Exponentially Enhanced Two-Mode Multiboson Entanglement via Phase-Modulated Tunneling

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(Received 8 February 2026; accepted 22 July 2026; published 8 September 2026)

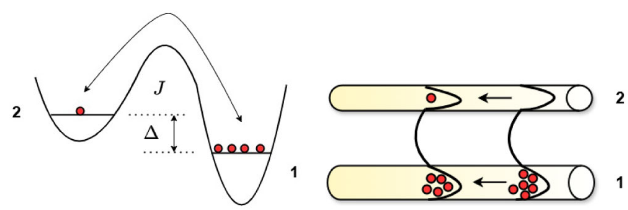
The entanglement of quantum systems is commonly restricted by their coupling Hamiltonian and initial state properties. Here, we prove by *exact analysis* of tunnel-coupled bosonic field modes that *factorized multiboson* two-mode states can become *fully entangled* via stroboscopic sign flips of the two-mode coupling. Their entanglement can exponentially grow with the number of flips. With the exception of specific prohibitive states that are invariant under such sign-flip control, this effect universally applies to two-mode state preparation. Remarkably, this *linear control* may provide entanglement resources for diverse quantum technological applications by readily available means.

DOI: 10.1103/mxnp-zvth

Introduction—Since its discovery by Gamow [1], tunneling has been found to be key to a vast scope of quantum processes [1–13]. Its importance for quantum information processing has been recently recognized by the Nobel Prize [14,15]. A prominent scenario that underlies diverse applications [5–9] concerns tunneling in a double-well potential. The tunneling rate is then hindered not only by the weak coupling through the barrier, but also by the *asymmetry or detuning* between the double-well minima. The degree of tunneling may be very small for large detuning, so that one has to invest a huge amount of energy comparable to the barrier height to activate such processes.

Here, we reveal a hitherto unexplored tunneling effect based on *linear dynamical control*: exponential enhancement of the tunneling rate and two-mode entanglement of multiple, noninteracting bosons in asymmetric double-well potentials or their optical analogs in tunnel-coupled waveguides. To this end, we take advantage of tunneling being *a coherent process that can be phase controlled*. Tunneling was previously shown to result from predominantly *destructive interference of Feynman paths* that cross the potential barrier [11,12]. It should, therefore, be possible to coherently enhance tunneling by phase modulation of interfering paths on timescales that are much faster than decoherence and dissipation, which tend to render the propagation classical [11,16].

We present an exact solution for controlled multiboson tunneling via a spin-$N/2$ mapping, based upon a theorem we prove for universal entanglement generation under phase-flip control. Undisturbed evolution in this model results in extremely slow tunneling when the two-mode coupling is ultraweak and *no entanglement generation* if the initial single-mode states are factorized (in the number basis) [Fig. 1(a)]. In stark contrast, our exact solution shows that periodic π-phase jumps (sign flips) of the coupling at



(a) (b)

FIG. 1. (a) Tunneling in an asymmetric double-well potential with a single level in each well. If well 1 is initially populated by N bosons and well 2 is empty, then well 2 will be increasingly populated at the expense of well 1. The process is coherent and reversible. (b) Realization of the above scenario for two single-mode ultraweakly coupled asymmetric waveguides of different widths.

appropriate intervals may exponentially enhance the entanglement and the tunneling rate with the number of phase jumps of initially factorized single-mode states, unless these states possess a symmetry that renders them invariant under the above control.

The most straightforward realization of this *entirely linear entanglement control* is envisaged in weakly coupled photonic waveguides [Fig. 1(b)] whose coupling is periodically phase modulated along the waveguide [Fig. 2(c)]. We foresee advantageous applications of this scheme as an entanglement resource of quantum information processing [17–22] and diverse quantum technologies [23–39] (see Discussion).

Undisturbed tunneling between two bosonic modes—The two bosonic modes in our scenario may be viewed as the energy-mismatched (mutually detuned) ground levels of two asymmetric potential wells that are coupled by tunneling through the potential barrier between the wells. We assume that the boson *interparticle interaction* is *negligibly small compared to the tunneling strength*, so

