discoveries originating from the minimal disorder and the noncentrosymmetry accompanying high crystallinity suggest that emerging 2D superconductors form a materials platform for new physics of superconductivity.

While the quantum phases [10–16,25] and the vortex dynamics [26–31] under magnetic fields have been intensively studied in 2D superconductors, the dynamical properties at high current and zero magnetic field have been largely unexplored. It has been well known that in 2D superconductors the zero-Ohmic-resistance state is achieved by the Berezinskii-Kosterlitz-Thouless (BKT) transition [32–34]. Above the BKT transition temperature (T_{BKT}), the spatial phase fluctuation of the order parameter causes the thermally excited unbinding of vortices and antivortices (V-AV), which can move freely and result in the Ohmic dissipation. When such vortices are bound in the form of V-AV pairs, the BKT transition to the zero-Ohmic-resistance state occurs at a temperature of $T = T_{\text{BKT}}$ in the low-current limit. Experimentally, the existence of the BKT transition has been confirmed by

the universal jump of the power α in the current (I)-voltage (V) characteristics ($V \propto I^\alpha$) at T_{BKT} and by the scaling behavior of linear resistance (R) obeying the Halperin-Nelson equation in the critical region just above T_{BKT} [35,36]. These transition features have been discussed in various systems such as disordered 2D superconductors [37–39] and high- T_c superconductors [40]. On the other hand, the zero-field V-AV dynamics beyond the low-current region in 2D systems, which would lead to understanding how the BKT state transfers to the normal state by increasing the current, have remained unsolved.

The dynamics of vortices, whether in zero or finite magnetic fields, has been a long-standing research field from the viewpoint of fundamental interest in interacting particles as well as application of superconductors at high current density. While the dynamics of slow Abrikosov vortices in magnetic field (B), containing the plastic flow state and the moving lattice state, has been thoroughly investigated in various 2D superconductors [26–28], there are still many ongoing discussions on the dynamics of ultrafast vortices under high current density [31,41]. One of the typical examples is the sudden jump of voltage induced by the vortex motion as observed in various superconductor films [29,30,42–44], which was interpreted based on the flux-flow instability predicted by Larkin and Ovchinnikov in the early stages [45] and was recognized later as the possible appearance of the phase slip lines [46], namely, flow channels of the Josephson-like (kinematic) vortices [47–49]. While such fast dynamical states of vortices were discussed, in many cases, with the dirty systems under B [30,48–50], the experiments in 2D superconductors under zero magnetic field, which indeed corresponds to the original condition of the theoretical prediction, are few [51]. In addition, whether the phase slip lines appear even in clean systems is unclear. In such a situation, exploring the dynamics of V-AV in clean 2D superconductors with weak pinning at zero magnetic field is highly desirable. Indeed, a discussion on

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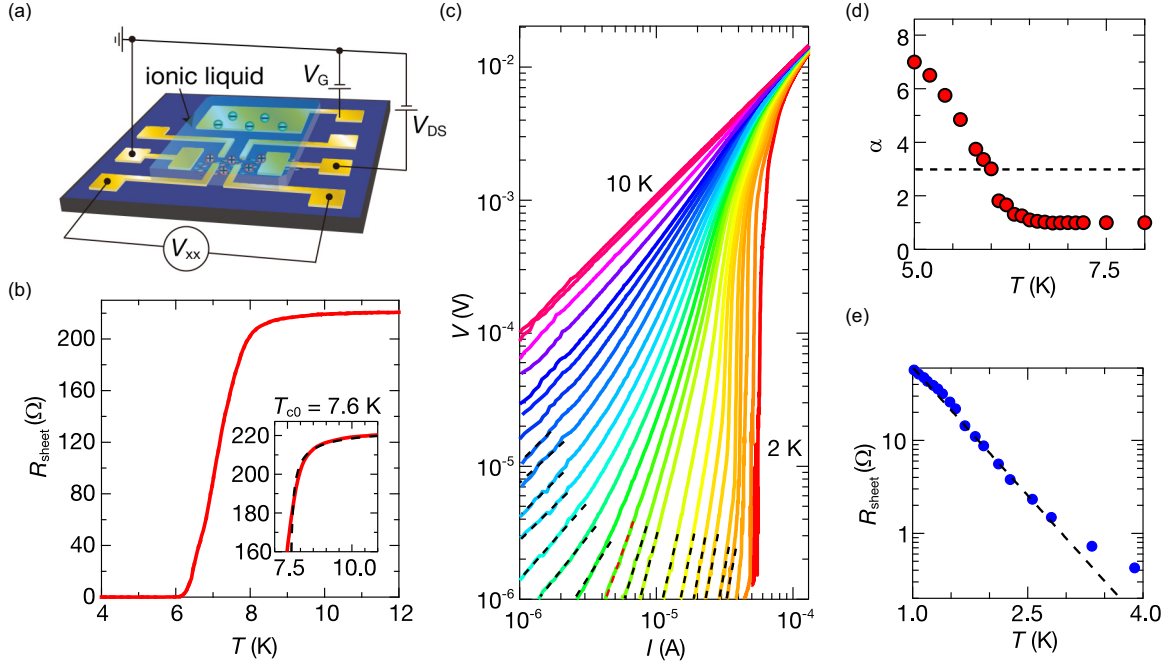


FIG. 1. Berezinskii-Kosterlitz-Thouless (BKT) transition in ion-gated MoS₂. (a) A schematic image of electric-double-layer transistor structure. (b) Superconducting transition in ion-gated MoS₂. The effective thickness of gate-induced superconductivity is around monolayer. Inset: A magnified view near the superconducting fluctuation regime. Black dashed curve shows Aslamazov-Larkin (AL) and Maki-Thompson (MT) fit. The superconducting transition temperature (T_{c0}) defined by AL and MT fit is 7.6 K. (c) Current-voltage curves on a logarithmic scale at various temperatures varying in 1-K steps from 2 to 4 K, in 0.2 K-K steps from 5 to 5.8 K; in 0.1 K steps from 5.9 to 7.2 K, and 4.2, 4.5, 5.7, 7.5, 8 and 10 K. The short black dashed lines are the log-log fit to the data in the low-current limit (below critical region) at each temperature. Red dashed line shows the same fit at the BKT transition temperature (T_{BKT}). (d) Temperature dependence of the power-law exponent α , as deduced from the fits shown in (c). From this plot, we defined the T_{BKT} is 6.0 K, where the α discretely becomes 3. (e) Plots of R_{sheet} as a function of $[(T_{c0}-T)/(T-T_{BKT})]^{1/2}$ at 5 μ A. The black dashed line indicates the fitting of the Halperin-Nelson equation, $R = R_0 \exp\{-2b(\frac{T_{c0}-T}{T-T_{BKT}})^{1/2}\}$. Here, R_0 and b , which are materials-dependent parameters, are 508 Ω and 1.06, respectively.

dynamical behavior has been recently reported on a few-layer NbSe₂ flakes and some data are interpreted using a model based on the phase slip phenomena [52].

