Noise-Symmetry Optimization of Quantum Error-Corrected Metrology

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Quantum error correction (QEC) code has emerged as a powerful tool to protect quantum-enhanced metrology against noise. However, the ability to correct errors alone does not guarantee high metrological sensitivity, as the encoded states may become insensitive to the parameter of interest. Here, we show that this limitation can be overcome by exploiting an intrinsic freedom of QEC codes: for a fixed set of correctable errors, the Knill-Laflamme conditions admit an equivalence class of encodings. When the correctable noise possesses unitary symmetries, these symmetries generate continuous transformations within this class, allowing systematic optimization of the encoding to increase the quantum Fisher information while preserving the correctable set of noise. Based on this observation, we develop a symmetry-based optimization approach and derive criteria identifying when such optimization can enhance metrological sensitivity. In particular, for stabilizer-sum Hamiltonians, it shows that the symmetry optimization can convert a code with vanishing quantum Fisher information into one achieving the standard quantum limit in general or even the Heisenberg scaling in specific cases, illustrating the power of symmetry optimization for QEC-assisted quantum metrology.

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Introduction—Quantum metrology employs quantum effects, such as quantum entanglement and squeezing, to enhance the precision of parameter estimation beyond the standard quantum limit (SQL) in noiseless systems [1–6]. These advantages have found broad applications ranging from gravitational wave detection [7–9] and quantum imaging [10–13] to quantum gyroscope [14–16], quantum target detection [17–19], etc.

In practical applications, noise may spoil the quantum effects and degrade measurement precision. To address these challenges, conventional approaches, e.g., adaptive measurements [20], feedback control [21], and stochastic measurement protocols [22], suppress or mitigate errors during the sensing dynamics or readout process. In contrast, quantum error correction (QEC) codes [23–26], which have been introduced to quantum metrology in recent years, exploit the structure of the correctable noise to restore noiseless precision and even recover the Heisenberg limit [27–32]. The QEC has been demonstrated for quantum metrology in experiments, e.g., signal field sensing by a room-temperature hybrid spin [33], single-mode phase estimation in a superconducting circuit [34], frequency measurements on trapped ions [35], magnetic field sensing by a 10-spin NOON state [36], etc.

While QEC code is a powerful tool to protect quantum systems against noise, the ability to correct errors does not by itself guarantee good metrological performance [37]. A QEC code that perfectly corrects the noise may remain weakly sensitive or even completely insensitive to the parameter of interest, depending on how the signal Hamiltonian acts within the code space. This reveals that error correctability and metrological sensitivity are fundamentally distinct requirements.

To address this issue, a key observation is that for a given set of errors, quantum error correction does not uniquely determine the encoding. In fact, the Knill-Laflamme theorem [38] allows the existence of multiple QEC codes that correct the same set of noise, which, nevertheless, may lead to drastically different precision for quantum metrology. In particular, if the set of correctable errors is invariant under specific unitary transformations, these transformations can generate a family of QEC codes that have the same error-correcting capability but different metrological sensitivity. Such unitary symmetries, therefore, provide the possibility to enhance the metrological sensitivity of QEC codes.

Inspired by the above idea, we develop a symmetry-based approach for optimizing the metrological sensitivity of QEC codes in this Letter. We characterize the symmetries of correctable noise and provide a constructive method to find their generators, which are then applied to maximize the quantum Fisher information (QFI) for an unknown parameter encoded in the Hamiltonian. We further derive criteria that determine when such optimization is possible. We demonstrate the power of this approach

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in stabilizer-sum Hamiltonians, where the logical code space coincides with the Hamiltonian ground space and yields zero QFI, but symmetry-based optimization can turn such metrologically "dark" codes into highly sensitive ones without sacrificing the error-correcting ability. Importantly, we clarify how the achievable precision scales with the number of physical qubits, a question that has remained largely unexplored in QEC-assisted metrology, and provide a general lower bound showing that symmetry-optimized stabilizer codes can attain at least SQL scaling, a seemingly modest scaling but actually nontrivial and not guaranteed *a priori*. Moreover, we show that more favorable scalings, including Heisenberg scaling, can be attained when only a finite number of symmetry transformations is required.

Preliminaries—In the theory of parameter estimation, Fisher information determines the fundamental precision limit of estimating an unknown parameter. For a parameter θ encoded in a probability distribution $P(X|\theta)$ of a random variable X, the Fisher information is defined as the second moment of the score function $\partial_\theta \log P(X|\theta)$ [39,40], and the precision of any unbiased estimator of θ is limited through the Cramér-Rao bound [3,41–43] by

$$\Delta^2\hat{\theta}\cdot F(\theta) \geq 1, \tag{1}$$

where Δ^2 denotes the variance.

