

Local analysis of the clustering, velocities, and accelerations of particles settling in turbulence

Mohammadreza Momenifar and Andrew D. Bragg*

Department of Civil and Environmental Engineering, Duke University, Durham, North Carolina 27708, USA



(Received 31 July 2019; accepted 26 February 2020; published 20 March 2020)

Using three-dimensional Voronoi analysis, we explore the local dynamics of small, settling, inertial particles in isotropic turbulence using direct numerical simulations. We independently vary the Taylor Reynolds number $Re_\lambda \in [90, 398]$, Froude number $Fr \equiv a_\eta/g \in [0.052, \infty]$ (where a_η is the Kolmogorov acceleration, and g is the acceleration due to gravity), and Kolmogorov scale Stokes number $St \equiv \tau_p/\tau_\eta \in [0, 3]$. In agreement with previous results using global measures of particle clustering, such as the radial distribution function, we find that for small Voronoi volumes (corresponding to the most clustered particles), the behavior is strongly dependent upon St and Fr , but only weakly dependent upon Re_λ , unless $St > 1$. However, larger Voronoi volumes (void regions) exhibit a much stronger dependence on Re_λ , even when $St \leq 1$, and we show that this, rather than the behavior at small volumes, is the cause of the sensitivity of the standard deviation of the Voronoi volumes that has been previously reported. We also show that the largest contribution to the particle settling velocities is associated with increasingly larger Voronoi volumes as the settling parameter $Sv \equiv St/Fr$ is increased. Our local analysis of the acceleration statistics of settling inertial particles shows that clustered particles experience a net acceleration in the direction of gravity, while particles in void regions experience the opposite. The particle acceleration variance, however, is a convex function of the Voronoi volumes, with or without gravity, which seems to indicate a nontrivial relationship between the Voronoi volumes and the sizes of the turbulent flow scales. Results for the variance of the fluid acceleration at the inertial particle positions are of the order of the square of the Kolmogorov acceleration and depend only weakly on Voronoi volumes. These results call into question the “sweep-stick” mechanism for particle clustering in turbulence which would lead one to expect that clustered particles reside in the special regions where the fluid acceleration is zero (or at least small). We then consider the properties of particles in clusters, which are regions of connected Voronoi cells whose volume is less than a certain threshold. The results show self-similarity of the clusters, and that the statistics of the cluster volumes depends only weakly on St , with a stronger dependence on Fr and Re_λ . Finally, we compare the average settling velocities of all particles in the flow with those in clusters, and show that those in the clusters settle much faster, in agreement with previous work. However, we also find that this difference grows significantly with increasing Re_λ and exhibits a nonmonotonic dependence on Fr . The kinetic energy of the particles, however, is almost the same for particles whether or not they are in clusters.

DOI: [10.1103/PhysRevFluids.5.034306](https://doi.org/10.1103/PhysRevFluids.5.034306)

*andrew.bragg@duke.edu

I. INTRODUCTION

Turbulent multiphase flows have been the subject of numerous studies for many years because of their broad range of applications in nature and industrial contexts such as distribution of greenhouse gases [1], cloud formation [2], volcanic eruptions [3], contaminant transport [4], heat exchangers [5], boiling heat transfer [6], combustion processes [7], air-lift pumps [8], cavitating flows [9], sedimentation process in settling tanks [10], thermosyphons [11], and fluidized bed reactors [12], to name a few. The present paper explores the class of multiphase flows involving the motion of small, settling inertial particles in turbulent flows.

A well-known and striking effect of particle inertia is that it leads to the spatial clustering of the particles in turbulent flows [4,13]. This has important implications for the rate at which particles collide in turbulent flows [14–17]. It is also important since regions with a high concentration of particles have the potential to significantly modify the underlying turbulent flow in those same regions, through momentum coupling with the fluid [18]. Several different methods and techniques have been used to analyze the particle clustering, such as box counting [19], Voronoi analysis [20], Lyapunov exponents [21], Minkowski functionals [22], and radial distribution functions (RDFs) [14,17,23]. These methods can provide different insights and perspectives into the spatial clustering, and two of the most commonly used methods are the RDF and Voronoi analysis.

The RDF, defined by the probability of finding a particle at a given distance from a reference particle, is unaffected by variation in the number of particles in the domain (except for statistical noise) and is directly related to the particle collision kernel [14]. However, since it is constructed using spatial averages, the RDF cannot be used to consider the properties of individual clusters of particles, and only provides global information on the clustering. In a Voronoi analysis, the domain is divided into cells associated with each individual particle, where any point in a given cell is closer to that particle than any other. This approach gives local insight into the particle clustering, with the local particle concentration field given by the inverse of the volume of the Voronoi cell. Using this technique enables characterization of the topology, kinematics, and dynamics of individual clusters in the turbulent flow. However, a disadvantage is that unlike the RDF, the results of a Voronoi analysis are sensitive to the number of particles in the flow, at least quantitatively [24]. As discussed in Ref. [25], since the RDF and Voronoi analysis provide different information on the particle clustering (e.g., global versus local information), they can both be used to provide a more complete understanding of the problem.

Recently, extensive investigations into the effects of the flow of the Taylor Reynolds number Re_λ and the Froude number Fr (quantifying the effect of gravity) on the particle clustering have been performed using the RDF computed from direct numerical simulations (DNS) [17,23,26]. These studies have shown that when the Kolmogorov scale Stokes number of the particle St is $\lesssim 1$, there is virtually no effect of Re_λ on the clustering, whereas the clustering increases with increasing Re_λ when $St > 1$. Concerning the effect of gravity, it was found that for $St \lesssim 1$, gravity leads to a reduction in the clustering, while for $St > 1$ it enhances the clustering. In contrast, it appears that such a systematic study of the parametric dependence of clustering as quantified by a Voronoi analysis has not been undertaken. We now summarize what has been done on this.

The experimental study of Ref. [27] considered the clustering of small water droplets in isotropic turbulence using a Voronoi analysis. They examined the range $200 \leq Re_\lambda \leq 400$ and $2 \leq St \leq 10$, and reported that the level of clustering was enhanced with increasing Re_λ . The experimental study of Ref. [28] considered $170 \leq Re_\lambda \leq 450$, $0.1 \leq St \leq 5$, and found that the clustering is strongly enhanced with increasing Re_λ , but with a weak dependence on St . These studies focused on the regime of large Fr , where the effects of gravity on the particle motion are weak. The recent experimental study of Ref. [25] also used a Voronoi analysis to quantify the clustering and explored the range $200 \leq Re_\lambda \leq 500$, $0.37 \leq St \leq 20.8$, and $0.73 \leq Fr \leq 3.7$. They found that the clusters exhibit a range of sizes (up to the integral scale of the flow), possess self-similarity, and are elongated

and aligned with the gravity direction [25]. They also found that the clustering is enhanced with increasing Re_λ , and in agreement with the RDF analysis in Refs. [17,23,26], that gravity can enhance the clustering of the inertial particles. A limitation of these experimental results is that their Voronoi analysis is only two dimensional, and it is known that at least in the context of the RDF, a two-dimensional analysis of particle clustering can differ significantly from the full three-dimensional results [29]. Moreover, and perhaps more importantly, in these experiments it was not possible to independently vary Re_λ and Fr , and so a systematic understanding of their individual effects on the clustering as quantified by a Voronoi analysis is lacking. DNS investigations into the effect of Re_λ on the Voronoi analysis of particle clustering have been undertaken, and in agreement with the experimental results, they show that the clustering is enhanced with increasing Re_λ [24]. However, only the limited range $75 \leq Re_\lambda \leq 180$ was considered, and they did not consider the effect of gravity.

In addition to an analysis of the local particle concentration using a Voronoi analysis, the DNS study of Ref. [30] explored the idea of coherent clusters of inertial particles, with $Fr = 0.1$ in a low Reynolds number flow, $Re_\lambda = 65$. They defined a coherent cluster as a group of connected Voronoi cells whose volumes are smaller than a certain threshold. Moreover, they specified that the total volume of the cluster of connected cells must exceed $8\eta^3$ (where η is the Kolmogorov length scale) in order to be considered a coherent cluster. They found that the coherent clusters exhibit a tendency to align with the direction of the local vorticity vector, and that the cluster volume depends strongly on St . Furthermore, in the presence of gravity, coherent clusters were found to be elongated and aligned with the gravity direction, as also found experimentally in Ref. [25]. Since they only considered a single set of values for Re_λ and Fr , the effects of these parameters on the coherent clusters are not known.

In addition to using a Voronoi analysis to quantify the particle clustering, this analysis has also been used to understand the settling rates of particles in turbulence and how this is related to the local particle concentration and turbulence properties [19,31–34]. Again, however, the effects of Re_λ and Fr on the results was not systematically explored. It has also been recently shown theoretically and numerically how the settling of particles in turbulence is affected by a range of flow scales that depends on St , Re_λ , and Fr [35]. It is therefore of interest to explore the effect of St , Re_λ , and Fr on the particle settle speeds using a Voronoi analysis in order to gain further insights into the multiscale nature of the problem.

A recent striking finding concerning the effect of gravitational settling is that it can dramatically enhance the fluctuating accelerations of inertial particles in turbulence [23,36], which in turn can profoundly affect the collision and mixing rates of settling particles in turbulence [26,37,38]. An analysis of how these enhanced accelerations might be related to the local particle concentration has not yet been undertaken, but would provide insights concerning the physical mechanism responsible for the effect, and also the scales at which it occurs.

In summary of this brief review, it is clear that there is a need to systematically explore the effects of Re_λ and Fr on particle clustering and the geometry of clusters as analyzed using a Voronoi analysis, and to understand how this might differ from the perspective gained using the RDF analysis in Refs. [17,23,26]. Moreover, there is a need to explore how the enhanced fluctuating particle accelerations due to gravitational settling might be connected to the properties of the local particle concentration in order to gain further insights into the mechanisms responsible for this effect. In the present paper we address these issues.

II. COMPUTATIONAL DETAILS

Since one of the main goals of this paper is to understand how the Re_λ , Fr dependence of particle clustering as determined by a Voronoi analysis might differ from that based on an RDF analysis in Refs. [17,23,26], we therefore consider the same description for the particles and fluid as that in Refs. [17,23,26]. In particular, we focus on the motion of a dilute suspension of one-way coupled, small, heavy, spherical inertial particles whose motion is governed by a simplified version of the

TABLE I. Simulation parameters for the DNS study of isotropic turbulence (arbitrary units). N is the number of grid points in each direction, $\text{Re}_\lambda \equiv u'\lambda/\nu$ is the Taylor microscale Reynolds number ($\text{Re}_\lambda \equiv \sqrt{15 \text{Re}}$ for homogeneous and isotropic flows), $\lambda \equiv u'/\langle(\nabla \mathbf{u})^2\rangle^{1/2}$ is the Taylor microscale, $\mathcal{L}$ is the box size, ν is the fluid kinematic viscosity, $\epsilon \equiv 2\nu \int_0^{\kappa_{\max}} \kappa^2 E(\kappa) d\kappa$ is the mean turbulent kinetic energy dissipation rate, $l \equiv 3\pi/(2k) \int_0^{\kappa_{\max}} E(\kappa)/\kappa d\kappa$ is the integral length scale, $\eta \equiv \nu^{3/4}/\epsilon^{1/4}$ is the Kolmogorov length scale, $u' \equiv \sqrt{(2k/3)}$ is the fluid rms fluctuating velocity, k is the turbulent kinetic energy, u_η is the Kolmogorov velocity scale, $T_L \equiv l/u'$ is the large-eddy turnover time, $\tau_\eta \equiv \sqrt{\nu/\epsilon}$ is the Kolmogorov timescale, $\kappa_{\max} = \sqrt{2}N/3$ is the maximum resolved wave number, and N_p is the number of particles per Stokes number.

Parameter	DNS 1	DNS 2	DNS 3	DNS 4	Parameter	DNS 5	DNS 6	DNS 7	DNS 8
N	128	128	1024	512	N	512	1024	1024	1024
Re_λ	93	94	90	224	Re_λ	237	230	398	398
Fr	∞	0.3	0.052	∞	Fr	0.3	0.052	∞	0.052
$\mathcal{L}$	2π	2π	16π	2π	$\mathcal{L}$	2π	4π	2π	2π
ν	0.005	0.005	0.005	0.0008289	ν	0.0008289	0.0008289	0.0003	0.0003
ϵ	0.324	0.332	0.257	0.253	ϵ	0.2842	0.239	0.223	0.223
l	1.48	1.49	1.47	1.40	l	1.43	1.49	1.45	1.45
l/η	59.6	60.4	55.6	204	l/η	214	213	436	436
u'	0.984	0.996	0.912	0.915	u'	0.966	0.914	0.915	0.915
u'/u_η	4.91	4.92	4.82	7.60	u'/u_η	7.82	7.7	10.1	10.1
T_L	1.51	1.50	1.61	1.53	T_L	1.48	1.63	1.58	1.58
T_L/τ_η	12.14	12.24	11.52	26.8	T_L/τ_η	27.36	27.66	43.0	43.0
$\kappa_{\max}\eta$	1.5	1.48	1.61	1.66	$\kappa_{\max}\eta$	1.62	1.68	1.60	1.60
N_p	262144	262144	16777216	2097152	N_p	2097152	16777216	2097152	2097152

equation of Maxey and Riley [39],

$$\ddot{\mathbf{x}}^p(t) \equiv \dot{\mathbf{v}}^p(t) = \frac{\mathbf{u}[\mathbf{x}^p(t), t] - \mathbf{v}^p(t)}{\tau_p} + \mathbf{g}, \quad (1)$$

where $\mathbf{x}^p(t)$, $\mathbf{v}^p(t)$ are the particle position and velocity vectors, $\mathbf{u}(\mathbf{x}^p(t), t)$ is the fluid velocity at the particle position, $\mathbf{g}$ is the gravitational acceleration, and τ_p is the particle response time. Furthermore, fluid particles are tracked by solving $\dot{\mathbf{x}}^p(t) \equiv \mathbf{u}[\mathbf{x}^p(t), t]$.

The relevant nondimensional parameters in this problem are the Stokes number ($\text{St} \equiv \tau_p/\tau_\eta$), the settling parameter ($\text{Sv} \equiv \tau_p g/u_\eta$, where the numerator represents the particle's terminal velocity in the laminar flow), and the Froude number [$\text{Fr} \equiv a_\eta/g = \epsilon^{3/4}/(\nu^{1/4}g)$] which characterize the particle's inertia, the effect of gravity on inertial particles, and the effect of gravity on the flow, respectively. Here, a_η is the Kolmogorov acceleration, τ_η is the Kolmogorov timescale, u_η is the Kolmogorov velocity scale, and ϵ is the mean turbulent kinetic energy dissipation rate.

Direct numerical simulations (DNS) of the incompressible Navier-Stokes equation are performed on a triply periodic cube of length $\mathcal{L}$, using a pseudospectral method on a uniform mesh with N^3 grid points. A deterministic forcing scheme is used to generate statistically stationary, isotropic turbulence. Details of the numerical setup and simulations can be found in Refs. [17,23,40], and information on the simulations is provided in Table I. In Table II we also provide information on the particle parameters considered in the DNS, to aid the reader in following the parameter values in the figures that follow.

As mentioned in the Introduction, a Voronoi analysis can be sensitive to the number of particles used in the analysis. Therefore, when comparing results from different DNS with different Re_λ and Fr we choose the number of particles such that the average distance between the particles is the same in each study, $\bar{r} \equiv \mathcal{L}/N_p^{1/3} = 7.9\eta$. This number is comparable to that in the studies of Refs. [20,24,41]. A smaller value for $\bar{r}$ would be desirable, especially for performing the coherent cluster analysis [30]. However, the computational expense for the higher Re_λ cases we consider

TABLE II. Particle parameters considered in the DNS. For each value of the Stokes number St , the associated settling number Sv is shown for both $Fr = 0.3, 0.052$.

