

Stochastic reduced-order model for the bimodal low-frequency dynamics of a turbulent separation bubble

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The low-frequency unsteadiness of turbulent separation bubbles is manifested in a contraction and expansion of the reverse-flow region. This study shows that, in a pressure-gradient-induced turbulent separation bubble developing in a half-diffuser, the dominant long-time behavior follows nonlinear stochastic dynamics. Planar time-resolved particle image velocimetry reveals a spanwise-organized coherent structure with anticorrelated core and sidewall regions. Its temporal evolution is quantitatively described using a Langevin equation, modeling both the deterministic drift and stochastic forcing of the dynamical system. This model suggests that the low-frequency unsteadiness may arise from stochastic forcing, resulting in transitions between two equilibrium states. The methodology and findings documented in the present paper therefore constitute a significant step toward understanding turbulent separation bubbles as probabilistic dynamical systems.

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I. INTRODUCTION

Turbulent separation bubbles (TSBs) are confined regions of near-wall separated flow that arise when a turbulent boundary layer separates due to a sufficiently strong adverse pressure gradient and subsequently reattaches downstream. Such flows occur in a wide range of aerodynamic configurations, including diffusers, airfoils near stall, and internal flows, where they induce losses, noise, and unsteady loading. Beyond their mean-flow characteristics, TSBs are known to exhibit unsteady behavior over a wide range of temporal and spatial scales [1–3]. Of particular interest is a low-frequency, large-scale unsteadiness, often referred to as breathing, yielding alternating contractions and expansions of the recirculation region. This motion has been consistently observed in pressure-gradient-induced TSBs across a variety of experimental configurations and facilities [4–7], with a characteristic nondimensional frequency of order $St = fL_b/U_\infty = 0.01$ where L_b is the characteristic separation length and U_∞ the reference velocity. Owing to its large spatial footprint and long timescale, this low-frequency motion dominates the long-term dynamics of the separated flow and governs the low-frequency statistics of wall-pressure and velocity fluctuations.

Early investigations of TSB breathing primarily relied on measurements along the symmetry plane or at isolated spanwise locations, using planar particle image velocimetry (PIV) or wall-pressure measurements [4,6,7]. These studies established the robustness of the low-frequency phenomenon and its apparent association with a large-scale coherent structure enclosing the

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separation bubble. Modal analyses based on proper orthogonal decomposition (POD) or spectral POD (SPOD) further demonstrated that the breathing motion can be captured by a dominant low-frequency mode that contains a substantial fraction of the turbulent kinetic energy [5]. However, such approaches were often limited to planar perspectives, leaving the spanwise organization of the low-frequency dynamics largely unresolved. A recent investigation incorporating spanwise wall-pressure measurements suggested a strong degree of coherence across the span, pointing toward a quasi-two-dimensional process [8]. In contrast, a spanwise cellular pattern was found recently based on volumetric velocity measurements [9], which is consistent with the standing-wave dynamics reported in Ref. [10]. At the same time, mean-flow studies of diffuser and ramp flows have highlighted a fundamentally three-dimensional topology, characterized by U-shaped separation lines and pronounced sidewall effects [11]. Thus, there appears to be some evidence that the low-frequency unsteadiness is not genuinely two-dimensional but is instead organized in a nontrivial spanwise manner.

A more fundamental challenge concerns the nature of the low-frequency dynamics themselves. While the term *breathing* may suggest a self-sustained oscillation, experimental time series typically exhibit broadband low-frequency spectra and irregular temporal behavior. Rather than showing a well-defined periodicity, the flow appears to alternate intermittently between states of increased and decreased separation extent, with strongly variable dwell times [12]. Similar phenomenology has been reported in a variety of turbulent flows, including bluff-body wakes and separated shear flows, where reduced-order descriptions reveal noise-driven bistability and stochastic switching between preferred flow states [13–15]. In these systems, the long-time dynamics can often be captured by low-dimensional stochastic models in which deterministic drift mechanisms are continuously perturbed by turbulent fluctuations. A common and well-established framework for such descriptions is provided by Langevin-type equations and their associated Fokker–Planck formulations [16–18].

Recent theoretical and data-driven studies have converged on the view that the low-frequency dynamics of turbulent separation bubbles are governed by a global mode of the mean flow with vanishing or near-vanishing frequency [19]. Mean-flow-based linear stability and resolvent analyses consistently identify such a zero-frequency mode as the dominant amplification mechanism at long timescales [10]. While this mode does not correspond to a self-sustained oscillation, it renders the flow highly receptive to very-low-frequency perturbations and provides a natural explanation for the broadband, slowly evolving character of the observed unsteadiness. Importantly, a zero-frequency global mode represents a nonoscillatory deformation of the mean flow rather than a periodic motion. If the corresponding eigenvalue becomes weakly unstable or marginally stable in the presence of turbulence, the expected nonlinear outcome is therefore not a limit cycle but a steady bifurcation.