Here, we report on the current-induced dynamical states of V-AV pairs at zero magnetic field in a gate-induced 2D superconductor, MoS₂, through the measurements of transport properties as a function of temperature and current. In addition to confirming the standard BKT transition in low current and high temperature, we observed the anomalous transport with the nonmonotonic increase against temperature and current in the high-current and low-temperature region. The present results suggest the existence of multiple dynamic vortex phases between the BKT state and the normal state at zero magnetic field. Including the present result, we propose a global magnetic field-current-temperature phase diagram, which offers a comprehensive understanding of vortex matter in clean 2D superconductors with weak pinning potentials.

II. RESULTS

A. Transport properties in ion-gated MoS₂

We prepared thin flakes (typically 20 nm thick) of MoS₂, which is a typical layered semiconductor, from bulk single crystals by mechanical exfoliation, and patterned Au (90 nm)/Cr (5 nm) electrodes onto an isolated thin flake in a standard four-terminal and Hall bar configuration with

a side gate electrode on a Si/SiO₂ substrate. To complete an electric-double-layer transistor (EDLT) structure [Fig. 1(a)], we put a drop of ionic liquid, *N,N*-diethyl-*N*-(2-methoxyethyl)-*N*-methylammonium bis (trifluoromethylsulphonyl) imide (DEME-TFSI) on the above device, as a gate medium. Here, the length between the voltage contacts L and width W are 2.0 and 4.0 μ m, respectively. By an application of a gate voltage of 4.5 V at 230 K, the MoS₂ EDLT showed the metallic T dependence of sheet resistance R_{sheet} ($dR_{sheet}/dT > 0$), followed by the superconducting transition, as shown in Fig. 1(b). Here, the current for the $R_{sheet}(T)$ measurement is 5 μ A. The residual-resistance ratio (RRR) of the sample, which is defined as $R(200\text{ K})/R(10\text{ K})$ here since the gate is applied 230 K and ionic liquid will be frozen around 200 K, is roughly 7–8. This value is indeed comparable to the RRR [$R(200\text{ K})/R(10\text{ K})$] of a bilayer (7–8) and monolayer NbSe₂ (8–9) [19] and much higher than the RRR of ~ 1.5 in chemical vapor deposition (CVD)-grown [53] and molecular beam epitaxy (MBE)-grown [54] NbSe₂ films, which shows high crystallinity of the present sample. To determine the critical temperature, we used the Aslamazov-Larkin (AL) [55] and the Maki-Thompson (MT) models [56,57], where the excess sheet conductance by the fluctuation ΔG is given by $\Delta G = \frac{e^2}{16h}(\frac{T_{c0}}{T-T_{c0}}) + \frac{e^2}{8h} \frac{T_{c0}}{T(1-\delta)-T_{c0}} \ln(\frac{T-T_{c0}}{\delta T})$, where $T_{c0} = 7.6\text{ K}$ (mean-field transition temperature) and $\delta = 0.06$ (pair-breaking parameter), which are used as fitting

parameters with e the elementary charge and $\hbar$ the Dirac's constant [the inset of Fig. 1(b)]. The sheet carrier density n_s of this device was $1.19 \times 10^{14} \text{ cm}^{-2}$, which was confirmed by the Hall measurements at 15 K. According to the previous studies [17,18], the application of such a high gate voltage leads to the accumulation of the carriers in the topmost layer with effective thickness $d \sim 1 \text{ nm}$. It is noted that the normal state sheet resistance R_N is around 220Ω , which results in $k_F l = \frac{1}{s s'} \frac{2\hbar}{e^2} \frac{1}{R_N} \sim 59$. This value is much larger than the Ioffe-Regel limit ($k_F l \sim 1$). Here, k_F is the Fermi wave number, l is the mean free path, and s and s' are spin and valley degree of freedom, respectively. In addition, in the superconducting state, the value l/ξ_0 is about 0.37, where l and ξ_0 (Pippard's coherence length) are 30 and 81 nm, respectively. All the properties mentioned above suggest the 2D superconductor on the surface of MoS_2 has the less-disordered nature far from the dirty limit.

Figure 1(c) shows the I - V characteristics of the ion-gated MoS_2 at various temperatures ranging from 2 and 10 K. We found that α , which is the exponent of the relation at low-current limit, $V \propto I^\alpha$, shows almost 1 above 6.5 K, gradually increases with decreasing temperature, and then abruptly jumps to 3 at 6.0 K [Fig. 1(d)]. From these data, we determined the BKT transition temperature $T_{\text{BKT}} = 6.0 \text{ K}$. Furthermore, the plot of $\ln(R_{\text{sheet}})$ as a function of $[(T_{c0}-T)/(T-T_{\text{BKT}})]^{1/2}$ at $I = 5 \mu\text{A}$ in the Ohmic region shows linear behavior [Fig. 1(e)], which is consistent with a Halperin-Nelson model $R = R_0 \exp\{-2b[(T_{c0}-T)/(T-T_{\text{BKT}})]^{1/2}\}$ [35,36,58]. This implies that the decrease in the resistance above T_{BKT} is caused by the development of the V-AV correlation with the characteristic length scale $\xi_+ \sim \xi \exp[b[(T_{c0}-T)/(T-T_{\text{BKT}})]^{1/2}]$, with b numerical constants of order of unity. To obtain the dashed line in Fig. 1(e), we used $T_{c0} = 7.6 \text{ K}$ and $T_{\text{BKT}} = 6.0 \text{ K}$ extracted from the AL and MT fit and I - V characteristic, and $R_0 = 508 \Omega$ and $b = 1.06$ as the materials-dependent fitting parameters. Both the abrupt change of α in I - V characteristic and the scaling of $R_{\text{sheet}}-T$ dependence is therefore in fair agreement with the standard BKT picture of 2D superconductors.

