

Self-excited acoustic parametric instability in downward-propagating premixed flames

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Flame-acoustics interactions are traditionally studied to understand and mitigate the occurrence of thermoacoustic instabilities. Combustion instabilities have been a huge problem in the development of high-performance combustors such as gas turbines, rocket motors, and industrial furnaces because they produce vibration and noise that compromise flow stability, structural integrity, and combustor efficiency, ensuing a shortened combustor lifetime or worse, a catastrophic failure [1]. Thermoacoustic instabilities arise from the complex dynamic interaction between combustor acoustics and unsteady heat release, presenting a challenging problem in combustion science and nonlinear dynamics. The Rayleigh criterion [2] states that pressure perturbations can either grow or decay depending on the phase relationship between pressure and heat release fluctuations, and the competition between acoustic energy generation and damping mechanisms from thermal, viscous, and acoustic losses.

Combustible mixtures ignited in confined geometries typically produce sound during propagation and are inherently susceptible to thermoacoustic instabilities. Propagating flames exhibit oscillatory behavior due to the interaction between the reactive surface, flow-field, and natural acoustic modes of the combustor. Acoustic flame instabilities can be fundamentally studied in a simple experiment of downward propagating premixed flames in open-closed combustion tubes [3–5] as shown in Fig. 1(a). In the experiments, described exhaustively in our previous works [4,5], two types of acoustic instabilities are observed during flame propagation: primary and secondary instabilities [3–5]. Primary instabilities are observed at low-amplitude acoustic fluctuations characterized by curved and flat flames that pulsate back and forth at the same frequency of acoustic fluctuations. Secondary instabilities occur upon the saturation of primary instability where vibrating planar flames spontaneously excite surface waves that increase in amplitude, leading to large-amplitude acoustic fluctuations and an explosive self-turbulization. Secondary instability is triggered by

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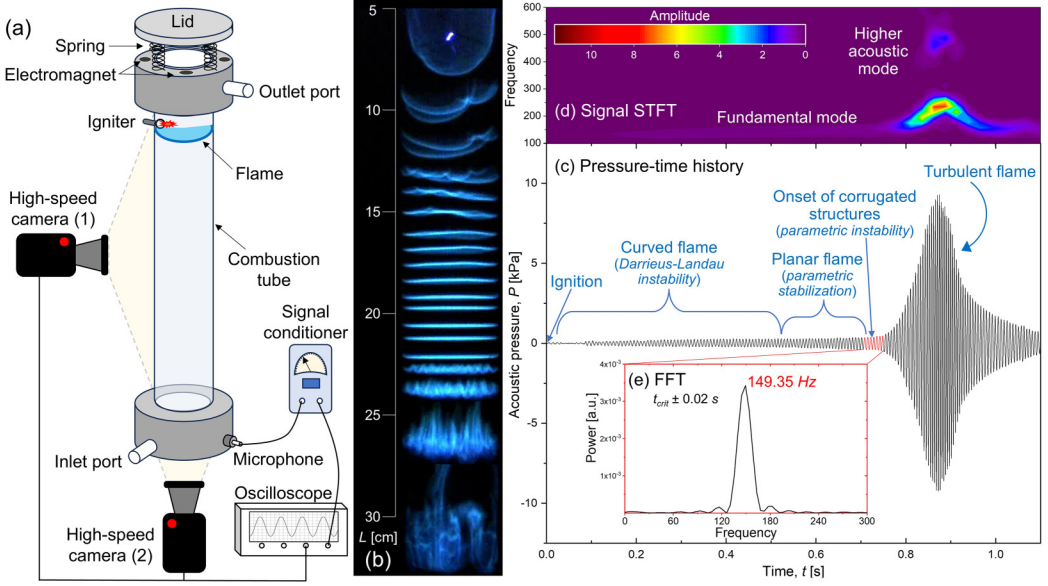


