where

$$g=0.10(\varphi'''r/\varphi'')^2,\quad \alpha=0.12(\varphi^{iv}r^2/\varphi''),\quad \text{and } r=\hbar^{1/2}(M\varphi'')^{-1/4}.$$

The denominator in Eq. (3) is about unity for neon. The constant $g$ may be found either by doing the self-consistent average or by relating $g$ to the Grüneisen constant. These give, respectively, $\delta a/a = -0.0012c$ and $-0.0014c$. Putting $c=0.095$, we can, in particular, find the difference in the lattice constants of natural neon and neon-20. The result is $\delta a/a = -0.000\,11$ and $-0.000\,13$ respectively, versus the experimental result[1,5] of $-0.000\,14 \pm 0.000\,03$. The agreement is within experimental error. However, more precise measurements are needed over a wider range of concentrations and for samples whose chemical purities are known to be identical.[6]

The solution of Eqs. (1) and (2) for $\delta\Gamma(kk')$ determines the change in the force constants of the crystal and therefore can be used to find the scattering rate for phonons from the mass defect plus strain field. The formula is

$$\tau(k)^{-1}=8\pi Nc\textstyle\sum_{k'}\delta(\omega-\omega')[2\delta\Gamma^0(kk')-\delta\Gamma(kk')]^2(1-\cos\theta_{kk'}), \tag{4}$$

where

$$\delta\Gamma^0(kk')=(\delta M/M)(2N)^{-1}(\omega\omega')^{-1/2}(\omega+\omega')^{-1}.$$

When there is no anharmonicity in the solid, $\delta\Gamma(kk')$ becomes $\delta\Gamma^0(kk')$. Then Eq. (4) gives the usual formula for mass-defect scattering.[7]

When this is applied to neon, we find that the scattering rate will be 1.4 times that for the mass defect alone. In view of the lattice-constant experiments, the scattering might be even larger than this. In any case, the effect should be sufficiently large to be observable in a thermal-conductivity experiment.

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## SELECTIVE MEASUREMENT OF WAVE-FUNCTION AMPLITUDES OF ELECTRONS ON SPECIFIC ORBITS ON FERMI SURFACES*

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A method of determining wave-function amplitudes on different sheets of the Fermi surface of pure metals through de Haas–van Alphen type oscillations in the Knight shift is given. Experimental results and an analysis for Cd are presented.

Measurements of the Knight shift $\sigma$ in the NMR signal in metals have been used for many years to determine the average amplitude of the conduction-electron wave functions. This is possible since, for constant frequency, the field at which the NMR line is observed in a metal is shifted from the observed field for the same nuclei situated in a salt by an amount

$$\frac{B-B_0}{B}=\sigma=\frac{8\pi}{3}\chi_p\langle|\psi(0)|_{E_F}^2\rangle_{\rm av}V, \tag{1}$$

where $B$ is the resonant field in the metal, $B_0$ is the resonant field in the salt, and $\chi_p$ is the paramagnetic susceptibility of the conduction elec-

trons. Diamagnetic corrections to this expression are found to be small. The wave-function amplitude $\langle|\psi(0)|_{E_F}{}^2\rangle_{av}$ refers to the average amplitude of electrons on the Fermi surface (FS) evaluated at the nuclear site, and $V$ is the volume in which the wave functions are normalized. This shift is due to the contact part of the conduction-electron nuclear hyperfine interaction.

The expression in Eq. (1) is the result of a summation over single-particle states of electrons at the FS. Thus, a more general form for $\sigma$ is given by[1]

$$\sigma=\frac{16\pi\mu_0{}^2}{3}\sum_{k,n}|\psi_{k,n}(0)|^2\delta(\zeta-E_n(k)), \tag{2}$$

where $\mu_0$ is the electronic Bohr magneton, $\psi_{k,n}(0)$ is the wave function of electrons of wave vector $k$ and band index $n$ evaluated at the nuclear site, $\zeta$ is the Fermi energy, and $E_n(k)$ is the band energy of electrons of wave vector $k$ and band index $n$. In the presence of high magnetic fields, where Landau quantization in the electronic states is appreciable, the extremal area orbits on the FS occurring in the above sum will give rise to an oscillatory component[2] $\tilde{\sigma}$. Accordingly, one may write the Knight shift as

$$\sigma+\tilde{\sigma}=\frac{8\pi}{3}\chi_p\langle|\psi(0)|_{E_F}{}^2\rangle_{av}V + \frac{8\pi}{3}\tilde{\chi}_p\langle|\psi(0)|_{E_F}{}^2\rangle_{o\,av}V, \tag{3}$$

where $\langle|\psi(0)|_{E_F}{}^2\rangle_{o\,av}$ means that the wave function is averaged over a particular orbit on the FS, and $\tilde{\chi}_p$ is the oscillatory part of the paramagnetic susceptibility due to electrons on the same orbit. By measuring the amplitudes of $\tilde{\sigma}$ one can determine values for the amplitudes of electrons on particular orbits on the FS. This gives rise to the interesting possibility of mapping through angular dependent studies the amplitudes of wave functions of electrons on the FS in a detailed manner much the same as the FS is mapped via the de Haas–van Alphen (dH-vA) effect.

