

Enhancing Quantum Metrology by Quantum Resonance Dynamics

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Quantum effects in metrology can in principle enhance measurement precision from the so-called standard quantum limit to the Heisenberg limit. Further advancements in quantum metrology largely rely on innovative metrology protocols that can avoid a number of known obstacles, including the challenge of preparing entangled states with sufficient fidelity, the readout noise in measuring highly entangled states, and no-go theorems for quantum metrology under noisy environments. In this Letter, exploiting some peculiar but experimentally feasible dynamical features of a collection of spins with all-to-all time-periodic interactions, we propose a metrology protocol that can circumvent all three mentioned obstacles and yet still make good use of time as a resource for metrology. Specifically, by mapping the dynamics of such a periodically driven spin system to that of a paradigm of quantum chaos but tuned to some high-order quantum resonance, it is shown that a simple SU(2) coherent state can, after evolving to highly entangled states in the ensuing dynamics, be dynamically brought back to the same initial coherent state. The associated quantum Fisher information is found to exhibit quadratic scaling with both the number of spins and the duration of the metrology protocol. The achieved Heisenberg scaling can also largely survive in the presence of Markovian noise. Representing a previously unknown strategy for quantum metrology, the protocol proposed here can be tested on available experimental platforms.

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Introduction—Quantum metrology has become an indispensable tool in advancing quantum technologies, driving breakthroughs in high-precision applications such as magnetometry [1–4], gravitational wave detection [5–8], atomic clocks [9–16], and navigation systems [17]. Classical noise imposes one constraint on a class of quantum metrology protocols, known as the standard quantum limit (SQL) such that measurement precision scales as $1/\sqrt{N}$ (with N representing the number of qubits or atoms) [18–20]. Surpassing the SQL requires to harness resourceful quantum effects without a classical analog. The resulting Heisenberg limit (HL) as a quantum precision limit scaling as $1/N$ epitomizes one big advantage of quantum metrology.

Achieving the HL experimentally remains a challenge despite substantial efforts [21–35]. The preparation and readout of highly entangled states represent two major obstacles. Indeed, current proposals towards the HL typically rely on the Greenberger-Horne-Zeilinger (GHZ) state—a maximally entangled state [31] that can be notoriously difficult to prepare and protect in current experimental platforms [36–40]. The rich information content of

GHZ states also makes them highly susceptible to readout noise [41,42]. A third hurdle to overcome is the unavoidable noise experienced by a quantum metrology protocol, prompting some no-go theorems for noisy quantum metrology [18,19,43–51]. It is now known that environment noise of the Markovian type severely restricts information encoding time and hence makes the HL scaling difficult to achieve [20,24,25]. If a quantum metrology protocol is subject to such noise for too long, its performance can even degrade below the SQL [18].

Building on previous work on chaotic sensors [52] and quantum recurrence dynamics [53], we advocate in this work an innovative metrology protocol that can go around all the three major issues mentioned above with quantum resonance dynamics [54–72]. In a nutshell, the quantum resonance dynamics of a collection of spin systems subject to periodic all-to-all interaction can evolve, on its own, a simple easy-to-prepare initial state to a highly entangled state, and yet at the end, feature a complete recurrence of the initial state. These dynamical features hence serve as a natural process to encode quantum information and also facilitate the quantum state readout at the end of the protocol. Furthermore, the quantum resonance dynamics featured below also allows us to analytically obtain the growth of the QFI with time, which is notably quadratic. The quantum resonance dynamics used below hence can exploit time very well as a resource for metrology.

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Specifically, we use spin coherent states with minimal quantum uncertainty as the initial state. Such “classical” initial states are of no quantum entanglement content to start with, hence experimentally friendly, more robust against noise [73–75] and also avoids readout noise [41,42] upon quantum recurrence at the end of the metrology protocol. Furthermore, the time-periodic interaction applied to the spin system, designed to generate the quantum resonance dynamics, also places the quantum system under a useful nonequilibrium situation to effectively fight against decoherence [50]. With these advantages, we shall show in this work that the quantum Fisher information (QFI) of the adopted quantum resonance dynamics exhibits quadratic scaling with both the number of spins and the duration of the metrology protocol. Even in the presence of Markovian noise, the QFI can sustain a near-Heisenberg scaling in terms of the number of spins, though saturated with time. Representing a previously unknown strategy for quantum metrology, we stress that the protocol proposed here can be tested on available experimental platforms.

