Engelund bedload transport formula for sparsely vegetated channels

Yihan Qu
College of Water Conservancy and Hydropower Engineering, Hohai University, Nanjing 210098, China

Zhiheng Ye
Guangdong Research Institute of Water Resources and Hydropower, Guangzhou 510635, China

Qingwei Lin
College of Water Conservancy and Hydropower Engineering, Hohai University, Nanjing 210098, China

Limo Tang*
College of Water Conservancy and Hydropower Engineering, Hohai University, Nanjing 210098, China

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The current research was conducted to investigate bedload transport in sparsely vegetated channels through theoretical derivation. Most research on bedload transport in vegetated channels has relied on theories developed for bare beds. The classical Engelund formula is highly effective in predicting bedload transport rate on bare beds, but is based on four oversimplified assumptions. Meng *et al.* [Int. J. Sediment Res. **31**, 251 (2016)] modified those oversimplified assumptions and proposed an improved version of the Engelund formula for bare beds. In this work, the improved Engelund formula was successfully extended to sparsely vegetated channels through the redefinition of key parameters. For instance, we adjusted the number of moving sediment particles per unit bed area and the effective Shields number (dimensionless bed shear stress acting on sediment movement in vegetated channels) to account for the impact of emergent rigid vegetation. Parameter calibration was accomplished using published data from a variety of reliable sources, ensuring the accuracy and applicability of the extended model. The prediction results correlated well with measured data across a broad range of bedload transport rates, particularly in scenarios of medium and high bedload transport rates.

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I. INTRODUCTION

Bedload transport refers to the movement of sediment particles by rolling, sliding, and saltating near the riverbed [1–2]. It has been extensively studied in open channel flows, and numerous formulas to predict the bedload transport rate were developed. Commonly used formulas in the literature include those by Meyer-Peter and Müller [3], Einstein [4], Bagnold [5], Engelund and Fredsøe [6], and Parker [7]. These models generally perform well within the respective ranges of applicability [8].

Recently, increasing attention has been directed toward bedload transport in vegetated channels [9–13]. The presence of vegetation was shown to significantly increase flow resistance [14], reduce flow velocity [15], alter flow velocity distribution [16], and decrease the flow's capacity to transport

*Contact author: tanglimo@hhu.edu.cn

sediment [17]. Studies of bedload transport in vegetated channels mostly rely on theories developed for bare beds, with modifications made to hydraulic parameters to account for the effect of vegetation. These approaches can be broadly categorized into three types. The first is to use shear stress, with the bedload transport rate correlated with the bed shear stress. Jordanova and James [9] studied bedload transport in emergent vegetated channels and predicted the bedload transport rate from the bed shear stress (τ_{bv}), which was the difference between the total flow resistance (τ_{tot}) and the vegetation drag force (τ_v). Kothyari *et al.* [10] derived a semiempirical formula for bedload transport in vegetated channels and suggested that the bed shear stress, rather than the total flow resistance, provides a better description of the variations in bedload transport rate. Wang *et al.* [18] noted that bedload transport in vegetated channels was only related to the grain shear stress (τ_{gv}), which was a component of the bed shear stress (i.e., $\tau_{gv} = \tau_{bv}–\tau_{sv}$, where τ_{sv} is the form drag caused by bedforms). They proposed a semiempirical formula to predict the bedload transport rate from the grain shear stress. This formula performed very well in predicting bedload transport rate for vegetated channels, particularly in cases of weak bedload transport rates. The second approach is to use turbulence kinetic energy (TKE), with the bedload transport rate correlated with the TKE generated by the vegetation stems. Yang and Nepf [19] discussed that traditional τ_{bv}-based formulas are not accurate for vegetated channels, mainly due to neglecting vegetation-induced turbulence. They proposed a turbulence-based bedload transport model for vegetated channels. In a subsequent study, Yang and Nepf [20] showed that near-bed TKE could be a more universally effective predictor of bedload transport than bed shear stress. They found that the τ_{bv}-based formula is effective at low vegetation densities ($\lambda < 0.011$), but its performance diminished at higher vegetation densities. Zhao and Nepf [12] noted that the bedload transport rate was influenced by TKE, independent of stem diameter and arrangement. They reformulated the Meyer-Peter and Müller (MPM) formula [3] to express it in terms of TKE. The third approach involves modifying key parameters in the classic bedload transport formula for bare beds, such as the dimensionless flow intensity parameter (Ψ) and the dimensionless bedload transport rate (Φ), to account for the influence of vegetation, thereby making the formula applicable to the calculation of bedload transport rate in vegetated channels. Armanini and Cavedon [21] proposed a probabilistic/deterministic bedload transport formula for vegetated channels by redefining Einstein's dimensionless parameters Φ and Ψ to be functions of vegetation characteristics. These studies have significantly enriched the research on predicting bedload transport rates in vegetated channels. They perform well within their respective domains of validity and offer valuable insights for future research.