In quantum mechanics, the parameter θ is usually encoded in a quantum state and can be inferred by measurements on the system. The Fisher information of the parameter depends on the measurement schemes, and maximizing the Fisher information over all possible measurement schemes yields the quantum Fisher information, which characterizes the ultimate estimation precision allowed by quantum mechanics [3,42,44]. For a pure state $\rho_\theta = |\psi_\theta\rangle\langle\psi_\theta|$ generated by unitary evolution under a Hamiltonian θH_0 for an evolution time t, $|\psi_\theta\rangle = e^{-i\theta H_0 t}|\psi\rangle$, where $|\psi\rangle$ is the initial state of the system, the QFI can be obtained as

$$\mathcal{F}_\theta = 4t^2\langle\psi|\Delta^2 H_0|\psi\rangle. \tag{2}$$

And it can be shown that $\mathcal{F}_\theta$ is maximized to $t^2(\lambda_{\max}-\lambda_{\min})^2$ when $|\psi_0\rangle$ is an equal superposition of the eigenstates of H_0 with the maximum and minimum eigenvalues $\lambda_{\max}$ and $\lambda_{\min}$.

We now briefly recall the theory of QEC code. The basic idea of QEC is encoding logical information into a subspace of the quantum system such that different errors map the code space to orthogonal subspaces. Let $\mathcal{E}$ be a set of error operators $\{\varepsilon_a\}$ correctable by an $[[n,l,d]]$ QEC code encoding l logical qubits into n physical qubits with code distance d. The correctability of $\mathcal{E}$ requires the existence of a trace-preserving quantum channel $\mathcal{R}$ that reverses $\mathcal{E}$ for any state ρ in the code space, i.e., $(\mathcal{R}\circ\mathcal{E})(\rho)\propto\rho$, where the proportionality factor is independent of ρ. This leads to the Knill-Laflamme (KL) theorem [38,45,46], which gives the necessary and sufficient condition for the existence of $\mathcal{R}$ that corrects $\mathcal{E}$,

$$P\varepsilon_a^\dagger\varepsilon_b P = C_{ab}P, \tag{3}$$

with P the projector onto the code space and C_{ab} the elements of a Hermitian matrix C.

Continuous symmetry of noise—Symmetry plays a fundamental role in physics [47–49]. If a system is invariant under a group of unitary transformations $U(q) = \exp[-\mathrm{i}Aq]$, where q is a real parameter and A is a Hermitian operator, it can be regarded to possess the symmetry generated by A. For a continuous symmetry group, any infinitesimal transformation $U(\delta q)\approx\mathbb{I}-\mathrm{i}A\delta q$ keeps the state of the system invariant.

Now we apply the continuous symmetry to QEC. Consider a set of errors $\{\varepsilon_a\}$, which is correctable by a QEC code with P as the projector onto the code space. P and $\{\varepsilon_a\}$ satisfy the KL theorem (3). When the noise is transformed by a general unitary operator U, the KL theorem is also satisfied by a transformed code $UPU^\dagger$,

$$UPU^\dagger(U\varepsilon_a U^\dagger)^\dagger(U\varepsilon_b U^\dagger)UPU^\dagger = C_{ab}UPU^\dagger. \tag{4}$$

Due to the discretization of errors in QEC, any combination of the correctable errors $\{\varepsilon_a\}$ can also be corrected by the same code, implying all the correctable errors span a linear space, which we denote as $\mathcal{L}_\varepsilon$. Now, if the correctable error space $\mathcal{L}_\varepsilon$ is invariant under U (though each $U\varepsilon_a U^\dagger$ can be different from ε_a), i.e., $U^\dagger\mathcal{L}_\varepsilon U = \mathcal{L}_\varepsilon$, the error set $\{\varepsilon_a\}$ is then also correctable by the transformed QEC code with the code space $UPU^\dagger$, which means a new code is generated by the symmetry of noise that can correct the same noise. The distinction between symmetry and asymmetry transformations of the correctable error space is illustrated in Fig. 1.