FIG. 1. Experimental setup and flame response during flame propagation. (a) Schematic of the combustion tube experiment. (b) High-speed flame images depicting the lateral spatial evolution of flame structures along the tube length, and the transition from laminar fronts to cellular and turbulent regimes of $\text{C}_3\text{H}_8/\text{O}_2/\text{N}_2$ flame at equivalence ratio of $\phi = 1.2$ with laminar flame speed of $S_L = 20 \text{ cm/s}$. (c) Pressure-time profile and corresponding major stages of instability, including initial ignition and intrinsic Darrieus-Landau instability, followed by flame front flattening, emergence of parametric instability, and transition to turbulent flame propagation. (d) Short-time Fourier transform (STFT) of pressure signal, revealing dominant frequencies and acoustic modes. (e) Fast Fourier transform (FFT) of pressure signal at the onset of parametric instability indicating characteristic excitation frequency.

parametric instability, a mechanism similar to Faraday instability first recognized by Markstein [6]. Figures 1(b)–(d) show the spatiotemporal evolution of the flame and corresponding acoustic pressure oscillations measured from the closed end of the tube. Resonant frequencies are dominated by fundamental mode; however, higher harmonics arise during turbulence. Before ignition, no acoustic oscillations are present in the tube. Weak perturbations are initially present due to the ignition process or the growth of curved flame caused by intrinsic flame instabilities such as Darrieus-Landau (D-L) hydrodynamic instability. Amplitude and frequency of oscillations gradually increase with time which cause the flame to evolve into different flame shapes. Amplification usually involves two processes: velocity coupling [6,7] and pressure coupling [8,9]. Velocity coupling occurs when pulsating velocity fluctuations on the flame front changes the flame shape and amplitude. The change in flame surface area induces heat release fluctuations that couple with acoustic oscillations. Pressure coupling, on the other hand, occurs when the flame front undergoes adiabatic compression and expansion as pressure waves pass through the combustion front. This periodic process causes small temperature variations and enthalpy fluctuations in phase with acoustic waves. Generally, velocity coupling dominates the instability mechanism [10]; however, when flame becomes planar, pressure coupling can also govern the amplification of oscillations [8,9].

A theory on parametric acoustic instability [11], derived from large activation energy asymptotics [12], accurately predicts the flame behavior observed in the experiments. Originally proposed by Markstein [6], Searby and Rochwerger [11] extended the dispersion relation describing intrinsic D-L instability [12] by accounting the effect of velocity fluctuations. The analytical relations are reduced into a Mathieu equation representing a parametric oscillator subjected to a sinusoidal