Within this stationary global-mode framework, the low-frequency dynamics of turbulent separation bubbles are expected to be strongly influenced by turbulent forcing. While previous studies have characterized this behavior primarily in terms of modal structure and spectral amplification, a quantitative description of the long-time amplitude dynamics of the dominant low-frequency mode is still lacking. In particular, it remains unclear how turbulent fluctuations shape the temporal evolution of the mode amplitude and give rise to the observed broadband, intermittent breathing-like behavior.

The present study addresses this gap by combining time-resolved, full-span PIV measurements with stochastic reduced-order modeling in a pressure-gradient-induced turbulent separation bubble developing in a half-diffuser. Building on a POD-based reduced-order representation, we attempt to model the temporal evolution of the leading mode amplitude using a nonlinear Langevin equation, allowing deterministic drift and stochastic forcing to be quantified separately.

The article is organized as follows: First, the experimental methods and stochastic modeling approach are explained in Sec. II. Then, in Sec. III, we introduce the main characteristics of the flow configuration at hand before seeking to identify the dominant low-frequency POD mode. Building on this reduced-order description, we model the temporal evolution of the leading mode amplitude using a nonlinear stochastic Langevin equation. Some final remarks are offered in Sec. IV.

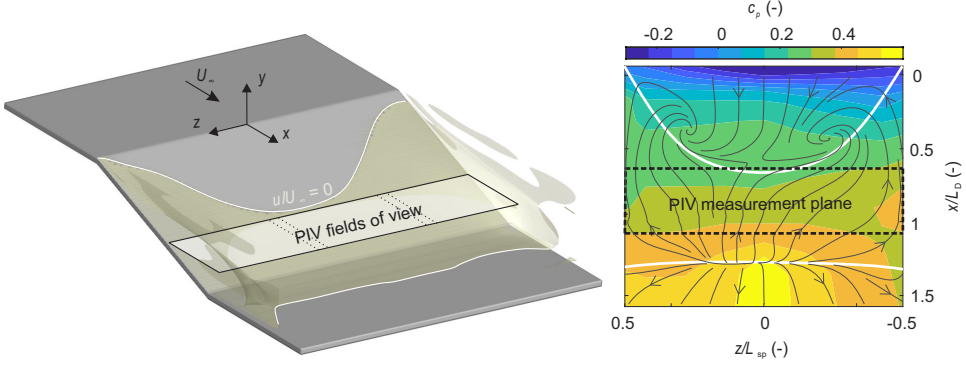


FIG. 1. Three-dimensionality of one-sided diffuser flow; mean TSB indicated by beige isosurface, three-camera velocity measurement plane indicated by parallelogram (left) top view of wall pressure field overlaid with wall shear-stress lines (right).

II. METHODS

This section serves as documentation of the experimental setup and measurement procedure. Furthermore, the stochastic modeling approach applied to the low-frequency signature will be explained.

A. Wind tunnel setup

The flow under consideration occurs in a two-dimensional test section with a span of $S = 0.6$ m installed in a low-speed closed-loop wind tunnel operated at $U_\infty = 20$ m/s. The Reynolds number, based on the momentum thickness of the incoming boundary layer $\vartheta \approx 0.79$ mm, is $Re_\vartheta \approx 1000$. The test section features a bottom surface segment of length $L = 0.34$ m that is inclined by $\varphi \approx 21^\circ$, resulting in an axial diffuser extent of $L_D \approx 0.32$ m. In this one-sided diffuser, or straight backward-facing ramp, the mean kinetic energy decreases in the main flow direction as dynamic pressure is converted to static pressure. Due to the resulting adverse static pressure gradient, the near-wall momentum flux is reduced and, as a consequence, the boundary layer separates from the diffuser surface before reattaching further downstream on the test section floor.

Considering the relative simplicity of the test section geometry, it is important to keep in mind that the diffuser flow is fully three-dimensional. As shown in Ref. [10], the presence of solid sidewalls enforces a discrete reflection symmetry in the spanwise direction, which plays a key role in the organization of the low-frequency dynamics discussed below. In terms of the mean flow, the separation line (white line in Fig. 1) is U-shaped as the boundary layer separates further upstream near the side walls, which is explained by the more substantial momentum deficit associated with the corner flow. Near the symmetry plane, the flow is accelerated to a greater extent at the diffuser entrance ($c_p < -0.2$, right-hand side in Fig. 1), and mean separation occurs approximately in the middle of the diffuser ramp. The curved separation line gives rise to a surface flow topology characterized by secondary flow vortex footprints with opposite signs of rotation at the top part of the diffuser ramp.