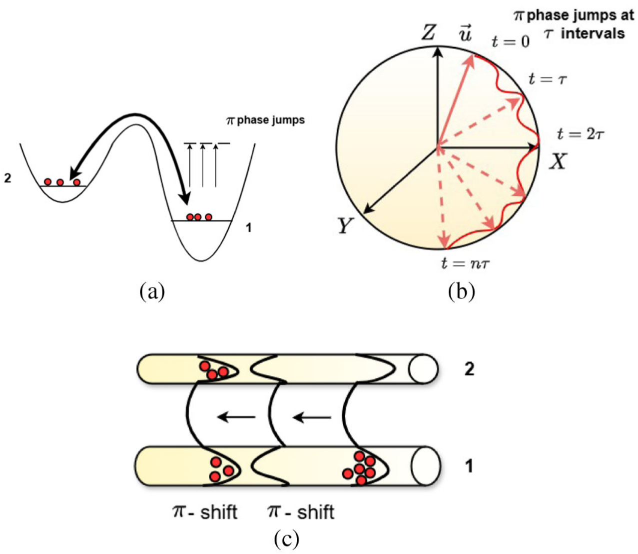


FIG. 2. (a) π phase flips in one well amplify the tunneling of the initially factorized $|N,0\rangle$ state, evolving it to the fully entangled $(|N,0\rangle + |0,N\rangle)/\sqrt{2}$ (NOON) state. (b) Phase jumps on the N-dimensional hypersphere. (c) Periodically π-shifted tunnel-coupled waveguides [modified Fig. 1(b)].

that the *dynamics is linear*, unlike that of commonly investigated Kerr-nonlinear Bose condensates [23–25]. Higher levels in these wells, associated with additional bosonic modes, are assumed to be much further detuned and are, therefore, excluded from our analysis.

Then the Hamiltonian is given by [23,40,41]

$$\hat{H} = J\hat{L}_x + \Delta\hat{L}_z, \tag{1}$$

where we assume for concreteness that $J > 0$ and $\Delta \geq 0$. Here, J is the resonant tunneling strength [42], Δ is the detuning (energy mismatch) between the modes, and $\hat{L}_i$ ($i = x, y, z$) are the collective N-boson angular momentum operators. In the Jordan-Schwinger representation, they are expressed in terms of the annihilation and creation bosonic operators, $\hat{a}_1$, $\hat{a}_1^\dagger$ and $\hat{a}_2$, $\hat{a}_2^\dagger$ of modes 1 and 2 [29],

$$\hat{L}_x = \frac{\hat{a}_1^\dagger\hat{a}_2 + \hat{a}_2^\dagger\hat{a}_1}{2}, \quad \hat{L}_y = \frac{\hat{a}_1^\dagger\hat{a}_2 - \hat{a}_2^\dagger\hat{a}_1}{2i}, \quad \hat{L}_z = \frac{\hat{a}_1^\dagger\hat{a}_1 - \hat{a}_2^\dagger\hat{a}_2}{2}.$$

As a simple case, consider N bosons shared by the two modes. There are $N+1$ collective states $|n\rangle$ ($n = 0, 1, \ldots, N$), which correspond to n (respectively, $N-n$) bosons in mode 1 (2) that occupy the ground levels of the two wells. These two-mode states $|n, N-n\rangle \equiv |m, l\rangle$ are eigenstates of $\hat{L}_z$:

$$|n, N-n\rangle \equiv |m, l\rangle; \qquad \hat{L}_z|m, l\rangle = m|m, l\rangle;$$
$$m = n - l, \quad l = N/2, \tag{2}$$

l being the value of the fictitious N-boson angular momentum. The above description holds regardless of the shape and the width of each potential well, as long as the higher levels of the double-well potential remain unpopulated.

The evolution of the initial N-boson wave function under Hamiltonian (1),

$$|\psi(t)\rangle = e^{-iHt}|N,0\rangle = \sum_{m=-l}^{l} c_{l,m}(t)|m, l\rangle, \tag{3}$$

is isomorphic to the rotation caused by the unitary evolution operator

$$\hat{U}(t) = e^{-i\Omega t(\hat{L}_x \sin\theta + \hat{L}_z \cos\theta)}, \quad \Omega = \sqrt{J^2 + \Delta^2},$$
$$\theta = \arctan(J/\Delta). \tag{4}$$

The initial product state $|N,0\rangle \equiv |m = l, l\rangle$, wherein all bosons are initially in mode 1, evolves via (4) to the two-mode *entangled form* (3) of superposed m states with fixed $l = N/2$ having the probability amplitudes [43,44]

$$c_{l,m}(t) = \langle l, m|\hat{U}(t)|l, l\rangle = \sum_{m'} d_{mm'}^{(l)}(\theta) e^{-im'\Omega t} d_{mm'}^{(l)*}(\theta), \tag{5}$$

where $d_{mm'}^{(l)}(\theta)$ are the real-valued Wigner d-matrix elements dependent on the rotation angle θ (see Supplemental Material [45]).

