Gallery of Fluid Motion

Fireworks of viscous fingering

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When a more viscous fluid is displaced by a less viscous one, which can be either miscible or immiscible to each other, in a porous medium or Hele-Shaw cell, the fluid-fluid interface develops peculiar structures in the form of fingers called viscous fingering, or the Saffman-Taylor instability [1]. This particular interfacial phenomenon has been thoroughly studied for many decades because of its intricate pattern formation and potential applications in many practical applications, including enhanced oil recovery [2], carbon segregation [3], and mixing enhancement [4]. Depending on the applications, the emergence of fingers can be either favorable or undesired. For instance, the emergence of fingers of the less viscous injected fluid is the leading cause of poor oil recovery. On the contrary, intense viscous fingering can be beneficial for mixing two miscible fluids because of the low Reynolds number in a porous medium or Hele-Shaw cell. Therefore, different strategies intended to manipulate the emergence of viscous fingering have been actively investigated. Instead of the conventional way to weaken viscous fingering, which mainly focuses on lowering viscosity contrast or increasing fluid miscibility, one particularly successful controlling technique has been applied through time-dependent injection schemes [5–9]. Similarly, to achieve a better mixing efficiency, injection of the two miscible fluids is alternated to enhance the fingering phenomena [10,11].

To demonstrate the pattern of viscous fingering in an immiscible binary fluids system, simulations based on the Cahn-Hilliard-Hele-Shaw model incorporating highly accurate numerical

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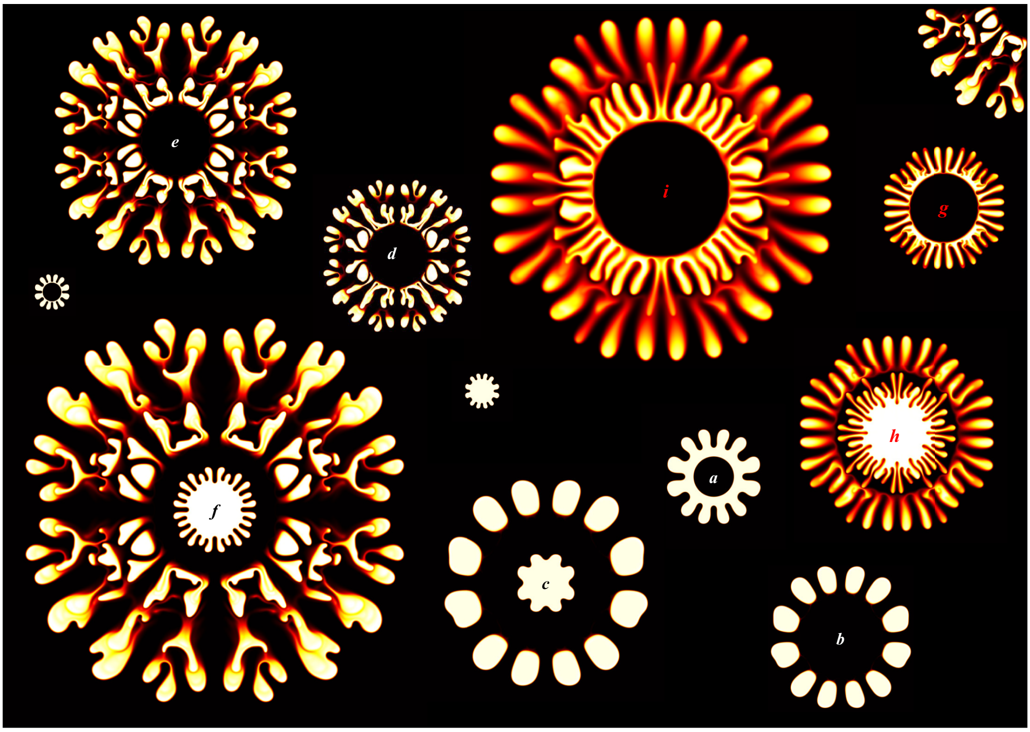

