

Altermagnetism in Quasicrystals

Rui Chen,¹ Bin Zhou¹, and Dong-Hui Xu^{2,3,*}

¹*Department of Physics, Hubei University, Wuhan 430062, China*

²*Department of Physics and Chongqing Key Laboratory for Strongly Coupled Physics, Chongqing University, Chongqing 400044, China*

³*Center of Quantum Materials and Devices, Chongqing University, Chongqing 400044, China*



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Altermagnets are a recently discovered class of magnetic materials that combine a collinear, zero-magnetization spin structure, characteristic of antiferromagnets, with spin-split electronic bands, a hallmark of ferromagnets. This unique behavior arises from the breaking of combined time-reversal and spatial symmetries (such as inversion or lattice translation), which are preserved in conventional antiferromagnets. To date, research has mainly focused on altermagnetic phases in periodic crystals, where the order is linked to rotational symmetries compatible with translational periodicity. In this Letter, we demonstrate that quasicrystals, which possess rotational symmetries incompatible with periodicity, can host exotic altermagnetic orders. Using symmetry analysis and self-consistent mean-field theory, we predict stable *g*-wave and *i*-wave altermagnetism in octagonal and dodecagonal quasicrystals, respectively. These phases are characterized by global C_8T and $C_{12}T$ symmetries and exhibit anisotropic spin-splittings in their spectral functions and spin conductance, with characteristic eight and twelvefold nodal structures that establish a theoretical framework for identifying these phases in future experiments. Our findings establish quasicrystals as a versatile platform for realizing unconventional altermagnetic orders beyond the constraints of periodicity.

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Introduction—Altermagnets represent a recently discovered class of magnetic materials, distinct from conventional ferromagnets and antiferromagnets [1–11]. They uniquely combine a collinear spin structure with zero net magnetization, characteristic of antiferromagnets, with spin-split electronic bands, a hallmark of ferromagnets. This unconventional behavior is rooted in the breaking of composite symmetries—specifically, time-reversal combined with spatial operations like inversion or translation—which remain intact in conventional antiferromagnets. It enables unique physical phenomena such as anomalous transport [12–16], giant tunneling magnetoresistance [17,18], and potential topological responses [19–22], thereby prompting widespread research both theoretically and experimentally [23–29]. To date, however, the exploration of altermagnetism has been almost exclusively confined to periodic crystals. Within this paradigm, altermagnetic phases are protected by specific crystal rotational symmetries (e.g., fourfold or sixfold). This reliance on crystallographic symmetry fundamentally forbids altermagnets with higher-order or rotational symmetries incompatible with periodicity. This limitation necessitates the search for alternative platforms beyond periodic crystals to realize more exotic altermagnetic states.

Concurrently, a second, independent paradigm shift has transformed our understanding of aperiodic systems. Quasicrystals were considered textbook examples of geometrically frustrated structures [30,31], presumed to support only disordered, spin-glass-like magnetic states [32–34]. However, subsequent theoretical studies ruled out the notion that geometrical frustration fundamentally forbids long-range magnetic order in quasicrystals [35–38]. Experimental realization of such phases remained elusive for decades. The subsequent observation of long-range ferromagnetic order [39,40] and, most strikingly, long-range antiferromagnetic order in a true icosahedral quasicrystal [41,42] has demonstrated that the complex aperiodic order of quasicrystals can support coherent magnetic ground states. These breakthroughs position quasicrystals as a fertile platform for discovering unconventional magnetism.

This Letter unites these two revolutionary frontiers by showing that the rotational symmetries of quasicrystals, which are incompatible with periodicity, can enable and stabilize forms of altermagnetic order unavailable in periodic crystals. This mechanism is distinct from recent proposals of altermagnetism in amorphous solids [43], where rotational invariance is statistical rather than an exact long-range symmetry of a deterministic quasiperiodic structure. In the quasicrystalline case studied here, the altermagnetic order is protected by magnetic rotational symmetries tied to the quasiperiodic long-range order itself.

*Contact author: donghuixu@cqu.edu.cn

Here we show that this principle enables altermagnetic phases with symmetry structures inaccessible in periodic crystals. Specifically, the Ammann-Beenker (AB) tiling, a canonical octagonal quasicrystal, supports a g -wave altermagnetism protected by a composite eightfold rotational symmetry C_8T , where T is time-reversal symmetry. Our analysis of the Fourier spectra reveals a momentum-dependent spin structure with a nodal g -wave configuration. We propose that this unique order can be detected via double-tip scanning tunneling microscopy (STM) [44–48] and show that transport measurements would exhibit a spin-polarized current whose anisotropy directly reflects the underlying g -wave symmetry. Furthermore, we extend our analysis to show that an i -wave altermagnetic phase can emerge in the Stampfli-tiling dodecagonal quasicrystal. Our findings establish a previously uncharted direction in magnetism, demonstrating that the rich symmetries of quasicrystals offer a new route to designing and discovering altermagnetic orders beyond periodic systems.

Model—We adopt a minimal t - J -like model [43] for the orbital altermagnet phase of an AB-tiling quasicrystal [Fig. 1(a)]. The AB-tiling quasicrystal consists of two types of primitive tiles: square tiles (red) and rhombus tiles (cyan) with an acute angle of 45° . Each site contains two orbitals connected by a C_8 rotation [Fig. 1(b)]. A concrete realization of this setup is achieved with the $\{d_{xy}, d_{x^2-y^2}\}$ orbitals. They form a symmetry-protected degenerate doublet belonging to the E_2 irreducible representation in the C_{8v} point symmetry of the Ammann-Beenker tiling [49,50]. The total Hamiltonian, $H = H_K + H_{\text{Int}}$, consists of a kinetic term and an interaction term. The kinetic term

$$H_K = \sum_{\langle jk \rangle} \Psi_j^\dagger [F(\eta\theta_{jk})] \sigma_0 \Psi_k \quad (1)$$

describes hopping between adjacent sites. Here, $\langle \dots \rangle$ denotes sites connected by the bonds of distance $a = 1$, as shown in Fig. 1(a). $\Psi_j^\dagger = (c_{j\alpha\uparrow}^\dagger, c_{j\beta\uparrow}^\dagger, c_{j\alpha\downarrow}^\dagger, c_{j\beta\downarrow}^\dagger)$ are electron creation operators at site j . α and β represent the two orbitals at one site, and spin degrees of freedom are denoted as $\uparrow$ and $\downarrow$. θ_{jk} is the polar angle of bond connecting sites j and k with respect to the horizontal direction. The orbital-dependent hopping matrix is given by

$$F(\eta\theta) = \begin{pmatrix} (t_1 - t_2)\cos^2\eta\theta + t_2 & (t_1 - t_2)\sin\eta\theta\cos\eta\theta \\ (t_1 - t_2)\sin\eta\theta\cos\eta\theta & (t_1 - t_2)\sin^2\eta\theta + t_2 \end{pmatrix}. \quad (2)$$