B. Anomalous transport at high excitation current

To investigate how the BKT state transfers to the normal state by the application of the high current, we measured the nonlinear sheet resistance R_{NL} at various bias currents ranging from 5 to $200 \mu\text{A}$ [Fig. 2(a)]. While the $R_{\text{NL}}-T$ curves strongly depend on I above $5 \mu\text{A}$, we note the region with R_{NL} smaller than the noise level (BKT state as discussed later) is maintained until $55 \mu\text{A}$ in the measured T range $> 2 \text{ K}$. This feature shows marked contrast to the case of application of out-of-plane magnetic fields, in which the BKT state is very fragile even in a very small B [10,16]. More importantly, we found that the anomalous steplike behavior in $R_{\text{NL}}-T$ curves appears at middle bias currents ($I = 35\text{--}65 \mu\text{A}$), followed by the T -independent behavior at high bias currents ($I > 70 \mu\text{A}$). On the other hand, such step-like behavior becomes obscure below $I = 30 \mu\text{A}$, suggestive of a single transition at low enough currents ($I < 20 \mu\text{A}$).

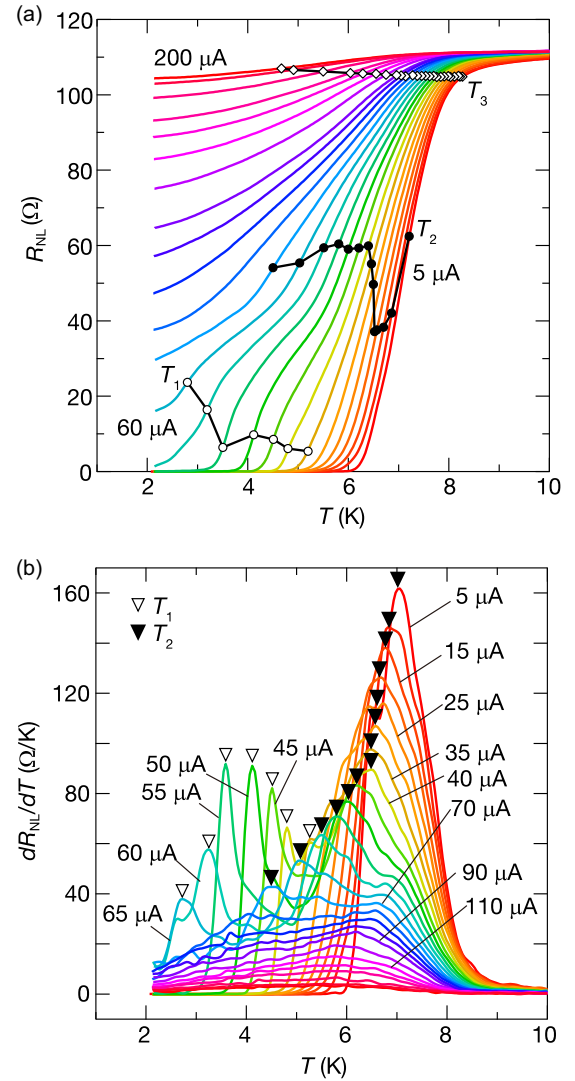


FIG. 2. Longitudinal nonlinear resistance R_{NL} and its derivative dR_{NL}/dT versus temperature at various currents. (a) R_{NL} at various bias currents varying in $5\text{--}\mu\text{A}$ steps from 5 and $90 \mu\text{A}$; in $20\text{--}\mu\text{A}$ steps from 110 and $170 \mu\text{A}$; and 100, 180, $200 \mu\text{A}$. The black and white circles show T_1 and T_2 , respectively, which are defined by white and black triangles in (b). Also, T_3 is defined by the temperatures where the resistance becomes 95% of the normal state resistance, which is indicated by white diamonds. (b) Temperature derivative of R_{NL} versus temperatures at various currents. Black and white triangles show the double ($35\text{--}65 \mu\text{A}$) and single ($5\text{--}30$ and $70\text{--}200 \mu\text{A}$) peak positions of dR_{NL}/dT versus T curve, respectively.

In order to quantify such features, we plot the temperature derivative of resistance dR_{NL}/dT at various currents as shown in Fig. 2(b). We found the two characteristic temperatures T_1 (white triangles) and T_2 (black triangles) with the peaks, corresponding to the inflection points in the $R_{\text{NL}}-T$ curves denoted by white and black circles in Fig. 2(a). So far, it has not been discussed intentionally how the BKT state below T_{BKT} or the flux-flow state above T_{BKT} ($< T_{c0}$) are extended along the current axis, although one notices that the dynamics of V-AV in both states plays a central role there [35,39]. Figure 2(b) suggests the existence of intermediate dynamical states

of V-AVs between the BKT and the fluctuation region of the order parameter amplitude with clear boundaries.

For further understanding of the current-induced anomalous behavior observed in Fig. 2, we extracted the $d \ln V / d \ln I$ against I from the data in Fig. 1(c). Figures 3(a)–3(c) show $d \ln V / d \ln I$ as a function of I at various temperatures, for the whole regime ($0 < I < 100 \mu\text{A}$) (a) and magnifications around 50 – $76 \mu\text{A}$ (b) and 45 – $95 \mu\text{A}$ (c), respectively, which represent the changes in the nonlinear behavior between the BKT state and the fluctuation and normal states. Here, we provide the data at representative temperatures for clarity and visibility. We found three distinct peaks at I_{c1} [black triangles in Fig. 3(a)], I_{c2} [white diamonds in Fig. 3(b)], and I_{c3} [white triangles in Fig. 3(c)]. As displayed in Figs. 3(a)–3(c), I_{c1} , I_{c2} , and I_{c3} shift toward the low current with increasing temperature. At the same time, these peaks become broad and eventually undefined around 6.5 , 3.0 , and 6.0 K for I_{c1} , I_{c2} , and I_{c3} , respectively. It should be noted that the I - V curves keep nonlinearities with $d \ln V / d \ln I > 3$ after the steplike increases of V (corresponding to the peaks of $d \ln V / d \ln I$ in Fig. 3) at I_{c1} and I_{c2} below T_{BKT} , implying these two anomalies are completely different from the thermal quench process to the normal state. Also, as shown in Fig. S1 in the Supplemental Material [59], the peaks of I_{c2} and I_{c3} become vanishingly small or washed out at 0.2 T (corresponding to 10% of the perpendicular upper critical field (~ 2 T)). These data suggest that there are three dynamical transitions or crossovers between the BKT (superconducting) state and the amplitude fluctuation (partly normal) state at low temperature. Because the dissipation of the superconductor at low temperature originates from the motion of vortices, especially that of V-AV pairs in zero magnetic field, the anomalies are attributed to the abrupt change in the velocity of dissociated V-AV pairs or to the transition from the V-AV flow state to the normal or fluctuation (partly normal) regime, as discussed later.