Quantum parameter estimation and dynamical sensitivity—Quantum parameter estimation theory focuses on the information extractable about an unknown parameter α , which is encoded in a quantum state represented by the density operator ρ_α . The quantitative measure of this information is the quantum Fisher information (QFI), which is directly related to the measurement precision through the well-known Cramér-Rao bound [76]. The QFI is defined as $I = \text{Tr}(\hat{L}_\alpha^2 \rho_\alpha)$, where $\hat{L}_\alpha$ is the symmetric logarithmic derivative satisfying $\partial_\alpha \rho_\alpha = (\hat{L}_\alpha \rho_\alpha + \rho_\alpha \hat{L}_\alpha)/2$ [77]. Because the precision limit is proportional to $1/\sqrt{I}$ [76], the QFI should scale as $I \propto N$ to achieve the SQL and $I \propto N^2$ to achieve the HL. On the other hand, a key measure of the dynamical sensitivity of a quantum system is the Loschmidt echo (fidelity) F_e . The Loschmidt echo thus defined quantifies the overlap between a state undergoing forward unitary evolution and the same state evolving backward under a slightly perturbed unitary operator [78]. Specifically, $F_e(t) = |\langle \psi_\alpha(0) | U_\alpha(t) U_{\alpha+\epsilon}(-t) | \psi_\alpha(0) \rangle|^2$ with $U_\alpha(t)$ the unitary operator of the evolution. It can be shown that the QFI can be expressed in terms of the Loschmidt echo as $I(t) = \lim_{\epsilon \rightarrow 0} 4[1 - F_e(t)]/\epsilon^2$ [43,52]. This makes it clear that the QFI is intrinsically linked to the dynamical sensitivity of quantum dynamics. For nonunitary evolution, the definition of the Loschmidt echo can be adjusted to $F_e(t) = \|\rho_\alpha^{1/2} \rho_{\alpha+\epsilon}^{1/2}\|_1^2$, where $\|Q\|_1 \equiv \text{Tr} \sqrt{QQ^\dagger}$ [43,79] and then a similar connection between dynamical sensitivity and the QFI can still be made.

It is then evident that having the rapid growth of the QFI with encoding time is also pivotal for high-precision measurements. Because this growth is closely connected with the dynamical sensitivity of a quantum system, previous researchers have gone beyond integrable models

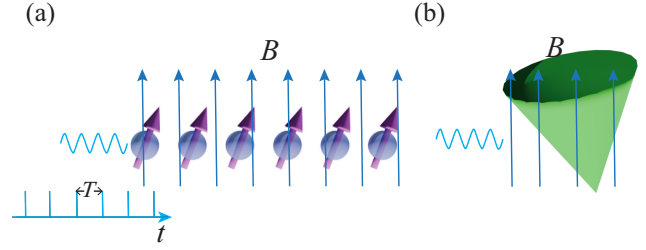


FIG. 1. Schematic of a quantum metrology protocol based on a system of N spins subject to periodic modulation in their all-to-all interaction. (a) A system of N spins under a magnetic field, featuring all-to-all time-periodic interactions, where $N = 2j$. (b) The setup of a quantum kicked top, describing the precession of a macro-spin of spin quantum number j subject to a magnetic field along the z -axis and time-periodic twisting about the y axis with period T .

[31] and started to explore the potential of quantum chaos models for enhancing the growth of the QFI [52]. Indeed, metrology protocols based on some quantum chaos models have the capability to build up quantum entanglement rapidly [52,80,81]. This rapid growth of the quantum entanglement and hence the QFI can be linked with the underlying chaotic classical trajectories of the classical counterpart system, but only up to the Ehrenfest time [82]. Because the Ehrenfest time is typically logarithmically short [82,83], the duration during which the QFI increases quickly is also rather limited. This motivates us to seek alternative means to make good use of time as a precious resource. In particular, the complete quantum recurrence in our metrology protocol below at particular timings indicates that the quantum dynamics simply repeats itself many times, and as such the growth of the QFI may be quadratic in terms of the sensing duration without saturation.