The survival probability of the initial $|N,0\rangle$ state $P_N(t)$ which is complementary to the tunneling probability, $P_{\mathrm{tun}}^{(N)} = 1 - P_N(t)$, can be evaluated to be (see Supplemental Material [45])

$$P_N(t) \approx \left(1 - \frac{J^2}{\Omega^2}\sin^2\frac{\Omega t}{2}\right)^N. \tag{6}$$

Off resonance, i.e., for $\Delta \neq 0$, complete transfer in the $|n, N-n\rangle$ basis is impossible. However, according to Eq. (6), the population of the initial state can decrease from 1 to a very small value near resonance: For $\Delta^2 \ll NJ^2$, it approximately exhibits periodic Gaussian revivals of the survival probability:

$$P_N(t) \approx e^{-NJ^2(t-t_{2k})^2/4}, \quad |t - t_{2k}| \ll \Omega^{-1} (k = 1, 2, \ldots), \tag{7}$$

where $t_k = k\tau$. This expression is independent of Δ, i.e., the effects of the detuning are insignificant in this limit. It is close to the resonant case wherein the population is $P_N(t) = \cos^{2N}(\Omega t/2)$, so that $P_N(t)$ vanishes at times when all the bosons are transferred to mode 2. In the opposite limit of large detuning, $\Delta^2 \gg NJ^2$, the decay of the survival probability is always small.

Equations (6) and (7) show that the effective strength of the coupling of the state $|N,0\rangle$ to the other states is *cooperatively enhanced* to the value $\sqrt{N}J$. This hitherto unknown *collective transfer enhancement* implies that the transfer rate can be amplified by a factor of N compared to a single-boson rate. This process is coherently reversible, unlike the well-known open-system anti-Zeno effect [46–49]. Since the total number operator $\hat{N} = a_1^\dagger a_1 + a_2^\dagger a_2$ commutes with the Hamiltonian (1), an arbitrary initial superposition of $|N,0\rangle$, i.e., N-boson numbers in one mode and 0 in the other having amplitudes A_N, evolves as

$$|\psi(t)\rangle = e^{-i\hat{H}t}\sum_N A_N|N,0\rangle = \sum_{l\in\{0,\frac{1}{2},1,\ldots\}} \tilde{A}_{2l}\sum_{m=-l}^{l(N)} c_{l,m}(t)|l,m\rangle, \tag{8}$$

where $\tilde{A}_{2l}$ originate from the initial A_N distribution whereas $c_{l,m}$ are the same as in (5). Equation (8) describes the independent evolution of each N sector via tunneling which is a *linear passive operation*.

Tunneling control by periodic phase jumps—For a fixed N and large Δ, $P_{\text{tun}} = 1 - P_N(t)$ is bounded by an exponentially small value, as can be easily seen from (6). Let us show that, irrespective of the detuning Δ, *the tunneling expressed by (8) can become complete under appropriate phase jumps. Since the undisturbed multi-boson tunneling process for a fixed initial N is a highly restricted rotation around an axis $\vec{u} = \sin\theta\hat{x} + \cos\theta\hat{z}$ of the N-dimensional hypersphere, we aim at lifting the restriction for $\Delta \gg J$.*

To this end, the mode detuning is modulated in time, so that the Hamiltonian (1) changes to

$$\hat{H} = J\hat{L}_x + [\Delta + \delta(t)]\hat{L}_z, \tag{9}$$

where $\delta(t)$ is the intermode detuning change due to a control field. In the frame rotating around the z axis at the frequency $\delta(t)$, we can write

$$|\psi(t)\rangle = e^{-i\phi(t)\hat{L}_z}|\chi(t)\rangle, \qquad \phi(t) = \int_0^t dt'\delta(t'), \tag{10}$$

Upon transforming to the frame that rotates by $\phi(t)$ around $\hat{L}_z$, the Hamiltonian acting on $|\chi(t)\rangle$ becomes

$$\tilde{H}(t) = J[\cos\phi(t)\hat{L}_x + \sin\phi(t)\hat{L}_y] + \Delta\hat{L}_z. \tag{11}$$