The key to formulating the bedload transport formula lies in understanding how sediment moves within vegetated channels. Several experiments have revealed that the incipient motion of sediment particles in vegetated channels significantly differs from that on bare beds [13,22]. Tang *et al.* [22] identified three stages of bedload transport in emergent vegetated channels. Initially, the flow intensity was weak, and no sediment was in motion (Stage 1). Subsequently, as the flow intensity increased, sediment particles near the vegetation stems began to move due to vegetation-generated turbulence, leading to scour holes formed around the stems (Stage 2). Eventually, as the flow intensity further increased, sediment particles away from the vegetation stems also began to move (Stage 3), which is indicative of sediment movement with a measurable bedload transport rate. Wu *et al.* [13] also observed this phenomenon and discovered that the sediment movement patterns during Stage 2 varied with different vegetation densities. With low vegetation density (sparse), scour holes around the stems and sand ridges behind the stems were observed. In contrast, with high vegetation density (dense), only scour holes were observed.

Recent studies indicate that bedload transport in sparsely vegetated channels is complex. In sparsely vegetated channels, sediment away from the stems is almost unaffected by them, and sediment movement can be divided into two stages based on the flow intensity. When the flow intensity is weak, sediment movement is confined to Stage 2, where sediment movement within scour holes is the primary contributor to the bedload transport rate. When the flow intensity is medium or high, sediment particles away from the vegetation stems begin to move (Stage 3), leading to a measurable bedload transport rate. The bedload transport rate in these two stages is completely different. Therefore, the impact of varying flow intensities on bedload transport in sparsely vegetated channels requires further investigation.

In this study, bedload transport in sparsely vegetated channels was investigated through theoretical derivation. The improved Engelund formula, which is applicable for calculating the sediment transport rate on bare beds, was extended to calculate the bedload transport rate in sparsely vegetated channels. This extension was achieved by reinterpreting key parameters such as the number of movable particles near the bed and the effective Shields number (dimensionless bed shear stress). Based on published experimental data, the parameters of the extended formula were calibrated and compared with other formulas for sparsely vegetated channels.

II. THEORETICAL CONSIDERATIONS

A. Basic formula

In this paper, it is assumed that bed shear stress is the primary cause of bedload transport in sparsely vegetated channels. The Engelund formula was chosen as the basic formula in this study due to its concise structure, clear physical meaning, and straightforward derivation procedure. Engelund and Fredsøe [6] simplified sediment particles into spheres and presented the basic structure of the bedload transport formula as:

$$q_b = \frac{\pi}{6} D^3 \rho_s N_b p u_b, \tag{1}$$

where q_b is the bedload transport rate per unit width, D is the sediment particle diameter, ρ_s is the sediment density, N_b is the number of sediment particles per unit bed area, p is the probability of sediment particles being entrained into motion in a single layer, and u_b is the mean velocity of moving particles in the streamwise direction. Eq. (1) can be transformed into a dimensionless form, as shown in Eq. (3), using Eq. (2):

$$\Phi = \frac{q_b}{\rho_s \sqrt{(\rho_s - \rho) g D^3 / \rho}} \tag{2}$$

and

$$\Phi = 11.6(\sqrt{\Theta_b} - 0.7\sqrt{\Theta_c})(\Theta_b - \Theta_c), \tag{3}$$

where Φ is the dimensionless bedload transport rate, ρ is the fluid density, g is the gravitational acceleration, $\Theta_b = \tau_b/[(\rho_s - \rho)gD]$ is the Shields parameter (the dimensionless bed shear stress), τ_b is the bed shear stress on bare beds, and Θ_c is the Shields parameter of sediment incipient (the dimensionless incipient shear stress). Meng *et al.* [8] pointed out that the Engelund formula oversimplifies parameters such as N_b, u_b, p, and Θ_c, which can lead to drawbacks. For instance, when the flow intensity is high, the probability of sediment incipient motion (p) may exceed 1. Consequently, they redefined these oversimplified parameters (N_b, u_b, p, and Θ_c) and proposed an improved version of the Engelund formula. This improved formula is presented in a dimensionless form as follows:

$$\Phi = kp(\sqrt{\Theta_b} - \sqrt{\Theta_c})(\Theta_b - \Theta_c), \tag{4}$$

where k is a constant coefficient calibrated by experimental data and p is the probability of sediment incipient on bare beds. The improved version of the Engelund formula proposed by Meng *et al.* [8] can be used to calculate the bedload transport rate in vegetated channels by modifying key parameters. The expression is as follows:

$$\Phi = k_v p_v(\sqrt{\Theta}_{bv} - \sqrt{\Theta}_{cv})(\Theta_{bv} - \Theta_{cv}), \tag{5}$$

where k_v is a constant coefficient calibrated by experimental data, p_v is the probability of sediment incipient, Θ_{bv} is the effective Shields parameter (the dimensionless bed shear stress in vegetated channels), and Θ_{cv} is the Shields parameter of sediment incipient. Subscript $'v'$ denotes the flow parameters specific to vegetated channels. The key parameters, such as the number of particles per unit bed area in vegetated channels (N_{bv}), effective Shields parameter [Θ_{bv}, as given in Eq. (5)], and Shields parameter of sediment incipient [Θ_{cv}, in Eq. (5)] need to be redefined to account for the presence of emergent vegetation.

