

Reflected Multientropy and Its Holographic Dual

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We introduce a mixed-state generalization of the multientropy through the canonical purification, which we call “reflected multientropy.” We propose the holographic dual of this measure. For the tripartite case, a field-theoretical calculation is performed using a six-point function of twist operators at large c limit. At both zero and finite temperature, the field-theoretical results match the holographic results, supporting our holographic conjecture of this new measure.

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Introduction—Quantum entanglement [1,2] has recently emerged as a key tool in understanding the AdS/CFT correspondence [3–5], which remains one of our most successful frameworks for probing quantum gravity. Remarkably, the Ryu-Takayanagi formula [6–9] provides a means to calculate entanglement entropy (EE) in conformal field theories (CFTs) via the minimal area of a codimension-2 surface in anti-de Sitter (AdS) space, establishing a precise link between spacetime geometry and quantum entanglement. However, EE exclusively captures quantum entanglement for pure states. To extend this connection to mixed states, holographic duals for several other measures, such as the entanglement of purification [10–12], reflected entropy [13–15], logarithmic negativity [16–20], and odd entanglement entropy [21,22], have recently been studied. Another important direction lies in the investigation of multipartite correlation measures [23–30], whose holographic duals play a critical role in understanding the emergence of bulk spacetime from the boundary many-body quantum entanglement. The aim of this Letter is to bridge the gap between multipartite correlations in mixed states and spacetime geometry.

For a given multipartite mixed state, we can canonically purify it to obtain a pure state [13]. Using the recent proposal for pure state multipartite entanglement measure [31] based on the replica trick and permutation invariance, one can compute the so-called “multientropy” for the purified state. We propose this as an intrinsic multipartite measure for the original mixed state, which we term “reflected

multientropy.” Unlike multientropy [32–35], reflected multientropy is intrinsically well-defined in the ultraviolet, thus circumventing the divergence issue that plagues EE as well as multientropy in quantum field theory. We note that while the multipartite generalizations of reflected entropy have been studied in [36,37] [38], these attempts are fundamentally distinct from our approach. Crucially, unlike these previous measures in [36,37]—which are defined through the von Neumann entropy on replica spaces—our proposed reflected multientropy is fundamentally rooted in multientropy, an intrinsic multipartite entanglement measure.

In the remainder of this Letter, we first introduce reflected multientropy as the mixed-state generalization of multientropy, and then propose its holographic dual. For the tripartite case, we perform a field-theoretical calculation using a six-point function of twist operators and find that it matches the holographic result, both at zero and finite temperature.

A generalization of multientropy—Review of multientropy: Let us recall some basic notions of EE and Rényi entropy. Given a pure state $|\psi\rangle_{AB}$ that describes two systems A and B , the EE between them is defined as

$$S(A) = -\text{Tr} \rho_A \log \rho_A, \quad (1)$$

where $\rho_A = \text{Tr}_B |\psi\rangle\langle\psi|_{AB}$ is the reduced density matrix. Rényi entropy is a one-parameter generalization of EE,

$$S_n(A) = \frac{1}{1-n} \log \text{Tr} \rho_A^n, \quad (2)$$

which reduces to EE (1) in the limit $n \rightarrow 1$. This is called “replica trick.” In order to introduce the multientropy, let us reformulate ρ_A^n through the permutation and contraction of the indices of the n copies of ρ_A ,

$$\begin{aligned} \rho_A^n &= (\rho_A)_{\alpha_1}^{\alpha_2} (\rho_A)_{\alpha_2}^{\alpha_3} (\rho_A)_{\alpha_3}^{\alpha_4} \cdots (\rho_A)_{\alpha_n}^{\alpha_1} \\ &= (\rho_A)_{\alpha_1}^{\alpha_{\sigma(1)}} (\rho_A)_{\alpha_{\sigma(2)}}^{\alpha_{\sigma(3)}} (\rho_A)_{\alpha_{\sigma(3)}}^{\alpha_{\sigma(4)}} \cdots (\rho_A)_{\alpha_n}^{\alpha_{\sigma(n)}}, \end{aligned} \quad (3)$$