We fix $t_1 = 1$, $t_2 = 1/2$, and $\eta = 2$ in the case of the AB-tiling quasicrystal. At $\theta = 0$, the hopping is dominated by the overlap between the first orbitals t_1 , while the second orbitals have a smaller overlap t_2 , with $t_2 < t_1$. At $\theta = \pi/4$, these hopping strengths are reversed. This angular dependence captures the rotating orbital texture illustrated in

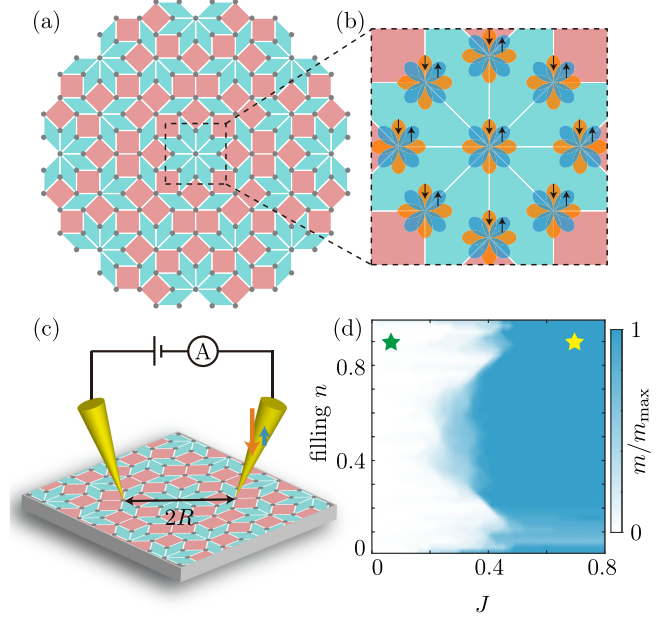


FIG. 1. (a) Schematic illustration of the AB-tiling octagonal quasicrystal, which consists of two types of primitive tiles: square tiles (red) and rhombus tiles (cyan) with an acute angle of 45° . (b) Each site of the AB-tiling quasicrystal features two orbitals (blue and orange) connected by a C_8 rotation. (c) We propose that the altermagnetic order in a quasicrystal can be probed by a double-tip STM setup. When a spin-unpolarized current is injected, the output current becomes spin-polarized due to the altermagnetic structure. (d) Normalized mean altermagnetization m/m_{max} as a function of the filling n and the interaction coupling strength J . Each data point is obtained by using the self-consistent mean-field method.

Fig. 1(b), where the orbital character naturally twists along different bond directions due to the underlying C_8 rotational symmetry.

We include a ferromagnetic Heisenberg-like combined spin-orbital interaction [43,51],

$$H_{\text{Int}} = -J \sum_{\langle jk \rangle, \zeta\zeta'} \left(\Psi_j^\dagger \tau^\zeta \sigma^\zeta \Psi_j \right) \left(\Psi_k^\dagger \tau^{\zeta'} \sigma^{\zeta'} \Psi_k \right) - n_j n_k, \quad (3)$$

where $n_j = \sum_{\mu,s} c_{j\mu s}^\dagger c_{j\mu s}$ is the total on-site occupation operator, with $s = \uparrow, \downarrow$ labeling spin and $\mu = \alpha, \beta$ labeling orbital. σ^ζ (τ^ζ) are Pauli matrices acting on the spin (orbital) subspace. In the single-fermion-per-site limit, Eq. (3) resembles a ferromagnetic Kugel-Khomskii interaction, expressed as $(\mathbf{S}_j \cdot \mathbf{S}_k)(\boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k)$ [52]. In Sec. SI of Supplemental Material [53], we provide a detailed group-theoretical derivation of the orbital selection and Slater-Koster hopping textures and discuss possible experimental directions. Mn-based octagonal quasicrystals and approximants [54] provide useful motivation for octagonal quasiperiodic environments, although the Ammann-Beenker model used here should be viewed as a minimal symmetry-based model rather than a material-specific description of Mn-Si-Al.

Mean-field decoupling and emergent altermagnetism—The interaction term is decoupled via a mean-field approximation, yielding an effective single-particle Hamiltonian,

$$H_{\text{Int}} = -J \sum_{\langle jk \rangle} \langle m_j \rangle m_k - \langle n_j \rangle n_k + (j \leftrightarrow k), \quad (4)$$

where n_j is the total occupation and $m_j = \Psi_j^\dagger \sigma^z \tau^z \Psi_j$ denotes the local magnetization. We assume that spontaneous symmetry breaking occurs along the σ^z and orbital τ^z directions, retaining only the diagonal components relevant to the symmetry-broken state. At each iteration, we compute site-resolved expectation values for the local charge density $\langle n_j \rangle$ and magnetization $\langle m_j \rangle$. These expectation values are then used to update the mean-field Hamiltonian for the next iteration. This self-consistent procedure is repeated until convergence is achieved (typically within 200 iterations) and allows for spatially inhomogeneous ordering without presupposing translational symmetry.

This interaction term in Eq. (4) lifts the degeneracy among the four spin-orbital configurations. For a positive m_j , the states $|\alpha\uparrow\rangle$ and $|\beta\downarrow\rangle$ on the j th site have lower energy, whereas $|\beta\uparrow\rangle$ and $|\alpha\downarrow\rangle$ have higher energy. Consequently, the spin and orbital degrees of freedom become locked, with electrons in the α -orbital predominantly carrying spin-up and those in the β -orbital carrying spin-down [see Fig. 1(b)]. This establishes the altermagnetic phase in quasicrystals. Although the magnetic term m_j individually breaks the eightfold rotation symmetry $C_8 = \tau_y \sigma_0 \mathcal{R}_8$ and time-reversal symmetry $T = i\tau_0 \sigma_y \mathcal{K}$, their combination $C_8 T$ remains a preserved symmetry of the system. Here, $\mathcal{R}_8$ is the orthogonal matrix that permutes the tiling sites to rotate the whole system by $\pi/4$, and $\mathcal{K}$ is the complex conjugation. This composite symmetry plays a central role in characterizing the altermagnetic phase.

We emphasize that the full Hamiltonian lacks global spin-rotational symmetry due to the locking of the spin degrees of freedom to the fixed, geometry-dependent orbital textures of the quasicrystalline lattice. Consequently, the total energy of the system is sensitive to the specific direction of the spin and orbital polarization. In Sec. SIV of Supplemental Material [53], we performed fully unrestricted self-consistent calculations and found that the altermagnetic state further breaks spin-rotational symmetry, with spin polarization stabilized along the principal axes (x , y , or z) concomitant with orbital polarization (τ_z). This specific alignment allows the system to preserve the combined $C_8 T$ symmetry, where the spatial rotation C_8 acts on the lattice and the time-reversal operation T acts on the spins, which protects the characteristic g -wave spectral splitting.