Let us take $\delta(t)$ to consist of identical short (*stroboscopic*) pulses that produce π-phase shifts (flips) at intervals τ [Figs. 2(a) and 2(b)] so that, on neglecting the width of the pulses, $\phi(t) = \pi[t/\tau]$, where $[x]$ denotes the integer part of x. At even- or odd-numbered intervals $t_{2k} \equiv 2k\pi/\Omega$, $t_{2k+1} \equiv (2k+1)\pi/\Omega$, the propagator $\hat{U}(t_{2k})$ rotates the system, respectively, in the x–z plane, rendering (see Supplemental Material [45])

$$\hat{U}(t_{2k}) = e^{i4k\theta\hat{L}_y}, \tag{12a}$$

$$\hat{U}(t_{2k+1}) = e^{-i\pi\hat{\vec{L}}\cdot\vec{u}_k}, \qquad \vec{u} = [\sin(2k+1)\theta, 0, \cos(2k+1)\theta], \tag{12b}$$

where θ is defined in (4). Since we have chosen $\cos\phi(t) = (-1)^{\lfloor t/\tau\rfloor}$ and $\sin\phi(t) = 0$, the Hamiltonian alternates in sign:

$$\tilde{H}(t) = J(-1)^{\lfloor t/\tau\rfloor}\hat{L}_x + \Delta\hat{L}_z. \tag{13}$$

Here, we assume identical switching intervals and exact π-phase flips. Errors can reduce the constructive interference and, hence, the attainable tunneling and entanglement enhancement. Details of the robustness to small deviations in the switching and the phase parameter are deferred to Supplemental Material [45].

Physically, this corresponds to *alternating tunneling direction* every τ. For the $|N,0\rangle$ component, the survival probability is then

$$P_N(t_k) = \cos^{2N}(k\theta) \quad (k = 0, 1, \ldots). \tag{14}$$

Thus, control by π phase shifts (sign flips) does not significantly affect the rate of decay of $P_N(t)$ for small detuning $\Delta^2 \ll NJ^2$. By contrast, in the large-detuning limit, $\Delta^2 \gg NJ^2$, where the unperturbed tunneling is insignificant, this phase-flip control can result in *almost complete decay* of the initial state. This hitherto unexplored solution is here deemed the closed-system *collective anti-Zeno effect* (CAZE), since it enhances the decay rate from zero to a finite value that scales with N. In particular, for large detuning $\Delta \gg J$, Eq. (14) yields approximately

$$P_N(t_k) \approx \exp\left(-k^2\frac{NJ^2}{\Delta^2}\right), \qquad k \ll \Delta/J, \tag{15}$$

which exhibits *collective k^2N scaling of the decay exponent*. It should be contrasted with the open-system anti-Zeno effect of accelerated decay via coupling to a thermal bath or continuum [46–48].

Generation and enhancement of entanglement—Equations (3)–(5) describe a state that becomes entangled in the $|n, N-n\rangle$ basis via tunneling, having started from the factorized state where only mode 1 is occupied. The general question we ask is, *under what conditions does the stroboscopic control (12) and (13) of an arbitrary pure two-mode state generate or enhance two-mode entanglement within a finite number of stroboscopic steps? The answer is given by the following theorem.*

Theorem 1—Any initial two-mode state that exhibits coherent off-resonant tunneling dynamics under the unperturbed evolution undergoes, under phase-jump control, stroboscopic SU(2) dynamics equivalent to resonant coherent oscillations or tunneling. In particular, for the initial