B. Acquisition of time-resolved velocity data

The analysis carried out in this article is based on time-resolved two-component PIV measurements inside a (zx) horizontal plane covering the entire test section span (see Fig. 1), spanning the range $x/L_D = 0.64, \dots, 1.00$. The vertical position $y/L_D \approx -0.26$, approximately 40 millimeters above the diffuser foot, was chosen to yield information on low-frequency fluctuations that are expected in this region.

Di-2-ethylhexyl sebacate (DEHS) seeding particles with a mean diameter of $d_{\text{DEHS}} = 1 \mu\text{m}$ were supplied to the flow downstream of the test section so that, after passing the closed-loop wind tunnel, a uniform tracer distribution was achieved inside the diffuser. The particles were illuminated by a Litron LD30–527 PIV laser, where the laser beam was transformed to a light sheet introduced through one side-wall with 3 mm maximum thickness throughout the span. The scattered light was recorded using three Phantom VEO 710 cameras (1280×800 px, $20 \mu\text{m}$ pixel pitch) with overlapping fields of view, installed above the acrylic glass test section ceiling. Snapshots were recorded at an acquisition rate of $f_s = 200$ Hz over a duration of 125 s, hence the time series spans approximately 25 000 instantaneous velocity fields. The sampling rate was chosen to resolve the slow dynamics of the dominant low-frequency mode while not resolving all faster turbulent fluctuations individually.

The snapshots were preprocessed by masking the light reflections on the diffuser surface and average-intensity background subtraction. In-plane velocity components in the streamwise and transverse directions (u and w) were computed using multigrid cross-correlation with final interrogation area sizes of $\Delta x = \Delta z = 3.2$ mm. By applying a 50% overlap, the vector pitch was halved. Relative to the incoming momentum thickness, this yields $\Delta x/\vartheta \approx 4$. While the present spatial resolution is not intended to resolve the smallest turbulent structures, it is adequate for the dominant large-scale modes targeted by the POD and stochastic modeling.

Instantaneous vector fields were validated using the universal outlier detection method [20]. Although this spatial filtering approach is standard in PIV processing, its performance may be affected in regions with strong gradients. In the present work, however, the subsequent POD analysis targets large-scale coherent structures and is therefore expected to be less sensitive to isolated vector replacement than local derivative-based statistics. More advanced multivariate outlier-detection strategies [21] could nevertheless be beneficial in future studies of strongly separated regions. Finally, to obtain a single velocity field from the three fields of view (one for each camera), snapshots were projected by means of cubic interpolation onto a common structured grid.

The uncertainty of velocity measurements

$$\sigma_u = \frac{\sigma_x s}{\Delta t} \quad (1)$$

can be estimated based on the uncertainty in pixel-image displacement σ_x , which is on the order of 0.1 pixels, the calibration factor $s \approx 0.19$ mm/px and the pulse separation $\Delta t = 150 \mu\text{s}$. This yields a conservative estimate of $\sigma_u \approx 0.13$ m/s for both in-plane velocity components (i.e., approximately 0.7% of the free stream velocity).

C. Stochastic modeling

The stochastic modeling approach adopted here relies on the assumption that the low-frequency dynamics of the flow can be described by a small number of slow degrees of freedom that are well separated in timescale from the remaining turbulent fluctuations. Under this assumption, the long-time evolution of the dominant mode amplitude can be approximated as a Markovian stochastic process, in which deterministic drift represents the effect of coherent flow physics while unresolved fast dynamics act as effective noise.

Recall that the low-frequency dynamics of TSBs are governed by one slow coherent structure, as evinced by the POD and SPOD analyses of pressure-gradient-induced separation bubbles [4,5,19]. Hence, the flow field can be thought of as

$$u(x, t) = u_0(x) + A(t)\Phi(x) + F, \quad (2)$$

where u_0 denotes the mean flow, Φ is the spatial structure of the dominant mode with time-dependent amplitude A , and fast turbulent noise is summarized in F . In the present study, the amplitude $A(t)$ is given by the temporal coefficient of the leading POD mode. Since the dynamics

of $A(t)$ dominate the long-term behavior [4,9,10], the flow unsteadiness can be described by a stochastic model of this scalar quantity.

The Langevin equation

$$\frac{dA}{dt} = \underbrace{\sigma A - \gamma A^3}_{\text{deterministic drift}} + \underbrace{\sqrt{2D} \xi(t)}_{\text{stochastic forcing}} \quad (3)$$

is ideal for this purpose, explicitly separating deterministic dynamics and stochastic forcing. The first term on the right-hand side σA represents linear growth or decay: for a positive effective growth rate of the reduced-amplitude dynamics ($\sigma > 0$), small amplitudes tend to grow, whereas they decay to zero for $\sigma < 0$. This term is stabilized by nonlinear saturation ($-\gamma A^3$), which prevents blowups by pushing back large amplitudes. In the saturated state, linear growth is balanced by nonlinear effects, such that the time derivative of the amplitude vanishes. This yields three stationary solutions: $A = 0$ and $A = \pm\sqrt{\sigma/\gamma}$. For $\sigma = 0$, the trivial solution ($A = 0$) loses stability, and two nontrivial finite-amplitude states emerge, which is characteristic of a supercritical pitchfork bifurcation.