In order to illustrate the possibility of obtaining values of these localized wave-function amplitudes, measurements have been made of the amplitude of $\tilde{\sigma}$ for electrons on an orbit on the first-zone "caps"[3] in high-purity single crystals of cadmium. Results of the field-independent measurements of $\sigma$ are shown in Fig. 1. All of the experimental technique used in the present investigation to measure $\tilde{\sigma}$ is described in Khan, Reynolds, and Goodrich.[4] Cadmium is a particularly convenient material to investigate since

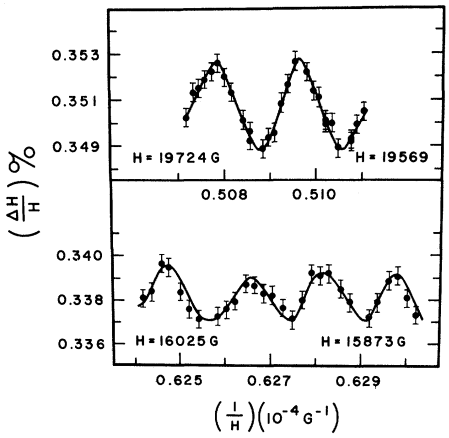


FIG. 1. Examples of the Knight shift in Cd as a function of applied field in two different field ranges.

$Cd^{113}$ is an abundant spin-$\frac{1}{2}$ isotope and the FS of Cd is well known.[3] The particular orbit investigated ($H$ oriented 30° from [0001] in the [10$\bar{1}$0] plane) is one on the first zone which in the range of fields used (14-20 kG) does not experience magnetic breakdown, and in this range no other oscillations are observed. In addition, recent calculations by Falicov and Kasowski[5] give values for $|\psi(0)|^2$ on various parts of the FS of Cd with which the results can be compared.

From Eq. (3) it is seen that

$$\frac{\tilde{\sigma}}{\sigma}=\frac{\tilde{\chi}_p}{\chi_p}\frac{\langle|\psi(0)|_{E_F}{}^2\rangle_{o\,av}}{\langle|\psi(0)|_{E_F}{}^2\rangle_{av}}. \tag{4}$$

Detailed measurements of the dH-vA amplitude in the magnetic susceptibility, for electrons on the same orbit for which $\tilde{\sigma}$ was observed, were made in order to determine the number of carriers contributing to the oscillations $\tilde{n}$ so that $\tilde{\chi}_p/\chi_p=\tilde{n}/n$ could be computed.[6] These measurements were performed in a force magnetometer (Faraday type)[7] on a thin slice taken from the NMR sample. The value of $\tilde{n}$ was obtained from these data by a nonlinear least-squares fit of approximately 80 field points to the expression for the dH-vA effect with corrections for the field inhomogeneity in the magnetometer and the Dingle factor.[8]

The value obtained for $\tilde{\sigma}/\sigma$ is $0.45\times10^{-2}$, and the value of $\tilde{n}/n$ is found to be $0.27\times10^{-2}$ using the data of Grenier, Efferson, and Reynolds[9] for $n$. This gives a value of $\langle|\psi(0)|_{E_F}{}^2\rangle_{o\,av}/\langle|\psi(0)|_{E_F}{}^2\rangle_{av}=1.67$. Although Kasowski[5] has not calculated values of $|\psi(0)|_{E_F}{}^2$ for the first zone in Cd, his values for points on the second zone range from 0.309 to 1.88 which when scaled by his value of

$\langle|\psi(0)|_{E_F}^2\rangle_{av} = 0.74$ gives a range for the ratio of 0.418 to 2.541, easily bracketing the measured value on the first zone.

A complete discussion of the method of analysis and the results of the entire field- and temperature-dependent study of $\sigma$ in Cd are to be reported later.

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## PHOTOEMISSION OBSERVATION OF SURFACE-PLASMON EXCITATION IN THE ALKALI METALS*

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Measured energy distributions of photoemitted electrons from K, Rb, and Cs show a high-energy peak identified as unscattered (primary) electrons, and an intermediate peak at somewhat lower energies which is believed to be due to a plasmon energy loss. This intermediate structure becomes more pronounced and closer to the primary peak on proceeding from K to Rb to Cs. This systematic trend, together with the absence of any clearly discernible intermediate structure in Na, is consistent with the variation of the corresponding plasmon energies. In Rb and Cs the intermediate peak is sufficiently pronounced to permit its identification as a surface rather than a volume-plasmon excitation.

The electron-electron interaction has an important influence on the photoemission process.[1] The photoemission process is conventionally envisaged in three distinct steps: (1) optical excitation of electrons in the interior of the material, (2) transport of some of these electrons to the surface, and (3) escape across the surface. In the transport to the surface, the excited electrons are quite likely to suffer an inelastic scattering event before they can escape into the vacuum. Scattering by electron-electron interactions is believed to be the dominant process, and the scattering length is a rapidly decreasing function of energy.[2] The number of electrons which escape without scattering (primary electrons) is therefore severely diminished. Moreover, electrons which have undergone an inelastic scattering event may still be sufficiently energetic to surmount the potential barrier at the surface, and escape from the material as slower secondary electrons. It is these secondary photoelectrons which will concern us here.

An excited electron may decay through the electron-electron interaction by either of two basic mechanisms, pair creation or plasmon creation.[3] We include in the latter both surface- and volume-plasmon losses. The energy distribution of secondaries is expected to differ for these two mechanisms. The contribution to the energy distribution curve (EDC) due to secondaries pro-