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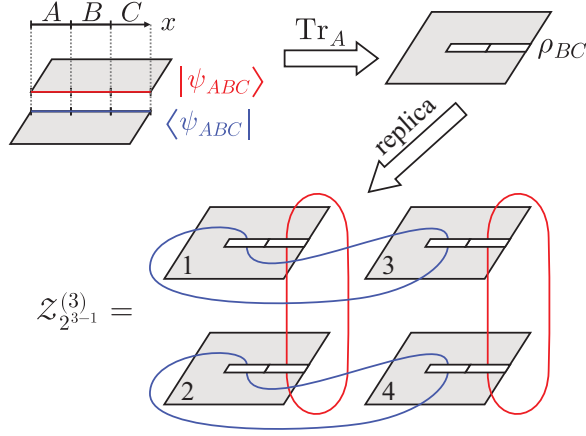


FIG. 1. The construction of the n^{q-1} -sheet Riemann surface in the special case $q = 3$ and $n = 2$.

with $\sigma = (123 \cdots n)$ the permutation acting on the index of replicas. Guided by this key observation, in [31] the authors defined the multientropy $S^{(q)}$, which generalizes the concept of EE to q -partite cases through the permutation and contraction of indices. Let us focus on tripartite pure state $|\psi\rangle_{ABC}$ to illustrate.

Like Rényi entropy, the entanglement measure $S_n^{(q=3)}$ should be determined by the choice of permutation $(\sigma_A, \sigma_B, \sigma_C)$ acting on the replica indices, i.e., the way of contracting multiple density matrices. Note that there is an equivalence relation $(\sigma_A, \sigma_B, \sigma_C) \sim (\sigma_A, \sigma_B, \sigma_C) \cdot g$ allowing us to set $\sigma_A = \text{id}$ by choosing $g = \sigma_A^{-1}$, reducing the problem to $(q-1)$ parties with the entanglement measure depending solely on (σ_B, σ_C) [41]. Both B and C have n replicas and (σ_B, σ_C) are chosen to be cyclic permutations of order n . In total, we have $n^{q-1} = n^2$ replicas, which form a $n \times n$ square lattice. The multientropy is defined by choosing the replica symmetry as $\mathbb{Z}_n \otimes \mathbb{Z}_n$ with the first or second $\mathbb{Z}_n$ acting on the subregion B or C and cyclically permuting the replicas located in the same row or column of the lattice.

For general n and q -partite entanglement, the Rényi multientropy is defined as [31,34,42]

$$S_n^{(q)} = \frac{1}{1-n} \frac{1}{n^{q-2}} \log \left(\mathcal{Z}_{n^{q-1}}^{(q)} / (\mathcal{Z}_1^{(q)})^{n^{q-1}} \right), \quad (4)$$

where $\mathcal{Z}_{n^{q-1}}^{(q)}$ is the partition function on the n^{q-1} -sheet Riemann surface with replica symmetry $\mathbb{Z}_n^{\otimes(q-1)}$ as described above. The multientropy is defined by taking the $n \rightarrow 1$ limit,

$$S^{(q)} = \lim_{n \rightarrow 1} S_n^{(q)}. \quad (5)$$

For $q = 3$ and $n = 2$, the partition function contains four replicas with the gluing $\sigma_B = (13)(24)$ and $\sigma_C = (12)(34)$ as illustrated in Fig. 1 via the Euclidean path integral [43–45].

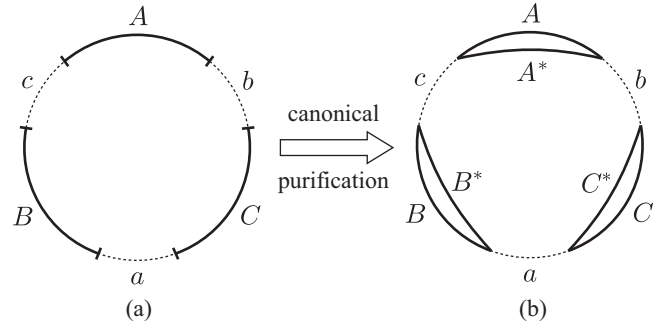


FIG. 2. Canonical purification of ρ_{ABC} . (a) Global pure state ψ_{AcBaCb} defined on a circle. Tracing out abc gives a mixed state ρ_{ABC} . (b) Picking up another copy of ρ_{ABC} and gluing these two copies along abc gives a big pure state $|\sqrt{\rho_{ABC}}\rangle$ (7), which is the canonical purification of ρ_{ABC} .

This results in a contraction of four density matrices (here we abbreviate ρ_{BC} as ρ),

$$\mathcal{Z}_{2^{3-1}}^{(3)} = \rho_{\beta_1 \chi_1}^{\beta_3 \chi_2} \rho_{\beta_2 \chi_2}^{\beta_4 \chi_1} \rho_{\beta_3 \chi_3}^{\beta_1 \chi_4} \rho_{\beta_4 \chi_4}^{\beta_2 \chi_3}. \quad (6)$$

The corresponding Rényi multientropy is then given by (4) with $\mathcal{Z}_1^{(3)} = \rho_{\beta \chi}^{\beta \chi}$ serving as the normalization factor.

