Phase Transition of the Kardar-Parisi-Zhang Equation in Four Substrate Dimensions

In a recent Letter [1], Lässig and Kinzelbach have studied the phase transition of the Kardar-Parisi-Zhang (KPZ) equation [2] and have claimed that the KPZ equation has an upper critical dimension $d_> \leq 4$, where the exponents in the strong coupling regime take the values $z = 2$ and $\chi = 0$, equal to those at the roughening transition. In this Comment, we have studied the finite temperature directed polymer on a random potential on four substrate dimensions and have found $z \approx 1.75$ and $\omega (= \chi/z) = 0.15 \pm 0.01$ in low temperature regime (strong coupling regime) and $z \approx 2$, $\omega \approx 0$ at the transition temperature.

The finite temperature directed polymer on a random potential problem in $d = 1$, 2, and 3 was studied in the literature [3]. Here we extend it to dimension $d = 4$. Consider a directed polymer on a discrete "hyper-pyramid" structure with random potential $\mu(\mathbf{r}, t)$ assigned to each site $(\mathbf{r}, t)$ where $\mathbf{r}$ is the d dimensional transverse vector and t is the longitudinal length of the polymer. The polymer starts from $\mathbf{0}$ and its path is restricted by $|\mathbf{r}(t) - \mathbf{r}(t+1)| = 0$ or 1. There is a bending energy γ against a transverse jump $|\mathbf{r}(t) - \mathbf{r}(t+1)| = 1$. The partition function $Z(\mathbf{r}, t)$ for the polymer ending at $(\mathbf{r}, t)$ can be obtained recursively. For example, in $d = 1$

$$\begin{aligned} Z(r,t) = {} & Z(r,t-1)\ \exp(-\tfrac{\mu(r,t-1)}{T}) \\ & + Z(r-1,t-1)\ \exp(-\tfrac{\mu(r-1,t-1)+\gamma}{T}) \\ & + Z(r+1,t-1)\ \exp(-\tfrac{\mu(r+1,t-1)+\gamma}{T}), \end{aligned} \quad (1)$$

where T is the temperature. The $\mu(r, t)$ are uncorrelated random variables between 0 and 1.

We define a total partition function $Z(t)$ as $\sum_{\mathbf{r}} Z(\mathbf{r}, t)$ and the related free energy $F(t)$ as $-T \ln Z(t)$. We are interested in both the free energy fluctuation

$$\Delta F(t) = [\overline{F^2(t)} - \overline{F(t)}^2]^{1/2}, \quad (2)$$

which is a similar quantity as the surface width in growth models [4], and the transverse fluctuation of the polymer $\overline{\langle \mathbf{r}^2 \rangle}$ where $\overline{A}$ is the sample average of $\mathbf{A}$ and the symbol $\langle A \rangle$ is the thermal average of a quantity $\mathbf{A}$ by

$$\langle A \rangle \equiv \frac{\sum_{\mathbf{r}} A Z(\mathbf{r}, t)}{\sum_{\mathbf{r}} Z(\mathbf{r}, t)}. \quad (3)$$

These quantities are characterized by the scaling exponents ω and z by

$$\Delta F(t) \sim t^{\omega} \quad \text{and} \quad \overline{\langle \mathbf{r}^2 \rangle} \sim t^{2/z}. \quad (4)$$

We have measured $\overline{\langle \mathbf{r}^2 \rangle}$ as a function of time t for various temperatures. As shown in Fig. 1, for $T = 0.30$, $\overline{\langle \mathbf{r}^2 \rangle}$ is linear to t implying $z = 2$. It increases as $t^{1.14}$ with $z \approx 1.75$ at $T = 0.05$. So, the value of z in the low temperature phase is less than two.

To look at the transition between these two phases more clearly, we monitor a dimensionless quantity [3]

$$g(T, t) \equiv \overline{\langle \mathbf{r} \rangle^2} / \overline{\langle \mathbf{r}^2 \rangle}, \quad (5)$$

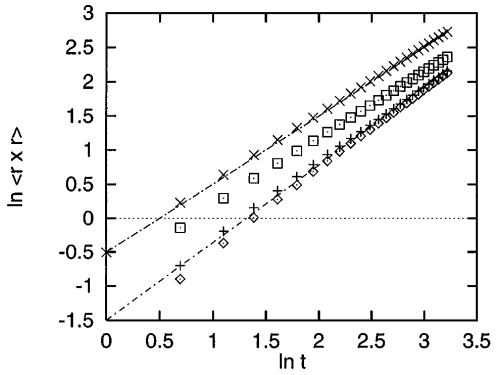

FIG. 1. $\overline{\langle \mathbf{r}^2 \rangle}$ as a function of t in log-log plot. The data are for $T = 0.30$, 0.18, 0.10, and 0.05 from the top to the bottom. The slope of the upper guide line is one, which should be $2/z$ ($z = 2$) and that of the lower guide line is 1.14 ($z \approx 1.75$ for $5 < t < 25$).

which depends on T and t. At $T = \infty$, since the randomness becomes irrelevant, we expect $g = 0$. On the other hand, at $T = 0$, only one path is dominant that $\langle \mathbf{r}^2 \rangle = \langle \mathbf{r} \rangle^2$ and g becomes unity. We have seen that g increases as a function of t for $T < 0.17$ and decreases for $T > 0.19$. Near $T = 0.18$, g is independent of the length t. So the critical temperature T_c is 0.18 ± 0.01.

We also measure the free energy fluctuation up to $t = 200$. At $T = 0.05$ (low temperature regime), ΔF shows a very good power law behavior t^{ω} with $\omega = 0.15 \pm 0.01$. However, at $T_c = 0.18$, $(\Delta F)^2$ grows as $\ln t$ and at $T = 0.30$ (high temperature regime), ΔF becomes saturated as t increases implying $\omega = 0$. The similar behavior is observed in both the directed polymer problem [3] and the surface width of a growth model on $d = 3$ [5].

Our results show that $z \approx 1.75$, $\omega \approx 0.15$ for $T = 0.05$ ($\ll T_c$), and $z \approx 2$, $\omega \approx 0$ for $T \geq T_c$ on $d = 4$. $T_c = 0.18 \pm 0.01$ ($d = 4$) is close to 0.23 of the critical temperature on $d = 3$ [3].

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