Finally, stochastic forcing, representing the cumulative effect of unresolved turbulent fluctuations and measurement noise, is modeled by $\sqrt{2D} \xi(t)$ where D is the diffusion coefficient representing the noise intensity and ξ is modeled as Gaussian white noise. The latter represents the influence of unresolved fast turbulent fluctuations on the slow mode dynamics under an assumed separation of timescales [16–18].

The stochastic model is established by determining the unknown parameters occurring in the Langevin equation, namely, σ , γ , and D , by fitting them to a time series of the first POD coefficient. To this end, the Euler discretization of the differential equation

$$A_{n+1} - A_n = (\sigma A_n - \gamma A_n^3) \Delta t + \sqrt{2D \Delta t} \eta_n \quad (4)$$

is considered where Δt denotes the temporal spacing of PIV snapshots. Then, linear regression is applied to obtain the deterministic skeleton. Specifically, σ and γ are chosen such that the prediction error of the discrete-time drift is minimized in a least-squares sense. With the drift parameters estimated based on the data, the residuals

$$r_n = A_{n+1} - A_n - (\sigma A_n - \gamma A_n^3) \Delta t \quad (5)$$

are assumed to be caused by the stochastic forcing term. For white noise, the residuals are expected to form a Gaussian distribution with variance $2D \Delta t$. The diffusion coefficient D is therefore obtained by computing the empirical variance of the residuals

$$\langle r_n^2 \rangle = \frac{1}{N} \sum_{n=1}^N r_n^2, \quad (6)$$

and rescaling it to

$$D = \frac{1}{2\Delta t} \langle r_n^2 \rangle. \quad (7)$$

This normalization yields a diffusion coefficient that describes the noise intensity per unit time and, for sufficiently small Δt , is independent of the temporal resolution. It should be interpreted as an effective unresolved forcing, which contains contributions from both physical turbulent fluctuations and experimental uncertainty.

Finally, once all parameters are estimated, the dynamics can be simulated by applying the Euler-Maruyama time-stepping scheme

$$A_{n+1} = A_n + (\sigma A_n - \gamma A_n^3) dt + \sqrt{2D} \Delta W_n, \quad (8)$$

where ΔW_n is the Gaussian random increment over one time step, which is implemented numerically by multiplying a normally distributed random number by $\sqrt{dt}$.

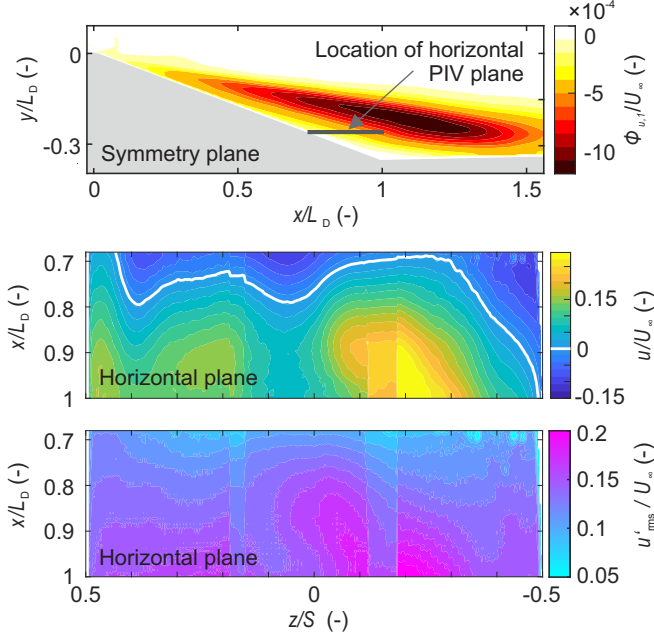


FIG. 2. The top panel shows the first POD mode inside the symmetry plane and indicates the location of the horizontal measurement plane; the two lower panels show the mean (center) and fluctuating (bottom) streamwise velocity component inside the horizontal measurement plane.

III. RESULTS

After providing an overview of the time-averaged and fluctuating velocity distributions, we turn our attention to the low-order stochastic model of the low-frequency dynamics.

A. Mean and fluctuating velocity field in horizontal plane

The mean and fluctuating parts of the velocity field inside the horizontal measurement plane are shown in Fig. 2. Since the presented data were merged from three camera views, minor discontinuities can be expected in the overlapping regions due to slight misalignments. The upstream end of this plane approximately coincides with the mean separation point on the centerline, whereas the downstream end reaches the streamwise location of the diffuser foot. Hence, low-magnitude streamwise velocities $|u/U_\infty| < 0.3$ are observed throughout the horizontal plane with mean reverse-flow occurring in the upstream part. Despite some asymmetry, the velocity deficit appears to be more distinct near the side-walls, especially at negative z , which is in agreement with the three-dimensional flow topology shown in Fig. 1.