FIG. 1. Fingering layers are produced by alternative injection of two fluids, i.e., a more viscous dark fluid and a less viscous transparent fluid. The patterns resemble fireworks in the dark sky.

schemes are carried out to solve the fluid concentration, i.e., mass fraction. Based on the model, the free energy profile governs the binary fluids' miscibility. A concave free energy profile is prescribed to simulate an immiscible fluid-fluid interface between the more and less viscous binary fluids. The continuity and Hele-Shaw equations are reformulated into the streamfunction-vorticity system. To solve the system numerically, time integration of the concentration equation utilizes a third-order Runge-Kutta procedure associated with spatial derivatives discretized by sixth- and fourth-order compact finite difference schemes for diffusion terms and convection terms, respectively. The updated concentration is discretized by a sixth-order compact finite difference scheme to calculate the vorticity. Subsequently, the Poisson equation of the streamfunction is solved by a pseudospectral method. The implementations and validation of the model and the numerical scheme are verified in various flow configurations by agreements with linear stability analysis, i.e., immiscible interface in a rational Hele-Shaw cell [12] or suction Hele-Shaw flow, i.e., negative injection [13], and partially miscible interface of a gravity-driven Hele-Shaw flow [14].

In the present situation of alternative injection, the Hele-Shaw cell is initially filled with a more viscous and dark fluid. A less viscous transparent fluid, which is immiscible to the more viscous fluid, is first injected into the cell through a small hole. Viscous fingers of the transparent fluid are triggered because of unfavorable viscosity contrast. Injection of the less viscous fluid lasts for a specific time period, allowing the viscous fingering to evolve. Then, the injected fluid is swapped to the more viscous fluid, in which the interface remains circularly stable, for the same period to complete a cycle of injection alternation. By alternating the injected fluids for several cycles, multiple radially expanded fluid-fluid interfaces of unstable (fingering) or stable (circular) in sequence, depending on whether the injected fluid is more or less viscous, result in typical patterns shown in Fig. 1. Shown in Figs. 1(a)–1(c), in which the letter is marked at the center of the image, is a pattern for an immiscible case with an insignificant viscosity contrast at times before, at, and after the completion of the first injection cycle, respectively. The outer fingering white layer of less viscous fluid is continuously pushed outward by the inner circular core of more viscous black fluid. As a result, the fingering layer is ruptured, forming fluid islands. More vigorous tip-splitting fingering is trigged in the case with a higher viscosity contrast. In that case, the fast-growing fingers of the inner layer follow the paths of fingers of the outer layer and catch their outer counterparts, as shown in Figs. 1(d)–1(f) at different times, e.g., completion of the second injection cycle, completion of the third injection cycle, and after the start of the fourth injection cycle, respectively. The same evolving paths of fingers at different layers are called the channeling effect. These unique features of rupture of injected fluid and channeling paths of fingers' growths observed in an immiscible alternation injection are qualitatively confirmed in similar experiments, e.g., Figs. 2 and 4 shown in Ref. [15].

On the other hand, similar highly accurate miscible simulations are also performed by solving the continuity and Hele-Shaw equations associated with the conventional advection-diffusion equation of concentration [11]. Even though the fingers of inner less viscous layers may still evolve following the paths of the outer fingers, which has also been observed experimentally [16], the absence of interfacial tension prevents the rupture of fingers. Instead, fluid mixing is apparent so that the fluid layer and fingers may be gradually pinched off by diffusion. A distinct fingering pattern is shown in (g), (h), and (i) at the completion times of one, one and a half, and two injection cycles, respectively.

