Erratum: Variance of the velocity in suspensions of particles does not diverge [Phys. Rev. Fluids 9, L102301 (2024)]

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It has come to my attention that the assumption for the correlations of the Langevin forces, $\boldsymbol{\xi}$, that I used in the original paper do not satisfy the fluctuation-dissipation theorem [1,2]. In the original paper, it was assumed that the frequency space correlations for the forces between particle m and n were given by $\langle \xi_{m,i}\xi_{n,j}\rangle = c^2\delta_{ij}\delta_{mn}/2\pi$, where c^2 was a constant that was determined from equipartition. However, to be consistent with the fluctuation-dissipation theorem, these correlations should be given by $\langle \xi_{m,i}\xi_{n,j}\rangle = (2k_BT/\pi)\mathrm{Re}[\hat{\boldsymbol{\gamma}}(\omega)]$, where k_BT is thermal energy and $\mathrm{Re}[\hat{\boldsymbol{\gamma}}(\omega)]$ is the real part of the Fourier transform of the resistance matrix [1,2]. The consequences of this error do not affect the overall result that consideration of the unsteady motion of the suspended particles leads to a finite variance of the velocity in a suspension of spheres; however, it does impact some of the quantitative results.

For the case of a single suspended sphere of radius a in a fluid of density ρ_f and viscosity η that is governed by the unsteady Stokes' equations, the frequency-space force correlations should be

$$\langle \xi_i\xi_j\rangle = \frac{2\zeta k_BT}{\pi}(1+k)\delta_{ij}, \tag{1}$$

where $\zeta = 6\pi\eta a$ is the Stokes' drag coefficient and $k = \sqrt{\rho_f a^2\omega/2\eta}$. It is then convenient to define $y = k/\sqrt{\alpha}$ with $\alpha = \rho_f/(\rho_s + \frac{1}{2}\rho_f)$ and ρ_s the density of the sphere. The variance of the sphere's velocity is then given by

$$\begin{aligned}\langle v_{p,i}(t)v_{p,j}(t)\rangle &= \frac{16kT\delta_{ij}}{9\pi m_e}\int_0^\infty \frac{y(1+\sqrt{\alpha}y)dy}{1+2\sqrt{\alpha}y+2\alpha y^2+\frac{8\sqrt{\alpha}}{9}y^3+\frac{4}{81}y^4}\\ &= \frac{kT\delta_{ij}}{m_e},\end{aligned} \tag{2}$$

where $m_e = (\rho_s + \frac{1}{2}\rho_f)V_p$ is the effective mass of the sphere and V_p is the sphere volume. That is, the correct application of the fluctuation-dissipation theorem naturally leads to the variance of the sphere's velocity depending inversely on the effective mass of the sphere, which was previously just assumed to apply in Eq. (5) of the original equation. It is also interesting, and not obvious, that the integral in Eq. (2) does not depend on the value of α.

One other important correction is that the relationship between the Green's function strength and the applied force on the mth particle, Eq. (3) in the original paper, should have been

$$\hat{\mathbf{B}}_m^0 = \mathbf{F}_m + \frac{2}{9}\left(1-\frac{\rho_s}{\rho_f}\right)\zeta\lambda^2a^2\mathbf{v}_m. \tag{3}$$

That is, for neutrally buoyant particles the zeroth Green's function strength is the net force on the particle.

In a suspension of particles, the resistance is affected by the presence of the other particles. However, here we are only considering the dilute limit and the leading order effects from the zeroth-order Green's functions. At this level of approximation, a sphere only dissipates energy from the motion produced by the force that is applied to it. Forces applied to the other spheres do not lead to additional dissipation from that sphere, because each sphere is merely advected by the flow produced by the other spheres. Consequently, in this scenario we can treat the forces on different particles as being uncorrelated and assume that the force correlations in the suspension are given by the generalization of Eq. (1),

$$\langle \xi_{m,i}\xi_{n,j}\rangle = \frac{2\zeta k_B T}{\pi}(1+k)\delta_{ij}\delta_{mn}. \tag{4}$$

These changes to the Green's function strength and the force correlations then simplify the frequency-dependent factor $h(\omega)$ in the definition of the fluid velocity variance that follows Eq. (12) in the original paper to

$$h(\omega) = \frac{2\zeta k_B T}{\pi}(1+k), \tag{5}$$

and modifies the definition of the particle velocity variance [Eq. (14) in the original paper] to

$$\langle \tilde{v}_{p,i}\tilde{v}_{p,j}\rangle = \frac{2k_B T(1+k)\delta_{ij}}{\pi\zeta DD^*} + \frac{9a^2 k_B T(1+k)}{8\pi\zeta}\sum_{n\neq m}\langle \hat{S}^0_{ik}\hat{S}^{0*}_{jk}\rangle. \tag{6}$$

Because of this change, the function in Eq. (16) of the original paper should have been multiplied by $1+k$ before being integrated with respect to frequency. Making this alteration and numerically integrating the expression over frequency, the result is found to be approximately equal to $(35.7\eta/\rho a)\delta_{ij}$. Consequently, the variance of the particle velocity to first order in ϕ for neutrally buoyant particles is

$$\langle v_{p,i}v_{p,j}\rangle \approx \frac{kT\delta_{ij}}{m_e}(1+0.68\phi), \tag{7}$$

and the fluid velocity variance is

$$\langle v_i v_j\rangle_f \approx \frac{2.23\phi kT\delta_{ij}}{m_e(1-\phi)}. \tag{8}$$

The alteration here is on the coefficients that multiply the ϕ dependence of the variances. In the original paper these terms were 1.07ϕ and 5.11ϕ instead of the corrected values of 0.68ϕ and 2.23ϕ, respectively.

[1] H. B. Callen and T. A. Welton, Irreversibility and generalized noise, Phys. Rev. **83**, 34 (1951).
[2] R. Kubo, The fluctuation-dissipation theorem, Rep. Prog. Phys. **29**, 255 (1966).