Mixed-state generalization of multientropy through canonical purification: A multipartite mixed state can be obtained by tracing out some part from a pure state. Again we will focus on tripartite case with $q = 3$ but it can be generalized to larger q straightforwardly. Starting from a pure state $\psi_{AcBaCb} \in \mathcal{H}_{AcBaCb}$ and tracing out abc , we obtain a mixed state ρ_{ABC} , which can be canonically purified by doubling the Hilbert space (see Fig. 2 for a schematic diagram of this purification process). For more discussions on canonical purification we refer to [13]. The purified state can be expressed as

$$|\sqrt{\rho_{ABC}}\rangle = |\sqrt{\text{Tr}_{abc} |\psi_{AcBaCb}\rangle \langle \psi_{AcBaCb}|}| \in (\mathcal{H}_A \otimes \mathcal{H}_A^*) \otimes (\mathcal{H}_B \otimes \mathcal{H}_B^*) \otimes (\mathcal{H}_C \otimes \mathcal{H}_C^*). \quad (7)$$

We define the reflected multientropy as

$$S_R^{(q=3)}(A; B; C) = S^{(q=3)}(AA^*; BB^*; CC^*)|_{\sqrt{\rho_{ABC}}}, \quad (8)$$

with $S^{(q)}$ the original multientropy (5). The subscript R in $S_R^{(q)}$ denotes “reflected” [46].

Here, we list some properties of reflected multientropy. (A) It reduces to twice the multientropy for pure state $\rho_{AB\dots} = |\psi\rangle\langle\psi|_{AB\dots}$,

$$\text{pure state: } S_R^{(q)}(A; B; \dots) = 2S^{(q)}(A; B; \dots). \quad (9)$$

(B) It reduces to the reflected entropy [13] for bipartite system ρ_{AB} , i.e., $S_R^{(q=2)}(A; B) = S_R(A; B)$. (C) It vanishes

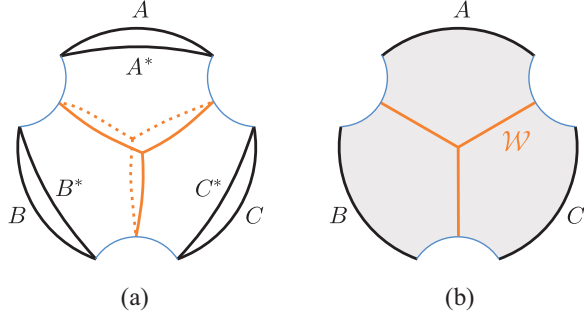


FIG. 3. (a) Canonical purification of ρ_{ABC} and its holographic dual. Tracing out abc corresponds to gluing along the RT surface for ρ_{ABC} (blue curves). The orange web $\mathcal{W}$ is the minimal web in the bulk as the holographic dual of $S^{(3)}(AA^*; BB^*; CC^*)$. (b) The single copy picture. The shadow area is the entanglement wedge of $A \cup B \cup C$ and the orange web is the holographic dual of $S_R^{(3)}(A; B; C)$.

for factorized density matrix $\rho_{AB\dots} = \rho_A \otimes \rho_B \otimes \dots$,

$$\text{product state: } S_R^{(q)}(A; B; \dots) = 0. \quad (10)$$

However, $S_R^{(q)}(A; B; \dots)$ can be nonzero for separable states. Consider a simple separable state,

$$\rho_{ABC} = \frac{1}{2}|000\rangle\langle 000| + \frac{1}{2}|111\rangle\langle 111|. \quad (11)$$

Its canonical purification is a GHZ state $\frac{1}{2}|000000\rangle + \frac{1}{2}|111111\rangle$ with nonzero $S_R^{(3)}(A; B; C) = \frac{3}{2}\log 2$. (D) It is bounded from below by multientropy of purification,

$$\text{lower bound: } S_R^{(q)}(A; B; \dots) \geq M_P(A; B; \dots), \quad (12)$$

where the multientropy of purification M_P is defined by minimization of multientropy over all possible purifications. (E) It is bounded from below by half the multipartite reflected entropy [37] for holographic states,

$$\text{lower bound: } S_R^{(q)}(A; B; \dots) \geq \frac{1}{2}\Delta_R(A; B; \dots). \quad (13)$$