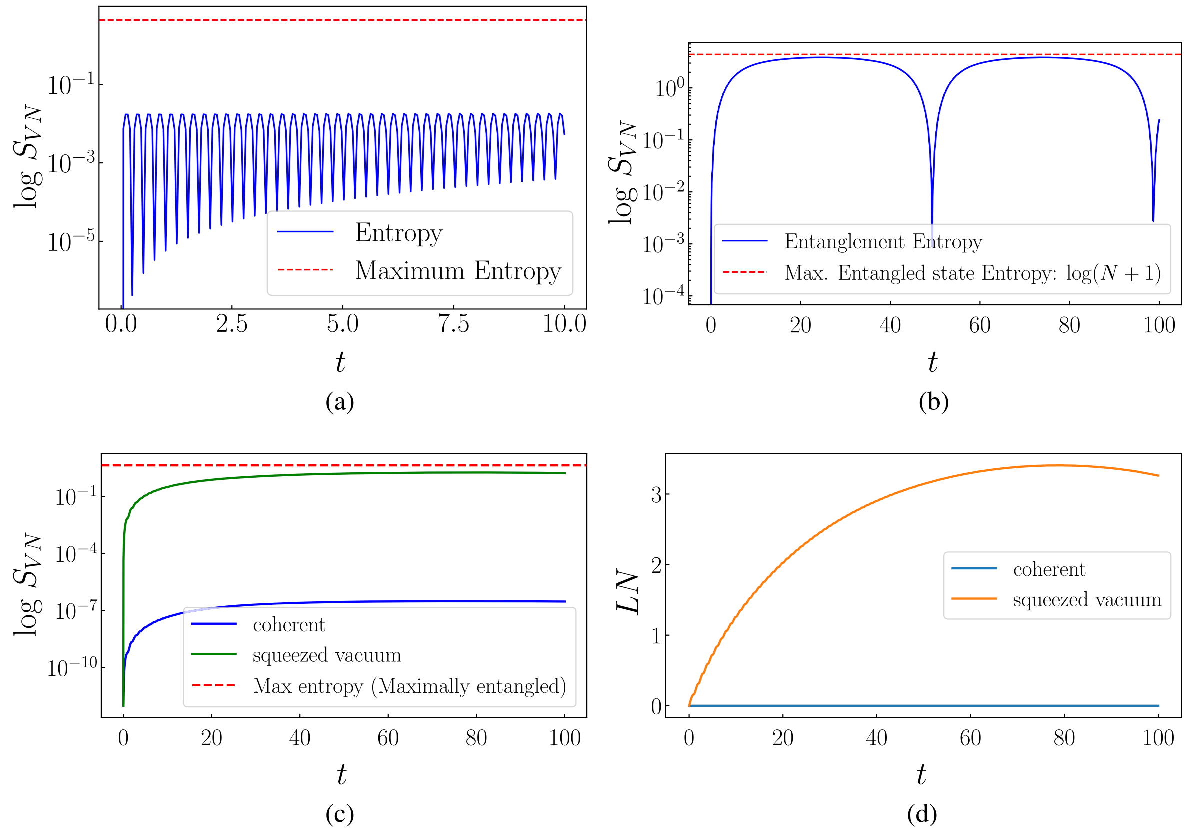


FIG. 3. (a) von Neumann entanglement entropy (S_{VN}) evolution for undisturbed tunneling with $J = 1$, $\Delta = 100$, and $N = 20$, in time units of $\pi/\sqrt{\Delta^2 + J^2}$, is restricted to values well below full entanglement. (b) Entanglement entropy as a function of time under phase flips. For (a) and (b) the initial state is $|N, 0\rangle$. (c) Comparison between the evolution of entanglement entropy under phase flips of coherent state with amplitude $\alpha = 2$ and squeezed vacuum states (squeezing parameter $r = 2$) with $J = 1$ and $\Delta = 100$. (d) The same as (c) for logarithmic negativity LN as a function of time under phase flips. Both (c) and (d) show that entanglement is severely restricted for spin-coherent states but exponentially enhanced for squeezed states.

Fock state $|N, 0\rangle$, the evolution yields $P_N(t_k) = \cos^{2N}(k\theta)$, mimicking resonant oscillations. Moreover, a necessary condition for the generation or enhancement of two-mode entanglement is that the initial state is nonclassical and that the stroboscopic evolution populates both modes.

The proof is given in End Matter.

To *quantify the two-mode entanglement*, we resort to the von Neumann entropy of the reduced density matrix for mode 1 or mode 2. This entropy grows due to tunneling-induced population mixing. An elaborate calculation (see Supplemental Material [45]) yields the entanglement entropy bound of the N-sector state

$$S_N(t) = -\sum_m P_N(m, t) \ln P_N(m, t) \approx \frac{1}{2} \ln(2\pi\sigma_{\mathrm{N}}^2) + \ln \eta_N, \tag{16}$$

where the effective variance σ_{N}^2 is fixed for unperturbed evolution [Fig. 3(a)] but scales with k under sign-flip modulation and $\eta_N \sim O(1)$ accounts for finite support corrections (see Supplemental Material [45]).

Let us assume, for example, that the initial state is $|r, 0\rangle$: squeezed vacuum with squeezing parameter r in mode 1 and the vacuum state in mode 2. Under stroboscopic sign-flip modulation control, there is a drastic monotonic increase in the entropy [Fig. 3(b)]. Subtracting the unperturbed entropy bound from its perturbed (modulated) counterpart yields the entropy enhancement due to modulation after k cycles (see Supplemental Material [45]):

$$\begin{aligned} \Delta S_k &= S_{\mathrm{mod}}(t_k) - S_{\mathrm{unmod}}(t) \\ &\gtrsim \frac{1}{2} \ln k + \frac{1}{2} \ln\left(\frac{\sin^2\theta}{\sin^2\theta + \cos^2\theta(\sinh^2 r + 1)}\right), \end{aligned} \tag{17}$$

where θ is as in (4).