A periodically driven spin system as a quantum sensor—Consider an N -spin system with the following Hamiltonian [52,84–86]:

$$H_{\text{spin}} = \sum_{k=1}^N \frac{\alpha}{2T} \sigma_k^z + \sum_{k=1}^N \sum_{l=1}^N \frac{\beta}{8j} \sigma_k^y \sigma_l^y \sum_{n=-\infty}^{\infty} \delta(t - nT), \quad (1)$$

where $\sigma_i^{x,y,z}$ are the usual Pauli matrices for the spin indexed by i . This Hamiltonian depicts a collection of spin-1/2 systems subject to periodic all-to-all interaction in the form of δ kicks with period T , in addition to a common field along the z direction. Here the external field parameter α could serve as the strength of a weak magnetic field to be probed. We refer to Fig. 1 for a schematic illustration of this model. If we restrict the wave function to the symmetric-invariant subspace, we may describe the quantum dynamics by introducing collective operators [52]. By defining $J_\alpha = \sum_{i=1}^N (\sigma_i^\alpha/2)$ and assuming an initial collective state, the above Hamiltonian can be rewritten in the following form:

$$H_{\text{QKT}} = \frac{\alpha J_z}{T} + \frac{\beta J_y^2}{2j} \sum_{n=-\infty}^{\infty} \delta(t - nT), \quad (2)$$

where J_x , J_y , and J_z are collective angular momentum operators satisfying the commutation relations $[J_\alpha, J_\beta] = i\epsilon_{\alpha\beta\gamma}J_\gamma$, with $(J_x^2 + J_y^2 + J_z^2)|j, j_z\rangle = j(j+1)|j, j_z\rangle$ and $J_z|j, j_z\rangle = j_z|j, j_z\rangle$, where $|j, j_z\rangle$ represents angular momentum eigenstates, and $j = N/2$. The unitary evolution operator of H_{QKT} associated with one period T is given by $U_{\text{QKT}} = \exp[-i(\beta J_y^2)/(2j)] \exp(-i\alpha J_z)$. Throughout we set $\hbar = 1$. Clearly then, our periodically kicked spin system as a quantum sensor is nothing but the celebrated quantum kicked top (QKT) model as one paradigm of quantum chaos [87]. Experimental realizations of the QKT are well established, with implementations ranging from cold atoms [88] to trap ions [89] and digital quantum simulators [85].

Achieving the HL via quantum resonance dynamics— Other than the correspondence between quantum dynamics and the underlying classical chaos, quantum chaos studies had uncovered other exotic aspects of quantum dynamics without a classical analog, with quantum resonance dynamics being one prominent example. Quantum resonance dynamics was first investigated in another quantum chaos model—the so-called quantum kicked rotor (QKR) model [56,59–70], where energy absorption from the driving field may show ballistic growth albeit with corresponding classical trajectories being chaotic. The concept of quantum resonance does apply to the QKT, if the resonance condition $\beta = 4\pi jr/s$ is satisfied, with r and s being two coprime integers. Qualitatively, resonance arises due to the dynamical phase generated by the pulsed interaction. Specifically, the term $\exp[-i(\beta J_y^2)/(2j)]$ generate dynamical phases with a special pattern (e.g., in the representation of the eigenstates of J_y) if the system parameter β assumes values commensurate with π . Importantly, it is known that the quantum resonance dynamics of the QKT remains stable under a small perturbation δ to β [71]. A full understanding the dynamics for general choices of r and s , especially when they taking large integer values (hence very high order resonances) is challenging due to intricate number-theoretic properties, but a few special resonance cases were studied [53,71] previously. Fortunately, for our purpose here, certain special resonance cases already suffice to design useful metrology protocols.

The quantum resonance dynamics of the QKT can even exhibit complete quantum recurrence for some specific system parameters under the condition of $\alpha = \pi/2$, which was extensively studied in Ref. [53]: (i) For $\beta = 2\pi j$, we have $U_{2\pi j}^2 = I$, and so the dynamics are periodic with a period of $2T$. However, such dynamics generates no quantum entanglement during the time evolution. Indeed, a simple examination shows that the associated one-step unitary operator is composed of local unitaries only. From