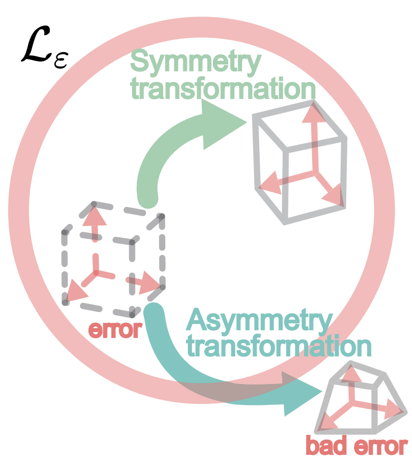


FIG. 1. Symmetry of correctable error space. A symmetry transformation (light green arrow) keeps the correctable errors within the linear space spanned by themselves (red circle). In contrast, an asymmetry transformation (deep green arrow) drives a correctable error out of $\mathcal{L}_\varepsilon$ and turns it into an uncorrectable error.

To exploit the continuous symmetry of noise to optimize the QEC code for quantum metrology, it is crucial to find all the continuous symmetries possessed by the correctable noise. It can be shown that for any continuous symmetry possessed by the error space $\mathcal{L}_\varepsilon$, a generator A of the symmetry must satisfy

$$[A, \mathcal{L}_\varepsilon] \subseteq \mathcal{L}_\varepsilon, \quad A = A^\dagger. \tag{5}$$

The proof is given in Supplemental Material, Sec. I [50].

In this Letter, we focus on the independent noise model in which errors act independently on different physical qubits and the noise rate is sufficiently low. In this case, it can be shown that the symmetry generators of the correctable noise space $\mathcal{L}_\varepsilon$ are all one-body local operators, i.e., spanned by single-qubit operators, and the correctability of the QEC code remains invariant under any local unitary transformation. The proof is provided in Supplemental Material, Sec. II [50].

QEC code optimization for QFI—The existence of noise symmetries implies that a family of QEC codes can correct the same set of errors but exhibit distinctive metrological performance. This freedom allows optimization of the QEC code to enhance the sensitivity of quantum measurements without sacrificing the error correcting ability.

We first illustrate this idea with the three-qubit bit-flip code as an example. Consider a Hamiltonian $H(\theta) = -\theta(Z_1Z_2 + Z_2Z_3 + Z_3Z_1)$, with an unknown parameter θ to estimate. The conventional choice of code space is spanned by $\{|000\rangle, |111\rangle\}$, which is actually a degenerate ground space of the Hamiltonian, so the variance of the Hamiltonian within the code space vanishes and the QFI for θ is zero, rendering the code metrologically inactive. Nevertheless, the bit-flip noise subspace is invariant under arbitrary single-qubit unitary transformations, so we can rotate the code space by a unitary operation, e.g., $\exp(\mathrm{i}\pi/4X_2)$, to $\{(|000\rangle + \mathrm{i}|010\rangle)/\sqrt{2}, (|111\rangle + \mathrm{i}|101\rangle)/\sqrt{2}\}$. This rotated code can correct exactly the same bit-flip errors as the original one, but the encoded states are no longer eigenstates of the Hamiltonian, and thus, acquire nonzero variances of $H(\theta)$. In fact, each of the two basis states can now lead to the highest QFI $16t^2$ after an evolution of time t according to Eq. (2), demonstrating that noise symmetry can activate a metrologically dark QEC code and restore Heisenberg scaling.

Motivated by this observation, we now formulate the general QEC code optimization problem for quantum metrology. Let $\mathcal{N}(\mathcal{L}_\varepsilon)$ denote the symmetry generator space of the error space $\mathcal{L}_\varepsilon$ with an orthonormal basis $\{F_a\}$. The symmetry transformations are generated by $\sum_i q_i F_i$ with $\{q_i\}$ real coefficients, which define a family of QEC codes with the same error correctability. To reach the highest precision for the parameter θ, one needs to optimize the coefficients $\{q_i\}$ to maximize the QFI.

In the following, we focus on two fundamental questions regarding the optimization of QEC codes: (i) for what kind of Hamiltonians the QEC codes can be optimized to increase the QFI; (ii) the scaling of QFI when the QEC codes are optimized. We will give general criteria for Hamiltonians that allow effective optimization of QEC codes and show that the scaling of QFI with the number of physical qubits can be significantly increased.