Such steplike behavior of the I - V curve has been reported in a 100-nm-thick Sn film in a narrow temperature range of 10 – 30 mK below T_{c0} [51] and in a wider temperature range in few-layer NbSe₂ flakes [52], which is ascribed to the occurrence of the phase slip line(s) with kinematic vortices running across the sample's plane [30,48–50]. This phenomenon seems closely related to our observation as discussed later. In most 2D superconductors, the abrupt jump of the I - V characteristic in the BKT state results in reaching the normal state with Ohmic behavior or the destruction of superconductivity [38,60]. One plausible reason for the successful observation of the multiple nonequilibrium states in this work is because the present 2D system belongs to relatively clean systems with weak pinning based on high crystallinity, which expands the I - V measurement window. In the case of superconductors with strong pinning, the pinning effect should decrease the measured voltage, blurring the subtle dynamical changes. In addition, if the system is in the dirty limit, the fast motion of unbound V-AV pairs, originating from the high-flux-flow resistance, narrows the region where the system shows nonlinear dynamic behavior against current. The present system with less disorder and minimal pinning potential does not suffer from such problems and thus provides an ideal playground for the V-AV dynamics, which makes easier access to the intrinsic nonequilibrium state of 2D superconductors.

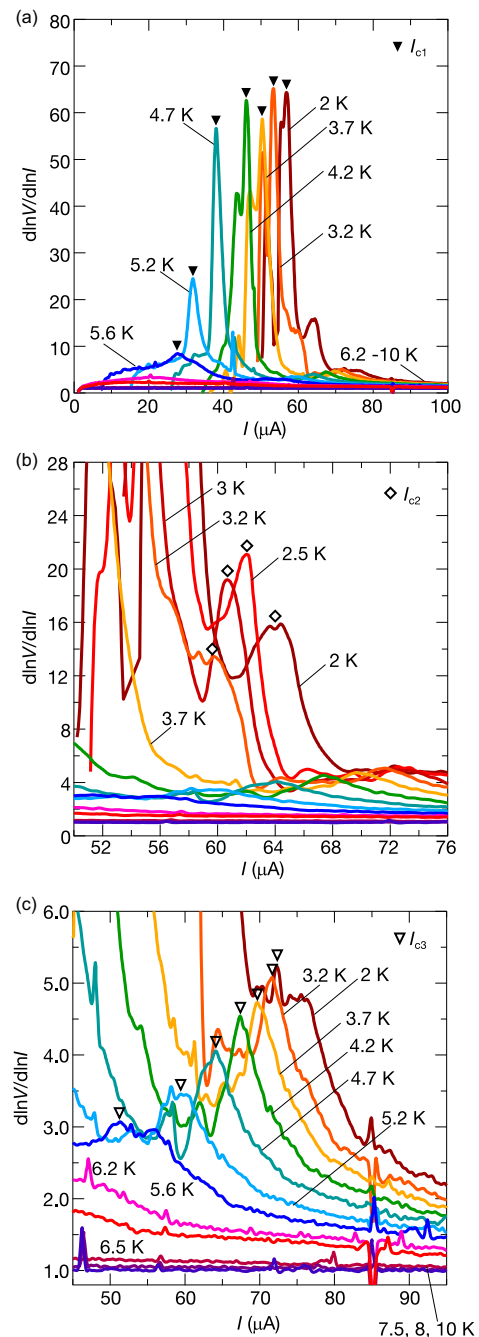


FIG. 3. Differential resistance $d \ln V / d \ln I$ at various temperatures. (a) $d \ln V / d \ln I$ as a function of I at various temperatures from 2 to 10 K. Here, we provided data at representative temperatures. We choose data at 2, 3.2, 3.7, 4.2, 4.7, 5.2, 5.6, and 6.2–10 K. Black triangles show the strongest peak positions below $60 \mu\text{A}$. We defined I_{c1} of each $d \ln V / d \ln I$ curve as the current where the $d \ln V / d \ln I$ curves show these strong peaks indicated by black triangles. (b) A magnification of (a) around 50 – $76 \mu\text{A}$. Here, we provide additional data at 2.5 and 3.0 K. White diamonds show the second strong peaks around 50 – $60 \mu\text{A}$. We defined I_{c2} of each $d \ln V / d \ln I$ curve as the current where the $d \ln V / d \ln I$ curves show these peaks indicated by white diamonds. (c) A magnification of (a) at the high-current regime above $50 \mu\text{A}$. White triangles show the broad peak positions at the high-current regime above $50 \mu\text{A}$. We defined I_{c3} of each $d \ln V / d \ln I$ curve as the current where the $d \ln V / d \ln I$ curves show these broad peaks indicated by white triangles.

