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CYCLOTRON WAVE TRANSMISSION ACROSS A PLASMA*

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The relativistic Vlasov dispersion relation is used to find the critical density $n_c = 3B_0^2\kappa T/8\pi m^2c^4$ below which the extraordinary wave can propagate freely across a mirror-field plasma into the cyclotron-resonance region.

It is well known that an electron can be accelerated by the extraordinary electromagnetic wave propagating in the direction perpendicular to a constant magnetic field. This wave has its electric-field vector in the plane of the Larmor orbit of the electron. When this single-particle model is extended to an electron in a plasma, the problem is, however, more complicated. Consider a plasma that is confined in a mirror field. If the wave frequency is equal to the electron cyclotron frequency in the interior of the plasma, the wave can in general no longer propagate freely into the resonance region. At some point before reaching the cyclotron-resonance region, the wave will be at least partially reflected in an evanescent region determined by the collective properties of the plasma, for which $\omega_{\mathrm{uh}} < \omega < \omega_{\mathrm{uc}}$, where $\omega_{\mathrm{uh}} = (\omega_p^2 + \omega_c^2)^{1/2}$ is the upper hybrid frequency and $\omega_{\mathrm{uc}} = \omega_c/2 + [(\omega_c/2)^2 + \omega_p^2]^{1/2}$ is the upper hybrid cutoff frequency. The local plasma and cyclotron frequencies are ω_p and ω_c, respectively.

In this paper, the relativistic dispersion relation for a thermal Vlasov plasma is used to analyze the propagation of these extraordinary waves as a function of density and temperature. The plasma dispersion relation[1] for the waves with $\vec{\mathrm{E}} \perp \vec{\mathrm{B}}_0$, $\vec{\mathrm{k}} \perp \vec{\mathrm{B}}_0$ is

$$N^2 = [1-\delta(R_{10}+R_{30})][1-\delta(R_{10}-R_{30})][1-\delta R_{10}]^{-1} \tag{1}$$

to the lowest order in temperature, where N is the index of refraction, $\delta = \omega_p^2/\omega^2$,

$$R_{10} = [\alpha^2/3K_2(\alpha)]\int_0^\infty \cosh t \exp(-\alpha \cosh t) \sinh^4 t P(t)dt = A_1 + iS,$$
$$R_{30} = [\alpha^2/3\nu K_2(\alpha)]\int_0^\infty \exp(-\alpha \cosh t) \sinh^4 t P(t)dt = A + iS, \tag{2}$$

$P(t) = [\cosh^2 t - \nu^{-2}]^{-1}$, $\alpha = mc^2/\kappa T$, $\nu = \omega/\omega_c$, and K_2 is the modified Hankel function. Note that $S = 0$ for $\nu \geq 1$.

If $\nu \neq 1$, the $\cosh^2 t$ term in $P(t)$ may be replaced by 1 for low-temperature plasmas. Thus, $R_{10} = (1-1/\nu^2)^{-1}$, $R_{30} = \nu^{-1}R_{10}$ and the dispersion relation reduces to the classical zero-temperature expression. The upper hybrid frequency is found by setting $1-\delta R_{10} = 0$ and the upper hybrid cutoff by setting $1-\delta(R_{10}+R_{30}) = 0$ in Eq. (1). As the plasma density is reduced, both the cutoff and the hybrid frequencies approach the cyclotron frequency since $N \propto \delta$. In order to treat this limit quantitatively, however, it is no longer correct to set $\cosh t = 1$ in $P(t)$. The integral must instead be evaluated more accurately either by numerical integration in higher temperature plasmas, or by expanding these hyperbolic terms to the first order in $\sinh^2 t$, when the temperature is low.