St	Sv (for $Fr = 0.3$)	Sv (for $Fr = 0.052$)
0	0	0
0.05	0.1667	0.96
0.1	0.3333	1.93
0.2	0.6667	3.86
0.3	1	5.79
0.4	1.3333	7.71
0.5	1.6667	9.64
0.6	2	11.6
0.7	2.3333	13.5
0.8	2.6667	15.4
0.9	3	17.4
1	3.3333	19.3
1.5	5	28.9
2	6.6667	38.6
3	10	57.9

make it unfeasible to consider significantly smaller values of $\bar{r}$. In order to provide some insight, however, concerning how our results might depend upon $\bar{r}$, in the Appendix we provide results from a simulation with $Re_\lambda = 90$, and a range of St, Fr and for $\bar{r} = 1.8\eta$, and we compare the results with those having $\bar{r} = 7.9\eta$ (see Figs. 18–26 in the appendix). Consistent with previous findings, we observe that while the results are quantitatively affected by the choice of $\bar{r}$, they are not affected qualitatively, with the dependence of the results on St, Fr being the same in each case.

Using our DNS data, we perform a three-dimensional (3D) Voronoi analysis of the particles, and then compute a range of statistical quantities using the information from the analysis.

Following Ref. [30], particles at the boundaries of the domain are removed for the cluster analysis, in order to ensure our cluster analysis is fully consistent with theirs. Nevertheless, we also compared the results to those obtained where periodic boundary conditions were applied to the particles. We did not find any noticeable difference when using these two methods, as expected, given the statistical homogeneity of the flow.

III. RESULTS AND DISCUSSION

A. Voronoi volume distributions

In Fig. 1, the probability distribution functions (PDFs) of the Voronoi cell volumes (normalized by the mean Voronoi cell volumes) for different St and with different Fr and Re_λ combinations are shown. The small (large) Voronoi volumes represent the high (low) concentration regions of the flow. For $St = 0$ the particles are randomly distributed, and their PDF follows that of a random Poisson process (RPP), described by Ref. [42]. Compared to the $St = 0$ case, the PDFs for $St > 0$ are significantly higher at small volumes, indicating an enhanced probability of finding regions with high local particle concentration, i.e., clustering. In the experimental study of Ref. [43] they observed a crossover, such that at sufficiently small volumes, the PDFs of the Voronoi volumes for inertial particles drop below that for fluid particles. They conjectured that this effect was due to interactions between the particles when they are sufficiently close. In our results we do not observe this crossover, and neither did the numerical studies of Refs. [24,30]. While this may be due to our neglect of the effect of particle interactions, we note that the crossover was also not observed in the recent experimental work of Ref. [25].

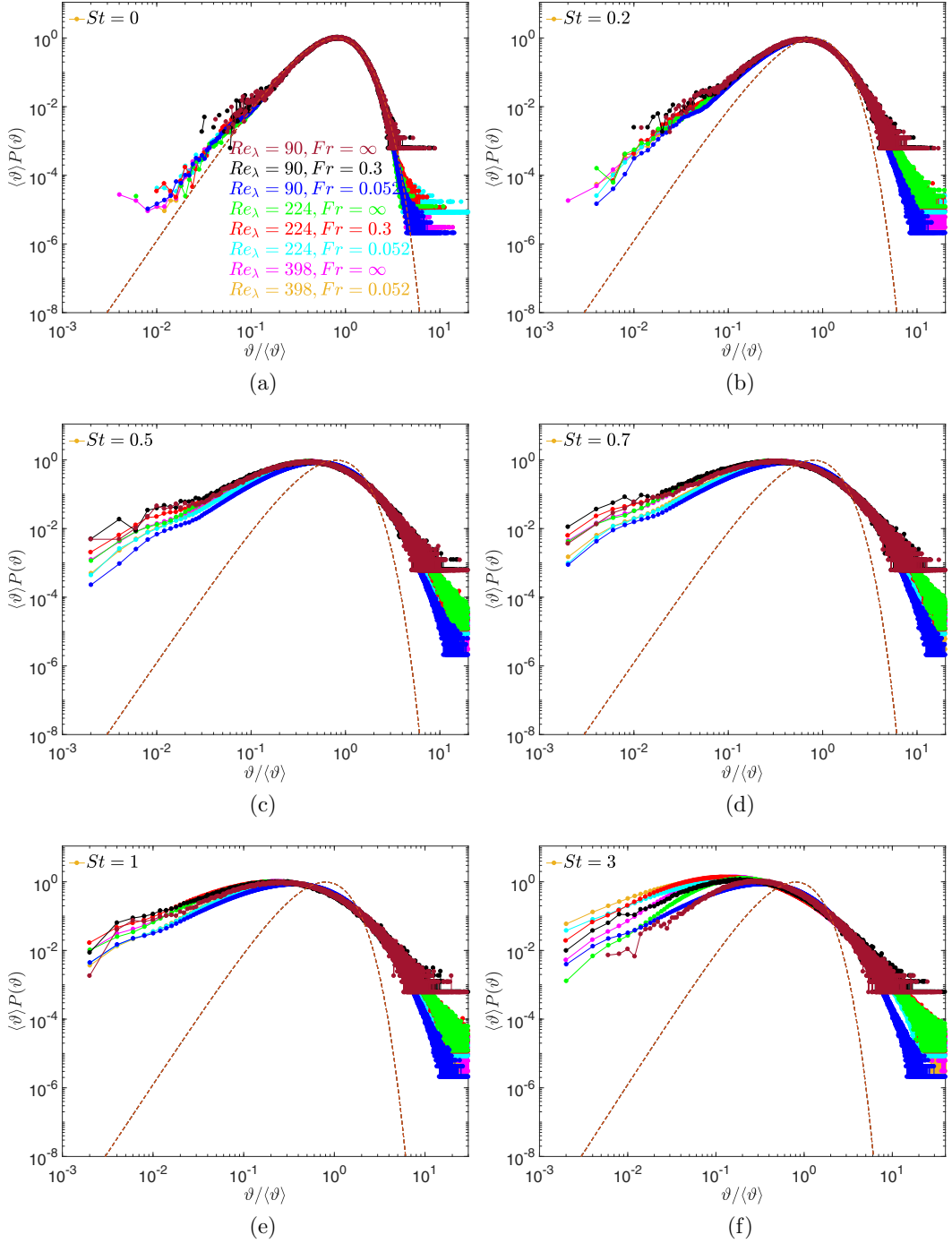


FIG. 1. PDF of the Voronoi volumes (normalized by the mean volume) at different cases of Fr and Re_λ combinations for (a) $St = 0$, (b) $St = 0.2$, (c) $St = 0.5$, (d) $St = 0.7$, (e) $St = 1$, and (f) $St = 3$. Different colors represent different cases. The dashed line represents the random Poisson distribution.

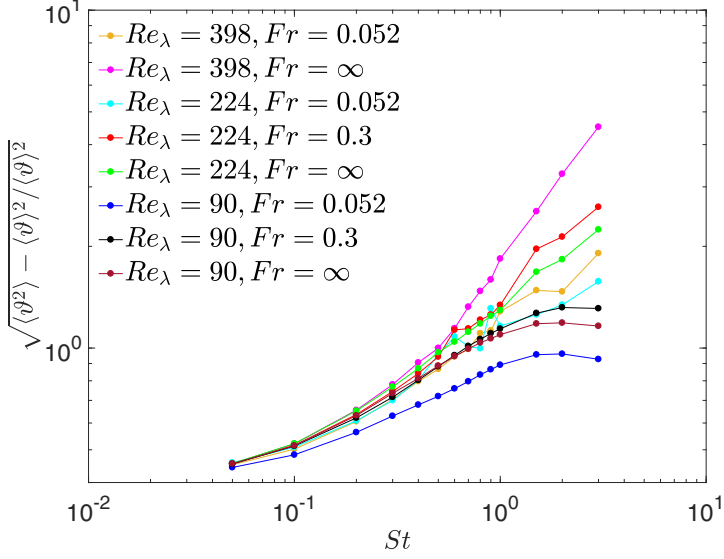
Concerning the effect of Re_λ , the results in Fig. 1 indicate that with respect to the behavior at small volumes, the effect of Re_λ depends on both St and Fr . When $St < 3$, the effect of Re_λ is quite weak, sometimes leading to a slight increase of the PDF at small volumes that then saturates as Re_λ is increased (e.g., for the case $St = 0.5$, $Fr = 0.052$), while in other cases it leads to a slight decrease of the PDF at small volumes (e.g., for the case $St = 0.7$, $Fr = 0.3$). However, for $St = 3$, the sensitivity of the Voronoi volume PDF to Re_λ becomes much more apparent, with the clustering becoming stronger as Re_λ is increased. This dependence of the clustering on Re_λ is similar to that found in Refs. [17,23] where the RDF was used to analyze the clustering. Arguments in Refs. [17,23,44] suggest that this behavior arises because unless St is sufficiently large, particles in the dissipation range are not able to remember their interaction with the inertial-range turbulence along their path history. In this case, their motion is dominated by the dissipate range dynamics of the flow, and their dissipation range motion is not affected by the changing size of the inertial range as Re_λ is increased. Nevertheless, even if their dissipation range clustering is not affected by the inertial range, the clustering could still be affected by the strong intermittency of the dissipation range turbulence [45]. In Refs. [17,46] it was argued that because the RDF is a low-order measure of the particle phase-space dynamics, it is not very sensitive to the flow intermittency that is most apparent in the high-order moments. However, the PDF of the Voronoi volumes is not confined to low-order information, and could in principle be sensitive to intermittent fluctuations in the particle motion, yet our results imply that the particle clustering is only weakly affected by the flow intermittency. One possible explanation for this surprising behavior is that due to the preferential sampling of the flow by the inertial particles, the inertial particles avoid the intermittent regions of the flow [17,23,47], and as a result their spatial clustering is not affected by the increased intermittency in the underlying turbulence as Re_λ is increased. The other possible explanation is that the average interparticle distance $\bar{r}$ in our simulations is too large for us to be able to obtain statistically reliable results at Voronoi volumes that are small enough to be strongly affected by dissipation range intermittency. To address this issue, future efforts would need to explore the problem using $\bar{r} \leq O(\eta)$, something that is not currently feasible for the higher Re_λ cases.

Concerning the effect of Fr , the results in Fig. 1 indicate that with respect to the behavior at small volumes, the clustering initially becomes stronger as Fr is reduced from ∞ (no gravity case) to 0.3, but then becomes weaker as Fr is further reduced. Therefore, gravity has a nonmonotonic effect on the clustering, and whether it enhances or reduces the clustering depends on St and Fr .

This effect of gravity on clustering agrees with the conclusions based on the RDF analysis in Ref. [23], although in that analysis the nonmonotonic dependence of the clustering on Fr was not so clear since only the values $Fr = \infty$, 0.052 were considered, and not the intermediate value $Fr = 0.3$. However, such nonmonotonic dependence was observed using an RDF analysis in our previous study [26]. Physical arguments for the effect of gravity on clustering can be found in Ref. [23].

Not shown here, concerning the effect of St we found, in agreement with previous studies, that for $Fr = \infty$, the probability of finding highly concentrated regions enhances with St , peaks at $St = 1$, and then drops for larger St . Furthermore, we observed that the presence of gravity can lead to the enhancement of clustering when $St \geq O(1)$, with its effect becoming more apparent at higher Re_λ . Interested readers can find the details in Ref. [48].

To obtain more insight into the Re_λ dependence, in Fig. 2 we plot the standard deviation (s.d.) of the Voronoi volumes as a function of St . In agreement with the experimental study of Ref. [43], we observe that for $Re_\lambda = 90$, the s.d. increases with increasing St until it reaches a peak around $St \approx 2$, after which it decreases. Also in agreement with their results we find that the s.d. becomes larger and the peak shifts to higher values of St as Re_λ is increased. This is essentially due to the fact that as Re_λ is increased, more flow scales are present in the turbulence at which the particles can cluster. However, for any finite St , the growth of the s.d. with increasing Re_λ must saturate, since at sufficiently large scales the particles behave as fluid particles and do not cluster [49]. The results in Fig. 2 reveal a stronger sensitivity to Re_λ than is implied by the results in Fig. 1 which may indicate that the sensitivity of the s.d. to Re_λ is mainly in the behavior of the PDF of Voronoi cells with large volume, whose behavior is not clear in a log-log plot. To consider this, in Fig. 3 we plot the PDF of


 FIG. 2. Standard deviation of Voronoi cells normalized by their mean as a function of St .

Voronoi volumes in a log-lin plot. Comparing the results with those in Fig. 1 confirms that it is the Voronoi cells with larger volumes that exhibit the strongest sensitivity to Re_λ and that the sensitivity to Re_λ is generally stronger in the presence of gravity.

The data in Fig. 1 show that the PDF for inertial particles intersects that for fluid particles at two points, denoted by ϑ_C and ϑ_v , respectively. These points are often used to define thresholds for detecting clusters and voids, with volumes less than ϑ_C denoting clustered regions, and volumes greater than ϑ_v denoting void regions. In view of these definitions, it is then apparent that the Re_λ dependence of the s.d. of the Voronoi volumes comes mainly from the Re_λ of the void regions, and not the clustered regions. In this way our conclusions are consistent with the findings in Refs. [17,23,26] based on the RDF that the small-scale clustering is quite insensitive to Re_λ when $St \lesssim 1$, and the strong Re_λ dependence of the s.d. of the Voronoi volumes does not contradict this.

A related point is that the results in Fig. 2 indicate that gravity suppresses the s.d. of the Voronoi volumes, while for the same cases (e.g., $St = 3$, $Re_\lambda = 398$), Fig. 1 indicates that gravity enhances the small-scale clustering. This is again because the s.d. is dominated by the behavior of the void regions rather than the clustered regions, and the void regions tend to be suppressed by gravity. This shows again the nontrivial effect of gravity on the particle clustering; while the probability of clustered and void regions both increases with increasing St , for a given St , decreasing Fr can enhance the probability of clustered regions while lowering the probability of void regions. These points illustrate that using the s.d. Voronoi volumes as a measure of clustering should be used with caution, since its properties are dominated by void, rather than clustered, regions of the flow.