The bottom diagram in Fig. 2 indicates that streamwise velocity fluctuations grow in x , which is explained by the measurement plane intersecting the shear-layer. Furthermore, a slight asymmetry is observed as the largest level of fluctuations appears to coincide with the region of largest mean velocity at $z/S = -0.3, \dots, -0.1$.

Figure 3 shows a selected interval of the time trace of velocity fluctuations as a function of the spanwise coordinate extracted at $x/L_D = 0.9$. For the sake of visualization, only every 10th time step is displayed.

Amidst the turbulent nature of the flow manifested in small-scale fluctuations, there are large-scale fluctuations (in terms of z) that persist for several seconds (i.e., much longer than the convective timescale of the flow). These correlated fluctuations appear to be organized in the

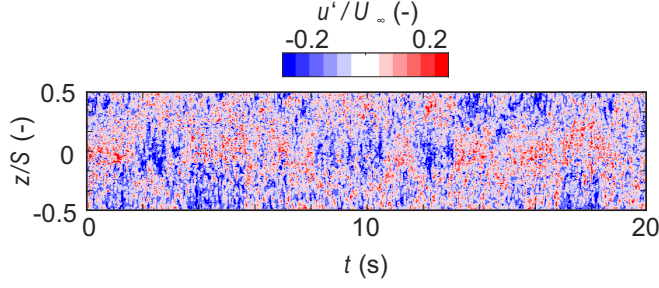


FIG. 3. Time trace of velocity fluctuations at $x/L_D = 0.9$ (only a selected interval of the entire 125 s time series is shown here).

spanwise direction inasmuch as regions of positive u' are accompanied by fluctuations of opposite sign in their spanwise neighborhood.

However, a deeper analysis and intuition are overwhelmed by the high-dimensional nature of the dynamics under consideration, spanning several orders of magnitude in spatiotemporal scales. This complexity motivates a reduced-order description.

B. Construction and validation of the stochastic reduced-order model

We utilize space-only snapshot POD to decompose the streamwise velocity fluctuations into spatial modes Φ_u^i and associated temporal coefficients a^i . This method has been proven to separate the low-frequency TSB unsteadiness from other higher-frequency dynamical events (e.g., in Ref. [4]).

The dominant spatial mode is presented in Fig. 4. Despite capturing only approximately 15% of the turbulent kinetic energy, this mode dominates the low-frequency variance, as indicated in Figs. 4(b) and 4(c).

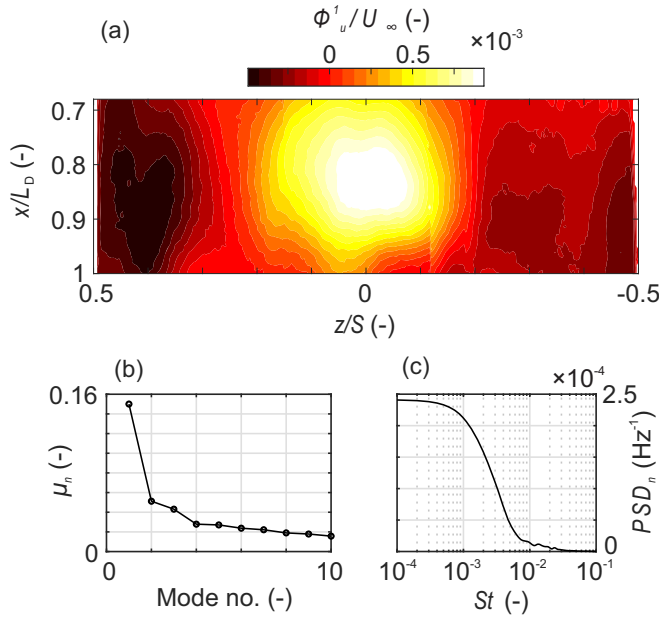


FIG. 4. (a) Leading space-only POD mode observed in the horizontal measurement plane, (b) Normalized POD eigenvalues, (c) normalized PSD of first POD coefficient.