B. Analytical tools

1. Determination of N_{bv}

In the classic Engelund formula, $N_b = 1/D^2$ was proposed to calculate the number of particles per unit bed area. It can only be applied to the single-layer sediment movement under conditions of weak flow intensity. Meng *et al.* [8] extended N_b to make it suitable for multilayer sediment movement:

$$N_b = m_b(\Theta_b - \Theta_c)\frac{1}{D^2}, \tag{6}$$

where $m_b(\Theta_b - \Theta_c)$ is a parameter that describes the multilayer sediment movement and m_b is a constant coefficient calibrated by experimental data. The presence of vegetation occupying the bed area and the influence of vegetation density should be taken into account when determining the number of sediment particles in vegetated channels (N_{bv}). Consequently, it is necessary to modify the number of sediment particles on bare beds (N_b) in Meng *et al.*'s [8] formula to make it applicable to vegetated channels:

$$N_{bv} = m_v(\Theta_{bv} - \Theta_{cv})\frac{1-\lambda}{D^2}, \tag{7}$$

where N_{bv} is the number of sediment particles per unit bed area in vegetated channels, m_v is a constant coefficient calibrated by experimental data, $\lambda = N\pi d^2/4$ is the vegetation density (area occupied by vegetation per unit bed area), d is the vegetation stem (cylinder) diameter, and N is the number of vegetation stems per unit bed area.

2. Determination of Θ_{bv}

In a steady, uniform open channel flow with emergent vegetation stems, the momentum balance in the streamwise direction can be expressed as follows [21]:

$$\tau_{\text{tot}} = \tau_{bv} + \tau_v = \tau_{bv} + \sum_{j=1}^{N} F_{Dj}, \tag{8}$$

where $\tau_{\text{tot}} = \rho g h J$ is the total flow resistance per unit bed area, h is the flow depth, J is the energy slope, τ_{bv} is the bed shear stress per unit bed area, τ_v is the vegetation drag per unit bed area, and F_{Dj} is the drag offered by a single vegetation stem, which can be expressed as [10]:

$$F_{Dj} = \tfrac{1}{2}\rho C_d d h {V_v}^2, \tag{9}$$

where C_d is the drag coefficient for a single vegetation stem and V_v is the average pore velocity through the vegetation.

Armanini and Cavedon [21] proposed an approach that utilizes the aforementioned momentum analysis to calculate the flow intensity parameter of homogeneously distributed, emergent vegetated channels (Ψ_{bv}):

$$\Psi_{bv} = \Psi_{\text{tot}}\left(1 + \beta_v \lambda \frac{h}{d}\left(\frac{h}{D}\right)^{b_v}\right), \tag{10}$$

where Ψ_{bv} is the flow intensity parameter for the vegetated channels (the effective flow intensity parameter), which is calculated with respect to the bed shear stress (τ_{bv}), Ψ_{tot} is the flow intensity parameter calculated with respect to the total flow resistance (τ_{tot}), and β_v and b_v are two constant coefficients calibrated by experimental data. Further, Armanini and Cavedon [21] took into account the possible effects of fluid viscosity and derived the final formula as follows based on their experiments:

$$\Psi_{bv} = \frac{\Delta D}{hJ}(1 - 1.5D_*{}^{-0.75})^{-1}\left(1 + 40\lambda\frac{h}{d}\right), \tag{11}$$

where $\Delta = (\rho_s - \rho)/\rho$ is the submerged relative density of the sediment, $D_* = D(\Delta g/\upsilon^2)^{1/3}$ is the dimensionless particle diameter, and υ is the kinematic viscosity of fluid. In many bedload transport formulas, the flow intensity parameter (Ψ) is replaced by its reciprocal; that is, the Shields parameter (Θ) is defined as $\Theta = 1/\Psi$, representing the dimensionless shear stress. In this study, $\Theta_{bv} = \tau_{bv}/[(\rho_s - \rho)gD]$ was used to represent the dimensionless bed shear stress in vegetated channels, which can be expressed as $\Theta_{bv} = 1/\Psi_{bv}$ as follows:

$$\Theta_{bv} = \frac{hJ}{\Delta D}(1 - 1.5D_*{}^{-0.75})\left(1 + 40\lambda\frac{h}{d}\right)^{-1}. \tag{12}$$

3. Determination of Θ_{cv}

The Shields parameter of sediment incipient (Θ_c) on bare beds is a function of the grain Reynolds number, meaning it is related to sediment particle diameter and friction velocity [23]. However, the presence of vegetation affects the sediment movement around the vegetation stems, complicating the determination of the Shields parameter of sediment incipient (Θ_{cv}) in vegetated channels. Therefore, Θ_{cv} is not only related to particle diameter and friction velocity but also to vegetation density and vegetation stem diameter. Kothyari *et al.* [10] determined Θ_{cv} equals 0.0588 through numerous experiments, which varied in vegetation density (λ) and sediment particle diameter (D). Zhao and Nepf [12] directly adopted the Shields parameter of sediment incipient (Θ_c) on bare beds as that for vegetated channels, that is, $\Theta_{cv} = \Theta_c = 0.03$. The presence of vegetation generally hinders the sediment transport over the bed; hence, Θ_{cv} in vegetated channels is expected to be greater than that on bare beds. In this paper, Θ_{cv} was determined through experimental parameter calibration.

C. Modified formula

Substituting Eqs. (7) and (12) into Eq. (5) yields the extended version of the improved Engelund formula:

$$\Phi = k_v p_v (1 - \lambda)(\sqrt{\Theta_{bv}} - \sqrt{\Theta_{cv}})(\Theta_{bv} - \Theta_{cv}). \tag{13}$$

The derivation of Eq. (13) is based on the improved Engelund formula proposed by Meng and colleagues [8]. It is assumed that the bed shear stress (τ_{bv}), rather than the grain shear stress (τ_{gv}), is more closely related to the bedload transport rate. In addition, it is hypothesized that only the bedload transport in Stage 3 (when sediment away from the stems begins to move) can be considered to have a measurable bedload transport rate, while the case of weak bedload transport rate (Stage 2) receives less consideration. In Eq. (13), model parameters such as k_v and Θ_{cv} can be calibrated using experimental data, while physical parameters such as p_v, λ, and Θ_{bv} can be calculated from experiment measurements such as h, J, D, d, and N.