The formation of isolated fluid fragments in the present situations of alternative injection, such as drops of pinched-off fingertips or ruptured fluid layers, are not observed in conventional viscous fingering by continuous injection of the less viscous fluid. It is noticed that similar broken patterns, e.g., isolated droplets, are recently observed in viscous fingering coupled with phase separation [17]. It suggests an additional mechanism is required to rupture the fingers, either the thermodynamic phase separation or hydrodynamic injection alternation. The ruptured multilayer fingering interfaces resemble the spectacular fireworks in the dark sky, and the images are called "Fireworks of Viscous Fingering." Comprehensive results and the underlying physics regarding the alternating injection for miscible and immiscible conditions are presented and discussed in Refs. [11,15], respectively.

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[1] P. G. Saffman and G. I. Taylor, The penetration of a fluid into a porous medium or Hele-Shaw cell containing a more viscous liquid, Proc. R. Soc. London A **245**, 312 (1958).

[2] G. M. Homsy, Viscous fingering in porous media, Annu. Rev. Fluid Mech. **19**, 271 (1987).

[3] H. E. Huppert, J. N. H. Huppert, and J. A. Neufeld, The fluid mechanics of carbon dioxide sequestration, Annu. Rev. Fluid Mech. **46**, 255 (2014).

[4] B. Jha, L. Cueto-Felgueroso, and R. Juanes, Fluid mixing from viscous fingering, Phys. Rev. Lett. **106**, 194502 (2011).

[5] C.-Y. Chen, C.-W. Huang, L.-C. Wang, and J. A. Miranda, Controlling radial fingering patterns in miscible confined flows, Phys. Rev. E **82**, 056308 (2010).

[6] E. O. Dias, E. Alvarez-Lacalle, M. S. Carvalho, and J. A. Miranda, Minimization of viscous fluid fingering: A variational scheme for optimal flow rates, Phys. Rev. Lett. **109**, 144502 (2012).

[7] C. Gin, Time-dependent injection strategies for multilayer Hele-Shaw and porous media flows, Phys. Rev. Fluids **6**, 033901 (2021).

[8] H. Ma and Q. Yuan, Control of viscous fingering: From the perspective of energy evolution, Phys. Rev. Fluids **6**, 023901 (2021).
[9] C. Cuttle, L. C. Morrow, and C. W. MacMinn, Compression-driven viscous fingering in a radial Hele-Shaw cell, Phys. Rev. Fluids **8**, 113904 (2023).
[10] B. Jha, L. Cueto-Felgueroso, and R. Juanes, Synergetic fluid mixing from viscous fingering and alternating injection, Phys. Rev. Lett. **111**, 144501 (2013).
[11] C.-Y. Chen, Y.-C. Huang, Y.-S. Huang, and J. A. Miranda, Enhanced mixing via alternating injection in radial Hele-Shaw flows, Phys. Rev. E **92**, 043008 (2015).
[12] C.-Y. Chen, Y.-S. Huang, and J. A. Miranda, Diffuse-interface approach to rotating Hele-Shaw flows, Phys. Rev. E **84**, 046302 (2011).
[13] C.-Y. Chen, Y.-S. Huang, and J. Miranda, Radial Hele-Shaw flow with suction: Fully nonlinear pattern formation, Phys. Rev. E **89**, 053006 (2014).
[14] Q. Li, W. Cai, C.-Y. Chen, and E. Meiburg, A diffuse interface model for low solubility binary flows in porous media, J. Comput. Phys. **470**, 111582 (2022).
[15] C.-C. Chou, W.-C. Huang, and C.-Y. Chen, Pattern rupture and channeling effect by alternating radial displacement, Int. J. Heat Mass Transfer **207**, 123983 (2023).
[16] C.-C. Chou, A. Kumar, P.-C. Lu, and C.-Y. Chen, Miscible fluids mixing via alternating injection in a radial Hele-Shaw cell: Experimental and numerical studies, J. Mech. **38**, 531 (2022).
[17] R. Suzuki, Y. Nagatsu, M. Mishra, and T. Ban, Phase separation effects on a partially miscible viscous fingering dynamics, J. Fluid Mech. **898**, A11 (2020).