Holography of reflected multientropy—Based on the canonical purification for holographic states [13] and the holographic proposal for multientropy [31], now we want to find the holographic dual for reflected multientropy. Consider a global pure state ψ_{AcBaCb} defined on a circle as show in Fig. 2(a). This pure state has a classical bulk solution as its holographic dual. Now we trace out abc , which corresponds to retaining only the entanglement wedge for ρ_{ABC} and prescinding the other portion of the bulk. To perform the canonical purification, we pick up another copy of the entanglement wedge and glue these two

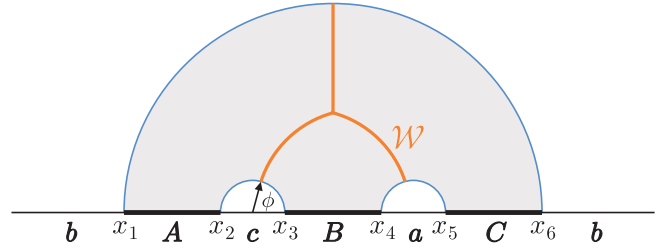


FIG. 4. The holographic dual of $S_R^{(q=3)}(A; B; C)$.

copies along the Ryu-Takayanagi surface (RT surface) for ρ_{ABC} . The resulting geometry is drawn in Fig. 3(a).

To figure out the geometry dual of the reflected multientropy, which now becomes the multientropy between AA^* , BB^* , and CC^* , we need to generalize the holographic proposal for multientropy [31] to multiboundary pure states. Recall that the original holographic proposal for multientropy is a minimal bulk web $\mathcal{W}$ consisting of minimal codimension-2 surfaces. For a single boundary pure state, $\mathcal{W}$ is anchored at the boundaries of all subsystems $A, B, \dots$. This condition is relaxed in the case of multiboundary, much like the RT surfaces for multiboundary states. Another important condition is that $\mathcal{W}$ should contain subwebs that are homologous to all the subsystems $A, B, \dots$. This can be easily satisfied. It is then natural to propose the holographic dual of the reflected multientropy as the minimal surface web $\mathcal{W}$ satisfying the following topological conditions: (1) $\mathcal{W}$ is anchored at the RT surface for $A \cup B \cup \dots$, and (2) $\mathcal{W}$ contains subwebs that are homologous to all the subsystems $A, B, \dots$. The holographic reflected multientropy is computed by

$$S_R^{(q)} = \frac{2}{4G_N} \min_{\mathcal{W}} L[\mathcal{W}], \quad (14)$$

with $L[\mathcal{W}]$ the area of the web $\mathcal{W}$ in a single copy as illustrated in Fig. 3(b).

Computation in $\text{AdS}_3/\text{CFT}_2$ —Let us study a simple example in $\text{AdS}_3/\text{CFT}_2$, as illustrated in Fig. 4. We work in Poincaré half plane with the metric

$$ds^2 = \frac{dx^2 + dy^2}{y^2}, \quad x \in \mathbb{R} \ \& \ y \in \mathbb{R}_+. \quad (15)$$

We consider the reflected multientropy $S_R^{(3)}(A; B; C)$ between subsystems A, B, C . ρ_{ABC} is a mixed state obtained by tracing out abc from the pure state ψ_{AcBaCb} defined on $\mathbb{R}$ as shown in Fig. 4. The subsystems A, B, C are chosen to be $A = [x_1, x_2]$, $B = [x_3, x_4]$, $C = [x_5, x_6]$ with $x_{i=1,\dots,6}$ parametrized by $\xi = a, b, c$,

$$\begin{aligned} x_6 &= -x_1 = b, \\ x_5 &= -x_2 = a + r, \quad x_4 = -x_3 = a - r. \end{aligned} \quad (16)$$

In the following we will calculate $S_R^{(3)}(A; B; C)$ using both holographic method and field-theoretical method and find the agreement between them.