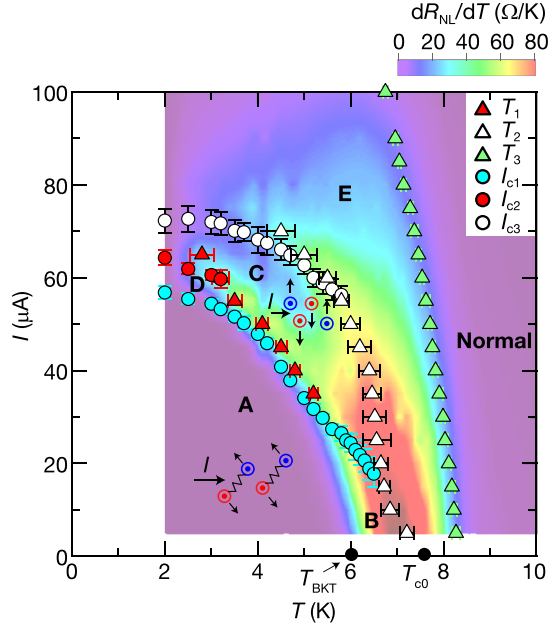


FIG. 4. Current-temperature phase diagram of ion-gated MoS_2 at zero magnetic field. Schematic images in the regime of the BKT state and vortex flow state show binding vortex-antivortex pairs (region A), and free vortices and antivortices in the current (region C). Red and white triangles show peaks of dR/dT - T curves at lower (T_1) and higher (T_2) temperatures in Fig. 2(b), respectively. Green triangles, which are defined by the temperatures where the resistance becomes 95% of the normal state resistance, show a boundary between fluctuation regime and normal state. Light blue, red, and white circles show I_{c1} , I_{c2} , and I_{c3} , respectively. The vertical error bars for I_{c1} , I_{c2} , and I_{c3} and the horizontal error bars for T_1 , T_2 , and T_3 represent uncertainties in determining each data point. The color map of dR_{NL}/dT values as a function of I and T is displayed in the measurement regime of $T > 2$ K and $T > 5$ μA . T_{c0} and T_{BKT} are determined from AL and MT fit and I - V characteristic in Fig. 1, respectively. Region A is the BKT state with zero resistance where the correlated vortices and antivortices are binding. Region B ($I < I_{c1}$, $T_{BKT} < T < T_2$) shows the BKT critical state with the finite V-AV correlation length. Region C ($I_{c2} < I < I_{c3}$, $T_1 < T < T_2$) shows the flux-flow region of kinematic vortices, which results in the intermediate state in nonequilibrium. Region D ($I_{c1} < I < I_{c2}$, $T < T_1$) shows another intermediate state possibly because of the fast flow of Abrikosov vortices called as the vortex street. Region E ($T_2 < T < T_3$, $I > I_{c3}$) shows the fluctuation state.

III. DISCUSSION

A. Current-temperature phase diagram

Figure 4 provides a current-temperature (I - T) phase diagram at zero magnetic field in ionic-gated MoS_2 , combined with a color map of dR_{NL}/dT values as a function I and T in the measurement regime of $T > 2$ K and $I > 5$ μA . Light blue, red, and white circles show I_{c1} , I_{c2} , and I_{c3} , respectively. Red and white triangles show peaks at lower (T_1) and higher (T_2) characteristic temperatures in Fig. 2, respectively. Green triangles, which are defined by the temperatures where the resistance becomes 95% of the normal state resistance (T_3) in Fig. 2(a), show an onset of amplitude fluctuation of order parameter. In Fig. 4, we note T_2 merges with I_{c3} , constructing

the same boundary. On the other hand, T_1 almost coincides with I_{c1} at high temperature, but separates from it at low temperature below ~ 3.5 K, forming an additional boundary together with I_{c2} .

Below I_{c1} , the system maintains the BKT state (region A) or the BKT critical state with the finite V-AV correlation length ξ_+ (region B). According to the BKT scenario, the electrical resistance is caused by flows of the unbound V-AV pairs [35,36]. In region A, such free vortices (dissociated V-AVs) are induced by applied current and the density of free vortices n_f is described as

$$n_f = CI^{\alpha-1}, \quad (1)$$

with C the temperature-dependent numerical constant. This relation is derived from the dissociation probability of the V-AV pairs following $\Gamma \propto n_f^2 \propto \exp[-U(r_c)/k_B T]$, where the energy barrier $U(r_c) = Kd \ln(r_c/\xi)$ and the threshold length for dissociation $r_c = KS/(I\phi_0)$ are determined by the r that maximizes the sum of the energy loss of V-AV interaction $Kd \ln(r/\xi)$ and the energy gain owing to the Lorentz force $I\phi_0 dr/S$ with the distance r of the thermally excited V-AV pair. Here, $K = \frac{1}{\mu_0} \left(\frac{\phi_0^2}{4\pi\lambda^2} \right)$, ϕ_0 is the flux quantum, ξ the Ginzburg-Landau (GL) coherence length, λ the magnetic field penetration depth, k_B the Boltzmann constant, $d (= 1.0 \text{ nm})$ the effective thickness, $S (= d \times W = 1.0 \text{ nm} \times 4.0 \text{ } \mu\text{m})$ the cross section of current flow, and $\alpha = Kd/2k_B T + 1$. By using Eq. (1) and the vortex velocity $v_f = \phi_0 I / \eta S$, which is determined by the balance of the viscous drag force $\eta v_f d$ and the Lorentz force $\phi_0 I d / \eta S$ working on the vortices, we obtain the I - V characteristic as

$$V = (n_f \phi_0 v_f) L = \frac{n_f \phi_0^2 I}{\eta} \left(\frac{L}{S} \right) = C \left(\frac{\phi_0^2 L}{\eta S} \right) I^\alpha, \quad (2)$$

with η the vortex flow viscosity and $L (= 2.0 \text{ } \mu\text{m})$ the distance between the voltage contacts, which is consistent with the experimental data in region A. In region B, on the other hand, the resistance in the low-current limit is Ohmic because of the thermally dissociated spontaneous (uncorrelated) V-AV pairs with the large distance $r_c > \xi_+$, but the non-Ohmic component appears at high current because of the unbinding of correlated V-AV with $r_c < \xi_+$, leading to additional dissipation according to Eq. (2).

As I increases from regions A or B, the I - V characteristics show diverging behavior at I_{c1} , corresponding to the peak of $d \ln V / d \ln I$, and then transfer to region C or D, depending on temperature. This diverging behavior reminds us of the occurrence of the flux-flow instability [45], where η at high current (at high v_f) is modified as $\eta_0 / \{1 + (v_f/v_0)^2\}$, where η_0 is viscosity at low-current limit and v_0 the critical velocity. In such a case, it is expected that the I - V curves jump to the different branches with the much smaller vortex flow viscosity (much faster vortex velocity) at $v_f = v_0$. Within the framework of existing theories, the formation of the phase slip line with kinematic vortices may be a leading candidate for the anomaly at I_{c1} . Another possibility is the failure of the thermally activated process in Eq. (1) because of the decrease in $U(r_c)$ with increasing I to the same order of $k_B T$, which will lead to the abrupt increase in n_f in Eq. (2). It is noted that because region B only exists in the high-temperature and