quantum metrology perspective, this dynamics is not resourceful since it cannot be used to encode quantum information with dynamics-generated entanglement. (ii) For $\beta = \pi j$, we have $U_{\pi j}^8 = I$. In this case the dynamics exhibits complete quantum recurrence with a period of $8T$. This case is drastically different because it generates several GHZ-like states during the middle steps of a complete recurrence process, thus involving entanglement heavily. Because of this key observation, we may simply pick the $\text{SU}(2)$ coherent state with minimum quantum certainty as the easy-to-prepare initial state, whereas the final state is guaranteed to be the same coherent state due to the perfect quantum recurrence. We thereby can exploit this case to facilitate straightforward state preparation and readout processes for quantum metrology. (iii) For $\beta = \pi j/2$, numerical studies indicate that $U_{\pi j/2}^{48} = I$, showing perfect recurrence behavior similar to case (ii) but with a longer recurrence time while generating highly entangled states during the dynamical process. This makes case (iii) another promising parameter choice for quantum metrology. In this Letter, we focus on the case (ii) with $\beta = \pi j$ to illustrate our central ideas. To make a better connection with experimental considerations, one may rewrite the time evolution operator U_{QKT} as $U_{\text{QKT}} = \exp[-i\chi t J_y^2] \exp(-i\alpha J_z)$ with $\chi t = \beta/2j$. The first term in this expression of U_{QKT} represents the standard one-axis twisting with twisting strength χt and twisting time t . Experimentally it is possible to have $\chi t \sim \pi$ [90,91], which suffices to place our system in the resonance regime considered in this work.

We have performed computer simulations starting from an initial coherent state $|\Theta, \Phi\rangle = \exp[i\Theta(J_x \sin \Phi - J_y \cos \Phi)]|j, j\rangle$. The Husimi representation $P(\Theta, \Phi) = \langle \Theta, \Phi | \rho | \Theta, \Phi \rangle$, where ρ is the density matrix, is used to visualize the wave packet dynamics [92]. The results are presented in Fig. 2, with additional simulations for $\beta = \pi j/2$ in Supplemental Material [93]. As shown in Fig. 2, the initial coherent state is split into a superposition of two well-separated wave packets at $t = 3T$ and a superposition of four well-separated wave packets at $t = 6T$, before a perfect quantum recurrence occurs at $t = 8T$. These superposition states represent highly entangled states involving all the N spins. Because the dynamical sensitivity is really about the sensitivity of one whole process, the generation of entangled states in the middle steps of a dynamical process suggests the possibility of Heisenberg scaling of the QFI, an insight that is indeed confirmed later.

One main difference between the resonance dynamics illustrated above and the classical-like chaotic wave packet dynamics is that quantum resonance dynamics exhibits the so-called *pseudoclassical stability* [71], a stability feature that can only be well explained by another effective classical system after technical treatments of quantum resonances. Without presenting such technical details, we highlight that unlike typical diffusive wave packet dynamics resembling

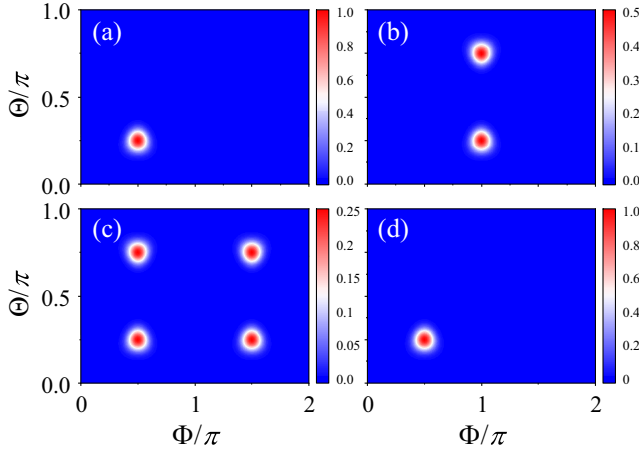


FIG. 2. The Husimi function representation $P(\Theta, \Phi)$ of the wavepacket dynamics of the QKT introduced in the main text, with $\alpha = (\pi/2)$, $\beta = \pi j$, $T = 1$, and $j = 56$ at different evolution times in one period: (a) $t = 0T$, (b) $t = 3T$, (c) $t = 6T$, and (d) $t = 8T$. The quantum recurrence features enable a highly efficient metrology protocol: with the initial state $|\Theta, \Phi\rangle = |(\pi/4), (\pi/2)\rangle$ (a) and the final state (d) staying as a coherent state, the intermediate quantum states (b) and (c) are GHZ-like, highly entangled states potentially yielding a rapid growth in the QFI.