III. RESULTS

A. Experimental datasets

To calibrate the parameters of Eq. (13), several literature experimental datasets [10,13,18,21,24] were collected. These datasets encompassed bedload transport rates ranging from weak to high and vegetation densities from sparse to dense, resulting in 164 experimental datasets. All experiments used uniform noncohesive sand and regular rigid cylinders to simulate emergent vegetation.

TABLE I. Experimental datasets of bedload transport in sparsely vegetated channels.

Investigators	D ($\times 10^{-3}$ m)	d ($\times 10^{-3}$ m)	λ (%)	Φ (-)	Runs (-)
Kothyari *et al.* [10]	0.55–5.9	2–5	0.17–1.2	0.000274–67.124	56
Yager and Schmeeckle [24]	0.5	13	0.8	0.00467–0.0333	3
Armanini and Cavedon [21]	0.5–0.55	10	0.39–0.8	0.00143–4.456	11
Wu *et al.* [13]	0.93	7.8–10	0.66–1.18	0.0000223–0.0121	21
Wang *et al.* [18]	0.6–1.45	5	0.39–1.09	0.000194–0.0181	15
Total	0.5–5.9	2–13	0.17–1.2	0.0000223–67.124	106

Yang and Nepf [19] pointed out that the τ_{bv}-based bedload model proposed by Kothyari *et al.* [10] performs well in sparsely vegetated channels. This is because, for low vegetation density ($\lambda < 0.012$), the bed is the main contributor to turbulence, and near-bed turbulence is strongly correlated with bed shear stress. The focus of this paper is bedload transport in sparsely vegetated channels; therefore, data points with vegetation density $\lambda > 0.012$ must be excluded. A total of 106 datasets were used in this paper, with vegetation density $\lambda = 0.0017-0.012$, stem diameter $d = 2-13$ mm, sediment particle diameter $D = 0.45-5.90$ mm, and dimensionless bedload transport rate $\Phi = 2.23 \times 10^{-5} - 67.124$, respectively, as shown in Table I.

B. Formulation

The measured datasets from the experiments of Kothyari *et al.* [10] were selected for parameter calibration of Eq. (13), which yielded $k = 22.92$ and $\Theta_{cv} = 0.066$. Consequently, Eq. (13) was transformed into Eq. (14) as follows:

$$\Phi = 22.92(1-\lambda)p_v(\sqrt{\Theta}_{bv} - 0.257)(\Theta_{bv} - 0.066). \tag{14}$$

The experimental data measured by Kothyari *et al.* [10], Yager and Schmeeckle [24], Armanini and Cavedon [21], Wu *et al.* [13], and Wang *et al.* [18] were plotted on a double-log plot of Φ versus Ψ_{bv}, as shown in Fig. 1. The experimental data are concentrated when $\Phi > 0.1$, while they are relatively dispersed when $\Phi < 0.1$, especially when $\Phi < 0.01$. The measured dimensionless bedload transport rate Φ was plotted against the predicted dimensionless bedload transport rate Φ using Eq. (14) in Fig. 2. The prediction of experimental data performed well when $\Phi > 0.1$, and the deviation between prediction and measurement was large when $\Phi < 0.1$, and more pronounced when $\Phi < 0.01$. Combined with Fig. 1, it can be observed that the predicted Φ is smaller than the measured Φ by Wu *et al.* [13] at the same flow intensity Ψ_{bv} (when $\Phi < 0.01$), that is, predicted values correspond to a higher flow intensity to achieve the same sediment transport rate as the measured values. Furthermore, a certain bedload transport rate has already been generated in Wu *et al.*'s [13] experiments before the critical state of sediment incipient assumed in the present paper is reached. This is because, at weak flow intensities, the main cause of sediment movement is turbulence generated by the vegetation stems rather than bed shear stress. Although the bed shear stress does not exceed the incipient shear stress, the sediment particles around the stems can still be set into motion (Stage 2), and the bedload transport rate at Stage 2 is contributed to by the sediment movement in the scour holes around the stems. Overall, among all datasets, the prediction of the present formula on experimental data by Wu *et al.* [13] was the least accurate, with other datasets mostly distributed on both sides of the prediction curve, while measured data by Wu *et al.* [13] was generally larger than the prediction.