Holographic calculation: The holographic dual of $S_R^{(3)}(A; B; C)$ is the minimal web denoted by the orange curves in Fig. 4. Because of the $\mathbb{Z}_2$ -symmetry about the $x = 0$ axis in Fig. 4, the junction I is expected to be on $x = 0$ and we denote it by $I(0, y_I)$. For general two points (x_1, y_1) and (x_2, y_2) on the Poincaré half plane, the distance between them is

$$D((x_1, y_1), (x_2, y_2)) = \text{arccosh} \frac{(x_1 - x_2)^2 + y_1^2 + y_2^2}{2y_1 y_2}. \quad (17)$$

Therefore, the distance between I and the RT surface of c is given by

$$\min_{\phi} D((0, y_I), (-a + r \cos \phi, r \sin \phi)), \quad (18)$$

with $(-a + r \cos \phi, r \sin \phi)$ a point on the RT surface of c . The distances between I and the RT surface of a, b can be worked out similarly and we finally end up with the length of the bulk dual web,

$$\begin{aligned} L[\mathcal{W}](a, b, r) = \min_{y_I} \left[\log(b/y_I) \right. \\ \left. + 2 \text{arccsech} \frac{2y_I r}{\sqrt{((a-r)^2 + y_I^2)((a+r)^2 + y_I^2)}} \right], \end{aligned} \quad (19)$$

where the first and second term in $[\dots]$ comes from the distance between I and the RT surface of b and $\{a, c\}$. The minimization over y_I in (19) will be evaluated numerically (see the blue curves in Fig. 5) when comparing with the field-theoretical results.

Field-theoretical calculation: Now we do the calculation in CFT_2 . Using the replica trick, we first represent the reflected multientropy as a path integral on a specific Riemann surface. Here, a new replica number m is introduced for canonical purification (see Ref. [13] for more details about this replica trick).

The n th Rényi version of reflected multientropy is

$$S_R^{(q)}[n] = \frac{1}{1-n} \frac{1}{n^{q-2}} \log \frac{\mathcal{Z}_{n^{q-1}, m}}{(\mathcal{Z}_{1, m})^{n^{q-1}}}, \quad (20)$$

and the reflected multientropy is obtained by taking the limit $n \rightarrow 1$ and $m \rightarrow 1$,

$$S_R^{(q)} = \lim_{m \rightarrow 1} \lim_{n \rightarrow 1} \frac{1}{1-n} \frac{1}{n^{q-2}} \log \frac{\mathcal{Z}_{n^{q-1}, m}}{(\mathcal{Z}_{1, m})^{n^{q-1}}}. \quad (21)$$

Now we use the twist operator correlation function to express $\mathcal{Z}_{n^{q-1}, m}$, the path integral on a glued Euclidean spacetime with mn^{q-1} copies. The conformal weights h_i of twist operators $\sigma_i(x_i)$ can be read from the representation of cyclic groups [47],

$$h_{i=1, \dots, 2q} = \frac{c}{24} \left(m - \frac{1}{m} \right) n^{q-1}. \quad (22)$$

We leave the derivation of the conformal weights (22) and (23) to Supplemental Material [48]. When $q = 2$, one can check that (22) reduces to the conformal weight of the twist operators for the reflected entropy as we expected [49]. The conformal weights h_a of the leading operators σ_a in operator product expansion $\sigma_i(x_i) \sigma_j(x_j) \rightarrow \sigma_{a=ij}(x_a)$ are

$$h_{a=ij} = \frac{c}{12} (n^{q-1} - n^{q-3}), \quad (23)$$

which is twice the conformal weight of the twist operators for the original multientropy as we expected [50]. It can also be checked that taking $q = 2$; (23) reduces to twice of the conformal weight of the twist operators for EE.

Using these twist operators, $\mathcal{Z}_{n^{q-1}, m}$ can be represented by the six-point function

$$\mathcal{Z}_{n^{q-1}, m} = \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \sigma_5 \sigma_6 \rangle_{\text{CFT} \otimes mn^{q-1}}, \quad (24)$$

with σ_i located at x_i . At large c limit, the calculation of (24) can be converted to solving a monodromy problem [51, 52]. Following the approach in [53] we compute the six-point function (24) at large c limit [48] and the partial derivatives of $S_R^{(3)}(A; B; C)$ with respect to a, b, r are plotted in Fig. 5.

On the other hand, the holographic result is given by minimizing $L[\mathcal{W}](a, b, r)$ in (19). The partial derivatives of $(c/3)L[\mathcal{W}](a, b, r)$ with respect to a, b, r are plotted in Fig. 5 and compared with the field-theoretical results. These two results match precisely. Notice that we are comparing the derivatives of the entropy. To match the entropy itself, we have to further include the contributions of the operator product expansion coefficients in the CFT calculation [48].