Interaction-induced altermagnetic ordering—Figure 1(d) presents the averaged magnetization normalized by its maximal value, m/m_{max} , as a function of the filling n and the interaction coupling strength J . The average magnetization is defined as $m = \sum_j \langle m_j \rangle / N$, where $N = 833$ is the

total number of lattice sites. The normalization factor $m_{\text{max}} = 2 - |4n - 2|$ represents the theoretical upper bound derived from the occupation constraints [43]. For small J , the system settles into a trivial metallic state, as the local altermagnetization $\langle m_j \rangle$ vanishes during the self-consistent calculation. For larger J , a stable altermagnetic phase emerges with a finite order parameter m , which approaches its theoretical maximum $m/m_{\text{max}} \rightarrow 1$ in the strong-coupling regime.

Spectral function—Direct visualization of spin-resolved momentum-space textures is experimentally challenging but has become increasingly feasible, even for complex materials like quasicrystals, due to the development of high-efficiency, multichannel spin detectors [66–68]. Although quasicrystals lack discrete translational symmetry, meaning crystal momentum is not a good quantum number, their momentum-space properties can still be probed via Fourier analysis. Specifically, one can project eigenstates onto plane-wave-like trial states of the form $|\mathbf{p}s\mu\rangle = N \sum_{\mathbf{x}_j} e^{i\mathbf{p}\cdot\mathbf{x}_j} |\mathbf{x}_j s \mu\rangle$. These trial states serve as Fourier components of the real-space basis, representing spatial modulations with wave vector $\mathbf{p}$. This Fourier-based projection thus provides a practical framework for analyzing the momentum-space structure of quasicrystalline systems in the absence of a Brillouin zone [69,70]. We therefore compute the spin-resolved spectral function [43]

$$\mathcal{A}_s(\mathbf{p}) = -\frac{\delta}{\pi} \sum_{\lambda\mu} \frac{|\langle \lambda | \mathbf{p} s \mu \rangle|^2}{(\epsilon_F - \epsilon_\lambda)^2 + \delta^2}, \quad (5)$$

where $\delta = 10^{-2}$ is a small broadening factor and ϵ_F is the Fermi level corresponding to the filling number n . This quantity is directly accessible in spin-resolved angle-resolved photoemission spectroscopy (ARPES) measurements on quasicrystalline samples [71,72].

Figures 2(a) and 2(b) show the numerically calculated spectra with $J = 0.05$ [marked by the green star in Fig. 1(d)]. For this weak coupling, the system remains a trivial metal, and the spin-up and spin-down spectral functions are degenerate. The spectrum exhibits an eightfold rotational symmetry, reflecting the underlying symmetry of the quasicrystal lattice. The spectrum also displays fine hierarchical structures characteristic of quasiperiodic electronic spectra [73,74]. In the following, we focus on the symmetry of the spin-resolved spectral weight, which provides the direct diagnostic of the altermagnetic phase.

Figures 2(c) and 2(d) show the spectra for a strong interaction coupling of $J = 0.7$ [marked by the yellow star in Fig. 1(d)]. In this regime, the system enters an altermagnetic phase, characterized by a finite self-consistent altermagnetic order. This results in a clear spin splitting in the spectral functions, accompanied by the breaking of C_8 symmetry while preserving a residual C_4 symmetry. Crucially, the spin-up and spin-down spectral functions are related by a C_8 rotation due to the preserved $C_8 T$ symmetry

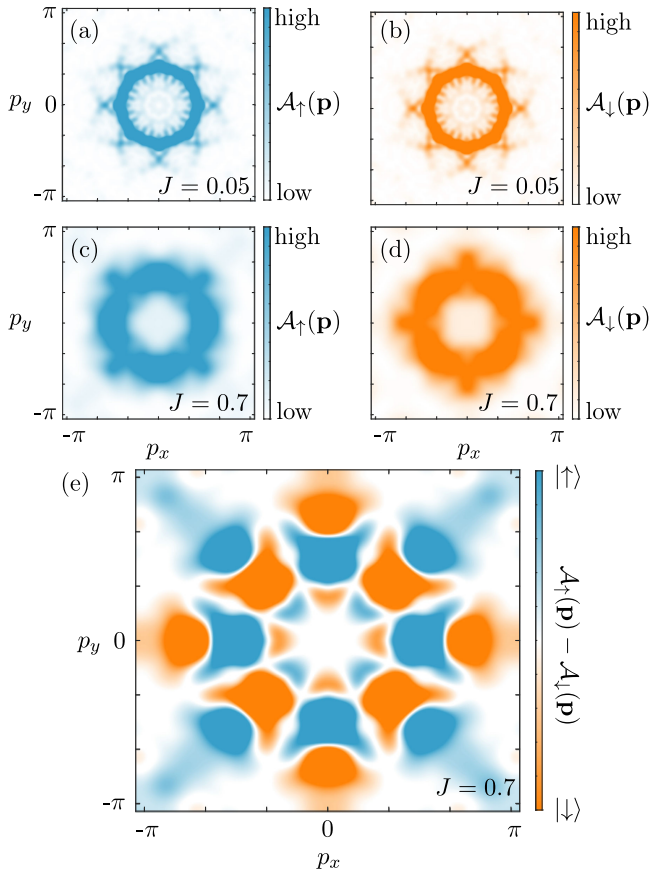


FIG. 2. Spectral density $\mathcal{A}(\mathbf{p})$ as a function of momentum $\mathbf{p}$. (a), (c) Spin-up spectral density $\mathcal{A}_\uparrow(\mathbf{p})$. (b), (d) Spin-down spectral density $\mathcal{A}_\downarrow(\mathbf{p})$. (e) Spin-resolved difference $\mathcal{A}_\uparrow(\mathbf{p}) - \mathcal{A}_\downarrow(\mathbf{p})$. We take the interaction coupling $J = 0.05$ in (a) and (b) and $J = 0.7$ in (c)–(e). The electron filling is fixed at $n = 0.9$ in all panels.

of the whole system. Consequently, the difference $\mathcal{A}_\uparrow(\mathbf{p}) - \mathcal{A}_\downarrow(\mathbf{p})$ transforms antisymmetrically under a C_8 rotation, meaning it changes sign and must vanish along the nodal line defined by $\theta = \pi/8 + n\pi/4$.