Hamiltonian criteria for symmetry optimization—For QEC codes, we find criteria determining whether a code can be effectively optimized for quantum metrology to increase the QFI via noise symmetry by a variational approach. Specifically, for a Hamiltonian $H(\theta) = \theta H_0$, the general first-order condition that allows effective optimization of a QEC code can be derived as

$$\Pi_{\mathcal{N}(\mathcal{L}_\epsilon)}[H_0, \rho\Delta H_0 + \Delta H_0\rho] \neq 0, \tag{6}$$

where ρ is the density matrix of the code state, $\Delta H_0 = H_0 - \langle H_0\rangle$, and $\Pi_{\mathcal{N}(\mathcal{L}_\epsilon)}$ is the projector onto the admissible symmetry generator space $\mathcal{N}(\mathcal{L}_\epsilon)$. We refer to this condition as the first-order Hamiltonian symmetry optimization (HSO) condition. For independent Pauli errors, this condition can be simplified so that the commutator $[H_0, \rho\Delta H_0 + \Delta H_0\rho]$ contains at least one single-body operator. When the first-order variation vanishes, a second-order optimization condition can be obtained. Detailed derivation and results are provided in Supplemental Material, Sec. III [50].

We now consider stabilizer codes as a typical example. Stabilizer codes are one of the most widely used classes of QEC codes based on stabilizer groups. A stabilizer group is an Abelian subgroup of a multiqubit Pauli group, and the code space of a stabilizer code is the simultaneous $+1$ eigenspace of all stabilizers. For a stabilizer-sum Hamiltonian [63],

$$H(\theta) = \theta H_0, \quad H_0 = -\sum_i g_i, \tag{7}$$

where $\{g_i\}$ are the generators of the underlying stabilizer group $\mathcal{S} = \langle g_1, \ldots, g_{n-l}\rangle$ that defines the code and θ is the unknown parameter to estimate; the code space coincides with the ground space of $H(\theta)$, and therefore, the logical code states remain invariant under $H(\theta)$, yielding vanishing QFI for θ. Nevertheless, it can be verified that $H(\theta)$ generally meets the second-order HSO criterion given in Supplemental Material, Sec. III [50] under independent errors, and hence, the noise symmetry can rotate the associated stabilizer code into a new, equivalent one whose logical states are no longer eigenstates of $H(\theta)$. As a result, the encoded states acquire nonzero variance of the Hamiltonian and become sensitive to the parameter θ, while preserving the same error-correction capability.

As an explicit example, the five-qubit QEC code $[\![5,1,3]\!]$, the smallest code that can protect against any single-qubit error with the stabilizers given in Table I(a), yields zero QFI for the stabilizer-sum Hamiltonian (7). Nevertheless, a local symmetry generator Y_4 anticommutes with all the stabilizer generators, so it can turn the ground space into the eigenspace associated with the largest eigenvalue of all the stabilizer generators. Consequently, $\exp(i\pi/4Y_4)$ turns a ground state to an equal superposition of the lowest and highest eigenstates, producing the maximum variance of $H(\theta)$, and hence, the maximum QFI, as shown in Fig. 2 and detailed in Supplemental Material, Sec. VII [50].

QFI scaling—Previous studies have shown that the QEC codes can restore Heisenberg-limited precisions with the number of *logical* qubits [31]. For large-scale QEC codes, the scaling of measurement precision with the number of *physical* qubits becomes a critical and nontrivial question. In the following, we show that the symmetry optimization of stabilizer QEC codes guarantees at least SQL scaling with the number of physical qubits and achieves the Heisenberg scaling in favorable scenarios.

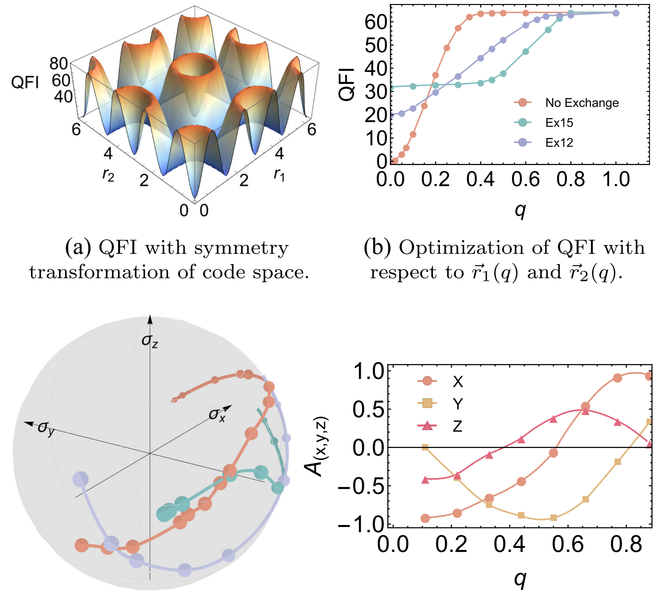
TABLE I. Stabilizer generators of typical QEC codes for symmetry-based metrological optimization. (a) The five-qubit $[\![5,1,3]\!]$ code. A single local noise symmetry generator Y_4 can anticommute with all stabilizer generators. (b) The nine-qubit Shor code. A single local operator is insufficient to anticommute with all stabilizers. Instead, a set of three mutually commuting operators, $\{X_2, Y_5, X_8\}$, is required to collectively anticommute with all stabilizer generators: X_2 anticommutes with g_1 and g_2, Y_5 anticommutes with g_3, g_4, g_7, and g_8, and X_8 anticommutes with g_5 and g_6. In both panels, the Pauli operators of the stabilizer generators that anticommute with the chosen local symmetry generators are underlined, which identify the specific physical qubits on which the symmetry transformation acts to rotate a metrologically dark code state into a highly sensitive one while preserving the error-correction capability.