We also examine the agreement between the field-theoretical and holographic results at finite temperature. From the CFT perspective, we compute the six-point function on an infinitely long cylinder with period β_{CFT} while on the gravity side, the bulk dual web length is evaluated in the Bañados-Teitelboim-Zanelli black hole background [48, 54]. We find that the reflected multientropy decreases with increasing temperature, indicating that thermal effects tend to wash out multipartite correlations. In the context of thermal systems, our proposal provides a

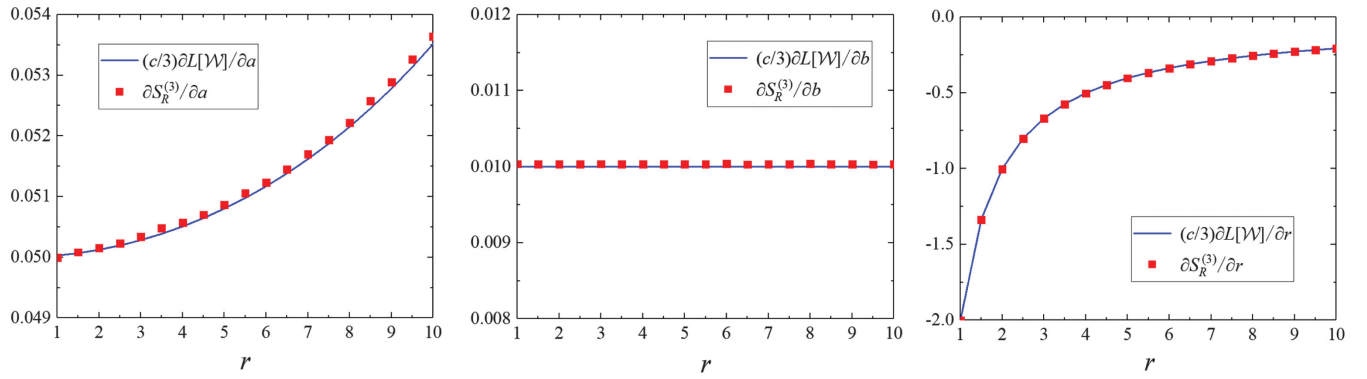


FIG. 5. Comparison of the partial derivatives of $(c/3)L[\mathcal{W}]$ and $S_R^{(q=3)}$ (divided by $c/3$) with respect to $\xi = a, b, r$ at zero temperature. We take $a = 20$, $b = 100$, and r ranging from 1 to 10. The holographic results and the field-theoretical results match well (the difference between them is within 0.4%).

concrete multipartite measure, whose potential connections to quantum chaos and information scrambling warrant further investigation. Another promising application of reflected multientropy is to explore whether it can serve as a diagnostic of topological order in mixed states [55,56].

Discussion—In this Letter we introduced reflected multientropy as a multipartite measure for mixed state by combining the recent proposal of multientropy and canonical purification and found good agreement between the CFT results and the holographic results. One may worry about the potential bulk replica symmetry breaking for multientropy. While the bulk replica symmetry was assumed in the original holographic derivation [31,32], recent work [27] shows this fails for $n > 2$ [57]. A similar issue arises for reflected multientropy with $n > 2$. Nevertheless, our holographic calculation focuses on the $n = 1$ case, evading the replica symmetry-breaking regime and ensuring the validity of our results. Our findings strongly support a new class of duality between multipartite measures for mixed states and spacetime geometry. Moreover, there are two natural extensions of our Letter. First, our discussion can be generalized straightforward to multiple canonical purifications. Second, similar to the covariant generalizations of reflected entropy and multientropy [13,31], we expect that the “covariant reflected multientropy” is obtained as the area of the minimal extremal surface web rather than the global minimal one.

Note added—Recently, the authors in [59] found a special quantum state where the reflected entropy is not monotonically decreasing under partial trace thus fails to be a correlation measure. However, the reflected entropy remains a valid correlation measure for holographic states, as emphasized in the same paper. See also Ref. [60] for the monotonicity property of reflected entropy in free fields.

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$$h_{g_A} = h_{g_A^{-1}} = \frac{cn(m^2 - 1)}{24m},$$

where g_A and g_A^{-1} are the twist operators located at the two endpoints of interval A .

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$$\frac{1}{m_{AB}} + \frac{1}{m_{BC}} + \frac{1}{m_{CA}} > 1, \quad m_{AB,BC,CA} \in \mathbb{Z}_+,$$

- with m_{AB} the order of $\sigma_A^{-1}\sigma_B$. For the n th Rényi multientropy ($m_{AB} = m_{BC} = m_{CA} = n$), this inequality is violated for $n > 2$, indicating replica symmetry breaking.
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