In a number of previous studies it has been reported that the PDF of Voronoi volumes follows a log-normal distribution [20,25,27,28,31]. In Fig. 4 the centered and normalized PDFs of the logarithm of the Voronoi volumes are presented. The results show that for $St \geq 0.5$ a Gaussian distribution can approximate the shape of $\log(\vartheta)$ in the interval $\pm 2\sigma_{\log(\vartheta)}$, while large deviations from the Gaussian PDF are found outside of this range. In particular, the tails of the PDF are heavier than a Gaussian distribution, as also observed in Ref. [31]. However, the PDF seems to approach a Gaussian shape as St increases. For smaller St the Gaussian PDF is not followed, which is to be expected since in the limit $St \rightarrow 0$ the PDF follows that for a RPP, as shown earlier. Therefore, the lognormal shape for the PDF of Voronoi volumes is only a reasonable approximation

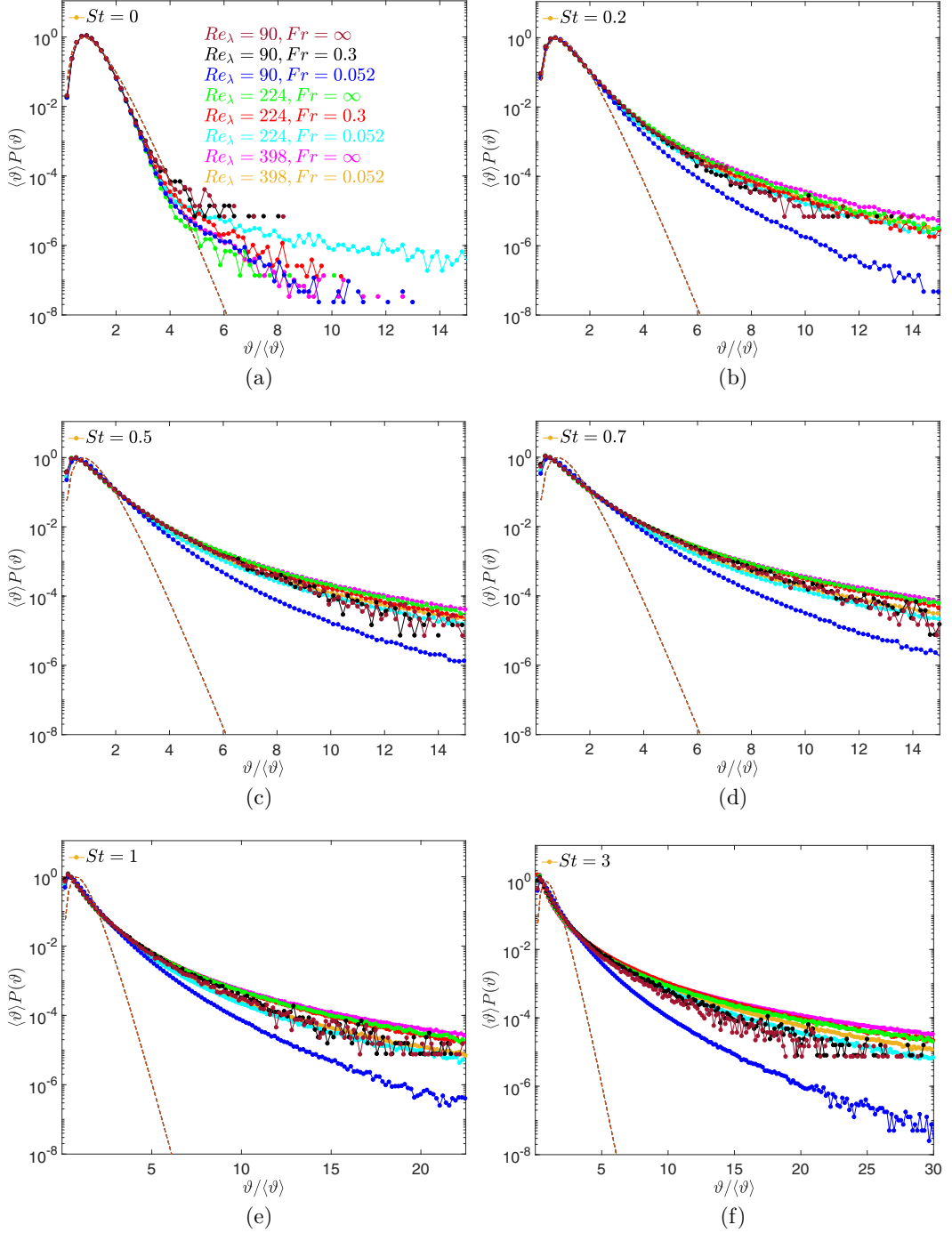


FIG. 3. PDF of the Voronoi volumes (normalized by the mean volume) at different cases of Fr and Re_λ combinations for (a) $St = 0$, (b) $St = 0.2$, (c) $St = 0.5$, (d) $St = 0.7$, (e) $St = 1$, and (f) $St = 3$. Different colors represent different cases. The dashed line represents the random Poisson distribution.

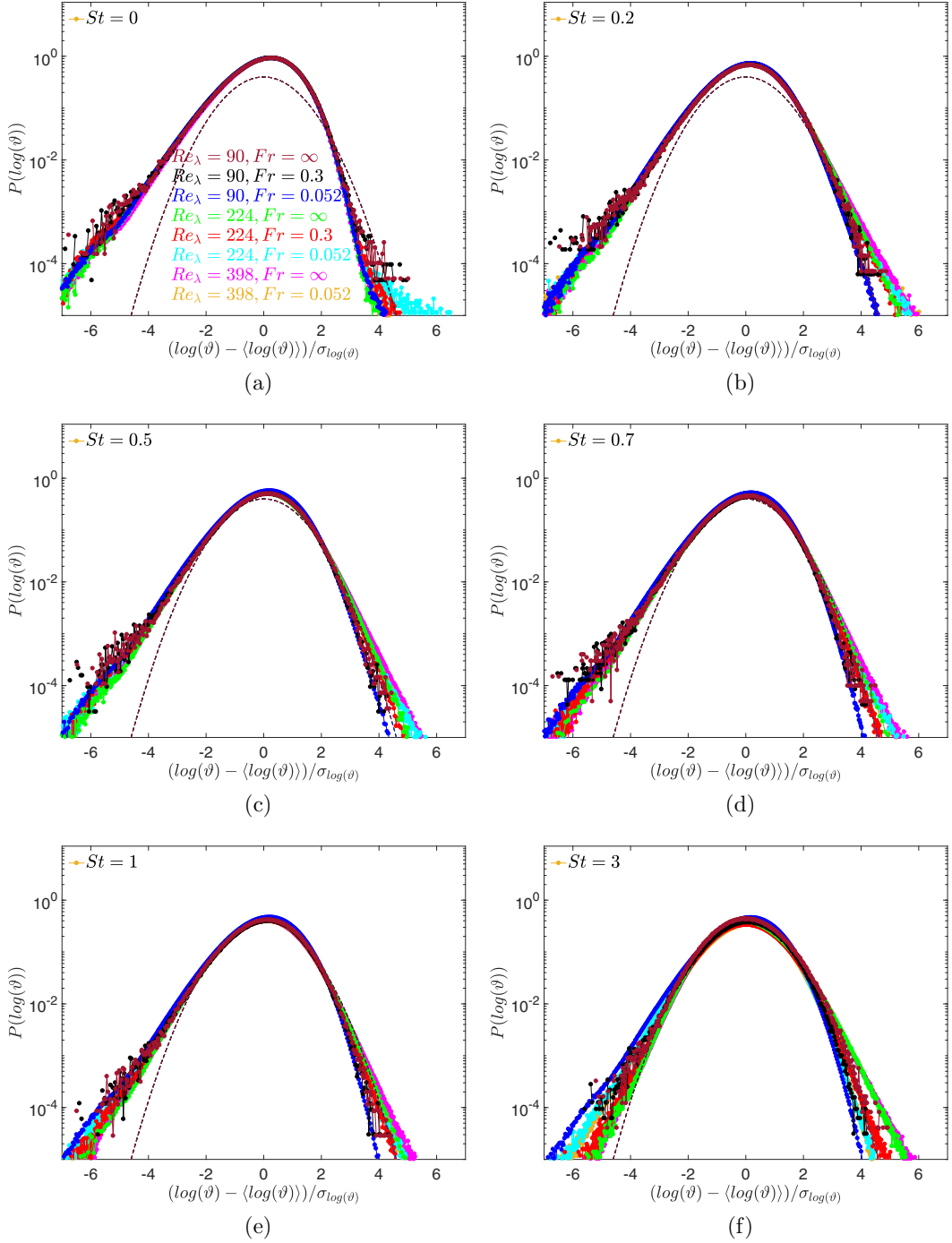


FIG. 4. PDF of the lognormal (natural logarithm) distribution of Voronoi volumes (centered at the mean and normalized by standard deviation) at different cases of Fr and Re_λ combinations for (a) $St = 0$, (b) $St = 0.2$, (c) $St = 0.5$, (d) $St = 0.7$, (e) $St = 1$, and (f) $St = 3$. Different colors represent different cases and the dashed line denotes the Gaussian distribution.

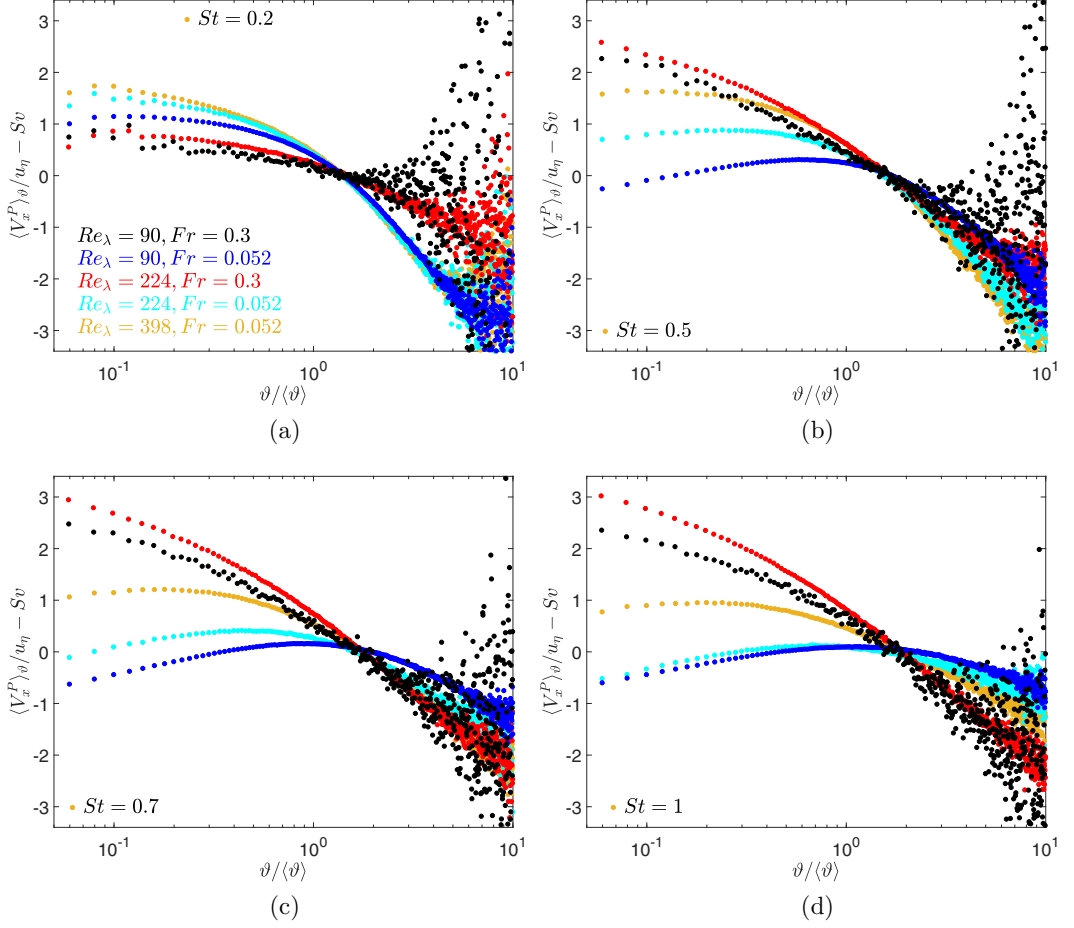


FIG. 5. Average velocity of particles conditioned on the PDF of the Voronoi volumes at different cases of Fr and Re_λ combinations for (a) $St = 0.2$, (b) $St = 0.5$, (c) $St = 0.7$, and (d) $St = 1$. Different colors represent different cases.

in certain regimes. Departures from the lognormal behavior exhibit different (and not entirely clear) dependencies on Re_λ and Fr for the left and right tails of the PDF, an explanation for which is not clear.

B. Particle settling velocities

We now turn to consider the average velocity of particles conditioned on the Voronoi volumes to explore how the particle settling velocities depend upon the local particle concentration. In our notation, the subscript x denotes the direction of gravity, so that positive velocities correspond to particle motion in the direction of gravity. We will also later use y to denote the direction perpendicular to gravity.

The results in Fig. 5 clearly illustrate the correlation between the particle settling velocity and the local concentration (inverse of the Voronoi volumes), as also observed over smaller ranges of Re_λ and Fr in the DNS studies of Refs. [30,32,50] as well as the experimental study of Ref. [19]. The recent experiments of Ref. [25] also considered this quantity under a wide range of particle loadings, to examine the influence of two-way coupling between the particles and fluid on the particle settling

velocities and its relation to the local concentration. Their results at low particle loadings are similar to ours, suggesting, as expected, that the one-way coupled assumption yields physically realistic results in that regime.

The results show that the Fr for which the settling velocity reaches the largest value depends upon St , which is simply because in the particle equation of motion, the important nondimensional number is the settling number $Sv \equiv St/Fr$. However, for $St > 0.2$, the settling enhancement is strongest for intermediate values of Fr , in agreement with the DNS study of Ref. [31]. For $Fr = 0.3$, the average settling velocity enhances almost monotonically with increasing particle concentration (the decrease observed at small volumes for $St = 0.2$ may be due to noise). However, for $Fr = 0.052$, there is a local concentration at which the mean settling velocity is maximum, and then decreases for higher concentrations. If we assume that larger Voronoi volumes are associated with larger flow scales, then this is consistent with the arguments in Ref. [35] that as Fr is decreased, the scales of the flow responsible for enhancing the particle settling velocity increase. The results also show that increasing Re_λ can significantly increase the settling velocities, especially for $St \geq 0.5$. This is explained by the recent theoretical analysis of Ref. [35] that shows that as Re_λ is increased, more and more energetic flow scales are introduced to the flow that are able to contribute to the particle settling velocity enhancement through the preferential sweeping mechanism described below.

Concerning the physical mechanism underlying the correlation between the settling velocity and concentration/clustering, since our DNS are one-way coupled, the effect cannot be explained in terms of the particles modifying the local flow field, but in terms of the way the particles passively sample the underlying flow field. Reference [4] provided a theoretical analysis and on the basis of this proposed that in the regime $St \ll 1$, the particles are preferentially swept around the downward (direction of gravity) moving side of vortices in the flow, and that as a result of this preferential sweeping, their average settling velocity exceeds the Stokes settling velocity. This is the so-called “preferential sweeping mechanism” [51]. This theoretical analysis was recently extended by Ref. [35] to arbitrary St , wherein the basic mechanism is the same as that proposed by Maxey, except that now the size of the vortices around which the particles are preferentially swept is shown to depend essentially on St , Fr , and Re_λ , and is a multiscale preferential sweeping mechanism. However, it was pointed out in Ref. [35] that unlike the regime $St \ll 1$, in the regime $St \geq O(1)$ the mechanism generating the clustering is distinct from the preferential sweeping mechanism. Indeed, for $St \geq O(1)$ the clustering is generated by a nonlocal mechanism and not the preferential sampling of the local flow field [44,49]. As a result, while the results in Fig. 5 for $St \ll 1$ can be directly explained in terms of the preferential sweeping mechanism, the same does not apply for $St \geq O(1)$. A direct and unambiguous test of the preferential sweeping mechanism would be to condition the average particle velocity not on the concentration, but on some measure of the local preferential sampling of the flow.

C. Local analysis of particle accelerations

In Refs. [23,36] it was shown that settling can significantly enhance the accelerations of inertial particles in turbulent flows, and in Ref. [26] it was also shown that the variance of the particle accelerations generally increases with increasing Re_λ and/or with decreasing Fr . To gain further insight into the behavior of the particle accelerations, we perform a local analysis by conditioning the particle acceleration behavior on the local Voronoi volume, to understand how the accelerations are linked to the concentration field.

Figures 6 and 7 show results for the particle acceleration $[a^p(t) \equiv \dot{v}^p(t)]$ conditioned on the local Voronoi volume, in the direction of gravity (which points along the positive x axis) and perpendicular to gravity (along y axis), respectively.