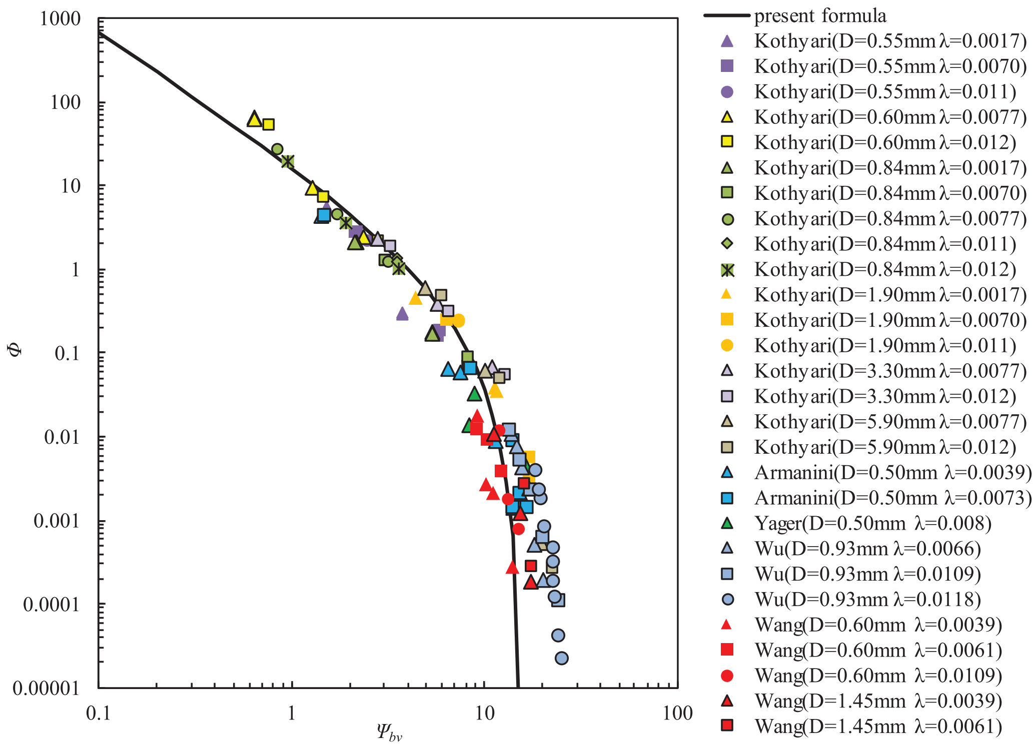

FIG. 1. Comparison of the prediction results and other experimental data.

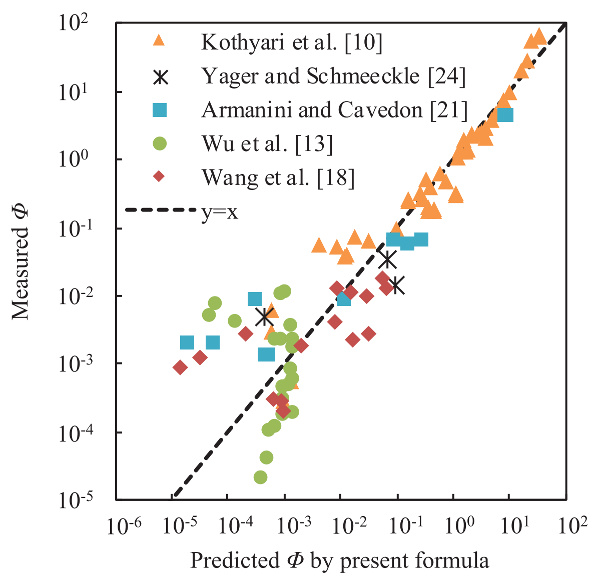

FIG. 2. Predicted dimensionless bedload transport rate versus the measured dimensionless bedload transport rate.

IV. DISCUSSION

A. Comparison with other models

To further evaluate the present formula, it was compared with formulas proposed by other researchers [10,18–19,21]. Those formulas are listed as follows:

1. *According to Kothyari et al. [10]*

$$\Phi = 5.37\left(\frac{{u_*}^2}{\Delta gD}\right)^{\frac{3}{2}}\left(1-\frac{0.06\Delta gD}{{u_*}^2}\right),$$

where u_* is

$$u_* = \sqrt{ghJ - \frac{1}{2(1-\lambda)}C_d N d h {V_v}^2},$$

where C_d is

$$C_d = 1.8\xi R_{ed}^{-3/50}[1+0.45\ln(1+100\lambda)]\times(0.8+0.2Fr-0.15Fr^2),$$

where $\xi = 0.8$ is a parameter representing the vegetation staggering pattern, $R_{ed} = (V_v d)/\upsilon$ is the stem Reynolds number, and $Fr = V_v/(gh)^{1/2}$ is the Froude number.

2. *According to Armanini and Cavedon [21]*

$$\Phi = \frac{1}{1-5\lambda}\frac{70}{\Psi_{bv}(1+4\Psi_{bv})}(1+0.45\Psi_{bv})\exp(-0.54\Psi_{bv}),$$

where Ψ_{bv} is

$$\Psi_{bv} = \frac{\Delta D}{hJ}(1-1.5{D_*}^{-0.75})^{-1}\left(1+40\lambda\frac{h}{d}\right).$$

3. *According to Yang and Nepf [19]*

$$\Phi = \begin{cases} 2.15\exp(-2.06/k_{t*}), & k_{t*} < 0.95 \\ 0.27k_{t*}^3, & 0.95 < k_{t*} < 2.74, \end{cases}$$

where $k_{t*} = k_t/(\Delta gD)$ and k_t is determined by

$$k_t = \frac{c_f}{0.19\rho} + 0.4\left(C_d\frac{\lambda}{(1-\lambda)\pi/2}\right)^{2/3}{V_v}^2,$$

where c_f and C_d are determined, respectively, by

$$c_f = \frac{1}{[5.75\log(2h/D)]^2}$$

and

$$C_d = \zeta^2\left[1+10\left(\frac{\zeta V_v d}{\upsilon}\right)^{-2/3}\right],$$

where the coefficient $\zeta = (1-\lambda)/(1-(2\lambda/\pi)^{1/2})$.