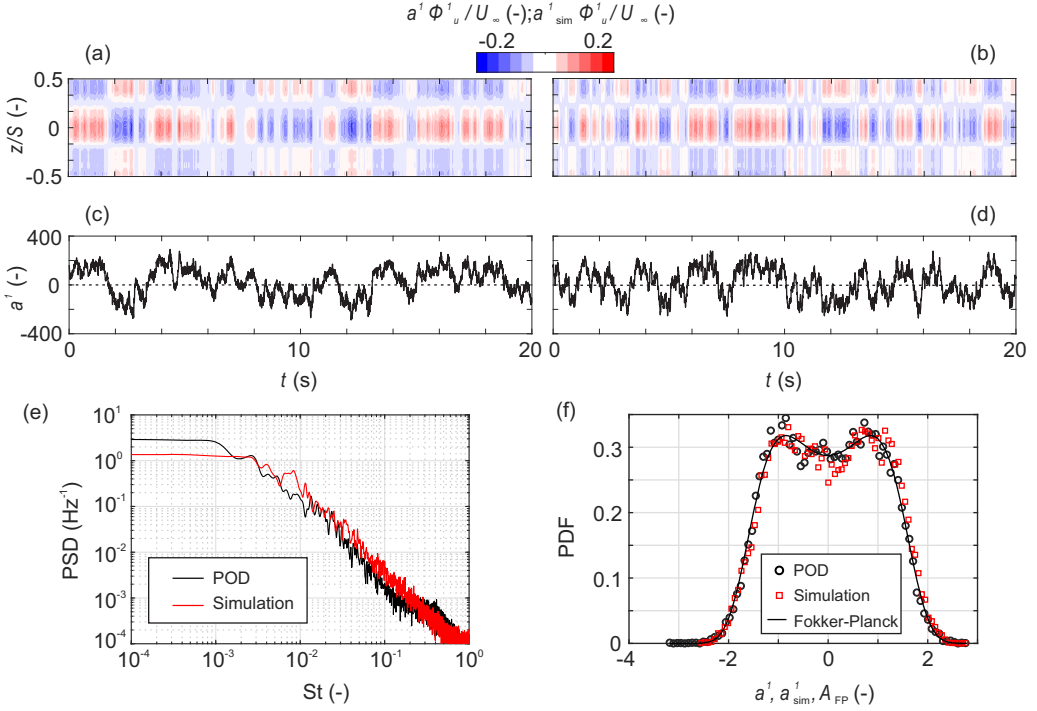


FIG. 5. Validation of stochastic model; low-order representation of velocity fluctuations at $x/L_D = 0.9$ according to (a) leading POD mode and (b) stochastic model with corresponding amplitudes in panels (c) and (d), respectively; (e) probability density functions of POD coefficient and modeled amplitude, along with the Fokker-Planck distribution; (f) frequency spectra of POD coefficient and stochastic model.

The structure is fully consistent with the leading modes revealed by spectral POD at low frequencies ($St < 0.05$), which were recently shown to correspond to a standing wave governing the spanwise dynamics [10]. Specifically, there are three distinct regions across the test section span featuring interrelated dynamics. The behavior near the two side walls appears to be correlated, whereas the velocity fluctuations near the centerline are of opposite sign. Hence, when the streamwise velocity in the center region is increased, negative velocity fluctuations are observed near the side walls, and vice versa.

By modulating the leading POD mode with the related temporal coefficient, a low-order model of the velocity fluctuations can be established [Fig. 5(a)].

The time trace shown in Fig. 5(a) covers the same time sequence as the high-dimensional state shown in Fig. 3. However, a much clearer picture unfolds due to the strong dimensionality reduction that exposes spanwise anticorrelated regions. It is also interesting to note that, according to the leading POD mode, the magnitude of fluctuations is smaller in the sidewall regions. This can be explained by continuity arguments, assuming that the bulk flow rate is approximately constant and considering that the sum of the sidewall regions makes up about two-thirds of the test section span (vs one-third of the core region). In other words, conservation of the bulk flow rate implies that the spanwise integral of the streamwise velocity fluctuations in the measurement plane is approximately zero at each time. As a result, positive fluctuations in the core region must be compensated by negative fluctuations near the sidewalls. This argument does not assume two-dimensional flow dynamics but reflects a redistribution of streamwise momentum across the span within the measured plane.

It is worth mentioning that the spanwise dynamics described by the dominant POD mode are consistent with previous studies that primarily rely on data obtained inside the symmetry plane. At $z/S = 0$, a low-frequency alternation between positive (red) and negative (blue) velocity fluctuations is observed, which yields the contraction and expansion of the TSB inside the symmetry plane.

The low-frequency nature of this process is underscored by the evolution of the leading mode coefficient in Fig. 5(c), which features approximately ten unambiguous zero-crossings within the presented 20 s interval.

The present POD analysis is based on planar PIV measurements and therefore does not provide the full three-dimensional flow state. The extracted leading POD mode should thus be interpreted as the dominant coherent structure in the measurement plane rather than as a complete three-dimensional mode. Recent volumetric measurements showed that the low-frequency TSB dynamics are inherently three-dimensional and governed by pronounced spanwise organization [9], while also indicating that, for a suitably chosen horizontal plane, the dominant low-frequency behavior can be recovered from planar measurements along the span. The present POD mode is therefore understood as a dynamically meaningful planar projection of the underlying three-dimensional motion.