According to these results, in the absence of gravity ($Fr = \infty$), the particle accelerations are zero (to within statistical noise) in both directions, independent of St and Re_λ . In the presence of gravity, this same behavior is observed for the accelerations in the direction normal to gravity, however,

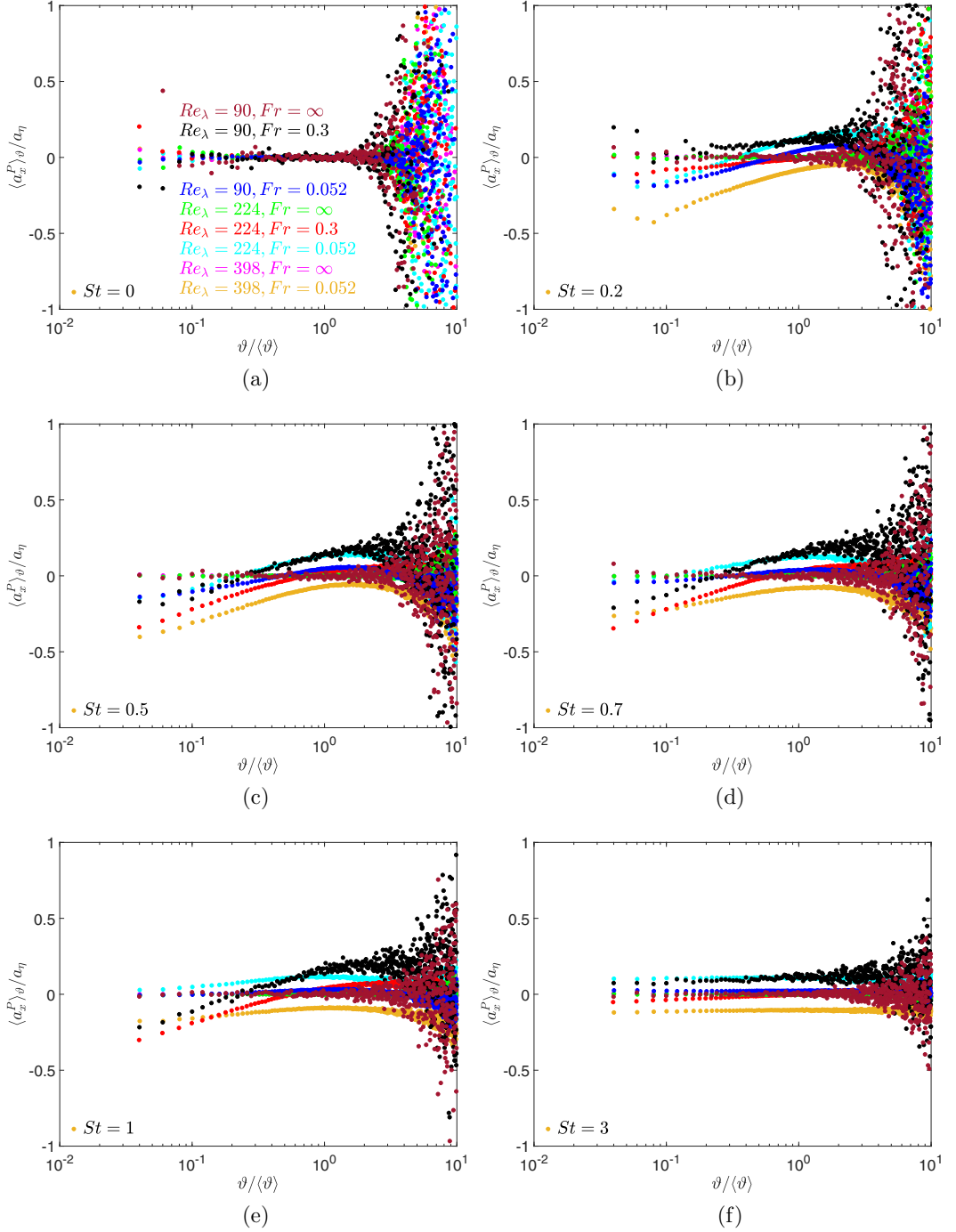


FIG. 6. Average acceleration of particles, in the gravity direction, conditioned on the PDF of the Voronoi volumes at different cases of Fr and Re_λ combinations for (a) $St = 0$, (b) $St = 0.2$, (c) $St = 0.5$, (d) $St = 0.7$, (e) $St = 1$, and (f) $St = 3$. Different colors represent different cases.

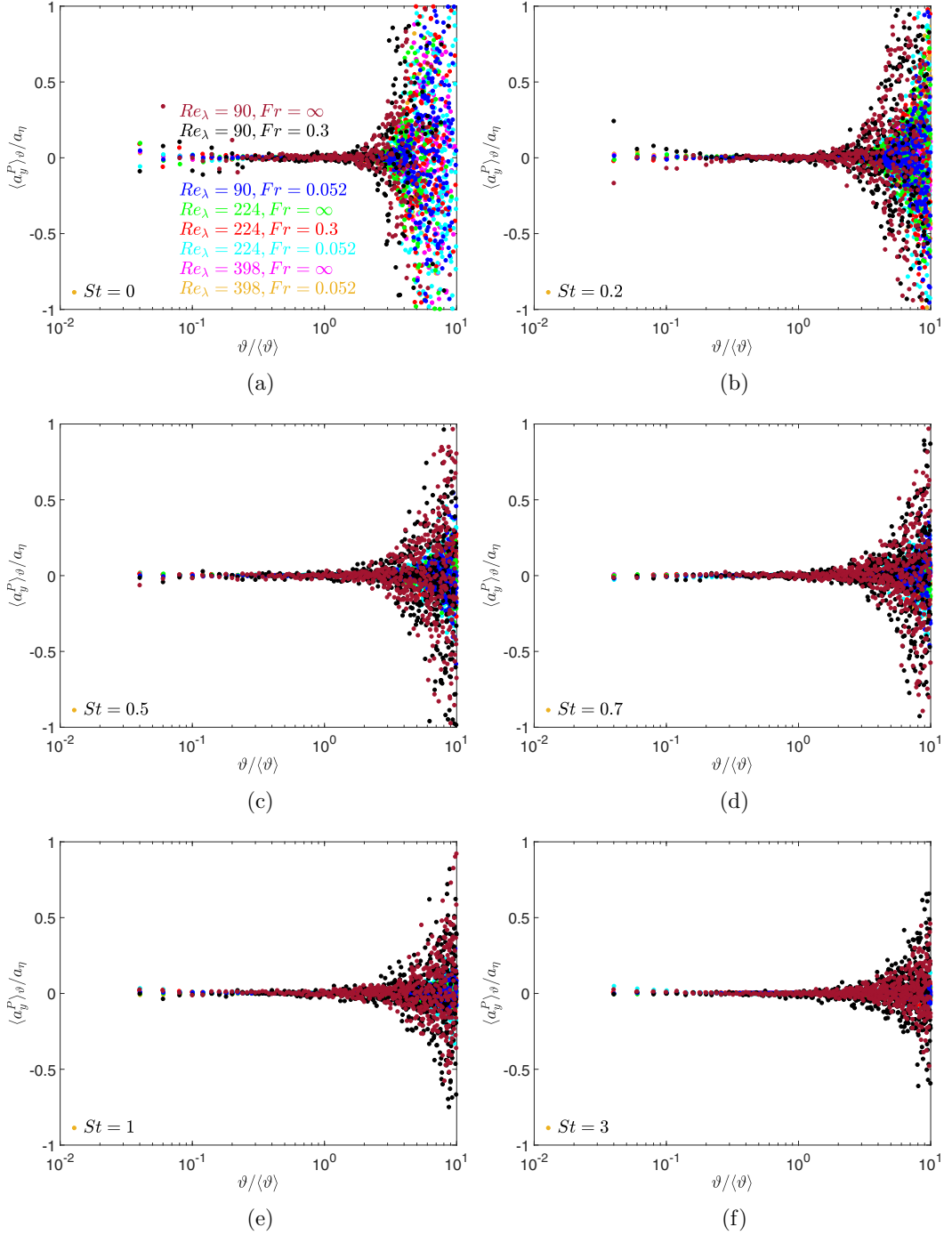


FIG. 7. Average acceleration of particles, in the plane normal to the gravity direction, conditioned on the PDF of the Voronoi volumes at different cases of Fr and Re_{λ} combinations for (a) $St = 0$, (b) $St = 0.2$, (c) $St = 0.5$, (d) $St = 0.7$, (e) $St = 1$, and (f) $St = 3$. Different colors represent different cases.

in the direction of gravity the quantity becomes finite for finite St . In particular, $\langle a_x^p \rangle_\vartheta$ becomes negative (denoting accelerations in the direction opposite to gravity) at small volumes and positive for large volumes. Moreover, $\langle a_x^p \rangle_\vartheta$ becomes increasingly negative at small ϑ as Re_λ is increased. Some of this behavior may be understood by considering the following identity,

$$\langle a_x^p \rangle \equiv \int_0^\infty \langle a_x^p \rangle_\vartheta P(\vartheta) d\vartheta. \quad (2)$$

Since $P(\vartheta) \geq 0$, and $\langle a_x^p \rangle = 0$ for statistically stationary, homogeneous turbulence, then the only way for (2) to be satisfied is if either $\langle a_x^p \rangle_\vartheta = 0 \forall \vartheta$, or else its sign varies with ϑ . The latter situation is what we observe in the presence of gravity, with particles in high concentration regions (small ϑ) exhibiting a net acceleration in the direction opposite to gravity even though the net acceleration of all the particles is zero, i.e., $\langle a_x^p \rangle = 0$. However, we are not able to explain why $\langle a_x^p \rangle_\vartheta$ is negative rather than positive at small ϑ .

Figures 8 and 9 show results for the second moment of the particle accelerations conditioned on the Voronoi volumes, in the direction parallel and perpendicular to gravity, respectively. The results reveal a surprising nonlinear dependence on ϑ , that is present irrespective of Fr , Re_λ , that disappears for sufficiently small or large St . Furthermore, the strong dependence of this behavior on St indicates that the nonlinear dependence of the acceleration variances on ϑ is related to the spatial clustering of the particles.

This nonlinear dependence of the acceleration variances on ϑ is not qualitatively affected by Fr , Re_λ , however, the results show that decreasing Fr and/or increasing Re_λ lead to an enhancement of the particle accelerations at all ϑ (to within statistical noise). This would seem to imply, in agreement with the physical arguments in Ref. [23], that the enhancement of the inertial particle accelerations due to gravitational settling are not fundamentally associated with a change in how gravity causes the particles to cluster. Rather, it is caused by the fact that gravity causes the particles to experience rapid changes in the fluid velocity along their trajectory, which in turn leads to large particle accelerations. This effect, described more fully in Ref. [37], would operate even if the particles were uniformly distributed throughout the flow. Indeed, our results in Figs. 8 and 9 for $St = 3$ reveal strong enhancements of the particle accelerations due to gravity, even though the acceleration variances are almost independent of ϑ .

In Fig. 10 we show results for the variances of the fluid acceleration (a^F) at the inertial particle positions, conditioned on ϑ , and for the direction of gravity. These results also exhibit, in general, a convex functional shape with respect to ϑ . They also show that as Fr is decreased, the accelerations become stronger, even for $St = 0$ cases which are independent of Fr (in general, the fluid particle accelerations should be independent of Fr , however, since the different Fr cases correspond to different simulations, the results can depend on Fr , for example, due to different domain sizes in the simulations). Since the fluid acceleration measured at the inertial particle positions can only differ from the fluid acceleration at fluid particle positions due to preferential sampling of the flow by the particles, this indicates that the settling causes the inertial particles to sample regions of the flow exhibiting strong accelerations.

The results in Fig. 10 also present a challenge to the “sweep-stick” mechanism that has been proposed to explain particle clustering in turbulent flows [52]. The sweep-stick mechanism states that inertial particles stick to points in the flow where the fluid acceleration is zero, and since these acceleration stagnation points are themselves clustered, this therefore explains the clustering of the inertial particles. In Ref. [49] a number of theoretical arguments were given that call into question the validity of this proposed mechanism for particle clustering, supported by results from DNS. If the sweep-stick mechanism were correct, we should expect to find that in regions where the particles are clustered (i.e., where ϑ is small), the fluid acceleration at the particle position should be zero, or at least very small compared with a_η . The results in Fig. 10 show, however, that in the regions where the particles are clustered, the fluid accelerations at the inertial particle positions are large, i.e., their magnitudes are $\geq O(a_\eta)$. Moreover, over a significant range of ϑ , the accelerations are almost

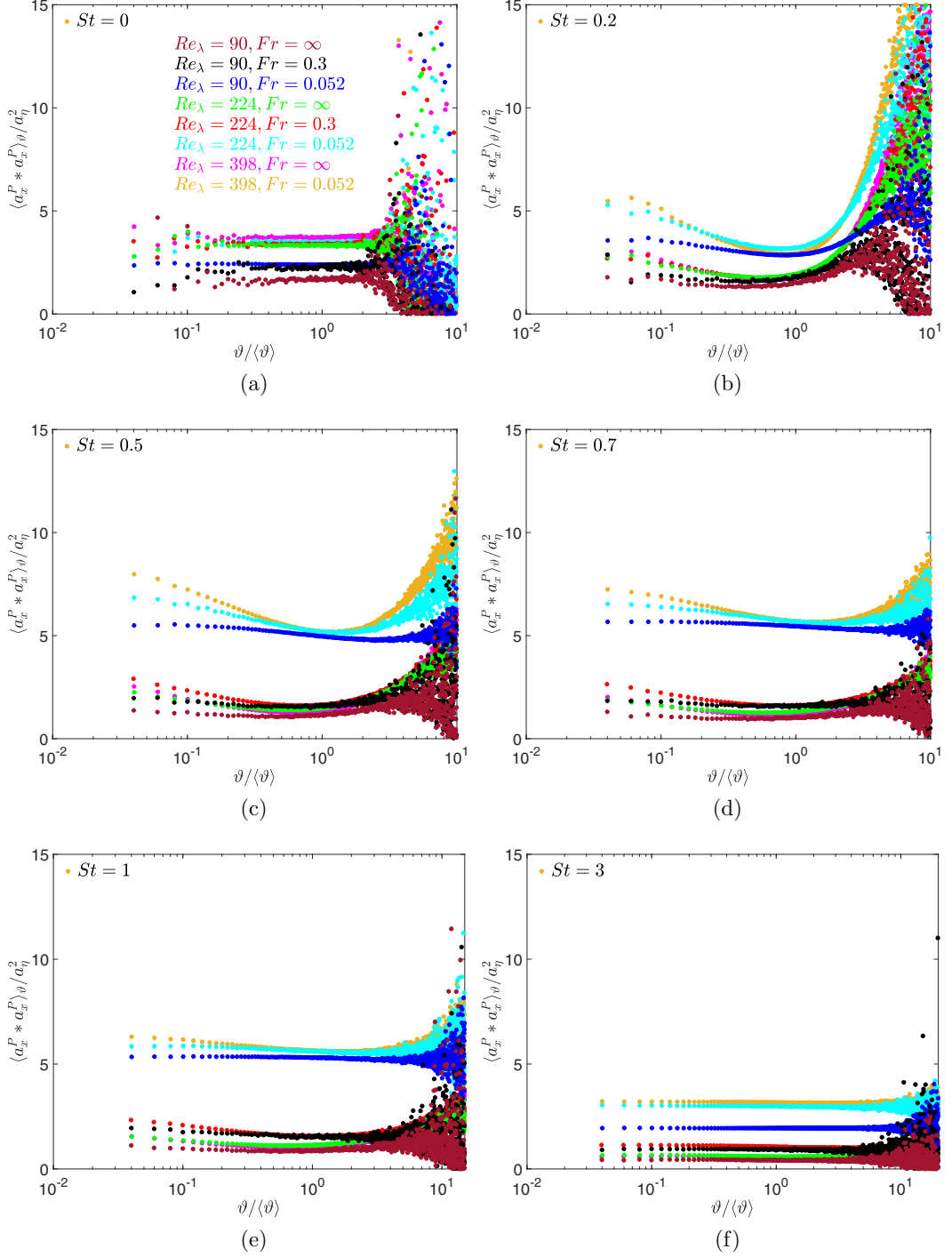


FIG. 8. Second moment of particle accelerations, in the gravity direction, conditioned on the PDF of the Voronoi volumes at different cases of Fr and Re_λ combinations for (a) $St = 0$, (b) $St = 0.2$, (c) $St = 0.5$, (d) $St = 0.7$, (e) $St = 1$, and (f) $St = 3$. Different colors represent different cases.