Spin-conductance—Another distinguishing experimental feature of altermagnetism is its unique spin transport properties [12–18]. We employ the two-terminal device to investigate the transport properties of the quasicrystalline system, where each lead is modeled as a tunable STM tip. One tip is positioned at $R(\cos \phi, \sin \phi)$, and the other is symmetrically located at $-R(\cos \phi, \sin \phi)$. By varying the probe separation R and angle ϕ , the setup allows for angle-resolved probing of the transport anisotropy.

Specifically, each STM tip is modeled as a one-dimensional chain with the Hamiltonian $H_{\text{tip}} = \sum_{\langle jk \rangle} \Phi_j^\dagger t \tau_0 \sigma_0 \Phi_k$, where Φ_j denotes the spinor at site j in the lead and $t = 1$. The coupling between the tip and the device is described by the tunneling Hamiltonian $H' = \sum_{jk} \Psi_j^\dagger t e^{-\mathbf{r}_{jk}} \tau_0 \sigma_0 \Phi_k + \text{H.c.}$, where Ψ_j denotes the spinor on the device [see Eq. (1)] and $\mathbf{r}_{jk}$ is the distance between the device site j and the lead site k . The exponential factor ensures that the coupling is

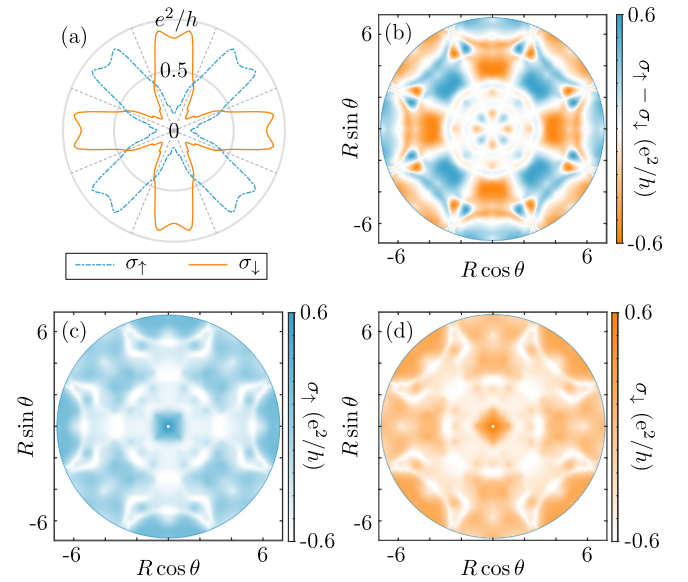


FIG. 3. (a) Spin conductances $\sigma_\uparrow$ and $\sigma_\downarrow$ as functions of the angle ϕ with $R = 4$. (b) $\sigma_\uparrow - \sigma_\downarrow$ as functions of the angle ϕ and the radius R . (c), (d) $\sigma_\uparrow$ and $\sigma_\downarrow$ as functions of the angle ϕ and the radius R . We take $J = 0.7$ and $n = 0.9$ in all panels.

spatially localized, restricting the tunneling to sites in close proximity to the tip position. The conductance between the two tips is calculated by using the Landauer-Büttiker formula [75–77] and the recursive Green’s function method [78,79].

Figure 3(a) shows the spin conductance $\sigma_{\uparrow,\downarrow}$ as functions of angle ϕ for $R = 4$. The spin-dependent conductance exhibits strong anisotropy, a phenomenon more clearly visualized in Figs. 3(c) and 3(d), where we plot $\sigma_{\uparrow,\downarrow}$ as functions of $R \cos \theta$ and $R \sin \theta$. Similar to the spectral function, the conductance for each spin component displays a clear C_4 symmetry. However, a C_8 rotation transforms the spin-up conductance into the spin-down conductance. As a result, the difference $\sigma_\uparrow - \sigma_\downarrow$ changes sign under an eightfold rotation and must vanish along the nodal line $\theta = \pi/8 + n\pi/4$ [see Fig. 3(b)]. These transport signatures further confirm that the system realizes a g -wave altermagnet. While experimentally demanding, multiprobe STM techniques provide a unique and powerful route to probe nonlocal transport anisotropies and correlation functions. Our calculations thus provide a clear theoretical target and a strong motivation for such advanced measurements, where the predicted symmetry of the spin-conductance offers an unambiguous signature of the underlying quasicrystalline altermagnetic order.

Stampfli-tiling dodecagonal quasicrystal—We extend our analysis to the Stampfli-tiling quasicrystal [Fig. 4(a)], which features a global twelvefold rotational symmetry [71]. This tiling consists of three primitive tiles: squares (red), regular triangles (purple), and rhombuses (cyan) with an acute angle of 30° . The Hamiltonian is analogous to the AB-tiling case, but with $\eta = 3$ in Eqs. (1) and (2) to reflect