The following asymptotic limits of (17) transpire: (a) For weak squeezing ($r \ll 1$), $\sinh^2 r \simeq r^2$ and $\Delta S_k \simeq \frac{1}{2} \ln k$, recovering the entropy scaling for the initial $|N, 0\rangle$ state. (b) For strong squeezing ($r \gg 1$), $\sinh^2 r \gg 1$, and the prefactor in the last logarithmic term of (17) decreases, indicating that the unperturbed (unmodulated) entropy is already high, while the modulation-induced growth retains the $\frac{1}{2}\ln k$ scaling, albeit with smaller relative enhancement. This result ensures reaching full two-mode entanglement after a finite (k) number of steps, regardless of how small the initial-state squeezing r was.

Indeed, at long times, as $k \to \infty$, the phase-modulated entanglement entropy approaches

$$S_{\mathrm{mod}}(t) \to \ln(2l + 1), \tag{18}$$

which is the maximal possible mode-entanglement entropy for a two-mode bosonic system with total boson number $N = 2l$, associated with the fully entangled state [28]. The maximal entropy in Eq. (18) is reached when the reduced density matrix becomes uniform, $P_N(t) = 1/(N + 1)$, corresponding to equal population of all $|n, N - n\rangle$ states. This requires initial states with sufficient support over the controlled eigenmodes and, in particular, exclusion of the symmetry-protected invariant (spin-coherent) states, which do not spread under the stroboscopic control dynamics.

An alternative measure of entanglement is logarithmic negativity [19,50–54] [Fig. 3(d)] that depends only on $P_N(n, t)$:

$$LN(t) = 2\log_2\left(\sum_{n=0}^{N} \sqrt{P_N(n, t)}\right). \tag{19}$$

The regime $\Delta \gg J$ highlights the strongest impact of the protocol: Whereas uncontrolled tunneling and entanglement are strongly suppressed, stroboscopic π flips restore substantial population transfer and entanglement growth. As Δ approaches J, this advantage diminishes, because the uncontrolled tunneling and entanglement are then already significant and largely insensitive to Δ even without control. Thus, the protocol is most effective for large detuning.

Discussion—We have obtained an exact solution for tunnel-coupled mode-sharing multiple-boson dynamics. This solution has unraveled a hitherto unknown effect of giant entanglement amplification and tunneling speedup via π-phase flips of the state vector at the rate that corresponds to the frequency mismatch or detuning Ω of the two modes. The analysis resorts to the mapping of the problem onto that of the N-dimensional angular-momentum vector that precesses on the hypersphere under the action of a fictitious magnetic field. This precession can be dramatically amplified by k sign flips, a case that can be *solved exactly*, although the Hamiltonian consists of two noncommuting terms. This comes about since the precession axis of the state vector jumps around with each phase flip and eventually fills the entire surface of the N-dimensional hypersphere. The problem bears analogy to stepwise coherent excitation of a system with N equidistant levels [41].

In general, the entanglement of two-mode states via unitary evolution is restricted by the strength and nonlinearity of their coupling [55] or, alternatively, by *nonunitary* measurement-induced effects [56,57]. Here, remarkably, we resort to linear control via periodic phase flips to exponentially boost entanglement of the initial factorized state. Our exact SU(2)-based treatment and entanglement theorem are restricted to the two-mode case, leaving genuine multipartite extensions as an interesting direction for future work.

Importantly, the results of our analysis differ drastically from those of the classical analog: Two coupled waveguides of different widths redistribute the intensity ratio as $I \sim d^2$, d being the spatial modulation period [58,59]. Thus, our predicted entanglement generation and exponential enhancement with k sign flips is a distinctly quantum effect, without classical counterpart.

Our description goes well beyond the domain of applicability of the universal Kofman-Kurizki formula that describes, among many processes [46], the perturbative dynamical control of bound-state tunneling to a continuum of states [47]. Depending on the spectrum of the control process, the Kofman-Kurizki formula yields either decay and tunneling slowdown (alias the quantum Zeno effect) or, conversely, its speedup (dubbed the anti-Zeno effect) [48]. This formula has been shown to describe macroscopic tunneling control in SQUIDS (superconducting quantum interference devices) [10] and in bosonic junctions [60]: It treats tunneling from a bound state to the continuum as a *nearly irreversible process*. By contrast, we here study tunneling between two modes of a *closed system*, which is a *reversible process*. Nevertheless, the exact solution for the dynamical control of tunneling has been shown here to yield giant analogs of the anti-Zeno effect. These analogs have been found to be amplified by the many-body collective (bosonic) character of tunneling, which is a hitherto unknown feature.