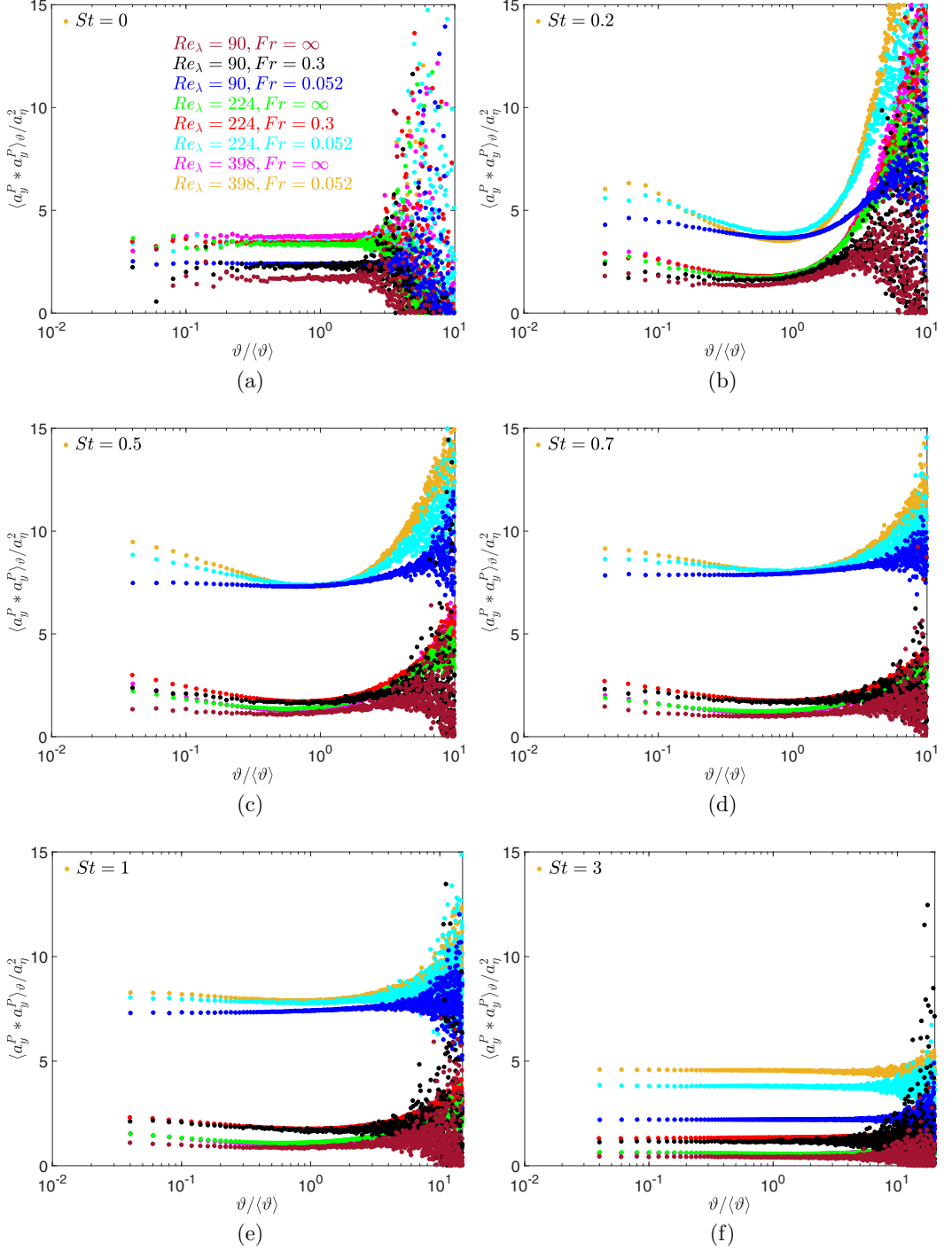


FIG. 9. Second moment of particle accelerations, in the plane normal to the gravity direction, conditioned on the PDF of the Voronoi volumes at different cases of Fr and Re_λ combinations for (a) $St = 0$, (b) $St = 0.2$, (c) $St = 0.5$, (d) $St = 0.7$, (e) $St = 1$, and (f) $St = 3$. Different colors represent different cases.

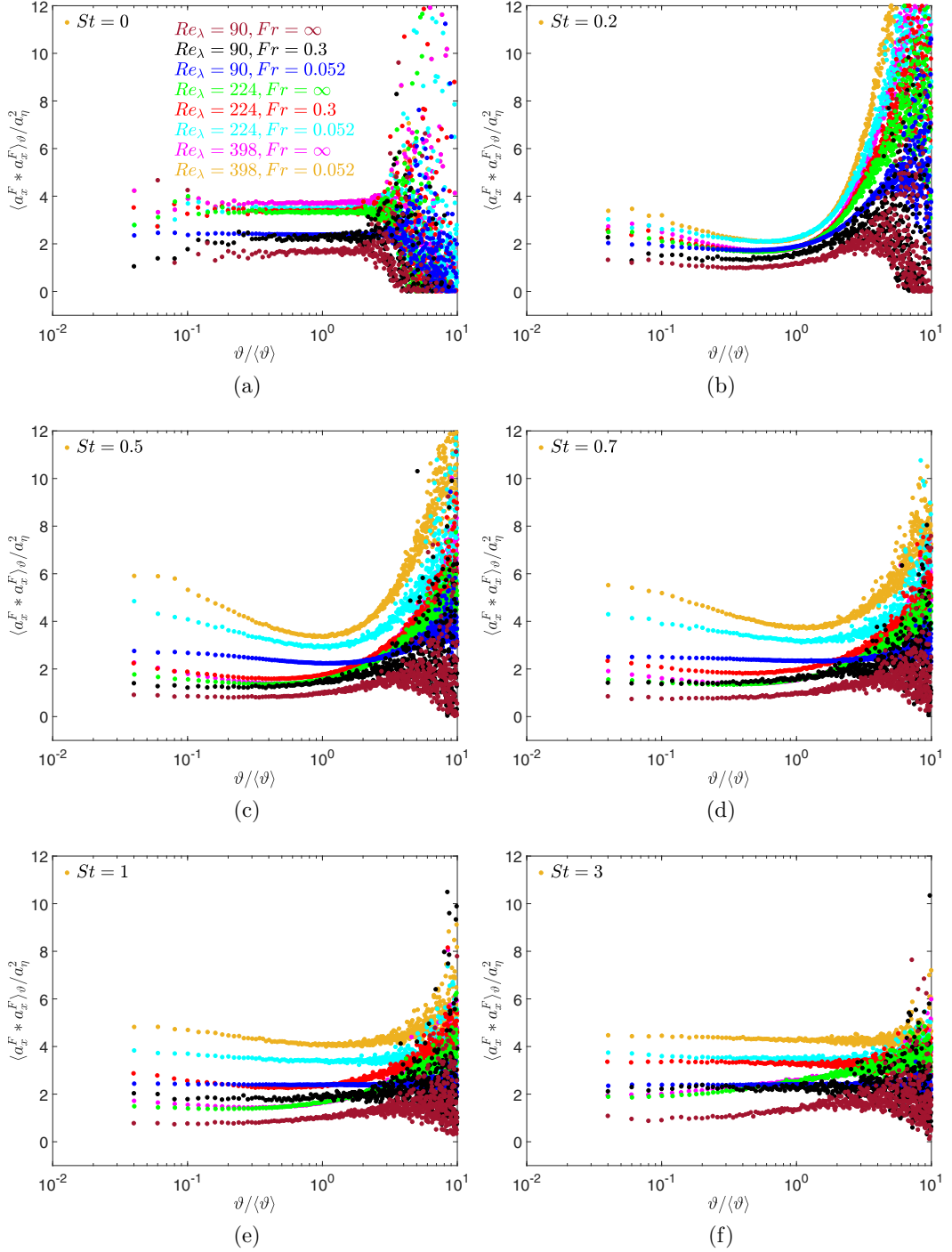


FIG. 10. Second moment of fluid accelerations at the particle position, in the gravity direction, conditioned on the PDF of the Voronoi volumes at different cases of Fr and Re_λ combinations for (a) $St = 0$, (b) $St = 0.2$, (c) $St = 0.5$, (d) $St = 0.7$, (e) $St = 1$, and (f) $St = 3$. Different colors represent different cases.

independent of ϑ . These results therefore strongly call into question the validity of the sweep-stick mechanism. We refer the reader to Ref. [49] for a detailed discussion concerning theoretical issues with the sweep-stick mechanism.

IV. CLUSTER ANALYSIS

We now turn our attention to the results for clusters of particles, which are defined as connected Voronoi cells that have volumes $\leq \vartheta_C$ [30]. Figure 11 shows the PDF of cluster volumes normalized by the Kolmogorov scale. The results show that cluster sizes are distributed over a wide range of scales, from the dissipation up to integral range scales. In agreement with previous studies [20,27,30,32,50], we find that the right-hand side of the PDF is well described as a power law with exponent -2 , which implies self-similarity of the clusters, in line with results in Refs. [20,53]. This slope was found to be $\sim -5/3$ in the experimental study of Ref. [28].

The results in Fig. 11 reveal a much weaker dependence on St , Re_λ , and Fr than the PDF of the Voronoi volumes considered earlier. Figure 12 shows the same results in a log-lin plot to emphasize the behavior for larger clusters, and again we find a weak dependence on St , Re_λ , and Fr . One possible reason for the weak dependence is related to the fact that given that the average interparticle distance is 7.9η , the clusters are quite large and so the effects of inertia are weaker than they would be for smaller clusters. Indeed, the study of Ref. [30] had a much smaller average interparticle distance of $\approx 2.2\eta$, and in their results the strongest effects of the particle inertia were found for the smallest clusters, as expected.

To obtain clearer quantitative insight, the standard deviation (s.d.) of the coherent cluster volumes is shown in Fig. 13. The results reveal a quite weak dependence on St , which was also reported in the experimental study of Ref. [28]. This is in striking contrast to the results for the s.d. of the Voronoi volumes shown in Fig. 2, for which a very strong dependence on St was observed. However, the effect of Fr and Re_λ on the s.d. of the coherent cluster volumes is much stronger. In particular, the s.d. increases strongly with increasing Re_λ , while it increases only slightly in going from $Fr = \infty$ to 0.3 , and then increases substantially in going from $Fr = 0.3$ to 0.052 .

In Fig. 14, the average percentage of particles within clusters compared with the number in the whole domain is shown. In agreement with Ref. [30], we observe that this percentage increases with increasing St , though for lower (or $Re_\lambda = 90$) Re_λ this reaches a maximum at $St \approx 2$ and then reduces. We expect that a maximum would also be observed for the higher Re_λ , but at larger St (a maximum must occur since in the limits $St \rightarrow 0$ and $St \rightarrow \infty$ the percentage must go to zero). The results show that, generally, increasing Re_λ leads to an increase of the percentage, while the dependence on Fr is nonmonotonic, with the percentage generally being largest for the intermediate value $Fr = 0.3$. It is important to note, however, that the percentages we observe are much larger than those observed by Ref. [30]. This difference is likely due to the fact that in their simulations the average interparticle distance was 2.2η , while in ours it has the significantly larger value 7.9η . If this is indeed the explanation, then it shows that the significance of the particle clusters, in terms of the number of particles contained within them as a fraction of the total number in the flow, depends essentially on the particle loading in the flow.

Figure 15 shows results for the average cluster size as a function of the number of particles in the clusters, which provides information on the relationship between the coherent cluster sizes and the particle concentrations within them. (For brevity, we only show the results for $St = 1$, but the behavior is the same for the other St cases.) This relationship is found to be approximately linear, and is independent of all the control parameters over the range we have explored. Such a linear relationship, and its lack of sensitivity to St and Fr , were already reported in Ref. [30] for $Re_\lambda = 65$, however, our results confirm that this quantity is also independent of Re_λ .

Finally, we turn to consider the velocity statistics of the particles in coherent clusters as compared with those in the entire domain. In Fig. 16 we show results for the difference in the average velocity of particles in clusters to that based on all particles in the flow. In agreement with those of Ref. [30], our results show that the mean settling velocity of particles in clusters can be significantly larger than

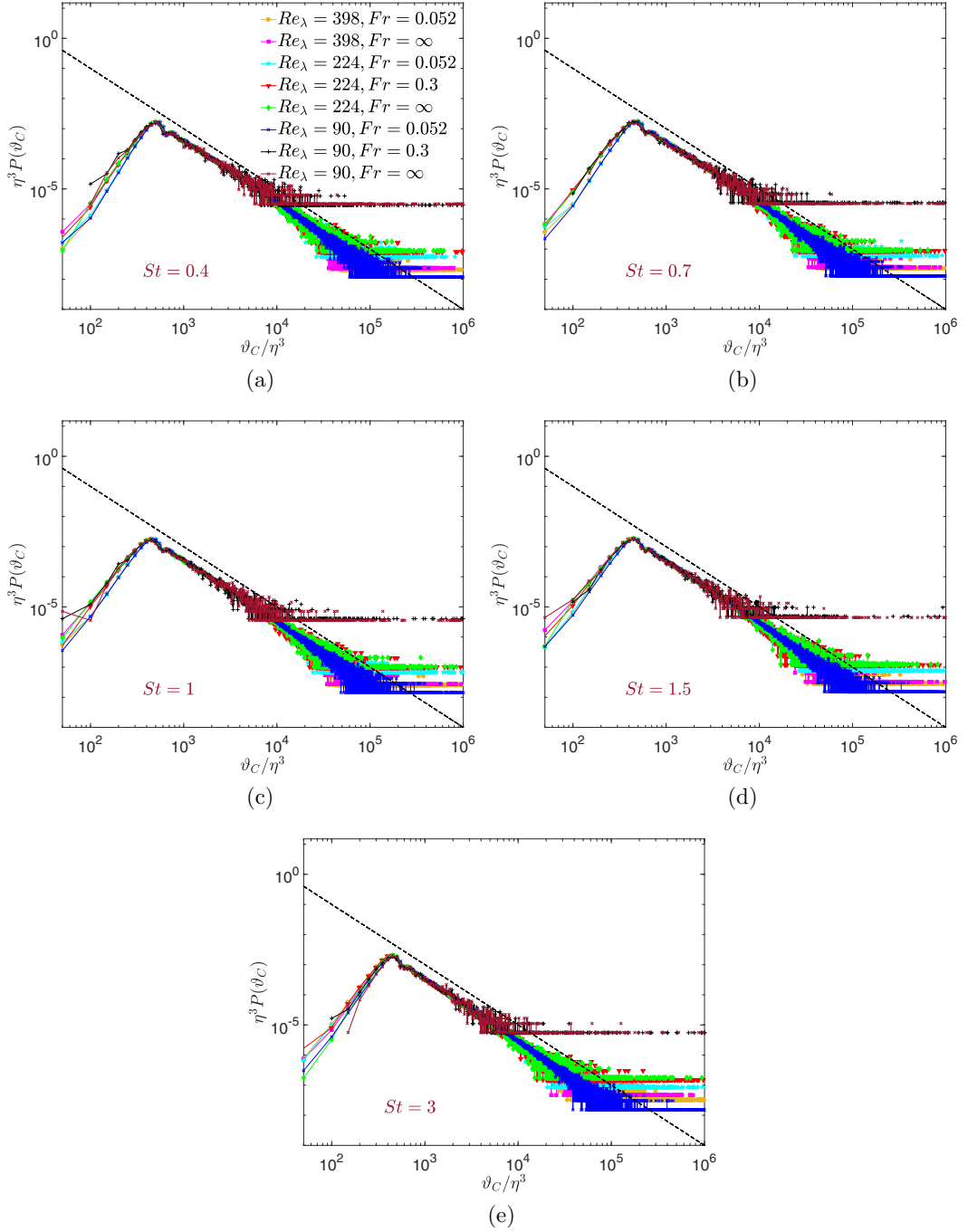


FIG. 11. PDF of the cluster volumes (normalized by the Kolmogorov scale) at different cases of Fr and Re_λ combinations for (a) $St = 0.4$, (b) $St = 0.7$, (c) $St = 1$, (d) $St = 1.5$, and (e) $St = 3$. Different colors represent different cases. The dashed line represents a power law with exponent -2 .