The low-frequency dynamics, represented by the dominant POD mode, are now modeled by the nonlinear stochastic Langevin equation (Sec. II C). Before elaborating on the model parameters, it is necessary to conduct a validation by comparing the simulated dynamics a_{sim}^1 with a^1 . To this end, the simulated time trace of fluctuations is presented in Fig. 5(b) along with the evolution of the corresponding amplitude [Fig. 5(d)]. It is important to keep in mind that an accurate stochastic model is not reflected in matching trajectories (a^1 versus a_{sim}^1) but rather in statistical matching. This is because, even with identical initial conditions, the stochastic term (i.e., the noise realization) differs; hence, a unique trajectory does not exist. Nonetheless, the underlying statistical laws are meaningful and comparable.

Comparing the POD coefficient and the simulated amplitude of the model in Figs. 5(a) through 5(d), good agreement is observed. Both the range of values and the nonperiodic, low-frequency behavior appear to be similar.

The latter aspect is confirmed by the power spectral density (PSD) distributions of both amplitudes presented in Fig. 5(e). Both spectra match reasonably well, showing the typical low-pass filter behavior [19] with a cutoff frequency of approximately $St \approx 0.003$.

Furthermore, the probability density functions (PDFs) of a^1 and a_{sim}^1 are compared in Fig. 5(f). Note that the amplitudes were normalized by the respective standard deviation.

Along with the PDFs for the empirical (POD) and simulated amplitude, the Fokker–Planck equation is drawn upon [22], representing the probability-level counterpart of the Langevin equation. Here, its analytical stationary solution

$$p_{\text{FP}}(A) \propto \exp\left(\frac{\sigma A^2}{2D} - \frac{\gamma A^4}{4D}\right). \quad (9)$$

is evaluated using the fitted parameters.

The stochastic model reproduces both the empirical data and the Fokker–Planck probability distribution; hence, a successful validation can be attested. Interestingly, all three PDFs exhibit a clear bimodal structure, indicating that the flow topology spends most of the time in or near two preferred amplitude states separated by a low-probability region. This is explained mathematically by a positive growth rate (Table I), yielding a stable nonzero amplitude (i.e., two fixed points of the drift term)

$$A_* = \pm\sqrt{\sigma/\gamma} \approx \pm 0.85. \quad (10)$$

Recall from Sec. II C that, in the case of negative linear growth, the sole stable fixed point would be $A_* = 0$. Here, however, the positive linear growth rate also indicates that small amplitudes are, on average, amplified rather than damped at a characteristic growth time of $1/\sigma \approx 0.46$ s.

TABLE I. Fitted parameters of the stochastic model.

Parameter	Symbol	Value
Linear growth rate	σ	2.19 s^{-1}
Nonlinear saturation	γ	3.06 s^{-1}
Noise intensity	D	3.89 s^{-1}

Finally, to estimate the extent to which the inferred stochastic forcing is affected by measurement noise reported in Sec. II B, the velocity-noise variance is propagated to the first POD coefficient and, subsequently, to the residuals. Under the snapshot POD normalization used here, the first modal coefficient is obtained by projecting the fluctuation field onto a unit-norm spatial mode. Assuming spatially uncorrelated zero-mean measurement noise, this implies that the induced coefficient variance is of the same order as the per-grid-point velocity-noise variance, i.e., $\text{Var}(a_1^{\sigma_u}) \approx \sigma_u^2$. The resulting contribution to the residual variance is $\langle (r_n^{\sigma_u})^2 \rangle \approx 0.017 \text{ (m/s)}^2$, compared with the total variance $\langle r_n^2 \rangle \approx 0.039 \text{ (m/s)}^2$. Experimental uncertainty therefore accounts for a non-negligible, but not dominant, fraction of the inferred stochastic forcing, indicating that the diffusion term should be interpreted as an effective noise level containing contributions from both measurement uncertainty and unresolved physical flow fluctuations.

C. Discussion of stochastic model parameters

The deterministic part of the Langevin equation can be expressed in terms of an effective potential $U(A)$, defined such that the drift corresponds to its negative gradient. Minima of this effective potential correspond to stable equilibria, while maxima correspond to unstable states [22]. The potential difference quantifies the barrier that must be overcome by stochastic forcing for transitions between the two stable states [23]. This quantity is given by

$$\Delta U = U(0) - U(A_*) = \frac{\sigma^2}{4\gamma} \approx 0.39, \quad (11)$$

which is small compared with the inferred effective noise intensity D (Table I), indicating that stochastic fluctuations can readily overcome the deterministic mechanisms. As a consequence, the system does not remain confined near one of the two stable states, and frequent noise-driven excursions away from A_* are observed.