The detuning in our model is simply the mismatch between the two mode frequencies or energies. In the two realizations discussed in the work, this corresponds, respectively, to the asymmetry of the two wells or to the propagation-constant mismatch of two asymmetric waveguides [Fig. 1(b)]. We assume that the modes are detuned because of well asymmetry or different waveguide widths, while the intermode coupling J is independently set by the barrier transparency or, in the photonic implementation, by the overlap of the evanescent tails. This makes the condition $\Delta \gtrsim J$, and, in particular, $\Delta \gg J$, physically natural. In the photonic-waveguide realization, moreover, the control does not require dynamically reaching resonance. One needs only weakly coupled asymmetric waveguides subject to periodically spaced π-phase shifts, separated by intervals $\pi c/\Omega$ (equivalently, π/Ω in time units). The required phase-coherent control and stroboscopic π-phase updates ($\tau \sim \pi/\Omega$) are within reach of existing Josephson-junction and photonic platforms [61–63].

This scenario may give further impetus to the emerging trend of exploring and implementing quantum electrodynamic effects in waveguides [64,65]. It is particularly promising in the context of quantum metrology [28], lithography [66], and quantum communication by means of entangled photonic NOON states, which can serve as *deterministic quantum repeaters*, unlike the usual probabilistic quantum repeaters [67]. Another intriguing potential application of the entangled NOON states obtained by the present scheme is for supersensitive microscopy [66,68,69].

The predicted effects bode well with novel designs of integrated solid-state and photonic structures, comprised of either asymmetric bosonic Josephson junctions or weakly coupled photonic waveguides with periodic phase modulation. In integrated photonics, programmable phase control is already standard. Reconfigurable quantum photonic circuits based on phase shifters and directional couplers have demonstrated programmable quantum interference with high visibility, showing that phase control can be exerted while preserving quantum coherence [62]. Gigahertz-frequency phase shifters have now been demonstrated to be faster than relevant coherence windows of on-chip photonic circuits [63]. Leveraging such advancements, the predicted effects can open a new route toward quantum multiphoton state control.

Realistically, some dissipation is always present in the system. In the simple non-Hermitian approach, the energy eigenvalues are then modified by $i\Gamma$, where Γ is the dissipation rate. This should lead to damping of the coherent oscillations in Eq. (14). In low-leakage optical waveguides, we may attain $\Gamma \lesssim$ MHz [63] that would lead to weak damping in Eq. (14) $\sim e^{-\Gamma t}$ for oscillations on a gigahertz scale of currently available phase shifters [62] against stochastic errors.

Acknowledgments—P. C. acknowledges the support from the International Postdoctoral Fellowship from the Ben May Center for Theory and Computation. G. K. acknowledges the support from a research grant from Magnus Konow.

Data availability—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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End Matter

Proof of Theorem 1—Let the initial state be an arbitrary two-mode pure state $|\psi(0)\rangle = \sum_{N=0}^{\infty} |\psi_N(0)\rangle$, where each $|\psi_N(0)\rangle$ belongs to the N-boson sector $\mathcal{H}_N = \mathrm{span}\{|n, N-n\rangle\}_{n=0}^{N}$. Since the Hamiltonian conserves the total boson number $\hat{N}$, the evolution decomposes into independent dynamics in each N sector:

$$|\psi(k\tau)\rangle = \sum_N \hat{U}_N(k\tau)|\psi_N(0)\rangle. \tag{A1a}$$

We now make explicit the structure of the evolved state in each N sector. Expanding in the Dicke basis $\{|l, m\rangle\}_{m=-l}^{l}$ with $l = N/2$, one has

$$\hat{U}_N(k\tau)|\psi_N(0)\rangle = \sum_{m=-l}^{l} c_{l,m}(t)|l, m\rangle, \tag{A1b}$$

where the amplitudes $c_{l,m}(t)$ are determined by the SU (2) evolution. Upon tracing out mode 2, the reduced density matrix of mode 1 in the N sector becomes diagonal in the number basis,

$$\rho_1^{(N)}(t = k\tau) = \sum_{n=0}^{N} P_N(n,t)|n\rangle\langle n|, \quad P_N(n,t) \equiv |c_{l,m}(t)|^2, \tag{A2}$$

where $l = N/2$ and $m = n - l$. The reduced state $\rho_1^{(N)}(t)$ is pure only if a single amplitude $c_{l,m}(t)$ is nonzero; otherwise, it is mixed. Since the global state remains pure under unitary evolution, a mixed reduced state implies two-mode entanglement. Therefore, entanglement is generated or enhanced if, in a sector $N_0 \equiv 2l_0$, the evolved state $\hat{U}_{N_0}(k\tau)|\psi_{N_0}(0)\rangle$ acquires at least two nonzero amplitudes with distinct m.