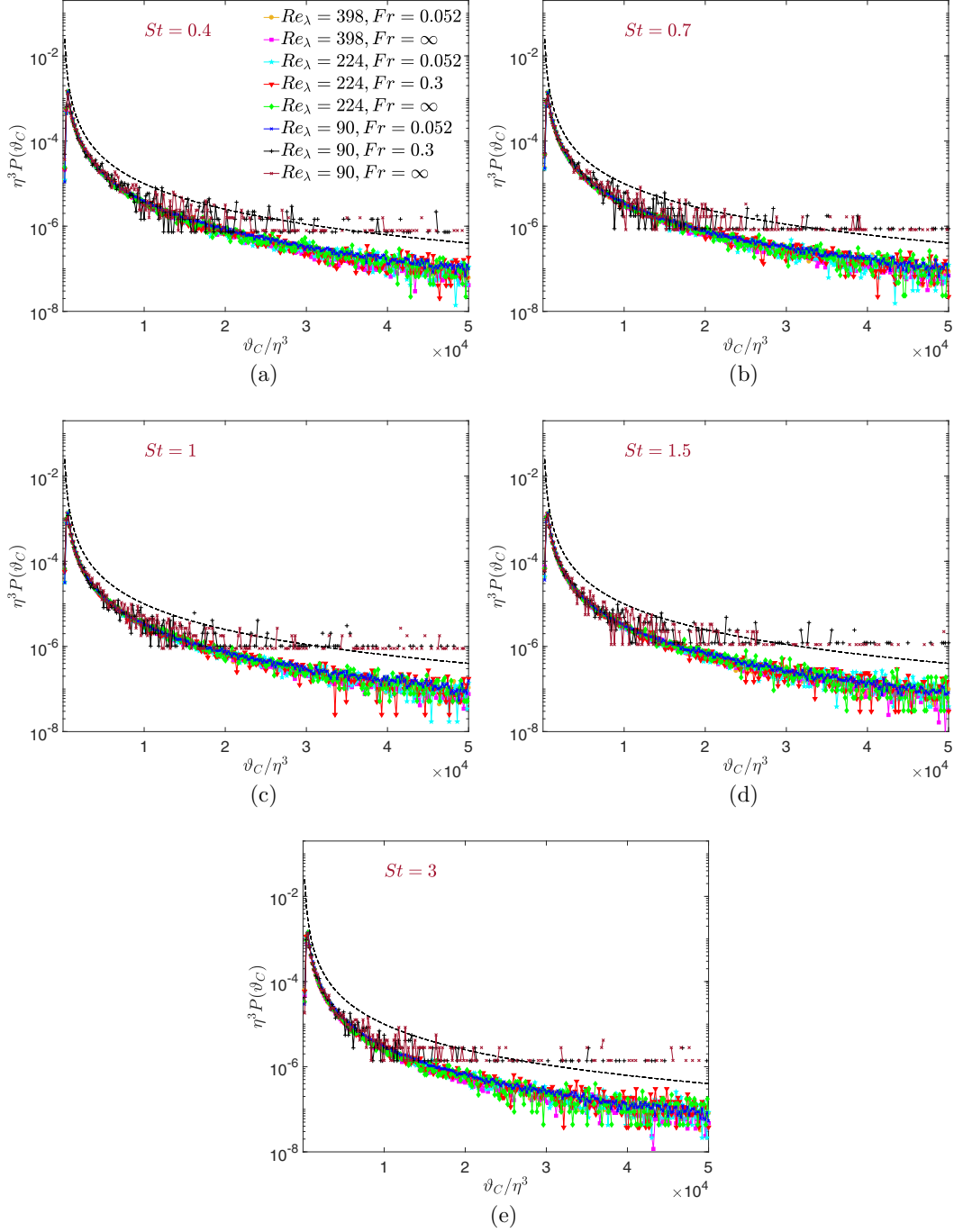
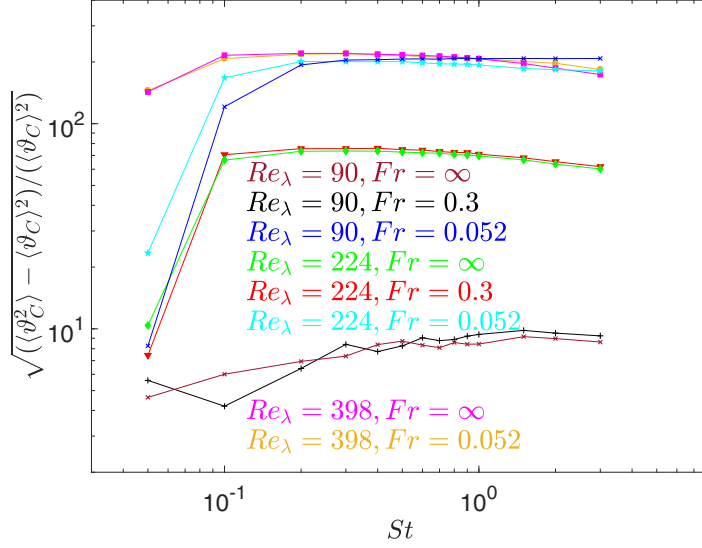
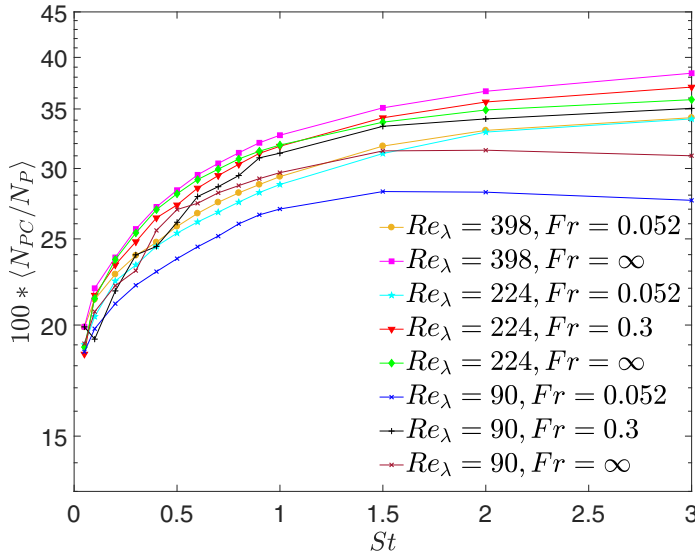


FIG. 12. PDF of the coherent cluster volumes (normalized by the Kolmogorov scale) at different cases of Fr and Re_λ combinations for (a) $St = 0.4$, (b) $St = 0.7$, (c) $St = 1$, (d) $St = 1.5$, and (e) $St = 3$. Different colors represent different cases. The dashed line represents a power law with exponent -2 .


 FIG. 13. Standard deviation of Voronoi cells of clusters as a function of St .

that based upon averaging over all particles in the flow. Our results also show that this difference increases with increasing Re_λ , with a nonmonotonic dependence on Fr (since the results are zero for $Fr = \infty$). The average settling velocity of all particles is known to increase with increasing Re_λ , which is principally due to the enhanced range of scales for the particles to interact with as Re_λ grows [35]. However, the Re_λ dependence of the results in Fig. 16 would seem to be due to something more subtle to this, since all particles in the flow, whether or not clustered, would be subject to the enhanced range of scales available for the particles to interact with as Re_λ grows.


 FIG. 14. Average percentage of total particles in clusters to the total number of particles in the whole domain, as a function of St and for different cases.

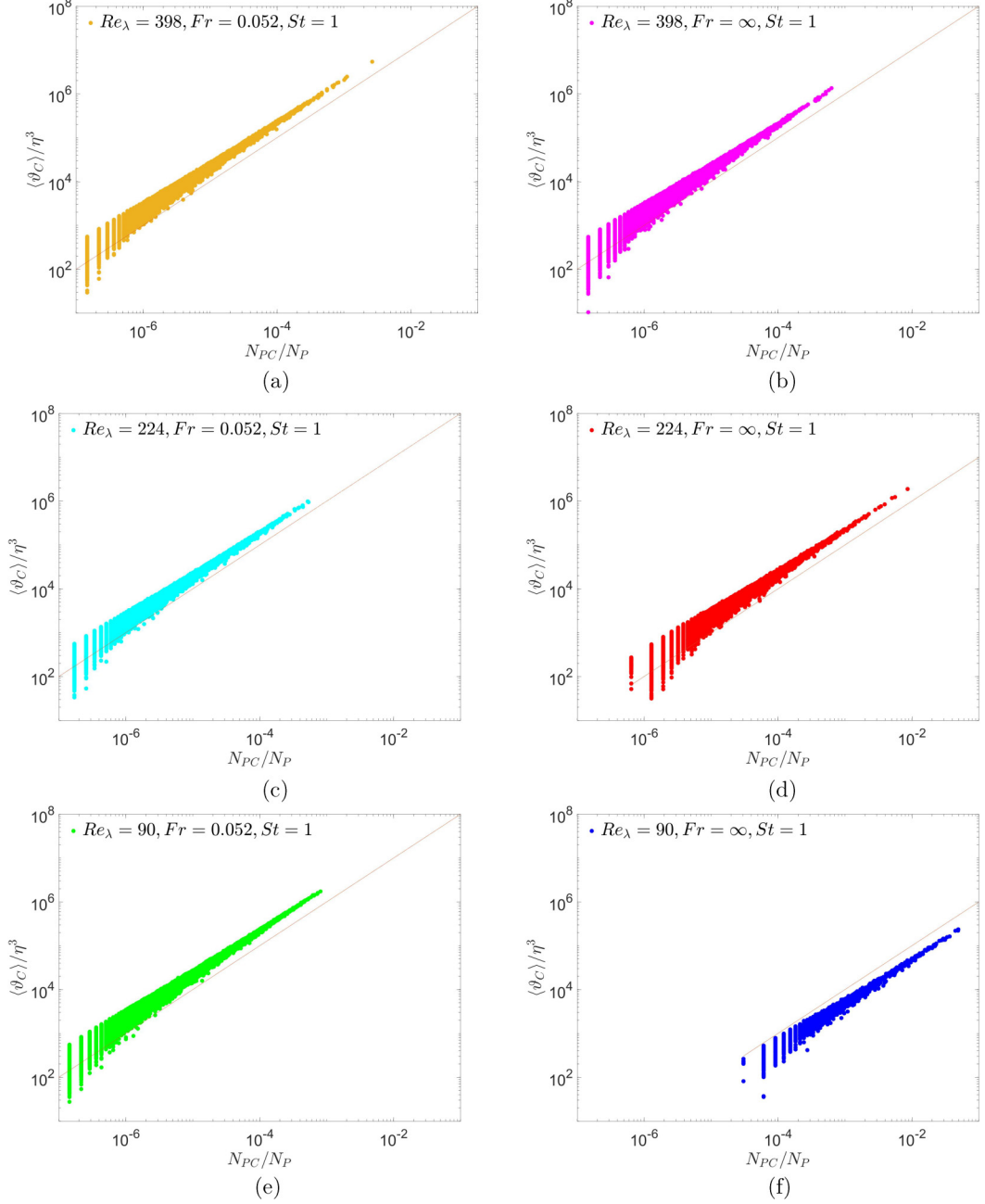


FIG. 15. The cluster volume normalized by the Kolmogorov scale vs particle counts in clusters normalized by the whole particles in the domain, for $St = 1$.

The results for the particle fluctuating kinetic energy $\mathcal{K}(St) \equiv (1/2)\langle \|\mathbf{v}^p(t) - \langle \mathbf{v}^p(t) \rangle\|^2 \rangle$ are shown in Fig. 17. The results show that while the settling velocity of particles in coherent clusters is significantly different from that of all the particles in the flow, there is only a small difference for their kinetic energy. In the absence of gravity, the particles in coherent clusters have slightly less kinetic energy compared with the average based on all particles in the flow. The most likely

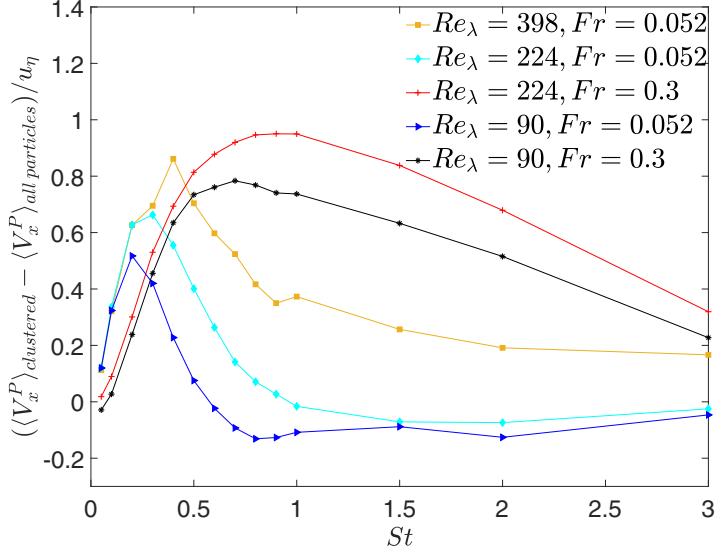


FIG. 16. Difference between the average velocity of particles within the clusters and the whole particles in the domain.

explanation for this is that the difference is caused by the preferential sampling of the turbulent flow, which is known to be suppressed in the presence of gravity [23]. As explained in Ref. [17], the decrease of the kinetic energy with increasing St is due to the inertial particles filtering out fluctuations in the turbulent flow, while the increase of the kinetic energy with increasing Re_λ (for a fixed St and Fr) is due to a reduction of the filtering effect owing to the increased timescale separation in the turbulent flow as Re_λ is increased. Finally, the decrease of the kinetic energy as Fr

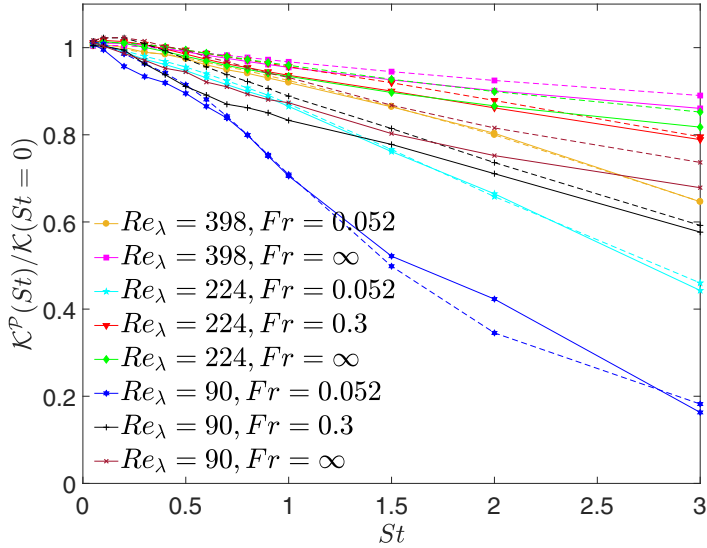


FIG. 17. Average kinetic energy of particles within the clusters (solid lines) and the whole particles in the domain (dashed lines).

is decreased (for a fixed St and Re_λ) is because the filtering effect is enhanced due to the timescale of the fluid velocity seen by the particle becoming shorter as the particles settle faster.

V. CONCLUSIONS

In this paper, we have used DNS and a three-dimensional Voronoi analysis to explore the local distribution, settling velocity, and acceleration of inertial particles (in the point particle limit) in statistically stationary, isotropic turbulence. In these simulations, we independently varied the Taylor Reynolds number $Re_\lambda \in [90, 398]$, Froude number $Fr \equiv a_\eta/g \in [0.052, \infty]$ (where a_η is the Kolmogorov acceleration, and g is the acceleration due to gravity), and Kolmogorov scale Stokes number $St \equiv \tau_p/\tau_\eta \in [0, 3]$. Independently varying these parameters was not possible in previous experimental investigations on the problem. The average interparticle distance was held fixed in all the DNS for consistency when comparing the Voronoi analysis results for different Re_λ .