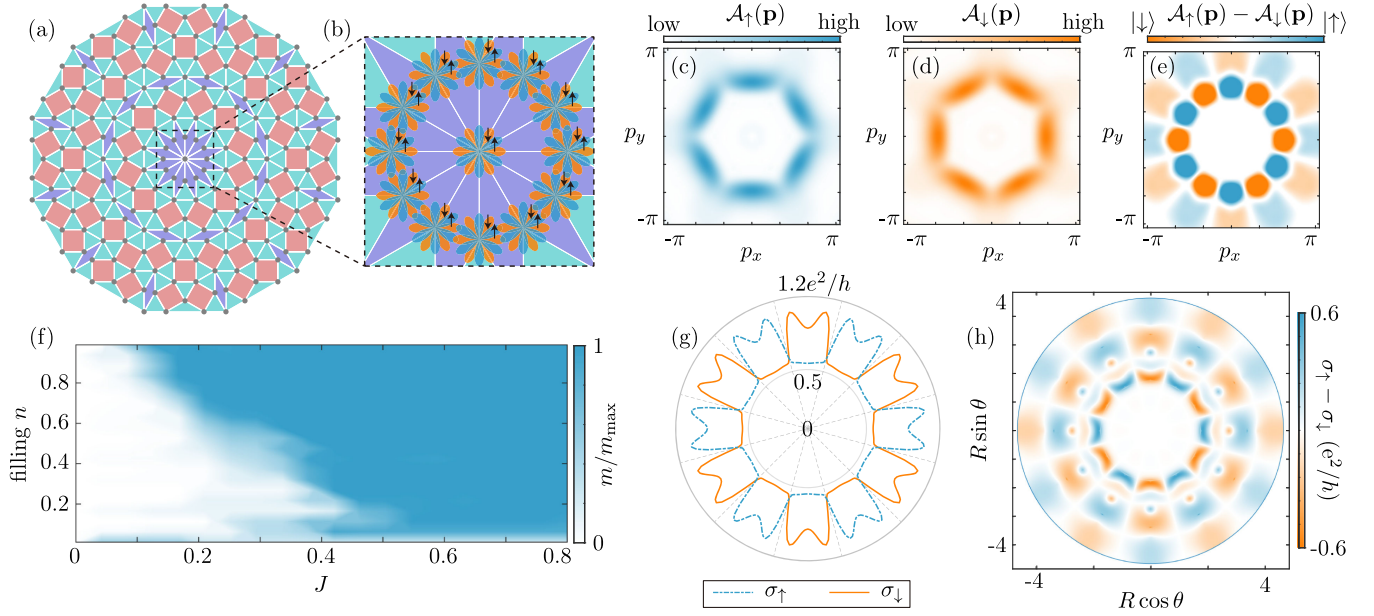


FIG. 4. (a) Schematic illustration of the Stampfli-tiling dodecagonal quasicrystals. (b) Each site of the Stampfli-tiling quasicrystal features two orbitals connected by a C_{12} rotation. (c)–(e) $A_{\uparrow}(\mathbf{p})$, $A_{\downarrow}(\mathbf{p})$, and $A_{\uparrow}(\mathbf{p}) - A_{\downarrow}(\mathbf{p})$, respectively. (f) Normalized mean altermagnetization $m/m_{\max}$ as a function of the filling n and the interaction coupling strength J . (g) Spin conductances $\sigma_{\uparrow}$ and $\sigma_{\downarrow}$ as functions of the angle ϕ with $R = 4$. (h) $\sigma_{\uparrow} - \sigma_{\downarrow}$ as functions of the angle ϕ and the radius R . The electron filling is fixed at $n = 0.9$ in (c)–(e), (g), and (h).

the C_{12} symmetry. This can be realized by choosing the $f_{x(x^2-3y^2)}$ and $f_{y(3x^2-y^2)}$ orbitals. Similarly, these two f orbitals form E_3 doublets in dodecagonal (C_{12v}) quasicrystals [49,50]. In this setup, for $\theta = 0$, hopping is dominated by the overlap t_1 between the first orbitals, and the second orbitals have a smaller overlap t_2 [Figs. 4(a) and 4(b)]. For $\theta = \pi/6$, the roles are reversed.

Figure 4(f) shows the normalized altermagnetization $m/m_{\max}$ as a function of filling n and interaction coupling strength J . We find that the altermagnetic phase in the Stampfli-tiling quasicrystal remains stable at large J , with m approaching its theoretical maximum. This behavior closely resembles our findings for the AB-tiling quasicrystal, indicating that the emergence of altermagnetism is a robust feature in quasicrystalline systems with higher-fold rotational symmetries.

Figures 4(c)–4(e), 4(g), and 4(h) present the spectral functions and transport properties of the Stampfli-tiling system, respectively. Because of the underlying C_{12} symmetry, the spin-up and spin-down components of the spectra and transport response are related by a twelvefold rotation. As a result, their differences, i.e., $A_{\uparrow}(\mathbf{p}) - A_{\downarrow}(\mathbf{p})$ and $\sigma_{\uparrow} - \sigma_{\downarrow}$, exhibit antisymmetric behavior under C_{12} rotations, changing sign upon a $\pi/6$ rotation. These signatures establish the presence of a $C_{12}T$ protected i -wave altermagnetic phase in this quasicrystal.

Conclusion and discussion—We have extended the concept of altermagnetism to quasicrystals, demonstrating that their unique higher-fold rotational symmetries,

incompatible with translational periodicity, can protect novel altermagnetic phases. By calculating symmetry-protected spectral textures and transport features, we have identified clear experimental signatures. The characteristic spectral textures are directly accessible via spin-resolved ARPES, and the anisotropic transport signatures can be probed using multitip STM.

We acknowledge that resolving these nodal structures via spin-ARPES or double-tip STM represents a significant challenge with current techniques applied to solid-state quasicrystals [12–18,66–68]. However, the identification of candidate materials like Au-In-Eu provides a clear motivation for future spectroscopic efforts. Furthermore, the predicted phases may be simulated in highly tunable platforms, such as optical quasicrystals for ultracold atoms [80,81] or moiré quasicrystals [82,83], where the specific orbital-dependent interactions can be engineered directly.

The proposed mechanism is experimentally motivated by two developments. First, transition-metal quasicrystals and approximants [54] provide quasiperiodic environments in which orbital doublets and direction-dependent hopping textures may arise. Second, the recent observation of long-range antiferromagnetic order in $\text{Au}_{56}\text{In}_{28.5}\text{Eu}_{15.5}$ [41] demonstrates that coherent magnetic order can be stabilized on quasiperiodic lattices. These results suggest that quasicrystalline magnets are promising systems in which to search for altermagnetic spin splitting, while a fully material-specific microscopic model remains a subject for future work.

Note added—During the preparation of this work, we became aware of two complementary studies on altermagnetism in quasicrystals [87,88], addressing hyperspatial-projective model construction and symmetry-based classification, respectively.

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Data availability—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