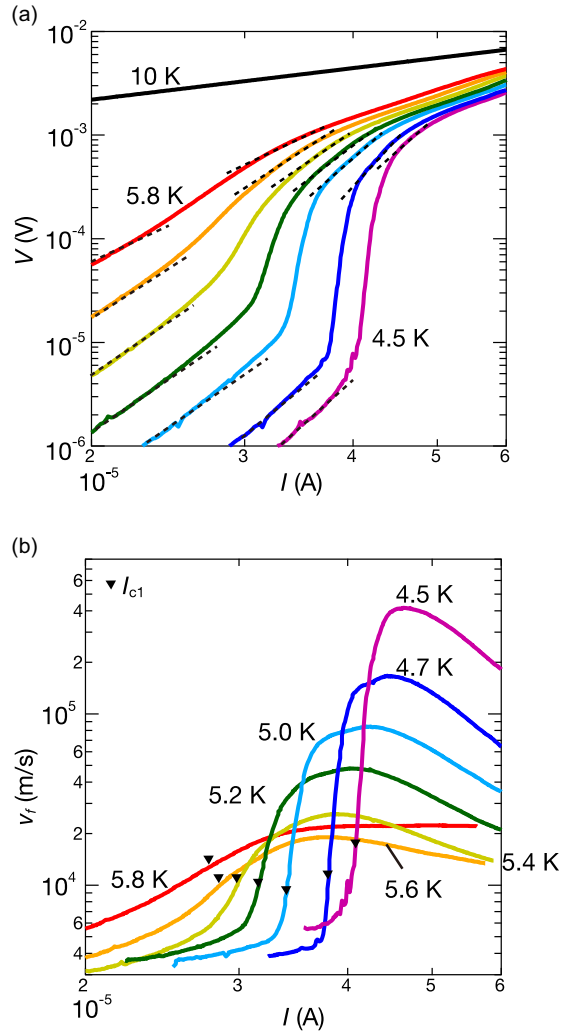


FIG. 5. The velocity of a single vortex versus current. (a) Current-voltage curves on a logarithmic scale at 4.5, 4.7, 5.0, 5.2, 5.4, 5.6, 5.8, and 10 K. Dashed lines show regions following $V \propto I^\alpha$ below and above I_{c1} . (b) Velocity of the single vortex as a function of the current calculated by $v_f = \frac{V/L}{C(T)I^{\alpha(T)-1}\phi_0}$, where L is the length between the four terminal contacts, ϕ_0 is a magnetic flux quantum. $C(T)$ is determined by the fit of the IV curve at the lowest-current limit using the relation $V = 2\pi\xi^2(T)C(T)R_N I^{\alpha(T)}$, where ξ is the GL coherence length; R_N is the normal state resistance of 220 Ω . Black triangles show the position of I_{c1} .

low-current region, the boundary between B and C disappears at low temperature and high current. Also, with the appearance of region D at low temperature and high current, two boundaries of A and D and of C and D appear, branching from the A-C boundary.

In order to understand the emergence of region C, we analyzed the I - V curves from 4.5 to 5.8 K in detail [Fig. 5(a)], for which a direct transition from region A to C is observed and the data show clearly the BKT-type power law at low-voltage limit explained by Eq. (2). As shown in Fig. 5(a), we note the slopes of I - V curves in log-log plots do not change so much by comparing the data before and after the step-like increase at I_{c1} [black dashed lines in Fig. 5(a)]. This implies that the nucleation process of dissociated V-AVs, namely, the

description of n_f in Eq. (1), is almost unchanged at least around the jump in V . Then, the transition from region A to C is ascribed to the abrupt increase in v_f because of the decrease in η in Eq. (2) rather than the increase in n_f . Indeed, the calculated $U(r_c) = Kd \ln(r_c/\xi) = Kd \ln(KS/\xi\phi_0 I)$ with $\xi = 12.0$ nm and $\lambda = 205$ nm [16,18] is 5–15 meV at I_{c1} , much larger than $k_B T$, meaning that the thermally activated process Eq. (1) holds at least around I_{c1} . Thus, the phase slip line with kinematic vortices likely occurs in region C.

To check this scenario, we directly calculated v_f from the data by using the relation

$$v_f = \frac{V/L}{n_f \phi_0} = \frac{V/L}{CI^{\alpha-1}\phi_0}, \quad (3)$$

assuming that Eq. (1) holds before and after the transition at I_{c1} . Here, C and α are obtained by fitting the linear part of the $\ln I$ - $\ln V$ data in the low-current region below I_{c1} (typically around 10–40 μ A) in Fig. 5(a) with Eq. (2) and $\eta = \eta_0 = \frac{\phi_0^2 d}{2\pi\xi^2 R_N}$ according to the Bardeen-Stephen law. Figure 5(b) shows v_f as a function of I between 4.5 and 5.8 K, where the black triangles in the figure display the position of I_{c1} . As can be seen in Fig. 5(b), $v_f \approx 10^3$ m/s below I_{c1} , which is the typical velocity of the Abrikosov vortex [61], abruptly increases to $v_f \approx 10^5$ – 10^6 m/s above I_{c1} and below 5 K. This value is indeed consistent with the picture of kinematic vortices previously reported [51]. We note that the decrease in v_f with further increasing current is unphysical. This may originate from the failure of the assumption of Eq. (1) in region C because of the interaction between the kinematic vortices, which limits the actual value of n_f and thus leads to underestimating v_f at the high-current region. With further increasing current in region C, the order parameter of the phase slip line decreases and becomes vanishingly small. Once the current increases over the I_{c3} , the flux-flow state becomes partially normal and then the system goes to the fluctuation state (region E; $T_2 < T < T_3$, $I > I_{c3}$). The peak of $d \ln V / d \ln I$ at I_{c3} may correspond to disappearance of the vortex picture. It is noted that in the geometry of EDLT, the gate electric field is not applied under electrodes and thus those regions are insulating at low temperatures, which might lead to the focused current flow between the Hall electrodes. Such an inhomogeneous current flow might assist the formation of the phase slip line and should be taken into account, as discussed in the previous work on NbSe₂ [52]. To confirm such inhomogeneity of the supercurrent and clarify the microscopic origin of the phase slip line there needs to be more direct probe such as a nano superconducting quantum interference device (nanoSQUID)-on-tip and thus it is left for future study.

In the dynamical phase diagram of Fig. 4, the appearance of region D below the boundary of I_{c2} and T_1 at low temperature lower than 3.5 K is also quite unexpected and its explanation remains an open question. It has been recognized that there are two types of the dissipative states due to fast vortex motion; one is the phase slip line with kinematic vortices as discussed above and the other is the vortex street with the fast flow of Abrikosov vortices, which is fast as compared to the homogeneous flux flow, but not as fast as the kinematic vortices [46,49]. Recently, the observation of the fast vortex

flow channels, where the vortices keep the nature of the Abrikosov vortices, has been reported through the scanning nano-SQUID imaging technique [41]. Although our experiments were performed at zero magnetic field, a similar state may occur in region D. As another candidate, the appearance of multiple phase slip lines might be possible. In this case, I_{c1} and I_{c2} , which correspond to the onset of the first and the second phase slip line, respectively, should be observed separately for all the temperature regions [52], which may not be the case in our experiments.