In agreement with previous results using the radial distribution function (RDF) to quantify particle clustering [17,23], which is a global measure, we find that for small Voronoi volumes (corresponding to the most clustered particles), the behavior is strongly dependent upon St and Fr , but only weakly dependent upon Re_λ , unless $St > 1$. However, larger Voronoi volumes (void regions) exhibit a much stronger dependence on Re_λ , even when $St \leq 1$, and we show that this, rather than the behavior at small volumes, is the cause of the sensitivity of the standard deviation of the Voronoi volumes that has been previously reported. Our results also show that the standard deviation of the Voronoi volumes is dominated by the behavior of the void regions, rather than the clustered regions. As a result, the results show that the standard deviation of the Voronoi volumes is reduced by gravity at all St , even though the probability density function (PDF) results show that gravity can enhance the small-scale clustering, as was also observed using the RDF analysis in Ref. [23]. This is because gravity seems to always suppress the void regions. This highlights that using the standard deviation of Voronoi volumes as a measure of particle clustering (which has been done in many previous studies) can be somewhat misleading, since its properties are dominated by void, rather than small-scale clustered, regions of the flow. We also show that the PDF of Voronoi volumes exhibits a quasilognormal behavior over a certain range of volumes, as previously reported. However, we show that this is only approximate, and does not describe well the Voronoi volumes that are sufficiently large or small. The validity of the lognormal assumption also depends in a rather complex way on Re_λ and Fr .

We find that the average settling velocities conditioned on the Voronoi volumes exhibit a non-monotonic dependence on the Voronoi volumes, with the largest contribution to the particle settling velocities being associated with increasingly larger Voronoi volumes as the settling parameter $Sv \equiv St/Fr$ is increased. This nonmonotonic behavior does not seem to have been previously reported, likely because the Sv considered were not sufficiently large. It can be explained by the recent work of Ref. [35] who show theoretically and numerically that as Sv is increased, the scales responsible for the particle settling velocities shift to larger scales.

Even though the globally averaged particle acceleration is zero in the system we are considering, our local analysis of the acceleration statistics of settling inertial particles shows that clustered particles experience a net acceleration in the direction of gravity, while particles in void regions experience the opposite. In the direction normal to gravity, and in the absence of gravity, the average particle accelerations are independent of the Voronoi volumes. The particle acceleration variance, however, is a convex function of the Voronoi volumes, with or without gravity, which seems to indicate a nontrivial relationship between the Voronoi volumes and the sizes of the turbulent flow scales. Results for the variance of the fluid acceleration at the inertial particle positions are of the order of the square of the Kolmogorov acceleration and depend only weakly on Voronoi volumes. These results call into question the “sweep-stick” mechanism for particle clustering in turbulence which would lead one to expect that clustered particles reside in regions where the fluid acceleration is zero [52].

We then consider the properties of particles in clusters, which are regions of connected Voronoi cells whose volume is less than a certain threshold. The results for the PDFs of the cluster volumes reveal significant self-similarity of the clusters. The standard deviation of the cluster volumes provides further insight and shows that the statistics of the cluster volumes depends only weakly on St , with a stronger dependence on Fr and Re_λ . The weak dependence upon St may, however, be due to the fact that given the average interparticle distance in our simulations, the cluster sizes are comparable to scales of the flow where one might expect the effects of particle inertia to be weak anyway. Finally, we compared the average settling velocities of all particles in the flow with those in clusters, and showed that those in the clusters settle much faster, in agreement with previous work. However, we also find that this difference grows significantly with increasing Re_λ and exhibits a nonmonotonic dependence on Fr . The kinetic energy of the particles, on the other hand, is almost the same for particles in and not in the clusters, especially in the presence of gravity.

ACKNOWLEDGMENTS

This work used the Extreme Science and Engineering Discovery Environment (XSEDE), which is supported by National Science Foundation Grant No. ACI-1548562 [54]. Specifically, the Comet cluster was used under allocation CTS170009. The Comet cluster and the Duke Computing Cluster (DCC) were used extensively for the computations in this study. The authors would like to express their sincere thanks and gratitude to Tom Milledge, Mahidhar Tatineni, and Roberto E. Gonzalez for their generous help with the computational challenges of this work. We also would like to thank Yuanqing Liu for all his input during the discussion on the coding issues.

APPENDIX

As discussed earlier, computational resources limit the values of the average interparticle distance $\bar{r}$ that we are able to explore in this study. Essentially, the value of $\bar{r}$ determines the smallest scales at which we are able to probe the particle motion in the flow. In this Appendix, we provide some preliminary results to consider how the choice of $\bar{r}$ might affect the results of the analysis. We performed new simulations (Table III) to study the impact of $\bar{r}$, where we increased the number of particles such that $\bar{r} \equiv \mathcal{L}/N_p^{1/3} = 1.8\eta$, almost four times smaller than that used in the results in the main body of this paper. The DNS was for $Re_\lambda = 90$, and for a range of St , Fr . This allows us to

TABLE III. Simulation parameters for new DNS simulations to study the impact of the average interparticle distance (arbitrary units).

Parameter	DNS 1	DNS 2	New DNS 1	New DNS 2
N	128	128	128	128
Re_λ	93	94	90.1	90.1
Fr	∞	0.3	∞	0.3
$\mathcal{L}$	2π	2π	2π	2π
ν	0.005	0.005	0.005	0.005
ϵ	0.324	0.332	0.3144	0.3142
l	1.48	1.49	1.45	1.45
l/η	59.6	60.4	55.6	57.87
u'	0.984	0.996	0.960	0.960
u'/u_η	4.91	4.92	4.82	4.82
T_L	1.51	1.50	1.51	1.52
T_L/τ_η	12.14	12.24	11.99	11.99
$\kappa_{\max}\eta$	1.5	1.48	1.52	1.52
N_p	262144	262144	2621440	2621440

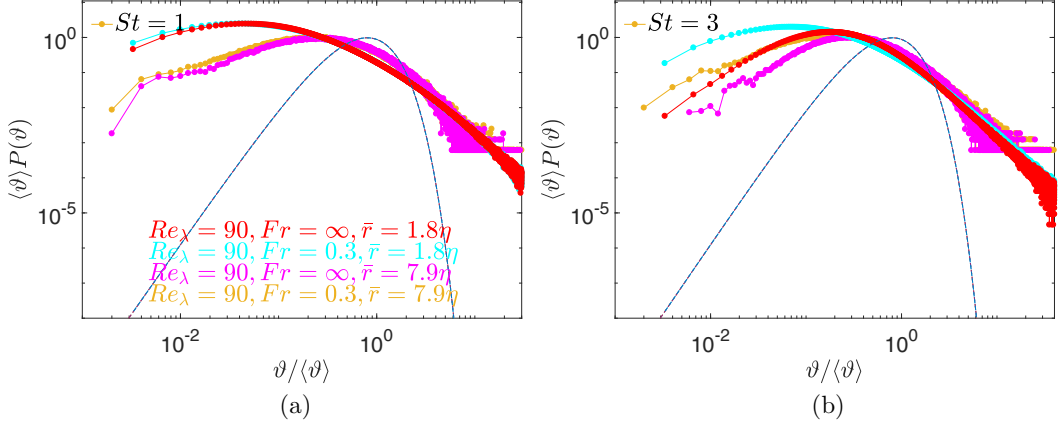


FIG. 18. PDF of the Voronoi volumes (normalized by the mean volume) at different cases of Fr and Re_λ combinations for (a) $St = 1$ and (b) $St = 3$. Different colors represent different cases. The dashed line represents the random Poisson distribution.

explore how the St , Fr dependencies of the results might be affected by $\bar{r}$. Unfortunately, due to the enormous computational expense of such simulations, we are not able to repeat this for higher Re_λ to see how the effect of Re_λ on the results might depend on $\bar{r}$. The results shown below demonstrate that the behavior for $\bar{r} = 1.8\eta$ and $\bar{r} = 7.9\eta$ is qualitatively the same. The quantitative differences are due to the well-known sensitivity of the Voronoi analysis to the number of particles in the domain (see Ref. [41]). Here, we briefly explain the results of these simulations corresponding to each section in the main body of the paper.

1. Voronoi volume distributions

According to Fig. 18, decreasing $\bar{r}$ does not change the trend of the PDF of Voronoi cell volumes qualitatively and still we observe that decreasing Fr enhances the probability of finding highly concentrated regions, especially for $St > 1$. Due to the formation of smaller Voronoi volumes with an increasing number of particles in domain, we obtain a well-converged statistics at the left tail of PDF which confirms the trend seen where $\bar{r} = 7.9\eta$.

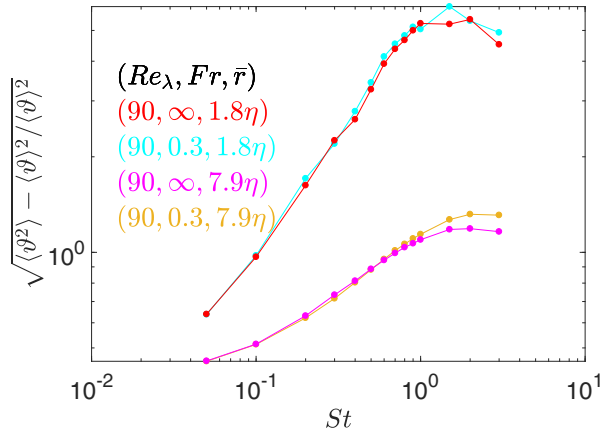


FIG. 19. Standard deviation of Voronoi cells normalized by their mean as a function of St .

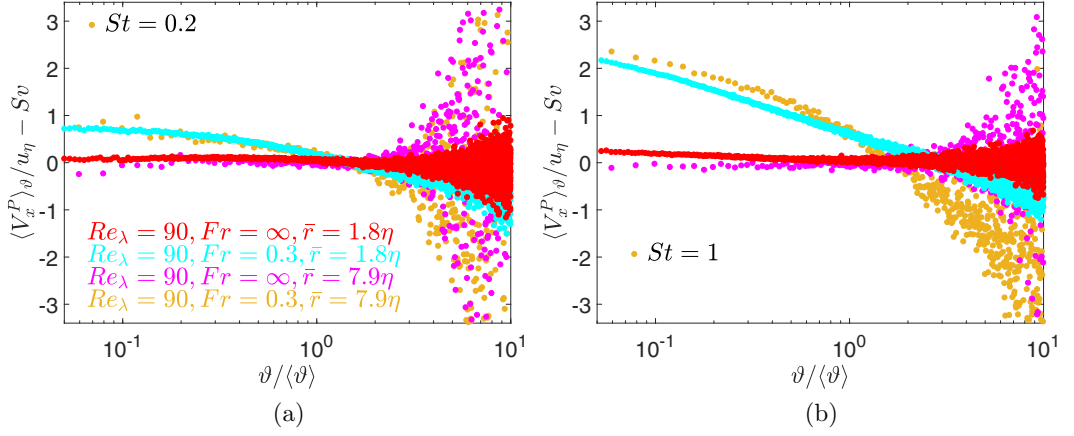


FIG. 20. Average velocity of particles conditioned on the PDF of the Voronoi volumes at different cases of Fr and Re_λ combinations for (a) $St = 0.2$ and (b) $St = 1$. Different colors represent different cases.

The standard deviation of the Voronoi volumes, shown in Fig. 19, also exhibits almost the same qualitative pattern, except the effect of Fr at $St \geq 1.5$ which is not so clear as the case with $\bar{r} = 7.9/\eta$. As reported in Ref. [41], increasing the subsampling ratio decreases this quantity (here we can consider the cases with $\bar{r} = 7.9\eta$ as a subsample of cases with $\bar{r} = 1.8\eta$).

2. Particle settling velocities

Looking at the average velocity of particles conditioned on the PDF of the Voronoi volumes, shown in Fig. 20, we again observe the same trend but now with a better statistical convergence in the clustered regions.

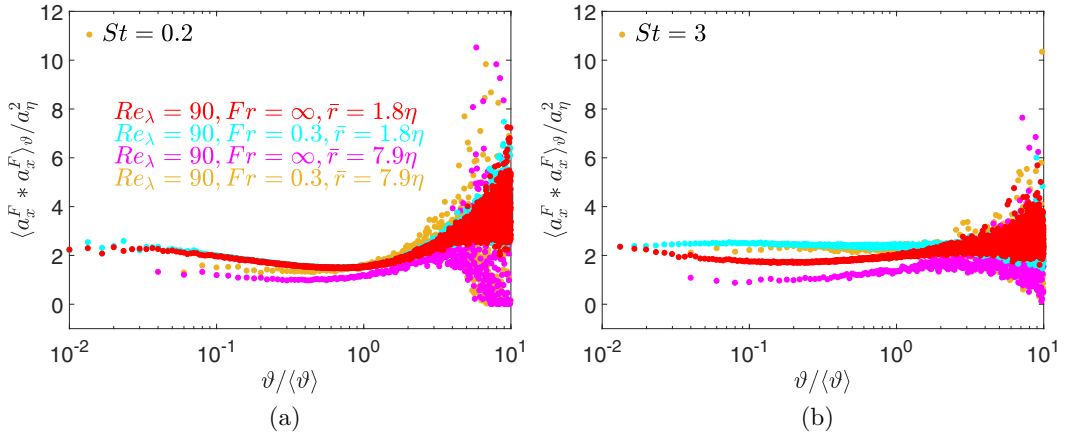


FIG. 21. Second moment of fluid accelerations at the particle position, in the gravity direction, conditioned on the PDF of the Voronoi volumes at different cases of Fr and Re_λ combinations for (a) $St = 0.2$ and (b) $St = 3$. Different colors represent different cases.

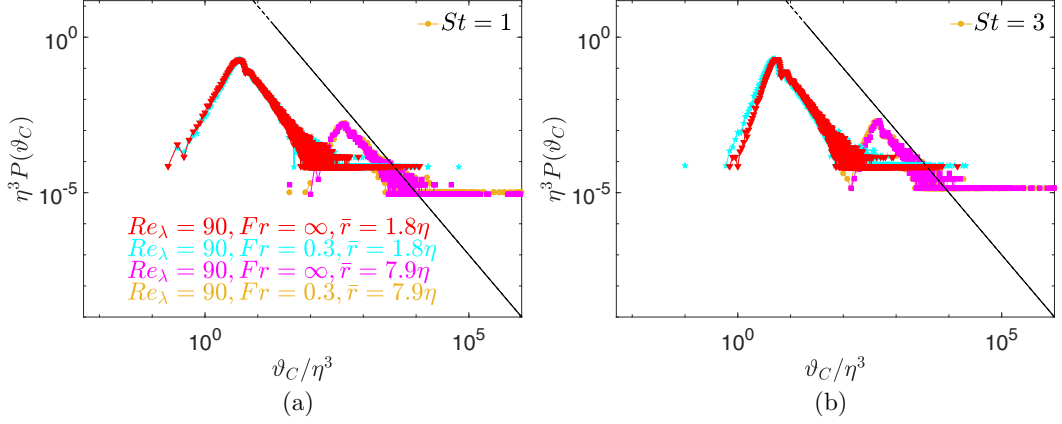


FIG. 22. PDF of the cluster volumes (normalized by the Kolmogorov scale) at different cases of Fr and Re_λ combinations for (a) $St = 1$ and (b) $St = 3$. Different colors represent different cases. The dashed line represents a power law with exponent -2 .

3. Local analysis of particle accelerations

The results of fluid acceleration at the inertial particles position conditioned on the Voronoi volumes, shown in Fig. 21, still call into question the validity of the sweep-stick mechanism and show a lack of a systematic relationship between the fluid accelerations and Voronoi volumes.

4. Cluster analysis

Looking at the PDF of cluster volumes in Fig. 22, we observe that smaller $\bar{r}$ leads to the formation of smaller clusters which still display the self-similarity behavior, a broad range of scales, and weak sensitivity to the control parameters. More importantly, our results for the standard deviation of cluster volumes shown in Fig. 23 confirm the weak sensitivity of this quantity to St and the quantitative difference can be explained by the same argument we brought for the standard deviation of Voronoi volumes (subsampling).