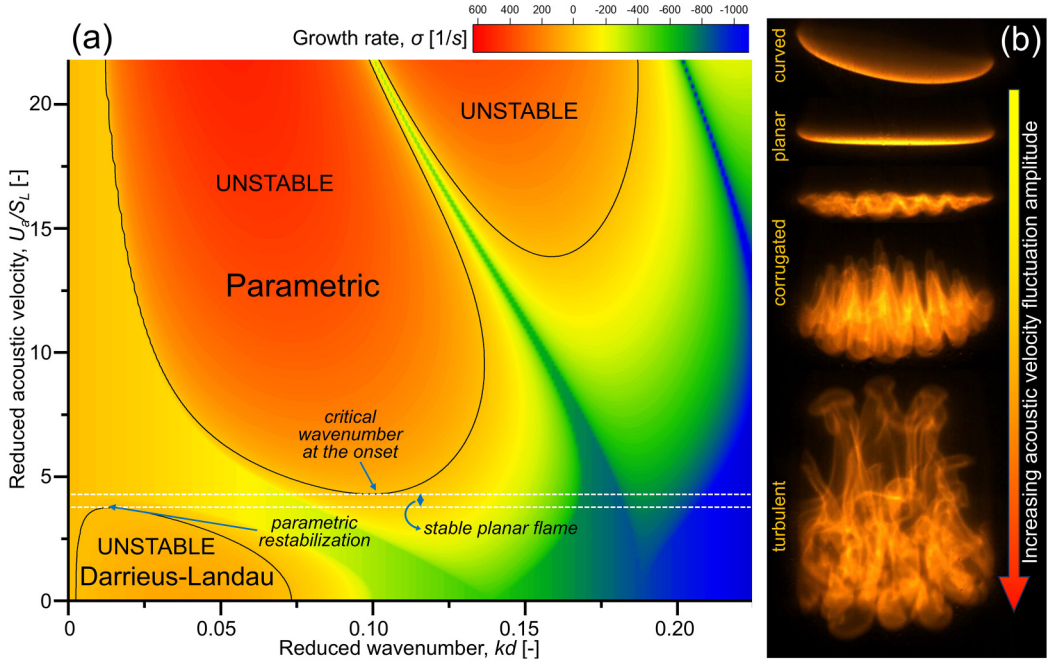


FIG. 2. Instability diagram of parametrically excited laminar premixed ammonia flame ($\text{NH}_3/\text{O}_2/\text{N}_2, \phi = 1.2$, $S_L = 20$ cm/s, $f = 175$ Hz) and corresponding flame morphologies during propagation. (a) Stability map showing tongues of instability (black) and growth rate (σ) of perturbations in reduced wavenumber-acoustic velocity plane. The system is stable ($\sigma < 0$), marginally stable ($\sigma = 0$, black line), and unstable ($\sigma > 0$). Intrinsic Darrieus-Landau instability, characterized by curved/wrinkled flames, persists at low-amplitude acoustic oscillations but is suppressed at sufficiently high acoustic velocity. Stable planar flame is observed for a range of acoustic velocities where growth rates remain negative shown in horizontal dashed lines. When another critical acoustic velocity is reached, parametric instability develops, leading to formation of oscillating cellular flames of certain wavenumber. (b) Corresponding flame structures demonstrate the transition from curved to planar, cellular, highly corrugated, and turbulent flames as amplitude of acoustic velocity fluctuations increases.

restoring force, whose solutions resemble those of a Kapitza pendulum as shown in Fig. 2(a). The abscissa refers to wavenumber of the flame (k) normalized by flame thickness (d), while the ordinate refers to acoustic velocity fluctuation amplitude (U_a) normalized by unstretched laminar burning velocity (S_L). Acoustic velocity is often confused with sound speed; however, U_a represents the velocity of a particle when a pressure wave passes through it. As shown in Figs. 1(b) and 2(b), the flame begins with smooth wrinkling and curvature at small wavenumbers (long wavelengths) because of intrinsic D-L instability shown in lower left unstable space in Fig. 2(a). When acoustic intensity increases as the flame freely propagates downwards, D-L instability weakens. At sufficiently high U_a , the presence of acoustic field effectively suppresses D-L instability and flame becomes stable and planar. The planar flame pulsates and propagates at a speed very close to S_L [3,4]. Planar flame regime corresponds to the band of U_a where growth rates are negative in Fig. 2(a). Symmetry breaking occurs when U_a acting on the planar flame reaches the critical parametric instability threshold where cellular flames at large wavenumbers (short wavelengths) emerge and oscillate at a subharmonic frequency like Faraday waves in vibrating pools [13]. Once parametric instability develops, amplitude of cellular flames drastically increases, and they break down into incoherent structures.

Strictly speaking, parametric instabilities only develop when a system is excited by external forcing of a well-defined frequency and amplitude like a Kapitza pendulum. Parametric instabilities in the case of freely propagating flames, however, are self-excited, with thermoacoustic coupling acting as an amplifying mechanism. The acoustic frequency and amplitude are results of a resonance phenomenon that vary at different locations of the tube [7,8] where heat losses can be significant [14] and higher acoustic modes can arise as seen in Fig. 1(d). In this work, excitation frequency is determined by syncing high-speed video with the pressure signal, visually determining the onset at t_{crit} (see Fig. 3), and performing fast Fourier transform (FFT) within a narrow time window around $t_{\text{crit}} \pm 0.02$ s, as shown in Fig. 1(e). The stability map in Fig. 2(a) is frequency sensitive. Higher (lower) frequencies shift the parametric instability tongues toward larger (lower) wavenumbers, and expands (shrinks) the band of U_a where stable flat flames can be sustained.