- [1] L. Šmejkal, J. Sinova, and T. Jungwirth, Beyond conventional ferromagnetism and antiferromagnetism: A phase with nonrelativistic spin and crystal rotation symmetry, *Phys. Rev. X* **12**, 031042 (2022).
- [2] L. Bai, W. Feng, S. Liu, L. Šmejkal, Y. Mokrousov, and Y. Yao, Altermagnetism: Exploring new frontiers in magnetism and spintronics, *Adv. Funct. Mater.* **34**, 2409327 (2024).
- [3] M. Naka, S. Hayami, H. Kusunose, Y. Yanagi, Y. Motome, and H. Seo, Spin current generation in organic antiferromagnets, *Nat. Commun.* **10**, 4305 (2019).
- [4] K.-H. Ahn, A. Hariki, K.-W. Lee, and J. Kuneš, Antiferromagnetism in RuO_2 as d -wave Pomeranchuk instability, *Phys. Rev. B* **99**, 184432 (2019).
- [5] S. Hayami, Y. Yanagi, and H. Kusunose, Momentum-dependent spin splitting by collinear antiferromagnetic ordering, *J. Phys. Soc. Jpn.* **88**, 123702 (2019).
- [6] L.-D. Yuan, Z. Wang, J.-W. Luo, E. I. Rashba, and A. Zunger, Giant momentum-dependent spin splitting in centrosymmetric low- z antiferromagnets, *Phys. Rev. B* **102**, 014422 (2020).
- [7] I. I. Mazin, K. Koepernik, M. D. Johannes, R. González-Hernández, and L. Šmejkal, Prediction of unconventional magnetism in doped FeSb_2 , *Proc. Natl. Acad. Sci. U.S.A.* **118**, e2108924118 (2021).
- [8] H.-Y. Ma, M. Hu, N. Li, J. Liu, W. Yao, J.-F. Jia, and J. Liu, Multifunctional antiferromagnetic materials with giant piezomagnetism and noncollinear spin current, *Nat. Commun.* **12**, 2846 (2021).
- [9] L. Šmejkal, J. Sinova, and T. Jungwirth, Emerging research landscape of altermagnetism, *Phys. Rev. X* **12**, 040501 (2022).
- [10] J. Krempaský, L. Šmejkal, S. D'souza, M. Hajlaoui, G. Springholz, K. Uhlířová *et al.*, Altermagnetic lifting of Kramers spin degeneracy, *Nature (London)* **626**, 517 (2024).
- [11] L. Šmejkal, A. H. MacDonald, J. Sinova, S. Nakatsuji, and T. Jungwirth, Anomalous Hall antiferromagnets, *Nat. Rev. Mater.* **7**, 482 (2022).
- [12] M. Naka, Y. Motome, and H. Seo, Perovskite as a spin current generator, *Phys. Rev. B* **103**, 125114 (2021).
- [13] D.-F. Shao, S.-H. Zhang, M. Li, C.-B. Eom, and E. Y. Tsymlal, Spin-neutral currents for spintronics, *Nat. Commun.* **12**, 7061 (2021).
- [14] R. M. Fernandes, V. S. de Carvalho, T. Birol, and R. G. Pereira, Topological transition from nodal to nodeless Zeeman splitting in altermagnets, *Phys. Rev. B* **109**, 024404 (2024).
- [15] R.-W. Zhang, C. Cui, R. Li, J. Duan, L. Li, Z.-M. Yu, and Y. Yao, Predictable gate-field control of spin in altermagnets with spin-layer coupling, *Phys. Rev. Lett.* **133**, 056401 (2024).
- [16] X. Zhou, W. Feng, R.-W. Zhang, L. Šmejkal, J. Sinova, Y. Mokrousov, and Y. Yao, Crystal thermal transport in altermagnetic RuO_2 , *Phys. Rev. Lett.* **132**, 056701 (2024).
- [17] H. Li, G. Wang, N. Ding, Q. Ren, G. Zhao, W. Lin *et al.*, Spectroscopic evidence of spin-state excitation in d -electron correlated semiconductor FeSb_2 , *Proc. Natl. Acad. Sci. U.S.A.* **121**, e2321193121 (2024).
- [18] L. Šmejkal, A. B. Hellenes, R. González-Hernández, J. Sinova, and T. Jungwirth, Giant and tunneling magnetoresistance in unconventional collinear antiferromagnets with nonrelativistic spin-momentum coupling, *Phys. Rev. X* **12**, 011028 (2022).
- [19] M. Ezawa, Detecting the Néel vector of altermagnets in heterostructures with a topological insulator and a crystalline valley-edge insulator, *Phys. Rev. B* **109**, 245306 (2024).
- [20] Y.-X. Li, Y. Liu, and C.-C. Liu, Creation and manipulation of higher-order topological states by altermagnets, *Phys. Rev. B* **109**, L201109 (2024).
- [21] R. Chen, X.-X. Yi, B. Zhou, and D.-H. Xu, Anomalous Hall effects in magnetic weak topological insulator films, *Phys. Rev. B* **111**, 045409 (2025).
- [22] R. Chen, Z.-M. Wang, K. Wu, H.-P. Sun, B. Zhou, R. Wang, and D.-H. Xu, Probing k -space alternating spin polarization via the anomalous Hall effect, *Phys. Rev. Lett.* **135**, 096602 (2025).
- [23] H. Bai, Y. C. Zhang, Y. J. Zhou, P. Chen, C. H. Wan, L. Han *et al.*, Efficient spin-to-charge conversion via altermagnetic spin splitting effect in antiferromagnet RuO_2 , *Phys. Rev. Lett.* **130**, 216701 (2023).
- [24] R. González-Hernández, L. Šmejkal, K. Výborný, Y. Yahagi, J. Sinova, T. c. v. Jungwirth, and J. Železný, Efficient electrical spin splitter based on nonrelativistic collinear antiferromagnetism, *Phys. Rev. Lett.* **126**, 127701 (2021).
- [25] L. Šmejkal, R. González-Hernández, T. Jungwirth, and J. Sinova, Crystal time-reversal symmetry breaking and spon-