It is emphasized that we can rule out the possibility of the spatial nonuniformity of the sample as the origin of the multiple peaks in $d \ln V / d \ln I$ and dR_{NL} / dT because those I_{c1} , I_{c2} , and I_{c3} (or T_1 and T_2) show different temperature (or current) dependence with each other. Especially, the T_1 line (or I_{c2} line) is merged into the I_{c1} line at high temperature in Fig. 4, which results in the limited observation of steplike features in $R_{NL}(T)$ in Fig. 2 only between 35 and 65 μA with the absence in the smaller-current region. In addition, even the I_{c1} line is connected to the T_2 line near T_{BKT} . In the case of inhomogeneous superconductors, such anomalies should appear rather in a wide range of temperature and current with scaled behavior. Furthermore, we found that in the normalized plot (Fig. S2 in the Supplemental Material [59]), I_{c3} (T_2) deviates from I_{c1} or I_{c2} (T_1) except for the temperature region near T_{c0} . This implies that I_{c3} (T_2) at low temperature has a different origin from I_{c1} and I_{c2} (T_1) at least. It is unexpected that T_3 shows a similar scaled curve to I_{c1} and I_{c2} (T_1). Although the origin of their similarity is unclear, this behavior is far from that for the depairing current scaled as $(1 - T/T_{c0})^{2/3}$ with a positive curvature. We also note that the temperature dependence of the normalized I_{c1} or I_{c2} is very similar to that of the normalized critical current in 2D NbSe₂ [52] with a negative curvature, which may be a common property of the characteristic current for the phase slip line in the clean superconductors, while the I_{c1} at the high temperature shows a more gradual slope than the normalized critical currents of 2D NbSe₂.

In the present study the Joule heating is fairly excluded because the dissipation power $P = IV$ is on the order of 1–100 nW (Fig. 6), which is small by a factor of more than 10^3 compared to the previous studies [30,62]. In addition, the temperature dependence of P at I_{c1} and I_{c3} in Fig. 6 is inconsistent with the Joule heating model, because P should decrease with increasing temperature and goes to zero with approaching T_c .

B. Magnetic field-current-temperature phase diagram

Finally, we discuss a B - I - T phase diagram of gate-induced superconductivity in MoS₂ which represents single-crystal-based clean 2D superconductors with weak pinning potentials. Figure 7 shows a schematic global phase diagram based on Fig. 4, incorporating the B - T phase diagram at zero-current limit [16] and the B - I nonequilibrium phase diagram at low T [21]. The zero resistance in the low-current limit is achieved by the BKT state at zero magnetic field, and pinned vortex states at small magnetic field (ZR in Fig. 7). When the magnetic field is increased below T_{BKT} at low current, there appears a “quantum metallic state” with T -independent

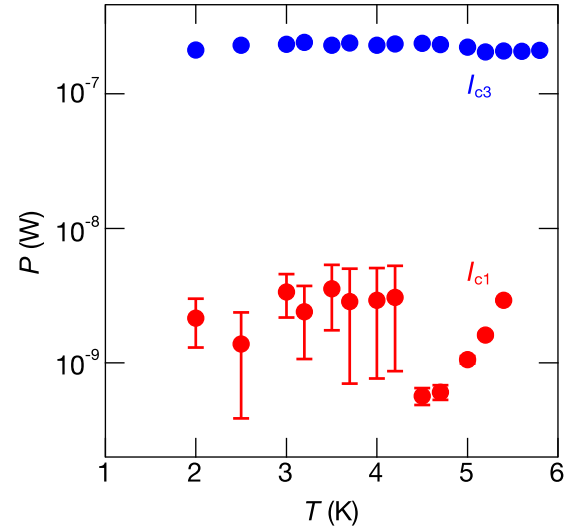


FIG. 6. Dissipated power. Temperature dependence of the dissipated power at I_{c1} and I_{c3} calculated by $P = IV$, where I is I_{c1} (red) or I_{c3} (blue), and V is the value at I_{c1} or I_{c3} at each temperature. The vertical error bars of P at I_{c1} represent uncertainties in determining the voltage at I_{c1} from the measured I - V characteristic.

resistance as reported in Refs. [10,11,16], which corresponds to a vortex liquid state as a ground state. In contrast to the conventional vortex liquid in quasi-2D bulk superconductors [63–66], the liquid state in the present system survives down to $T = 0$, and consequently, the zero-resistance state under magnetic field (vortex solid state) is very fragile. The origin of this state is still in debate [12] but the most plausible model for the finite Ohmic resistance, in other words, the energy dissipation, is the quantum collective creep of vortices due to the quantum fluctuation and weak pinning [10,16]. This quantum metallic state crosses over to the thermally activated creep state at higher-temperature regions, and to the free vortex flow state at higher magnetic field.

Another notable state is seen in the high magnetic field and low temperature region at the zero-current limit. Just before the superconductivity is completely destroyed by the application of magnetic field passing through the vortex flow state, the system becomes highly nonuniform, where the superconducting puddles (rare region) exist in the sea of the normal state with a rather long timescale and finite length scale due to the quantum fluctuation. This is called the quantum Griffiths state [13,16], where the slight residual disorder plays a central role. It is interesting that a phenomenon controlled by disorder appears in relatively clean systems, not in the conventional metallic thin film superconductors full of disorder. This is possibly because subtle phases like the quantum Griffiths state are masked in highly disordered systems.