that of an ensemble of classical chaotic trajectories, quantum resonance dynamics can be connected with stable pseudo-classical trajectories, and as such we can obtain stable time-evolving wave packets: they are not spreading with irregular patterns but instead split into multiple well-separated wave packets during the time evolution. This ensures that the resonance dynamics remain controlled and conducive to generating a rapid increase in the QFI throughout the time evolution [71]. Indeed, leveraging the solvable nature of quantum resonance dynamics, we have analytically obtained that the scaling of the QFI is given by $I(t) \propto t^2$ (see Supplemental Material [93]). Computer simulations further confirm this prediction of the QFI growth. The simulation results, shown in Fig. 3(a), reveal that this quadratic scaling remains robust under small perturbations δ to the value of β , thus making our protocol experimental feasible. More discussions on this point can be found in Supplemental Material [93]. In contrast to the Ehrenfest time limitation imposed on all sensors based on fast wave packet spreading, the unbounded long-time quadratic growth of QFI under quantum resonance dynamics is also confirmed in Fig. 3(a).

Computer simulations of the quantum dynamics with varying system sizes also confirm a perfect Heisenberg limit (HL) scaling, namely, $I(N) \propto N^2$, as shown in Fig. 3(b). As conjectured above, this desirable scaling can be connected with the generation of highly entangled states at times other than perfect quantum recurrence times, as illustrated in Fig. 1. Remarkably, if we now examine the dynamical sensitivity or the QFI over many periods of quantum

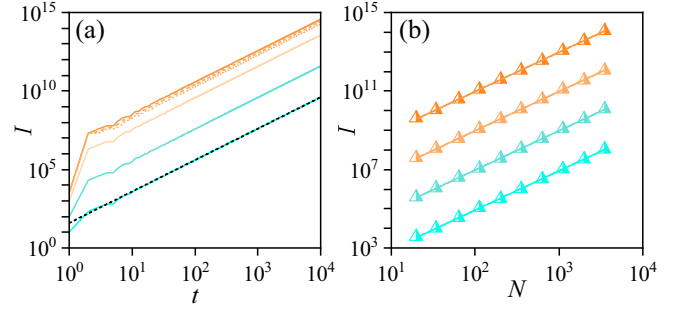


FIG. 3. Dynamics of the QFI associated with the quantum resonances of the QKT. (a) The time evolution of the QFI. Cyan to orange curves represent different system sizes ranging from $N = 20$ to $N = 6324$, with $\beta = \pi j$. The dotted line corresponds to $\beta = \pi j + \delta$ with $\delta = 1.5$, and the dashed line represents $\delta = 2.0$. The black dashed line represents a power-law fit, showing $I \sim t^a$ with $a \approx 2$. (b) The dependence of the QFI on the number of spins for $\beta = \pi j$ at different times $t/T = 10, 10^2, 10^3, 10^4$ (from cyan to orange). The solid lines represent a fitting with $I \sim N^a$, yielding $a \approx 2$, demonstrating the Heisenberg scaling.

recurrence, we find that this Heisenberg scaling persists, and hence in terms of both time and the number of spins, we have $I_{\text{QKT}} \propto t^2 N^2$, the ultimate Heisenberg scaling of the QFI that can be generated by unitary time evolution [94]. This combination of advantageous scaling for an initial coherent state really highlights the power of quantum resonance dynamics in enhancing quantum metrology. Indeed, as a comparison, the chaotic kicked top dynamics can only generate tN^2 scaling of the QFI, up to the Heisenberg time, followed by a behavior of $t^2 N$ scaling due to the saturation of the spreading of the time evolving state [52].

Noisy quantum metrology—Dissipative noise remains a significant challenge in modern quantum metrology experiments by limiting the information encoding time. Existing no-go theorems for noisy quantum metrology [18,19,43–51] best illustrate this point. However, because the time-periodic interaction to engineer the quantum resonance dynamics also places our quantum system under a nonequilibrium condition, previous no-go theorems may not apply [50]. We thus examine the dissipative dynamics of the QKT under Markovian noise to examine the possible quantum advantages of our metrology protocol under an adverse environment. The quantum resonance dynamics under Markovian noise can be modeled by the Master equation $\dot{\rho}(t) = -i[H, \rho(t)] + \mathcal{D}[\rho(t)]$. Here we consider superradiant damping [95,96], which allows simulations for large system sizes up to $N = 2 \times 10^3$. The superradiant damping can be expressed analytically as $\mathcal{D}[\rho(t)] = \gamma([J_-, \rho(t)J_+] + [J_-, \rho(t), J_+]) \equiv \Gamma\rho(t)$ with $J_{\pm} = J_x \pm iJ_y$ and γ denoting the dissipation strength [97]. Consequently, the evolution of the density operator from t to $t + T$ is given by