- taneous Hall effect in collinear antiferromagnets, *Sci. Adv.* **6**, eaaz8809 (2020).
- [26] Z. Feng, X. Zhou, L. Smejkal, L. Wu, Z. Zhu, H. Guo *et al.*, An anomalous Hall effect in altermagnetic ruthenium dioxide, *National electronics review* **5**, 735 (2022).
- [27] O. Fedchenko, J. Minar, A. Akashdeep, S. W. DSouza, D. Vasilyev, O. Tkach *et al.*, Observation of time-reversal symmetry breaking in the band structure of altermagnetic RuO₂, *Sci. Adv.* **10**, eadj4883 (2024).
- [28] S. Karube, T. Tanaka, D. Sugawara, N. Kadoguchi, M. Kohda, and J. Nitta, Observation of spin-splitter torque in collinear antiferromagnetic RuO₂, *Phys. Rev. Lett.* **129**, 137201 (2022).
- [29] P. Keßler, L. Garcia-Gassull, A. Suter, T. Prokscha, Z. Salman, D. Khalyavin *et al.*, Absence of magnetic order in RuO₂: Insights from μ SR spectroscopy and neutron diffraction, *npj Spintronics* **2**, 50 (2024).
- [30] B. Grünbaum and G. C. Shephard, *Tilings and Patterns* (Courier Dover Publications, New York, 1987).
- [31] W. Steurer and S. Deloudi, Fascinating quasicrystals, *Acta Crystallogr. Sect. A Found. Crystallogr.* **64**, 1 (2007).
- [32] C. Berger and J. J. Prejean, Evidence of a spin-glass transition in the quasicrystal Al₇₃Mn₂₁Si₆, *Phys. Rev. Lett.* **64**, 1769 (1990).
- [33] A. I. Goldman and M. Widom, Quasicrystal structure and properties, *Annu. Rev. Phys. Chem.* **42**, 685 (1991).
- [34] A. I. Goldman, T. Kong, A. Kreyssig, A. Jesche, M. Ramazanoglu, K. W. Dennis, S. L. Bud'ko, and P. C. Canfield, A family of binary magnetic icosahedral quasicrystals based on rare earths and cadmium, *Nat. Mater.* **12**, 714 (2013).
- [35] R. Lifshitz, Symmetry of magnetically ordered quasicrystals, *Phys. Rev. Lett.* **80**, 2717 (1998).
- [36] R. Lifshitz, Magnetic point groups and space groups, in *Encyclopedia of Condensed Matter Physics* (Elsevier, New York, 2024), pp. 1–10.
- [37] R. Lifshitz, Theory of color symmetry for periodic and quasiperiodic crystals, *Rev. Mod. Phys.* **69**, 1181 (1997).
- [38] R. Lifshitz, Magnetic quasicrystals: What can we expect to see in their neutron diffraction data?, *Mater. Sci. Eng. A* **294–296**, 508 (2000).
- [39] R. Takeuchi, F. Labib, T. Tsugawa, Y. Akai, A. Ishikawa, S. Suzuki, T. Fujii, and R. Tamura, High phase-purity and composition-tunable ferromagnetic icosahedral quasicrystal, *Phys. Rev. Lett.* **130**, 176701 (2023).
- [40] R. Tamura, A. Ishikawa, S. Suzuki, T. Kotajima, Y. Tanaka, T. Seki *et al.*, Experimental observation of long-range magnetic order in icosahedral quasicrystals, *J. Am. Chem. Soc.* **143**, 19938 (2021).
- [41] R. Tamura, T. Abe, S. Yoshida, Y. Shimozaki, S. Suzuki, A. Ishikawa *et al.*, Observation of antiferromagnetic order in a quasicrystal, *Nat. Phys.* **21**, 974 (2025).
- [42] F. Labib, T. Sugimoto, H. Takakura, A. Ishikawa, and R. Tamura, Identification of two distinct antiferromagnetic phases in the au-al-gd quasicrystal approximant, *Phys. Rev. B* **111**, 174409 (2025).
- [43] P. d'Ornellas, V. Leeb, A. G. Grushin, and J. Knolle, Altermagnetism without crystal symmetry, *Phys. Rev. B* **113**, 024426 (2026).
- [44] M. Leeuwenhoek, F. Groenewoud, K. van Oosten, T. Benschop, M. P. Allan, and S. Groblacher, Fabrication of on-chip probes for double-tip scanning tunneling microscopy, *Microsyst. Nanoeng.* **6**, 99 (2020).
- [45] A. Matsui and Y. Shigeta, Development of probe-to-probe approach method for an independently controlled dual-probe scanning tunneling microscope, *Rev. Sci. Instrum.* **78**, 106107 (2007).
- [46] T. Nakanishi and T. Ando, Conductance images between two STM probes in graphene, *Physica (Amsterdam)* **42E**, 726 (2010).
- [47] S. M. Hus, X.-G. Zhang, G. D. Nguyen, W. Ko, A. P. Baddorf, Y. P. Chen, and A.-P. Li, Detection of the spin-chemical potential in topological insulators using spin-polarized four-probe STM, *Phys. Rev. Lett.* **119**, 137202 (2017).
- [48] Y. Su, G. Zhang, D. Cao, and C. Zhang, Coincidence double-tip scanning tunneling spectroscopy, *arXiv:2507.17532*.
- [49] Joshua E. S. Socolar, Simple octagonal and dodecagonal quasicrystals, *Phys. Rev. B* **39**, 10519 (1989).
- [50] A. E. Madison, Symmetry of quasicrystals, *Phys. Solid State* **55**, 855 (2013).
- [51] S. Giuli, C. Mejuto-Zaera, and M. Capone, Altermagnetism from interaction-driven itinerant magnetism, *Phys. Rev. B* **111**, L020401 (2025).
- [52] K. I. Kugel and D. I. Khomskii, The Jahn-Teller effect and magnetism: Transition metal compounds, *Sov. Phys. Usp.* **25**, 231 (1982).
- [53] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/src1-b2xt> for details on the model (Sec. SI), symmetry analysis (Sec. SII), additional numerical results (Sec. SIII), and the effects of competing orders (Sec. SIV), which includes Refs. [1,41,54–65].
- [54] J. C. Jiang, N. Wang, K. K. Fung, and K. H. Kuo, Direct observation of domains and discommensurations in Mn-Si-Al octagonal quasicrystal by transmission electron microscopy, *Phys. Rev. Lett.* **67**, 1302 (1991).
- [55] J. C. Slater and G. F. Koster, Simplified LCAO method for the periodic potential problem, *Phys. Rev.* **94**, 1498 (1954).
- [56] I. V. Solov'yev, R. Ono, and S. A. Nikolaev, Ferromagnetic ferroelectricity due to the Kugel-Khomskii mechanism of orbital ordering assisted by atomic Hund's second rule effects, *Phys. Rev. B* **110**, 205116 (2024).
- [57] V. Leeb, A. Mook, L. Šmejkal, and J. Knolle, Spontaneous formation of altermagnetism from orbital ordering, *Phys. Rev. Lett.* **132**, 236701 (2024).
- [58] Q. N. Meier, A. Carta, C. Ederer, and A. Cano, Net and compensated altermagnetism from staggered orbital order: Layer-dependent spin splitting in Sr_{n+1}Cr_nO_{3n+1}, *Phys. Rev. Lett.* **136**, 116705 (2026).
- [59] M. Daghofer, K. Wohlfeld, and J. van den Brink, Altermagnetic polarons: The fate of altermagnetic band splittings at strong coupling, *Phys. Rev. Lett.* **136**, 146502 (2026).
- [60] S. Zeng and Y.-J. Zhao, Description of two-dimensional altermagnetism: Categorization using spin group theory, *Phys. Rev. B* **110**, 054406 (2024).
- [61] J. Kellendonk, D. Lenz, and J. Savinien, *Mathematics of Aperiodic Order* (Springer, Basel, 2015).