In the B - I plane at the lowest temperature, the zero-resistance state and the quantum metallic state in a magnetic field are rapidly suppressed with increasing the current [21]. In particular, MoS₂ has an in-plane broken inversion symmetry, and therefore the nonreciprocal response is observed under out-of-plane magnetic field. This allows us to distinguish the quantum metallic state for $B < B_1$ and the classical vortex flow (or plastic flow) for $B_1 < B < B_2$ due to the vortex

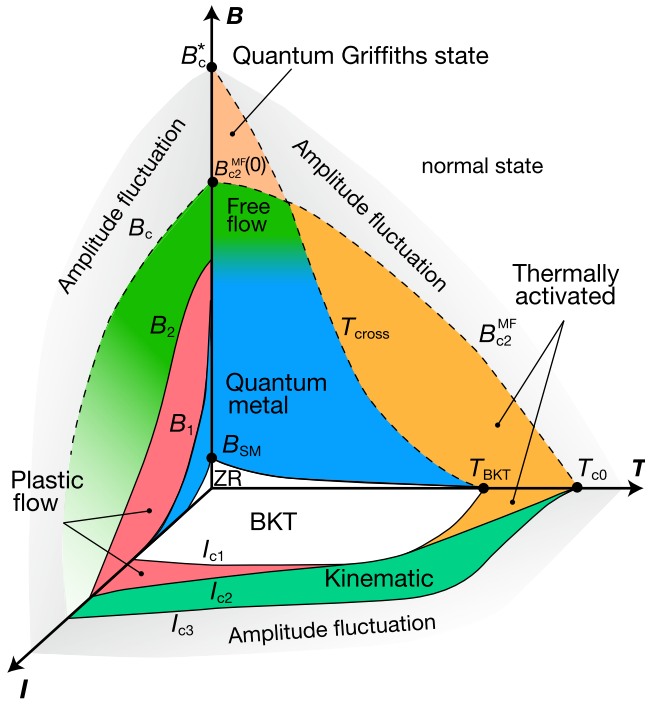


FIG. 7. Comprehensive B - I - T phase diagram in single-crystal-based clean 2D superconductors with weak pinning. The whole phase diagram is based on the results in Fig. 4 and previous works in Refs. [16,21]. In the B - T plane, white, blue, yellow, green, and orange regions show the zero resistance (ZR), the quantum metal, the thermally activated vortex creep, the vortex free flow, and the quantum Griffiths states, respectively. Here, T_{c0} is the transition temperature determined by the thermal fluctuation theories (Aslamazov-Larkin and Maki-Thompson models), and T_{BKT} is the BKT transition temperature. T_{cross} is the crossover temperature between the thermal activated vortex creep regime and the quantum creep regime, which are determined by the activation plot. B_{SM} is the hypothetical transition magnetic field from the ZR state to the quantum metal (vortex liquid). B_{c2}^{MF} is the mean-field upper critical field. With increasing magnetic field, the system goes to the quantum Griffiths state up to the characteristic critical magnetic field B_c^* through the free flow state (unpinned vortex). In the B - I plane, B_1 and B_2 are low- and high-threshold magnetic fields for nonreciprocal response [21], and B_c is the crossover magnetic field between free flow states and the fluctuation regime. In the I - T plane, white, yellow, green, and red regions correspond to A, B, C, and D in Fig. 4, respectively. A is the BKT state, and B, C, and D are attributed to the thermally activated V-AV, the kinematic vortex states, and the vortex street (plastic flow), respectively. Here, I_{c1} , I_{c2} , and I_{c3} are determined in Fig. 3. The outermost region of these states is governed by the amplitude fluctuation.

ratchet effect. The two boundaries B_1 and B_2 seemingly merge in the zero-current limit and thus, the quantum metallic state is dominating in the thermally equilibrium B - T plane [21].

The main subject of the present work was on the I - T plane, which can be reasonably connected to the B - T and the B - I plane to form a global phase diagram as shown in Fig. 7. The dynamics of rapidly moving vortices is attracting recent interest [31], since it is related to general questions on the stability of the topological defects. In this context, it is interesting to revisit the stability of the BKT state with

current-induced V-AV. Researchers have shown that the BKT state is destroyed and the fluctuation region or normal state is immediately recovered in conventional 2D metallic thin film superconductors. However, the present results on gated MoS₂ revealed that the transition from the BKT state to the normal or fluctuation state is much more complex, exhibiting at least three intermediate dynamical states, B, C, and D, in Fig. 4. We propose interpretations of the B and C states in terms of the thermally dissociated V-AV pairs and the kinematic vortex state with the phase slip line, respectively. D might be understood by the vortex street (which may develop into the plastic flow in a magnetic field).

IV. CONCLUSION

In conclusion, we investigated the dynamical behavior of vortices and antivortices at zero magnetic field in a 2D superconductor, ion-gated MoS₂. After confirming the BKT transition by the universal jump of the power α in the I - V characteristics ($V \propto I^\alpha$) at T_{BKT} , we established a dynamical phase diagram on the I - T plane. We proposed a scenario of the occurrence of ultrafast flow of vortices, which may be regarded as the kinematic vortices for a dynamic state based on the flux-flow instability of the current driven vortices and antivortices. Furthermore, we present a comprehensive phase diagram including nonequilibrium states on the B - I - T space. This phase diagram offers a global view on how the superconducting (BKT) state evolves to the normal state through multiple intermediate states by B , I , and T , which have never been observed in conventional disordered 2D superconductors described by the dirty boson picture [25]. We suggest that the unexpected richness of the dynamical vortex diagram is attributed to the combination of the weak pinning force, the small resistance, and enhanced fluctuations at the 2D limit.

Nonlinear properties of superconductivity are rarely observed due to Joule heating of samples and poor heat transfer between samples and substrates. This leads to a sharp jump from the BKT state directly to the normal state in a current-voltage characteristic without any intermediate states (i.e., the thermal instability). In marked contrast, the ionic gating induces superconductivity on a crystal surface, and thus the rest of the crystal may serve as a good heat bath, not only reducing thermal instability, but also making the dynamic properties at high currents accessible. The present results highlight the uniqueness and importance of the gate-induced 2D superconductivity, offering a challenge to construct a theoretical scheme for clean 2D superconductors.

V. OUTLOOK

Recently emerging 2D crystalline superconductors have been offering various opportunities for exploring transport properties reflecting intrinsic quantum phases and nonequilibrium vortex dynamics as well as crystal symmetry driven phenomena, which have been not found in disordered 2D superconductors. One possible direction for future discoveries is to investigate the real-space microscopic states by local microscopy such as scanning tunneling microscope (STM), angle-resolved photoemission spectroscopy (ARPES), and nanoSQUID as exemplified by the previous study [41].

Another direction is various applications, for example, for the superconducting transistors and superconducting quantum bit. In any case, further developments for the fabrication and characterization of 2D crystalline superconductors would be in high demand.

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Y.S., T.N., and Y.I. conceived the idea and wrote the manuscript. Y.S. and Y.M.I. fabricated the devices and analyzed the data. Y.S. designed the experiments and conducted the transport measurements. All authors discussed the results.

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