4. According to Wang et al. [18]

$$\Phi = 13\Theta_{gv}^{1.5}\exp\left(-0.05\Theta_{gv}^{-1.5}\right),$$

where Θ_{gv} is the grain Shields parameter (dimensionless grain shear stress), which is determined by

$$\Theta_{gv} = \frac{hJ}{\Delta D\left[28.49c_f C_d{}^{-\frac{2}{3}}\left(\frac{D}{d}\frac{\lambda}{1-\lambda}\right)^{-\frac{1}{3}} + \frac{15.4}{\pi}\frac{h}{d}C_d{}^{\frac{1}{3}}\left(\frac{D}{d}\right)^{-\frac{1}{3}}\left(\frac{\lambda}{1-\lambda}\right)^{\frac{2}{3}}\right]},$$

where c_f and C_d are determined, respectively, by

$$c_f = \frac{1}{[5.75\log(2h/D)]^2}$$

and

$$C_d = \frac{189}{R_{ed}} + 0.82 + 6.02\lambda + \left(\frac{D}{l_y}\right)^2,$$

where l_y is the lateral distance between adjacent stems at the same streamwise location.

In the aforementioned methods, the formulas proposed by Kothyari *et al.* [10] and Armanini and Cavedon [21] are based on bed shear stress (τ_{bv}), the formula by Yang and Nepf [19] is based on TKE, and the formula by Wang *et al.* [18] is based on grain shear stress (τ_{gv}). To facilitate further comparisons (details in the following text), we attempted to transform the grain flow intensity parameter ($\Psi_{gv} = 1/\Theta_{gv}$) proposed by Wang *et al.* [18] into the effective flow intensity parameter ($\Psi_{bv} = 1/\Theta_{bv}$) in this paper. In the paper by Wang *et al.* [18], a functional relationship τ_{gv} and τ_{bv} was established through momentum analysis [see Eq. (15)]. Similarly, a functional relationship between τ_{gv} and τ_{bv} could also be derived [see Eq. (16)]. We could obtain an approximate functional relationship between Ψ_{gv} and Ψ_{bv} by regression analysis of the experimental data [see Eq. (17)]

$$\tau_{gv} = \tau_{\text{tot}}\left[k\beta c_f C_d{}^{-\frac{2}{3}}\left(\frac{D}{d}\frac{\lambda}{1-\lambda}\right)^{-\frac{1}{3}} + \frac{2k}{\pi}\frac{h}{d}C_d{}^{\frac{1}{3}}\left(\frac{D}{d}\right)^{-\frac{1}{3}}\left(\frac{\lambda}{1-\lambda}\right)^{\frac{2}{3}}\right], \tag{15}$$

$$\tau_{bv} = \tau_{gv}\left[k\beta c_f C_d{}^{-\frac{2}{3}}\left(\frac{D}{d}\frac{\lambda}{1-\lambda}\right)^{-\frac{1}{3}}\right]^{-1}, \tag{16}$$

and

$$\log(\Psi_{bv}) = 1.157\log(\Psi_{gv}) - 0.4027. \tag{17}$$

To quantitatively compare these formulas, we adopted root mean-square deviation (RMSD) and mean relative error (MRE) as the error metrics, with RMSD serving as the primary evaluation criterion. Here, the two parameters are defined as follows:

$$RMSD = \sqrt{\frac{1}{m}\sum_{j=1}^{m}(\log(\Phi_m) - \log(\Phi_p))^2} \tag{18}$$

and

$$MRE = \frac{1}{m}\sum_{j=1}^{m}|(\Phi_m - \Phi_p)/\Phi_m|, \tag{19}$$

where m is the total number of experimental data, Φ_p and Φ_m are the prediction of the dimensionless bedload transport rate and measurements of the dimensionless bedload transport rate, respectively.

The measured dimensionless bedload transport rate Φ_m was compared with the dimensionless bedload transport rate Φ_p calculated using the aforementioned formulas, as shown in Fig. 3. The