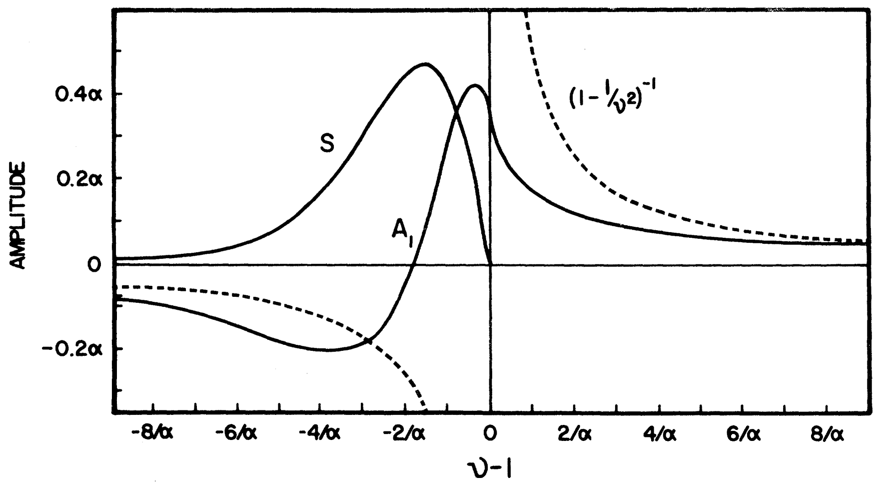

FIG. 1. The solid curves are numerical evaluations of the real, A_1, and imaginary, S, parts of the dispersion term R_{10} at nonrelativistic temperatures. The dashed curve represents the corresponding term as predicted by the classical theory.

The real and imaginary parts of R_{10} given by Eq. (2) have been calculated numerically. In Fig. 1, they are plotted as functions of frequency in the neighborhood of the cyclotron frequency for $\nu \approx 1$. The dashed curve represents the classical dispersion term $R_{10} = [1-1/\nu^2]^{-1}$. Note that the principal-part integral A_1 approaches the classical curve away from the resonance. However, the relativistic dispersion relation which includes the energy dependence of the cyclotron frequency must be used in the immediate neighborhood of the resonance even at low temperatures. At nonrelativistic temperatures $R_{30} \approx R_{10}$ in this region.

The classical dispersion theory predicts the existence of an upper hybrid frequency and upper hybrid cutoff frequency for some $\nu > 1$ independent of the density since R_{10} becomes arbitrarily large as $\nu \to 1$. However, the relativistic term A_1 is finite for all ν so that these two frequencies in fact only exist above some minimum density for which the equations $1-\delta R_{10} = 0$, $1-\delta(R_{10}+R_{30}) = 0$ can be satisfied.

At $\nu = 1$, the integrals (2) can be evaluated analytically giving $R_{10} = \alpha/3$ and $R_{30} = R_{10}K_1(\alpha)/K_2(\alpha)$. Since Eqs. (2) decrease monotonically with ν for $\nu \geqslant 1$, the condition $\delta(R_{10}+R_{30}) \leqslant 1$ at $\nu = 1$ insures that there is no upper hybrid cutoff frequency for $\nu > 1$. At a given nonrelativistic temperature and magnetic field, $n_c = 3B_0^2/8\pi mc^2\alpha$ is the critical density below which the waves can propagate freely into the cyclotron-resonance region. In the highly relativistic limit, $\alpha \to 0$, this critical density is increased by a factor of 2.

For $\nu < 1$, Eqs. (2) have simple poles. Now the principal-part integrals $A_{1,2}$ give the classical dispersion relation. The pole terms S, however, add an imaginary part to R_{10} and R_{30}, representing the cyclotron-resonance coupling of the electrons. Drummond and Rosenbluth[2] summed over similar pole contributions at higher harmonics of the cyclotron frequency in calculating the synchrotron radiation from a relativistic plasma.

In low-temperature laboratory plasmas $n \gg n_c$ and the interaction between a cyclotron wave and the plasma is dependent upon its transmission