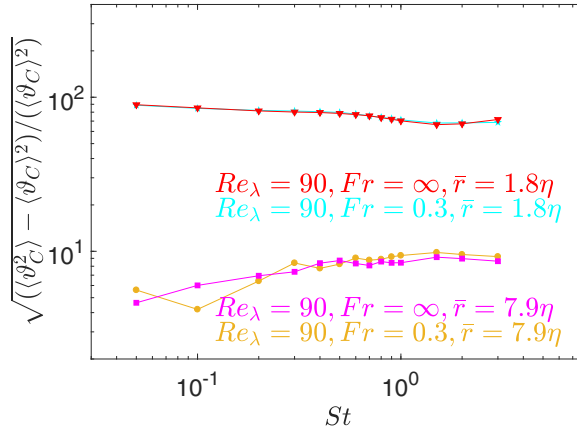


FIG. 23. Standard deviation of Voronoi cells of clusters as a function of St .

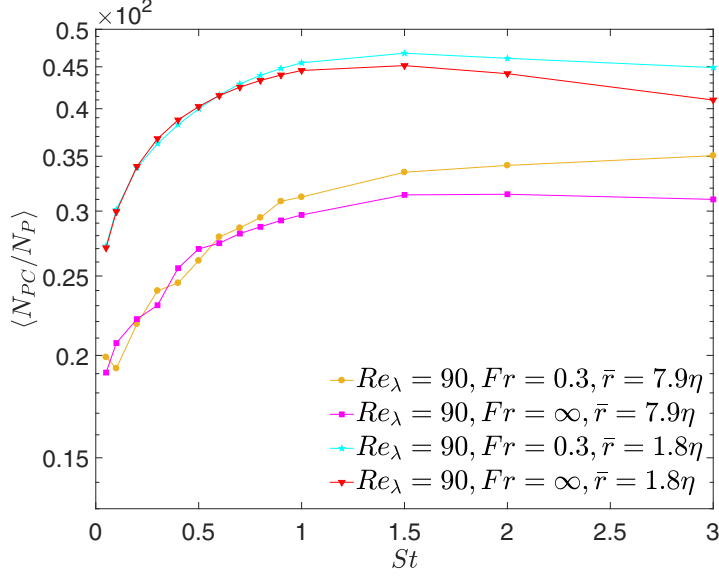


FIG. 24. Average percentage of total particles in clusters to the total number of particles in the whole domain, as a function of St and for different cases

Regarding the average percentage of particles that form clusters among all the particles in the domain, our results, presented in Fig. 24, show the same qualitative trend as seen in the larger $\bar{r}$ but a larger percentage of total particles in clusters due to the presence of a larger number of particles in the domain.

The results of the average settling velocity of particles in cluster shown in Fig. 25 again confirms that the particles settle faster in clusters and shows the same qualitative behavior with increasing St . The results of the kinetic energy of particles in clusters, shown in Fig. 26, also indicate the same sensitivity to Fr and St .

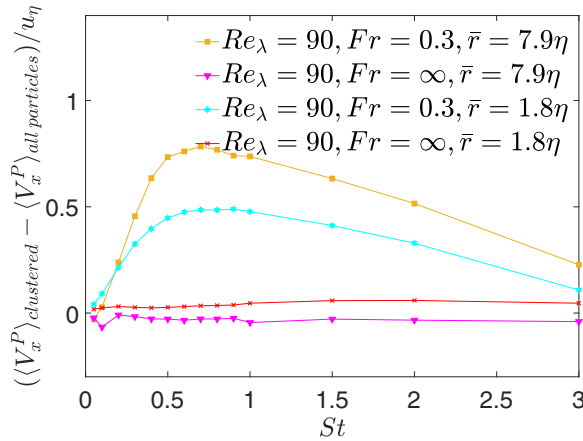


FIG. 25. Difference between the average velocity of particles within the clusters and the whole particles in the domain.

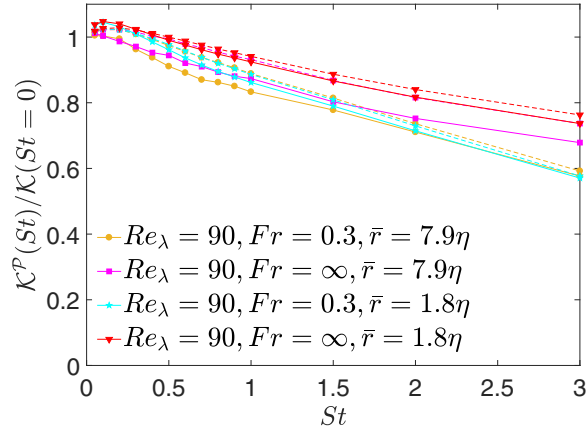


FIG. 26. Average kinetic energy of particles within the clusters (solid lines) and the whole particles in the domain (dashed lines).

- [1] E. A. D'Asaro, Turbulence in the upper-ocean mixed layer, *Annu. Rev. Marine Sci.* **6**, 101 (2014).
- [2] K. V. Beard and H. T. Ochs III, Warm-rain initiation: An overview of microphysical mechanisms, *J. Appl. Meteorol.* **32**, 608 (1993).
- [3] T. E. Ongaro, C. Cavazzoni, G. Erbacci, A. Neri, and M.-V. Salvetti, A parallel multiphase flow code for the 3D simulation of explosive volcanic eruptions, *Parallel Comput.* **33**, 541 (2007).
- [4] M. R. Maxey, The gravitational settling of aerosol particles in homogeneous turbulence and random flow fields, *J. Fluid Mech.* **174**, 441 (1987).
- [5] V. Bianco, O. Manca, and S. Nardini, Numerical investigation on nanofluids turbulent convection heat transfer inside a circular tube, *Int. J. Therm. Sci.* **50**, 341 (2011).
- [6] M. R. Momenifar, M. A. Akhavan-Behabadi, M. Nasr, and P. Hanafizadeh, Effect of lubricating oil on flow boiling characteristics of R-600a/oil inside a horizontal smooth tube, *Appl. Therm. Eng.* **91**, 62 (2015).
- [7] K. K. Kuo and R. Acharya, *Fundamentals of Turbulent and Multiphase Combustion* (Wiley, Hoboken, NJ, 2012).
- [8] P. Hanafizadeh, M. Momenifar, A. N. Geimassi, and S. Ghanbarzadeh, Void fraction and wake analysis of a gas-liquid two-phase cross-flow, *Multiphase Sci. Technol.* **26**, 261 (2014).
- [9] M. H. Arabnejad, A. Amini, M. Farhat, and R. E. Bensow, Numerical and experimental investigation of shedding mechanisms from leading-edge cavitation, *Int. J. Multiphase Flow* **119**, 123 (2019).
- [10] S. Vahidifar, M. R. Saffarian, and E. Hajidavalloo, Introducing the theory of successful settling in order to evaluate and optimize the sedimentation tanks, *Meccanica* **53**, 3477 (2018).
- [11] P. Hanafizadeh, M. R. Momenifar, M. Bastankhah, and M. H. Saidi, Thermosyphon analysis using the particle image velocimetry method (PIV), *Mech. Eng. Sharif* **32-3**, 11 (2016).
- [12] H. T. Bi, N. Ellis, I. A. Abba, and J. R. Grace, A state-of-the-art review of gas-solid turbulent fluidization, *Chem. Eng. Sci.* **55**, 4789 (2000).
- [13] J. Bec, L. Biferale, M. Cencini, A. S. Lanotte, S. Musacchio, and F. Toschi, Heavy Particle Concentration in Turbulence at Dissipative and Inertial Scales, *Phys. Rev. Lett.* **98**, 084502 (2007).
- [14] S. Sundaram and L. R. Collins, Collision statistics in an isotropic, particle-laden turbulent suspension I. Direct numerical simulations, *J. Fluid Mech.* **335**, 75 (1997).
- [15] F. Toschi and E. Bodenschatz, Lagrangian properties of particles in turbulence, *Annu. Rev. Fluid Mech.* **41**, 375 (2009).
- [16] R. A. Shaw, Particle-turbulence interactions in atmospheric clouds, *Annu. Rev. Fluid Mech.* **35**, 183 (2003).

- [17] P. J. Ireland, A. D. Bragg, and L. R. Collins, The effect of Reynolds number on inertial particle dynamics in isotropic turbulence. Part 1. Simulations without gravitational effects, *J. Fluid Mech.* **796**, 617 (2016).
- [18] S. Balachandar and J. K. Eaton, Turbulent dispersed multiphase flow, *Annu. Rev. Fluid Mech.* **42**, 111 (2010).
- [19] A. Aliseda, A. Cartellier, F. Hainaux, and J. C. Lasheras, Effect of preferential concentration on the settling velocity of heavy particles in homogeneous isotropic turbulence, *J. Fluid Mech.* **468**, 77 (2002).
- [20] R. Monchaux, M. Bourgoïn, and A. Cartellier, Preferential concentration of heavy particles: a Voronoï analysis, *Phys. Fluids* **22**, 103304 (2010).
- [21] J. Bec, L. Biferale, G. Boffetta, M. Cencini, S. Musacchio, and F. Toschi, Lyapunov exponents of heavy particles in turbulence, *Phys. Fluids* **18**, 091702 (2006).
- [22] E. Calzavarini, M. Kerscher, D. Lohse, and F. Toschi, Dimensionality and morphology of particle and bubble clusters in turbulent flow, *J. Fluid Mech.* **607**, 13 (2008).
- [23] P. J. Ireland, A. D. Bragg, and L. R. Collins, The effect of Reynolds number on inertial particle dynamics in isotropic turbulence. Part 2. Simulations with gravitational effects, *J. Fluid Mech.* **796**, 659 (2016).
- [24] Y. Tagawa, J. M. Mercado, V. N. Prakash, E. Calzavarini, C. Sun, and D. Lohse, Three-dimensional lagrangian Voronoï analysis for clustering of particles and bubbles in turbulence, *J. Fluid Mech.* **693**, 201 (2012).
- [25] A. J. Petersen, L. Baker, and F. Coletti, Experimental study of inertial particles clustering and settling in homogeneous turbulence, *J. Fluid Mech.* **864**, 925 (2019).
- [26] M. Momenifar, R. Dhariwal, and A. D. Bragg, Influence of Reynolds number on the motion of settling, bidisperse inertial particles in turbulence, *Phys. Rev. Fluids* **4**, 054301 (2019).
- [27] M. Oblgado, T. Teitelbaum, A. Cartellier, P. Mininni, and M. Bourgoïn, Preferential concentration of heavy particles in turbulence, *J. Turbulence* **15**, 293 (2014).
- [28] S. Sumbekova, A. Cartellier, A. Aliseda, and M. Bourgoïn, Preferential concentration of inertial sub-Kolmogorov particles: The roles of mass loading of particles, Stokes numbers, and Reynolds numbers, *Phys. Rev. Fluids* **2**, 024302 (2017).
- [29] G. L. Holtzer and L. R. Collins, Relationship between the intrinsic radial distribution function for an isotropic field of particles and lower-dimensional measurements, *J. Fluid Mech.* **459**, 93 (2002).
- [30] L. Baker, A. Frankel, A. Mani, and F. Coletti, Coherent clusters of inertial particles in homogeneous turbulence, *J. Fluid Mech.* **833**, 364 (2017).
- [31] A. Dejoan and R. Monchaux, Preferential concentration and settling of heavy particles in homogeneous turbulence, *Phys. Fluids* **25**, 013301 (2013).
- [32] R. Monchaux and A. Dejoan, Settling velocity and preferential concentration of heavy particles under two-way coupling effects in homogeneous turbulence, *Phys. Rev. Fluids* **2**, 104302 (2017).
- [33] T. Bosse, L. Kleiser, and E. Meiburg, Small particles in homogeneous turbulence: Settling velocity enhancement by two-way coupling, *Phys. Fluids* **18**, 027102 (2006).
- [34] A. Dejoan, DNS experiments on the settling of heavy particles in homogeneous turbulence: Two-way coupling and Reynolds number effects, *J. Phys.: Conf. Ser.* **333**, 012006 (2011).
- [35] J. Tom and A. D. Bragg, Multiscale preferential sweeping of particles settling in turbulence, *J. Fluid Mech.* **871**, 244 (2019).
- [36] H. Parishani, O. Ayala, B. Rosa, L.-P. Wang, and W. W. Grabowski, Effects of gravity on the acceleration and pair statistics of inertial particles in homogeneous isotropic turbulence, *Phys. Fluids* **27**, 033304 (2015).
- [37] R. Dhariwal and A. D. Bragg, Small-scale dynamics of settling, bidisperse particles in turbulence, *J. Fluid Mech.* **839**, 594 (2018).
- [38] R. Dhariwal and A. D. Bragg, Enhanced and suppressed multiscale dispersion of bidisperse inertial particles due to gravity, *Phys. Rev. Fluids* **4**, 034302 (2019).
- [39] M. R. Maxey and J. J. Riley, Equation of motion for a small rigid sphere in a nonuniform flow, *Phys. Fluids* **26**, 883 (1983).
- [40] P. J. Ireland, T. Vaithianathan, P. S. Sukheswalla, B. Ray, and L. R. Collins, Highly parallel particle-laden flow solver for turbulence research, *Comput. Fluids* **76**, 170 (2013).

- [41] R. Monchaux, Measuring concentration with Voronoi diagrams: The study of possible biases, [New J. Phys.](#) **14**, 095013 (2012).
- [42] J.-S. Ferenc and Z. Néda, On the size distribution of poisson Voronoi cells, [Physica A](#) **385**, 518 (2007).
- [43] M. Obligado, M. Missaoui, R. Monchaux, A. Cartellier, and M. Bourgoïn, Reynolds number influence on preferential concentration of heavy particles in turbulent flows, [J. Phys.: Conf. Ser.](#) **318**, 052015 (2011).
- [44] A. D. Bragg, P. J. Ireland, and L. R. Collins, On the relationship between the non-local clustering mechanism and preferential concentration, [J. Fluid Mech.](#) **780**, 327 (2015).
- [45] U. Frisch, *Turbulence: The Legacy of A. N. Kolmogorov* (Cambridge University Press, Cambridge, UK, 1995).
- [46] A. D. Bragg and L. R. Collins, New insights from comparing statistical theories for inertial particles in turbulence: I. Spatial distribution of particles, [New J. Phys.](#) **16**, 055013 (2014).
- [47] J. Bec, L. Biferale, G. Boffetta, A. Celani, M. Cencini, A. S. Lanotte, S. Musacchio, and F. Toschi, Acceleration statistics of heavy particles in turbulence, [J. Fluid Mech.](#) **550**, 349 (2006).
- [48] M. Momenifar and A. D. Bragg, The influence of Reynolds and Froude number on the local distribution of settling, inertial particles in turbulence, in *AIAA Scitech 2020 Forum* (AIAA Scitech, Orlando, FL 2020), p. 1571.
- [49] A. D. Bragg, P. J. Ireland, and L. R. Collins, Mechanisms for the clustering of inertial particles in the inertial range of isotropic turbulence, [Phys. Rev. E](#) **92**, 023029 (2015).
- [50] A. Frankel, H. Pouransari, F. Coletti, and A. Mani, Settling of heated particles in homogeneous turbulence, [J. Fluid Mech.](#) **792**, 869 (2016).
- [51] L. P. Wang and M. R. Maxey, Settling velocity and concentration distribution of heavy particles in homogeneous isotropic turbulence, [J. Fluid Mech.](#) **256**, 27 (1993).
- [52] S. W. Coleman and J. C. Vassilicos, A unified sweep-stick mechanism to explain particle clustering in two- and three-dimensional homogeneous, isotropic turbulence, [Phys. Fluids](#) **21**, 113301 (2009).
- [53] H. Yoshimoto and S. Goto, Self-similar clustering of inertial particles in homogeneous turbulence, [J. Fluid Mech.](#) **577**, 275 (2007).
- [54] J. Towns, T. Cockerill, M. Dahan, I. Foster, K. Gaither, A. Grimshaw, V. Hazlewood, S. Lathrop, D. Lifka, G. D. Peterson, R. Roskies, J. R. Scott, and N. Wilkins-Diehr, XSEDE: Accelerating scientific discovery, [Comput. Sci. Eng.](#) **16**, 62 (2014).