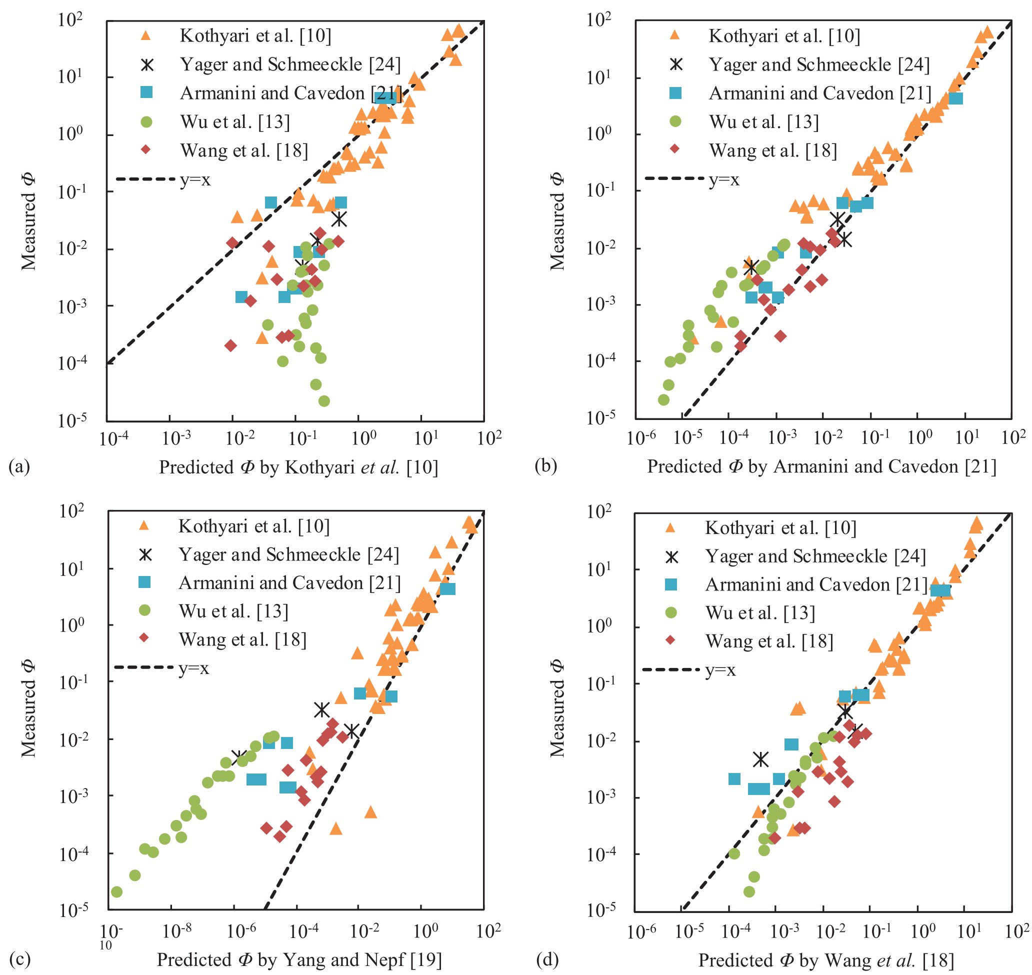

FIG. 3. Measured Φ versus the predicted Φ by other formulas: (a) Kothyari *et al.* [10], (b) Armanini and Cavedon [21], (c) Yang and Nepf [19], and (d) Wang *et al.* [18].

prediction errors from the different models are listed in Table II. For the prediction of all datasets, the present formula, the formula by Armanini and Cavedon [21], and the formula by Wang *et al.* [18] performed well, while the formula by Kothyari *et al.* [10] and the formula by Yang and Nepf [19] did not yield predictions as accurate as the first three. In addition, the formula by Wang *et al.* [18] had the best overall performance (RMSD = 0.35, MRE = 1.61), and the performance of the present formula (RMSD = 0.68, MRE = 1.38) is almost the same as that of the formula by Armanini and Cavedon [21] (RMSD = 0.69, MRE = 0.61). The formula by Kothyari *et al.* [10] and the formula by Yang and Nepf [19] only perform well in the cases of high bedload transport rates ($\Phi > 1$). Therefore, the formula by Kothyari *et al.* [10], which is based on bed shear stress (calculated by subtracting the vegetation drag from the total flow resistance), was not applicable to predict the bedload transport rate in sparsely vegetated channels. The formula by Yang and Nepf [19], which is based on TKE, is also not suitable.

Combined with Fig. 3 and Table II, the three formulas with good performance were further compared. These formulas can provide reliable predictions at medium bedload transport rates ($0.1 < \Phi < 10$). For high bedload transport rates ($\Phi > 10$), the present formula (RMSD = 0.25, MRE =

TABLE II. Prediction errors of different bedload transport formulas.

	All data		$\Phi < 0.1$		$0.1 < \Phi < 10$		$\Phi > 10$	
Formulas	RSMD	MRE	RSMD	MRE	RSMD	MRE	RSMD	MRE
Kothyari *et al.* [10]	1.33	223.19	2.06	567.29	0.53	1.72	0.23	0.65
Armanini and Cavedon [21]	0.69	0.61	0.94	0.86	0.46	0.67	0.29	0.67
Yang and Nepf [19]	2.01	1.16	3.05	2.09	0.81	0.75	0.51	0.77
Wang *et al.* [18]	0.35	1.61	0.65	3.29	0.33	0.73	0.45	0.78
Present paper	0.68	1.38	0.97	2.36	0.39	0.88	0.25	0.62

0.62) can provide better prediction than the formula by Armanini and Cavedon [21] (RMSD = 0.29, MRE = 0.67) and the formula by Wang *et al.* [18] (RMSD = 0.45, MRE = 0.78). However, in cases of weak bedload transport rates ($\Phi < 0.1$), all three formulas perform poorly. Among them, the formula by Wang *et al.* [18] was more reliable, with RMSD = 0.65 and MRE = 3.29, compared with the present formula (RMSD = 0.97, MRE = 2.36) and the formula by Armanini and Cavedon [21] (RMSD = 0.94, MRE = 0.86).