The transition from flat to corrugated flames at the critical threshold of parametric instability unfolds smoothly and strikingly in the experiments. Figure 3 synchronizes the flame evolution with acoustic pressure and velocity acting on the flame front. Cellular flames are not vividly defined at the onset of parametric instability. The flame front initially displays merged cellular patterns with outlines resembling concentric lines or rings (frames 1–5). As U_a increases, the amplitude of cellular structures grows (frames 6–15), eventually forming distinct, well-defined cells (frames 16–17). After one acoustic period, the centroid of local maxima (cells) aligns with the local minima (dark troughs) from the previous cycle and vice versa. Here, Faraday waves occur on a reactive interface separating the burned and unburned regions with unequal densities. At higher U_a , cellular structures begin to merge and lose coherence (frame 18), resulting to a chaotic behavior and violent turbulence.

Wavelengths of cellular flames can be determined by measuring centroidal distances between adjacent cells [4,5]. To obtain the spatial scale for these measurements, tube cross section is traced by adjusting image brightness. Ideally, experimental critical kd and U_a/S_L at the onset of parametric instability should correspond with theoretical predictions, shown in Fig. 2(a). However, the reliability of stability maps derived from the Mathieu equation heavily depends on the accuracy of the Markstein number (Ma), which must be known *a priori* [11]. Ma characterizes the sensitivity of local flame velocity to local stretch rate arising from flame curvature and/or flow-field strain, and it strongly influences flame stability [4,5,11,15–20]. Measuring the frequency and critical kd or U_a/S_L at the onset has been used to retrospectively infer Ma values [5,16]. Ma plays a role analogous to surface tension in Faraday waves observed on vertically vibrating liquid surfaces. Surface tension stabilizes interface by resisting deformation, similar to how Ma resists flame wrinkling and suppresses cellular instabilities.

Varying the combustor geometry alters the flame response during propagation due to the change in frequency, and heat, wall, and acoustic losses [10,14,16]. Moreover, changes in fuel type, equivalence ratio, and diluent composition influence key flame parameters such as the Zeldovich number (nondimensional activation energy quantifying the reaction rate sensitivity to temperature) [4,9], Lewis number (Le , ratio of thermal to mass diffusivity indicating how heat and mass species diffuse within the flame) [4,15–18,20], burning speed [4,9,15], and flame temperature (thermal expansion) [4,12]. Among these, Le significantly affects Ma [15,20]. Such low- Le effects are notorious for characterizing the highly unstable nature of lean hydrogen-air flames [18,21].

Parametric instabilities have also been observed in other burner configurations [16–19], where external forcing or intrinsic dynamics trigger flame-flow interactions. This raises a fundamental and thought-provoking question: do similar self-turbulization thresholds, like the onset of parametric instability in Fig. 2, exist in more complex environments such as rocket engines, gas turbines, and industrial furnaces? These systems involve a more complex interaction between turbulence, acoustics, and combustion dynamics. Identifying instability mechanisms similar to parametric instability could provide valuable insights on their stability limits, operational range, and failure mechanisms. Controlled studies, such as the present work on downward propagating premixed flames, offer a simplified yet systematic approach for uncovering the fundamental behavior of such instabilities.

