more slowly than S_x or Q. This may be shown analytically from a more complete set of equations of motion (which would include an equation for S_z), or may be seen from a picture of the precessing spin system. Moreover, while Q and S_x oscillate through zero with approximately sinusoidal motion, S_z need not change by much compared to its average value. The value of S_z is determined by the total energy of the spin systems, which is $\alpha S_z H_0$.

Consider now a situation in which the total energy in the cavity (that of the spins and the rf field) is less than half the maximum spin energy. Then, S_z is negative. Replace S_z in Eq. (2) by its average value, $\bar{S}_z$. Eqs. (1) and (2) now become the linear equations for two coupled harmonic oscillators, the field and the spins. It is well known, and can also be calculated easily from the equations of motion, that in such a situation the energy transfers back and forth from one oscillator to the other, the modulation frequency of the energy of each oscillator being ν_e, where $\nu_e^2 = -\pi^{-1}\omega^{-1}C^2\alpha^2 a_x^2 \bar{S}_z$. The accuracy of this expression increases as the total energy is reduced, since then $\bar{S}_z$ is a better approximation for S_z, and the coupling between the two oscillators becomes more linear. But it seems evident that the explanation for the modulation is the well known periodic transfer of energy between two coupled oscillators.[5] Damping caused by relaxation effects and cavity losses may destroy the modulation only if it is large enough to dissipate all the energy in one modulation cycle. Otherwise, it will affect the modulation phenomenon mainly through the (slight) dependence of the modulation frequency on the total energy, because of the nonlinear coupling. Nonuniformities in the dc field might conceivably blur the modulation phenomenon; and, according to the above theory, it would seem that the more uniform the magnetic field is, the more clear-cut the modulation would be.

Finally, we compare the calculated modulation frequency ν_e, above, with the experimental frequency, which can be obtained easily from the display of power output vs time in Fig. 1 of reference 2. For the lower-energy portion of the curve it is about 2×10^6 cycles per second. The authors also give the following data: $\omega/2\pi = 9\times10^9$ cycles/sec, the total number of electron spins is $\sim10^{18}$, and the temperature of the crystal is 4.2°K. The last two facts imply that $\bar{S}_z \sim 10^{16}\hbar$. For a_x^2 we assume the value of $|\vec{a}|^2$ averaged over the volume V of the cavity, $\langle a^2\rangle_{\mathrm{Av}} = \omega^2 C^{-2} V^{-1}$, where V is 12 cm^3.[3] Bearing in mind that $\alpha = 2\mu_0/\hbar$ where μ_0 is a Bohr magneton, we obtain $\nu_e = 2\times10^6$ cycles/sec, in agreement with the experimental data.

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DIRECT OBSERVATION OF SPIN-WAVE RESONANCE*

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It has been generally understood that the effects of exchange interactions on ferromagnetic resonance in metals can be observed experimentally provided the rf field gradient is sufficiently great.[1-6] A theoretical treatment for the case of a thin metallic film leads to the prediction of "body resonances" due solely to such an rf gradient.[7,8] However, Kittel[9] has now proposed a boundary condition which permits excitation of these resonances even by a uniform rf field and he assigns to them the more appropriate term "spin-wave resonances." Here the spins are simply assumed to be pinned down at the boundaries of the specimen due to a surface anisotropy. Kittel points out that thin films should be particularly well suited for observation of spin-wave resonance.

Figure 1 shows the microwave absorption spectrum at 8890 Mc/sec for a 80% - 20% Permalloy film of 5600 A thickness. The dc field is perpendicular to the film surface. The various

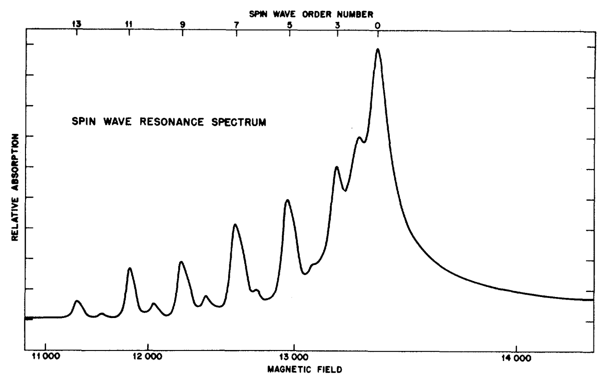

FIG. 1. Experimental observation of spin wave resonances in a 5600 A film of permalloy.

resonances are interpreted as arising from excitation of spin waves obeying the dispersion law

$$\omega = (2A\gamma/M)\,k^2 + \omega_0, \qquad (1)$$

and the Kittel condition for spin wave resonance

$$k = p\pi/L, \qquad (2)$$

where A is the exchange coupling constant, M the magnetization, L the film thickness and p an integer. A value for A of $(2.0 \pm 0.03) \times 10^{-6}$ erg/cm is obtained by fitting the resonance field data to the above expressions (1) and (2) for the odd modes $p = 7$ to 13. The small peaks between the large ones in the figure may be interpreted as even-mode resonance (even number of half-wavelengths) which in the absence of an rf field gradient would not be excited.

The exact exchange-effect theory worked out by the authors[7] when applied to this particular film yields highly damped peaks at the field values predicted by (1) and (2) for $p > 3$. The large magnitudes which are actually observed can thus be explained on the basis of the Kittel boundary condition, which permits odd-mode excitation even by a uniform field. The exact calculation,[7] however, shows that for $p < 3$ the dispersion relation (1) does not hold and the Q (proportional to $\mathrm{Re}k/\mathrm{Im}k$) of the spin waves[10] becomes small. In fact the mode $p = 1$ can be shown to fall on the high-field side of the main resonance and to be damped out in a distance less than one-half wavelength. Hence the first spin-wave resonance in the figure corresponds to $p = 2$. It should also be pointed out that the exact theory yields a line width for the main resonance of ~150 oe, a value quite close to the observed one.

The resonance field data only approximately follow the p^2 law for $p > 3$; a better experimental fit could be obtained by using an exponent less than two. This may be explained qualitatively by considering the strength of the spin pinning. It can be seen from Eq. (15) of reference 9 that the pinning at the boundaries gets weaker with increasing spin wave number thus causing a progressive shift of the resonance peaks.

The direct observation of spin-wave resonances thus provides a straightforward way of obtaining the exchange constant A from the location of the spin-wave peaks. Furthermore, the damping of isolated spin waves of a given wave number is obtained simply by measuring the width of the peaks. The nature of the surface anisotropy may be evidenced by deviations from the p^2 law.

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