Figure 4 shows the comparison between the three well-performing formulas and the published data. The predicted values for the present formula and the formula by Armanini and Cavedon [21] both performed well on the datasets provided by Kothyari *et al.* [10], Yager and Schmeeckle [24], Armanini and Cavedon [21], and Wang *et al.* [18], but both predict poorly based on the data of Wu *et al.* [13]. The present formula more accurately predicted the experimental data, as measured by Kothyari *et al.* [10], compared with the formula proposed by Armanini and Cavedon [21]. The formula proposed by Wang *et al.* [18] had very good prediction of Wu *et al.*'s [13] data, but the prediction values of his own experimental data are not as accurate as those of Wu *et al.*'s [13] data.

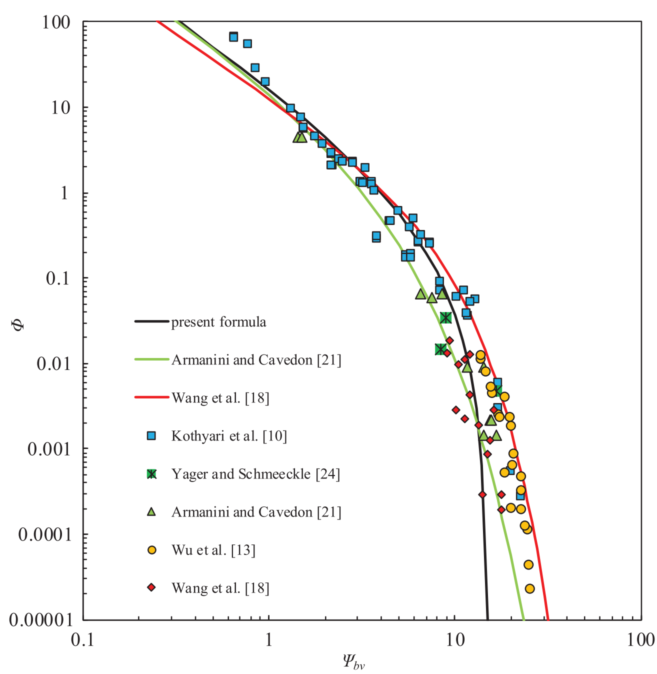

FIG. 4. Comparison between the well-performing formulas and published data.

The bedload transport formula proposed by Wang *et al.* [18], based on grain shear stress, exhibits different performance at weak and high bedload transport rates. This is because, at weak bedload transport rates, only a small amount of sediment around the vegetation stems gets into motion, the river bed is approximately flat, and there is no significant difference between the grain shear stress for the movable bed and that for the fixed bed. Therefore, the bedload transport formula based on grain shear stress performs well at weak bedload transport rates. In contrast, at high bedload transport rates, a large amount of sediment is in motion, there is a significant difference between the grain shear stress for the moveable bed and that for the fixed bed, and because of this, the formula by Wang *et al.* [18] does not perform well. On the contrary, other formulas based on bed shear stress have better prediction performance at high bedload transport rates.

B. Formula limits

The present formula is applicable to the prediction of the bedload transport rate in sparsely vegetated channels, where the bed shear stress can act directly on sediment particles on the bed. It can be observed from the prediction results that the predicted values are in good agreement with the measured values at medium and high bedload transport rates. However, the agreement is not as good at weak bedload transport rates. This is because, when the bedload transport rate is weak, the corresponding flow intensity is also weak, and the bed shear stress is insufficient to force the sediment into motion. In such cases, the main cause of sediment movement is the flow turbulence induced by the stems. Most of the moving sediment particles are found in the scour holes around the stems, while sediment in other regions remains nearly motionless. When the sediment particles in the scour holes are forced into motion by the above flow turbulence, even though the bed shear stress is low, it can still support the sediment movement that has been set in motion, allowing it to maintain movement. Therefore, it can be considered that, in local regions, the turbulence induced by the stems lowers the threshold of sediment incipient and produces bedload transport rates (Stage 2) that are not predicted by the present formula.

Another important issue is the determination of Θ_{cv}. For bare beds, the Shields parameter of sediment incipient (Θ_c) is a function of the grain Reynolds number, which is related to the particle diameter and friction velocity. The range of Θ_c on bare beds is 0.017 to 0.076 [23]. However, the presence of vegetation affects the sediment movement around the stems, complicating the determination of Θ_{cv} in vegetated channels. This determination is not only related to particle diameter and friction velocity but also to vegetation density and stem diameter. There is little research on this topic, and no consensus has been reached [10,12]. From a macro perspective, the presence of vegetation generally reduces sediment movement, so Θ_{cv} in vegetated channels should be larger than that on bare beds, except in cases of weak bedload transport rates. When the bedload transport rate is weak, the presence of vegetation lowers the threshold of sediment incipient, so Θ_{cv} in Stage 2 is also lower than that in Stage 3. As mentioned, further research is needed on Θ_{cv} in vegetated channels in the future.

V. CONCLUSION

The improved Engelund formula, which performed well on bare beds, was extended to sparsely vegetated channels to calculate the bedload transport rate. This extension involved refining key parameters, such as the number of moving sediment particles per unit bed area and the effective Shields number corresponding to the bed shear stress in vegetated channels. Using published data, parameter calibration of the extended Engelund formula was accomplished. The prediction results were in good agreement with the measured data at medium and high bedload transport rates, indicating that the formula based on bed shear stress is more desirable for these conditions. Additionally, the results also indicate that the prediction performance of the formula based on grain shear stress is better than that of the formula based on bed shear stress at weak bedload transport rates. In general, the present formula is applicable to cases with sparse vegetation density, but further investigation is needed for conditions beyond those discussed.

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