- [62] J. Jeon and S. B. Lee, Interaction-tuned pattern-selective superconductivity: Application to the dodecagonal quasicrystal, *Phys. Rev. B* **112**, 174511 (2025).
- [63] A. Reinhardt, J. S. Schreck, F. Romano, and J. P. K. Doye, Self-assembly of two-dimensional binary quasicrystals: A possible route to a dna quasicrystal, *J. Phys. Condens. Matter* **29**, 014006 (2016).
- [64] E. H. Lieb, Two theorems on the Hubbard model, *Phys. Rev. Lett.* **62**, 1201 (1989).
- [65] C.-K. Chiu, Jeffrey C. Teo, A. P. Schnyder, and S. Ryu, Classification of topological quantum matter with symmetries, *Rev. Mod. Phys.* **88**, 035005 (2016).
- [66] B. Lv, T. Qian, and H. Ding, Angle-resolved photoemission spectroscopy and its application to topological materials, *Nat. Rev. Phys.* **1**, 609 (2019).
- [67] J. A. Sobota, Y. He, and Z.-X. Shen, Angle-resolved photoemission studies of quantum materials, *Rev. Mod. Phys.* **93**, 025006 (2021).
- [68] Y. Wang and M. Dendzik, Recent progress in angle-resolved photoemission spectroscopy, *Meas. Sci. Technol.* **35**, 042002 (2024).
- [69] O. Lesser and R. Lifshitz, Emergence of quasiperiodic bloch wave functions in quasicrystals, *Phys. Rev. Res.* **4**, 013226 (2022).
- [70] D. Varjas, A. Lau, K. Pöyhönen, A. R. Akhmerov, D. I. Pikulin, and I. C. Fulga, Topological phases without crystalline counterparts, *Phys. Rev. Lett.* **123**, 196401 (2019).
- [71] S. J. Ahn, P. Moon, T.-H. Kim, H.-W. Kim, H.-C. Shin, E. H. Kim *et al.*, Dirac electrons in a dodecagonal graphene quasicrystal, *Science* **361**, 782 (2018).
- [72] V. A. Rogalev, O. Gröning, R. Widmer, J. H. Dil, F. Bisti, L. L. Lev, T. Schmitt, and V. N. Strocov, Fermi states and anisotropy of brillouin zone scattering in the decagonal Al–Ni–Co quasicrystal, *Nat. Commun.* **6**, 8607 (2015).
- [73] A. Jagannathan, The fibonacci quasicrystal: Case study of hidden dimensions and multifractality, *Rev. Mod. Phys.* **93**, 045001 (2021).
- [74] M. A. Bandres, M. C. Rechtsman, and M. Segev, Topological photonic quasicrystals: Fractal topological spectrum and protected transport, *Phys. Rev. X* **6**, 011016 (2016).
- [75] R. Landauer, Electrical resistance of disordered one-dimensional lattices, *Philos. Mag.* **21**, 863 (1970).
- [76] M. Büttiker, Absence of backscattering in the quantum Hall effect in multiprobe conductors, *Phys. Rev. B* **38**, 9375 (1988).
- [77] D. S. Fisher and P. A. Lee, Relation between conductivity and transmission matrix, *Phys. Rev. B* **23**, 6851 (1981).
- [78] A. MacKinnon, The calculation of transport properties and density of states of disordered solids, *Z. Phys. B* **59**, 385 (1985).
- [79] G. Metalidis and P. Bruno, Green’s function technique for studying electron flow in two-dimensional mesoscopic samples, *Phys. Rev. B* **72**, 235304 (2005).
- [80] A. Jagannathan and M. Duneau, An eightfold optical quasicrystal with cold atoms, *Europhys. Lett.* **104**, 66003 (2013).
- [81] K. Viebahn, M. Sbroscia, E. Carter, J.-C. Yu, and U. Schneider, Matter-wave diffraction from a quasicrystalline optical lattice, *Phys. Rev. Lett.* **122**, 110404 (2019).
- [82] A. Uri, S. C. de la Barrera, M. T. Randeria, D. Rodan-Legrain, T. Devakul, P. J. D. Crowley *et al.*, Superconductivity and strong interactions in a tunable moiré quasicrystal, *Nature (London)* **620**, 762 (2023).
- [83] Y. Li, F. Zhang, V.-A. Ha, Y.-C. Lin, C. Dong, Q. Gao *et al.*, Tuning commensurability in twisted van der Waals bilayers, *Nature (London)* **625**, 494 (2024).
- [84] X. Chen, J. Ren, Y. Zhu, Y. Yu, A. Zhang, P. Liu, J. Li, Y. Liu, C. Li, and Q. Liu, Enumeration and representation theory of spin space groups, *Phys. Rev. X* **14**, 031038 (2024).
- [85] Z. Xiao, J. Zhao, Y. Li, R. Shindou, and Z.-D. Song, Spin space groups: Full classification and applications, *Phys. Rev. X* **14**, 031037 (2024).
- [86] Y. Jiang, Z. Song, T. Zhu, Z. Fang, H. Weng, Z.-X. Liu, J. Yang, and C. Fang, Enumeration of spin-space groups: Toward a complete description of symmetries of magnetic orders, *Phys. Rev. X* **14**, 031039 (2024).
- [87] Y. Li, M. Pan, J. Leng, Y. Chen, H. Huang, preceding Letter, Unconventional altermagnetism in quasicrystals: A hyper-spatial projective construction, *Phys. Rev. Lett.* **137**, 046704 (2026).
- [88] Z.-Y. Shao, C. Lu, Z. Pan, Y.-B. Liu, and F. Yang, Classification of Magnetism and Altermagnetism in Quasicrystals, [arXiv:2508.15702](https://arxiv.org/abs/2508.15702).

End Matter

The g -wave and i -wave altermagnetic phases found here can be understood within the magnetic or black-and-white group classification of quasiperiodic crystals developed by R. Lifshitz [35–37]. This framework classifies magnetic structures by combining spatial operations with time reversal or spin reversal. In the present context, it provides a symmetry-based basis for analyzing which magnetic symmetries of a quasiperiodic

crystal can allow for altermagnetic spin splitting in electronic spectra.

In Sec. SII D of Supplemental Material [53], we adapt the spin-group language used for periodic altermagnets [1,60,84–86] to two-dimensional quasiperiodic point groups. We identify the admissible altermagnetic phases through the decomposition of the real-space symmetry group $\mathcal{G}$:

$$R_3 = [\mathcal{E}||\mathcal{H}] + [\mathcal{C}_2||\mathcal{G} - \mathcal{H}]. \quad (\text{A1})$$

Here, the spin group R_3 is constructed by pairing spatial operations in the halving subgroup $\mathcal{H}$ with the spin identity $\mathcal{E}$, while operations in the coset $\mathcal{G} - \mathcal{H}$ are coupled with a spin flip $\mathcal{C}_2$.

Based on this formalism, we categorize quasicrystals into three classes distinguished by the order of their rotational symmetry: odd-, $4n$ -, and $(4n + 2)$ -fold. Specifically, we show that, while mirror-driven altermagnetism is universally accessible, pure rotation-driven altermagnetism is allowed for $4n$ -fold systems (like the octagonal case studied here). This distinction arises

because only $4n$ -fold symmetries allow the vertical 180° rotation (C_{2z}) to be sequestered within the spin-preserving subgroup $\mathcal{H}$, thereby avoiding the enforced spin degeneracy that forbids rotation-driven phases in odd and $(4n + 2)$ -fold systems. The Ammann-Beenker and Stampfli tilings used in this Letter possess the full dihedral point symmetries $8mm$ and $12mm$, respectively. Accordingly, the magnetic point groups of the altermagnetic phases realized in these tilings are $8'm'm$ and $12'm'm$. In the cases studied here, the twofold rotation can be placed in the spin-preserving subgroup, allowing for altermagnetic spin splitting protected by C_8T and $C_{12}T$.