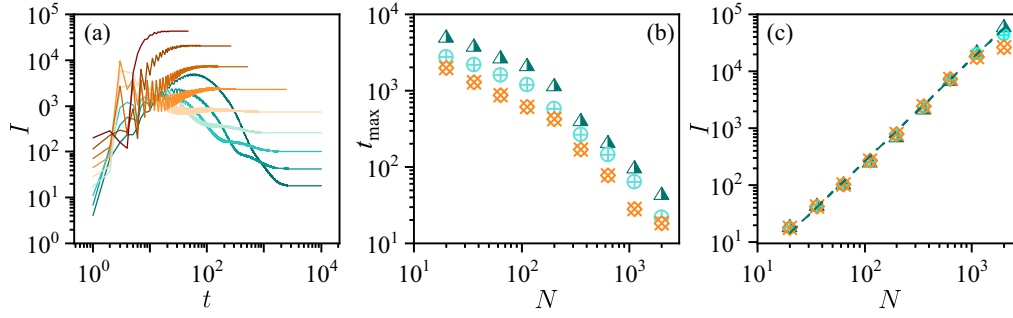


FIG. 4. The QFI in quantum resonance dynamics with dissipation. (a) Time evolution of the QFI in the quantum resonance dynamics of the QKT. Curves from cyan to orange represent different system sizes ranging from $N = 20$ to $N = 2000$, with $\beta = \pi j$. (b) Dependence of $t_{\max}$ on the number of spins, at dissipation rates $\gamma = 0.0003, 0.0005, 0.0007$ (from dark green to red). (c) Dependence of the saturated QFI on the number of spins for $\beta = \pi j$. The dashed line represents a fit with $I \sim N^a$, yielding $a \approx 1.8$, which shows a near-Heisenberg scaling. System parameters are chosen to be the same as in panel (b) for different symbols.

$$\rho(t + T) = U[\exp(\Gamma T)\rho(t)]U^\dagger. \quad (3)$$

We present in Fig. 4(a) our simulation results of the QFI dynamics, again starting from a coherent state. As expected, a saturation of the QFI is observed after a certain time $t_{\max}$. This time threshold $t_{\max}$ certainly reflects a balance between dynamics-generated quantum features and decoherence. Figure 4(b) presents a quantitative relation between $t_{\max}$ and the number of spins. Of more interest is the scaling of the saturated QFI with the number of spins. Notably, we find a near-Heisenberg scaling with $I \propto N^a$, where $a \rightarrow 1.8$, as shown in Fig. 4(c). Such near-HL scaling, achieved with an initial coherent state in a noisy environment, represents significant progress as compared to many other metrology protocols that require the use of a GHZ state as the initial state. The use of quantum resonance dynamics is hence useful in enhancing quantum metrology even in the presence of an environment.

Conclusion—We have shown a number of advantages in using one class of quantum resonance dynamics for quantum metrology. The approach advocated in this work has capabilities to overcome current key challenges in pushing the precision limit of quantum metrology, including the inefficient QFI scaling of initial coherent states under unstable time evolution, the difficulty in preparing GHZ-like states, the limitations imposed by existing no-go theorems in noisy metrology, and the readout noise of highly entangled states emerging at the end of a metrology protocol. The quantum recurrence intrinsic to the quantum resonance dynamics examined here enables highly efficient encoding and readout procedures. As a result, the QFI of even analytically solvable quantum resonance dynamics exhibits the Heisenberg scaling with the number of spins. Furthermore, both numerical and analytical investigations for closed systems show that the growth of the QFI can be quadratic in time without a bound, even for times far beyond the Heisenberg time. For the quantum resonance dynamics with dissipation, thanks to the nonequilibrium

nature of our system, near-Heisenberg scaling persists. This work is of immediate experimental interest because we only need to place the already experimentally realized dynamical model in a parameter regime different than before. In the future we plan to extend our approach to other settings, such as atomic spin-precession magnetometers [98–102].

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Any opinions, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not reflect the views of National Research Foundation, Singapore.

Data availability—The data that support the findings of this Letter are openly available [103].

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