In summary, the stochastic amplitude model reveals that the symmetric equilibrium at $A = 0$ has become unstable, and the system features two nonzero, dynamically equivalent equilibrium states that correspond to opposite spanwise redistributions of momentum. In the presence of stochastic forcing, these equilibria do not correspond to fixed steady states but to probabilistic attractors around which the system fluctuates due to turbulent noise. The resulting dynamics are characterized by a bimodal stationary PDF and intermittent, noise-driven switching between the two states on timescales that are large compared with the characteristic scales of the flow. Importantly, this behavior does not imply a periodic instability or deterministic oscillation, but rather a zero-frequency (steady) bifurcation blurred by stochastic forcing, where turbulence continuously perturbs the system across the unstable equilibrium.

Finally, the velocity fields corresponding to the two equilibrium states are constructed by superposing the mean field with the first POD mode modulated by A_* , see Fig. 6.

They clearly highlight the core vs sidewall dynamics that are expressed by the dominant mode. When only the symmetry plane is considered, the two states amount to an effective expansion ($-A_1$) and contraction ($+A_1$) of the separated region, which is fully consistent with the commonly observed breathing motion.

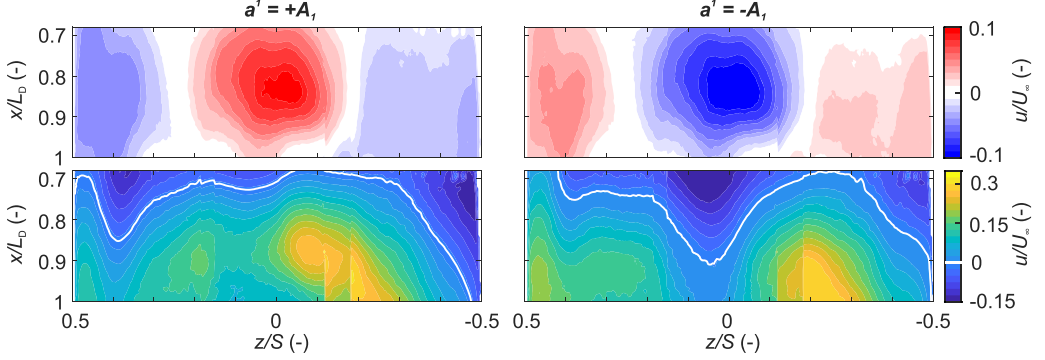


FIG. 6. The top row shows the streamwise velocity fluctuations associated with the two amplitude equilibria; the bottom row shows the corresponding reduced-order models of the velocity field.

IV. CONCLUSIONS

The objective of this study was to characterize and model the low-frequency spanwise dynamics of a turbulent separation bubble developing in a fully turbulent half-diffuser flow. By analyzing time-resolved PIV measurements across the full span, we sought to determine whether the classical breathing motion is genuinely two-dimensional and to what extent its temporal evolution can be described using stochastic reduced-order modeling. Modal decomposition of the streamwise velocity fluctuations revealed a leading POD mode that organizes the low-frequency unsteadiness in the spanwise direction. This mode exhibits anticorrelated dynamics between the core region and the sidewall regions, indicating a global redistribution of momentum rather than a uniform spanwise expansion. In agreement with recent studies [9], these findings underline that realistic TSBs, even in nominally two-dimensional geometries, possess spanwise degrees of freedom that govern their long-term evolution.

A nonlinear Langevin equation was identified from the time series of the leading POD coefficient using linear regression and residual-variance matching. The model reproduces the empirical power spectra and probability density functions and reveals that the flow is in a regime of noise-driven bistability. The two preferred amplitude states correspond to contracted and expanded TSBs, while turbulent forcing continuously perturbs the system across the unstable equilibrium at zero amplitude. The inferred noise intensity exceeds the effective potential barrier between the equilibrium states, explaining the observed intermittent switching between the two states or, in other words, the lack of locking onto one branch of the pitchfork. Beyond the present study, the relative effective noise intensity may provide a plausible explanation for why relatively persistent spanwise structures have been reported in other flow configurations (e.g., for some stall cell setups [24]). Rather than a fundamentally different instability mechanism, such behavior could reflect a comparatively weaker stochastic forcing.

Notably, recent linear analyses of TSB flows have identified a stationary global eigenmode and interpreted low-frequency breathing as a forced modal response, where turbulent forcing drives the receptivity of the flow to low-frequency disturbances [10,19]. The present results are consistent with the view that such a stationary mode may lead to steady mean-flow deformations. The nonlinear stochastic model provides a dynamical interpretation in terms of noise-driven switching between preferred separation-bubble states consistent with a stochastic pitchfork-type process rather than self-sustained periodic instability.

In summary, this study shows that the low-frequency dynamics of turbulent separation bubbles are governed by a spanwise-organized coherent structure whose amplitude follows nonlinear stochastic laws. The resulting bimodal behavior reconciles earlier observations of broadband breath-

ing motions with a physically grounded dynamical model and provides a quantitative basis for interpreting long-time unsteadiness in pressure-gradient-induced TSBs.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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