(a) Stabilizer generators of five-qubit code.

g_i	Operator				
g_1	X	Z	Z	$\underline{X}$	I
g_2	I	X	Z	$\underline{Z}$	X
g_3	X	I	X	$\underline{Z}$	Z
g_4	Z	X	I	$\underline{X}$	Z

(b) Stabilizer generators of nine-qubit code.

g_i	Operator								
g_1	Z	$\underline{Z}$	I	I	I	I	I	I	I
g_2	I	$\underline{Z}$	Z	I	I	I	I	I	I
g_3	I	I	I	Z	$\underline{Z}$	I	I	I	I
g_4	I	I	I	I	$\underline{Z}$	Z	I	I	I
g_5	I	I	I	I	I	I	Z	$\underline{Z}$	I
g_6	I	I	I	I	I	I	I	$\underline{Z}$	Z
g_7	X	X	X	X	$\underline{X}$	X	I	I	I
g_8	I	I	I	X	$\underline{X}$	X	X	X	X



(a) QFI with symmetry transformation of code space.

(b) Optimization of QFI with respect to $\vec{r}_1(q)$ and $\vec{r}_2(q)$.

(c) Optimal control paths of $\vec{r}_1(q)$ with and without qubit exchange.

(d) Pauli components of optimal control path $\vec{r}_1(q)$ without qubit exchange

FIG. 2. Symmetry optimization of five-qubit code for stabilizer-sum Hamiltonian. (a) QFI is initially zero, but can rise to the maximum via local symmetry transformation $U(q) = \otimes_k e^{i\vec{r}_k(q)\cdot\vec{\sigma}}$, where $\vec{r}_1(q)$ and $\vec{r}_2(q)$ correspond to the first and second qubits, respectively. (b) The efficacy of continuous symmetry optimization can be enhanced by qubit exchange operations, such as exchanging the first and fifth qubits (labeled as "Ex15") or exchanging the first and second qubits (labeled as "Ex12"). (c) The optimal control paths $\vec{r}_1(q)$ are plotted for different scenarios, and share the legend with (b). The radius of each point on the control paths represents the estimation error. (d) The Pauli components of optimal control path $\vec{r}_1(q)$ are plotted without qubit exchange.

Consider a stabilizer-sum Hamiltonian associated with a stabilizer QEC code $[\![n,l,d]\!]$. While the symmetry transformation is generally generated by the linear combination of the symmetry generators, it is instructive to take a step-by-step approach to optimize the QEC codes, which will clearly show how the optimization proceeds.

We first consider the ideal case that there exists a Hermitian operator A satisfying the noise symmetry condition (5) and anticommuting with all stabilizers simultaneously. In this case, A can flip the eigenvalues of all stabilizers and map a ground state of the Hamiltonian to the highest excited state. So, the unitary transformation $\exp[-i(\pi/4)A]$ converts an initial code state into an equal superposition of the lowest and highest levels of the Hamiltonian and attains the highest estimation precision for the parameter θ of the Hamiltonian, with the error-correction capability invariant.