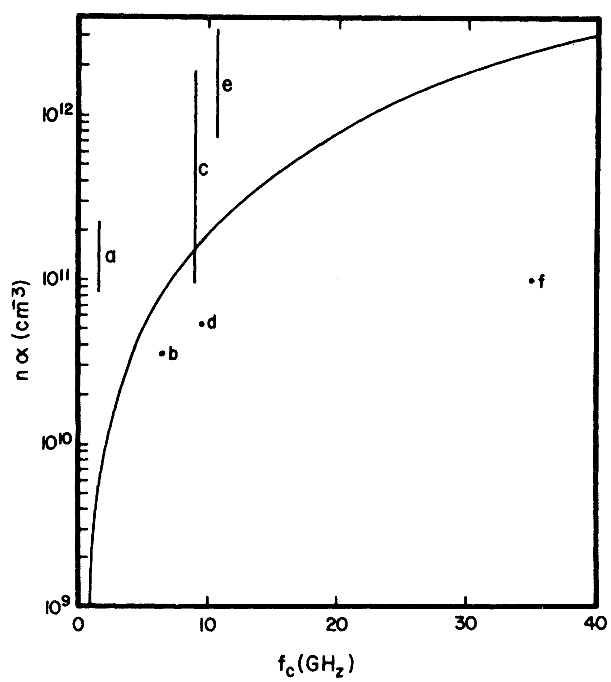

FIG. 2. The experimental values of $n\alpha$ attained in electron-cyclotron heating experiments by (a) Alikaev,[3] (b) Ikegami,[4] (c) Hartmen,[5] (d) Golant,[6] (e) Ard,[7] and (f) Ard[8] compared with the theoretical curve for $n_c\alpha$ as a function of the cyclotron frequency.

properties through the upper hybrid frequency regime. In the cyclotron resonance regime where $\nu \lesssim 1$, $\mathrm{Re}(N^2) = 2$, and $\mathrm{Im}(N^2) \propto (\delta\alpha)^{-1} \ll 1$ for these waves.

In the limit of very low densities, $n \ll n_c$, the interaction at the cyclotron frequency can be described as a sum of single-particle interactions. $\mathrm{Re}(N^2) = 1$ and $\mathrm{Im}(N^2) \propto \delta\alpha \ll 1$ for the waves.

When $n \lesssim n_c$, the waves can propagate freely into the cyclotron-resonance region. Then $\mathrm{Re}(N^2) \approx \mathrm{Im}(N^2) \propto 1$ for $1-\nu \propto \alpha^{-1}$. The density of the plasma is great enough under these conditions to permit an appreciable exchange of energy between the wave and the particles, without being so great as to affect the penetration of the waves into the cyclotron-resonance region. It may be shown that in this density regime the absorption coefficient per plasma particle, which is proportional to $\mathrm{Im}(N)/\delta$, may be appreciably larger than that predicted by the single-particle model. Under such conditions the coupling of the wave energy into the plasma may be most effective.

A number of experiments[3-8] have been performed in the last few years on the microwave heating of electrons by waves propagating perpendicular to a mirror field. In such steady-state plasmas, densities $n \approx n_c$ are achieved, as may be seen from Fig. 2. The theory predicts that the hot electron density can attain this value relatively easily. In order for it to increase further, however, the cyclotron wave must tunnel through the evanescent upper hybrid frequency region. Destructive turbulence induced by the wave in the evanescent region could provide a mechanism that limits the density in cyclotron-resonance plasmas.

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MEASUREMENT OF SHORTING CURRENTS FROM A THETA-PINCH PLASMA*

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Shorting currents from a theta-pinch plasma were measured by observing the azimuthal magnetic field, B_θ, outside the compression coil. The total current and the radial distribution of the current agree well with values predicted by a theoretical rigid-rotor model.

In a theta pinch of finite length, where the magnetic field lines from the plasma region intersect the discharge-tube wall, it is expected that any radial electrostatic fields, E_r, in the plasma region would be shorted out by electric currents which would flow along the magnetic field lines and the discharge-tube wall. These currents would give rise to an azimuthal magnetic field, B_θ. This paper reports the results of an experiment which measured the magnitude and spatial distribution of the B_θ field in the region outside the compression coil of a short, high-voltage theta pinch.

The experiment was performed on the Scylla-IA theta pinch,[1] a 90-kJ (at 50 kV main bank charging voltage) theta pinch with a compression coil 10 cm in diameter and 20 cm long. The main compression field rises to its maximum in 1.75 μsec. At this time it is crowbarred with a modulation of 30%. The experiment was run with an initial filling pressure of 24 mTorr D_2 and a main bank charging voltage of 40 kV. Under these conditions[1] the plasma at peak compression is characterized by the following parameters: ion temperature $T_i = 1.9$ keV, electron temperature $T_e = 300$ eV, $n(\text{on axis}) = 2.5\times10^{16}/\text{cm}^3$, and $\beta(\text{on axis}) = 0.4\text{-}0.7$.

The magnetic probe was a four-turn coil, 0.16