The unitary evolution (12a) and (12b) corresponds to SU(2) rotations in each sector:

$$\hat{U}_N(k\tau) = \hat{R}_k \cdots \hat{R}_2 \hat{R}_1, \qquad \hat{R}_p = \exp(-i\theta_p \hat{L}_{u_p}). \tag{A3}$$

The subgroup $G \subset \mathrm{SU}(2)$ is generated by rotations about two noncollinear axes (due to alternating $\hat{L}_x$ sign and $\hat{L}_z$ evolution). Because the spin-l representation of SU(2) is irreducible, it admits no nontrivial invariant subspaces. However, irreducibility alone does not guarantee that a given initial state spreads over multiple $|l, m\rangle$ components under a finite sequence of rotations. To establish spreading, we note that the stroboscopic evolution is generated by rotations about at least two noncollinear axes. Therefore, the unitary operators $\hat{R}_p$ do not commute and generate a non-Abelian subgroup of SU(2). For any initial state $|\psi_N(0)\rangle$ that is not an eigenstate of all generators $\hat{L}_{u_p}$, there exists at least one rotation $\hat{R}_p$ such that $\hat{R}_p|\psi_N(0)\rangle \not\propto |\psi_N(0)\rangle$. Hence, the state acquires components along multiple $|l, m\rangle$ basis states. Iterating this procedure, the successive action of noncommuting rotations produces a superposition involving at least two distinct m values after a finite number of steps, unless the initial state is invariant under the generated subgroup (e.g., symmetry-protected spin-coherent states aligned with the rotation axes). Therefore, for all noninvariant initial states, there exists a finite k such that the evolved state $\hat{U}_N(k\tau)|\psi_N(0)\rangle$ has at least two nonzero amplitudes $c_{l,m}(t)$.

In particular, for the initial Fock state $|N, 0\rangle$, which corresponds to the highest-weight state $|l, l\rangle$, the rotations $\hat{R}_p$ are not aligned with $\hat{L}_z$. Therefore, the state is not invariant and undergoes nontrivial redistribution, yielding $P_N(t_k) = \cos^{2N}(k\theta)$, which reproduces the functional form of resonant coherent oscillations with accumulated angle $k\theta$.

Explicitly, the amplitudes evolve as

$$\mathbf{c}(k\tau) = D^{(l)}(\hat{R}_k) \cdots D^{(l)}(\hat{R}_1)\mathbf{c}(0), \tag{A4}$$

where $D^{(l)}(\hat{R}_p)$ are SU(2) representation matrices. The noncommutativity of the rotations ensures that $\mathbf{c}(0)$ spreads over multiple components for generic initial states. Hence, for some finite k, at least two amplitudes become nonzero, implying that $\rho_1^{(N)}$ has rank ≥ 2 and, therefore, positive entropy.

However, this spreading leads to entanglement only under additional conditions. This distinction is illustrated in Fig. 3(c), where coherent states exhibit no entanglement growth under stroboscopic evolution, whereas squeezed states show rapid enhancement. The stroboscopic evolution implements a passive SU(2) linear transformation of the bosonic modes, $\hat{a}_1 \to u_k \hat{a}_1 + v_k \hat{a}_2$, $\hat{a}_2 \to -v_k^* \hat{a}_1 + u_k^* \hat{a}_2$, with $|u_k|^2 + |v_k|^2 = 1$. Such transformations preserve the class of classical states, namely, those with a positive Glauber-Sudarshan P representation. It is well established that passive linear transformations cannot generate entanglement from classical input states [19,70].

Therefore, entanglement generation requires that the initial state be nonclassical. In addition, if the evolution does not populate both modes, the state remains effectively single mode and no entanglement is produced.

This completes the proof that a necessary condition for entanglement generation is that the initial state is nonclassical and the stroboscopic evolution populates both modes (see Supplemental Material [45] for details). Under these conditions, correlations between the modes develop, and the reduced density matrix becomes mixed, implying two-mode entanglement.