This idea can be further generalized to the scenario where several symmetry generators contribute to the optimization. Suppose there are k mutually commuting Hermitian operators $\{A_j\}$ satisfying the noise symmetry condition, and each A_j anticommutes with a distinct subset of the stabilizer generators. If these subsets are disjoint and collectively cover all the stabilizer generators, one can apply each $\exp(-\mathrm{i}q_j A_j)$ to the initial state, where q_j is a real number, and derive the composite symmetry transformation as

$$U = \prod_{j=1}^{k} \exp(-\mathrm{i}q_j A_j). \tag{8}$$

Let each A_j anticommute with r_j stabilizer generators of the QEC code, as shown in Fig. 3. Then, the stabilizers are partitioned into disjoint subsets with $\sum_{j=1}^{k} r_j = n - l = O(n)$. By applying the symmetry optimization to a ground state of the stabilizer-sum Hamiltonian (7), it can be shown that the QFI is lower bounded by

$$(\mathcal{F}_\theta)_{\max} \geq 4t^2 \frac{(n-l)^2}{k}. \tag{9}$$

Therefore, when k remains constant as the system size increases, the estimation precision reaches the Heisenberg limit with respect to the number of physical qubits n. Detail of the derivation is provided in Supplemental Material, Sec. IV [50].

This construction is particularly suitable for the stabilizer codes, of which all the stabilizer generators are composed of Pauli X and Z operators only. In this case, one can first choose $A_1 = Y_{i_1}$, acting only on the i_1 th qubit that has the most X and Z operators across all the stabilizer generators, and thus, A_1 anticommutes with the generators with X or Z in that qubit. Then, one can choose $A_2 = Y_{i_2}$, acting only on the i_2 th qubit that has the most X and Z operators in the remaining generators. One can iterate this process, and finally divide all the stabilizers into disjoint subsets anticommuting with different A_j's, and all A_j's commute with each other.

We illustrate this process with some typical stabilizer codes. For the five-qubit code, the stabilizer generators of which are shown in Table I(a), a single-qubit operator Y_4 suffices to anticommute with all the stabilizer generators. For the nine-qubit Shor code encoding one logical qubit, whose stabilizer generators are shown in Table I(b), it requires at least three operators $\{X_2, Y_5, X_8\}$ to anticommute with all the stabilizer generators.

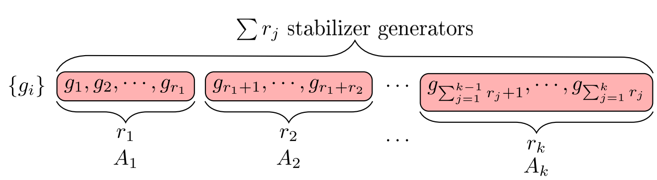


FIG. 3. Each A_j anticommutes with r_j stabilizer generators only. Together, the k symmetry generators A_j anticommute with $\sum_{j=1}^{k} r_j$ stabilizer generators jointly.

As the number k of the symmetry generators cannot exceed the number $n - l$ of the stabilizer generators, Eq. (9) guarantees at least SQL scaling $O(n)$ of QFI. Whenever a finite number of symmetry generators, independent of n, collectively flip all stabilizer generators, the QFI can further reach Heisenberg scaling $O(n^2)$. In more general cases, intermediate scalings are also possible. Supplemental Material, Sec. VII [50] presents an example with $\mathcal{F}_\theta = O(n^{\frac{3}{2}})$. Thus, the achievable scaling is governed by how the number of required symmetry generators grows with the system size, which is of particular significance for large-scale QEC codes, e.g., the surface code [64–67], that have received growing attention in recent years.

In practice, symmetry transformations may be incorporated into the encoding circuit prior to sensing. When they are implemented by physical controls, their time cost, calibration errors, and induced noise should be included in the metrological resource count [68], making the achievable precision enhancement platform and implementation dependent. Meanwhile, although the present work is focused on independent noise, the approach is fundamentally formulated in terms of the correctable error space and can accommodate correlated, crosstalk, or higher-order errors by enlarging this space and identifying its corresponding symmetries [69]. The resulting optimization then depends on the symmetry structure of the enlarged error space, but the symmetry optimization principle remains unchanged.

Conclusions—In this Letter, we show that noise symmetry provides a useful resource for optimizing QEC-assisted quantum metrology. Although a QEC code may perfectly correct a given noise set, it can remain metrologically inactive. By characterizing the continuous symmetries of the correctable errors, we demonstrate that these symmetries generate families of QEC codes that are equivalent for error correction but inequivalent for metrology. Based on this observation, we develop a symmetry optimization framework and derive a criterion that determines when metrological sensitivity can be activated. For stabilizer codes, this approach turns metrologically dark code states into sensitive ones, guarantees at least SQL scaling of QFI with respect to the number of physical qubits, and enables enhanced scalings, including the Heisenberg scaling, in favorable scenarios.

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Data availability—There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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