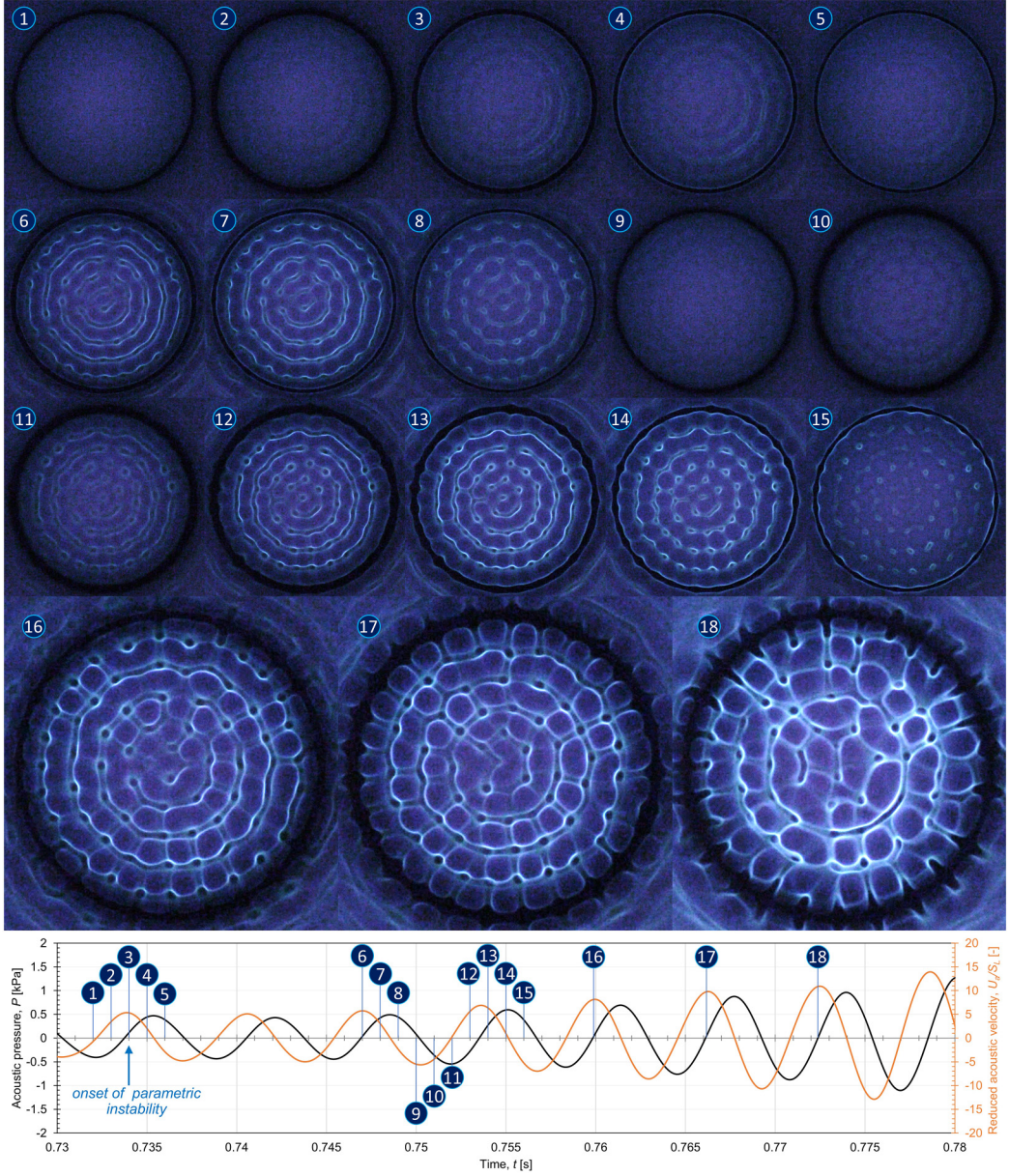


FIG. 3. Evolution of cellular flame structures during parametric instability for the same mixture in Fig. 1(b) captured from the bottom view using a Photron FastCam Mini AX200 at 3000 FPS with LUT postprocessing. The optical setup consisted of a Nikon 105 mm f/2.5 lens with a 10 mm extension ring for close-up imaging. (Top) Sequential flame images of the transition from laminar fronts to highly corrugated flame structures. Numbered frames correspond to specific time instances. (Bottom) Acoustic pressure P (black) and acoustic velocity U_a/S_L (orange) plotted against time, synchronized with flame evolution. Flame surface deformation follows periodic forcing at constant wavenumber, leading to the increase in flame amplitude and formation of more defined cellular boundaries as the instability intensifies.

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Data availability. The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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