

Predictive model for convective flows induced by surface reactivity contrast

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Concentration gradients in a fluid adjacent to a reactive surface due to contrast in surface reactivity generate convective flows. These flows result from contributions by electro- and diffusio-osmotic phenomena. In this study, we have analyzed reactive patterns that release and consume protons, analogous to bimetallic catalytic conversion of peroxide. Similar systems have typically been studied using either scaling analysis to predict trends or costly numerical simulation. Here, we present a simple analytical model, bridging the gap in quantitative understanding between scaling relations and simulations, to predict the induced potentials and consequent velocities in such systems without the use of any fitting parameters. Our model is tested against direct numerical solutions to the coupled Poisson, Nernst-Planck, and Stokes equations. Predicted slip velocities from the model and simulations agree to within a factor of ≈ 2 over a multiple order-of-magnitude change in the input parameters. Our analysis can be used to predict enhancement of mass transport and the resulting impact on overall catalytic conversion, and is also applicable to predicting the speed of catalytic nanomotors.

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I. INTRODUCTION

The ability of catalytic micropumps to generate motion from chemical energy has opened many possibilities [1–3]. By generating convection at the microscale, catalytic micropumps can enhance transport rates to surfaces where diffusion would otherwise be limiting [4]. Because of their use of local chemical energy, catalytic micropumps do not suffer from some of the issues associated with other methods of generating flow at the microscale. Pressure-driven flow is inefficient because of the rapid increase in hydraulic resistance with decreasing length scale. Meanwhile, other electrokinetic methods such as conventional electro-osmotic flow and induced-charge electro-osmosis [5,6] require imposition of external electric fields or, in the case of alternating-current electro-osmosis, electrical connection of microelectrodes [7–9].

A number of studies, which we review below, have contributed to identification of mechanisms leading to electrostatic potential gradients and fluid flow in systems involving surface reactivity contrasts. However, a quantitative understanding that would directly connect the outcome of induced potentials and flow fields to the key input parameters is yet to be developed. Typically, numerical simulation has been used to make quantitative connections, but numerical simulation is a costly approach in terms of computational time and resources. Ideally, it would be desirable to have an algebraic model predicting this outcome. As we shall see, flow fields in these systems can have a nonmonotonic dependence on input and therefore it is crucial to have a model that goes beyond simple scaling relations in large and small parameter limits. Our study presents such models for

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a canonical problem, directly relating input parameters to quantities of interest without using any tuning parameters.

The literature relevant to understanding of catalytic micropumps can be placed into several categories, each of which we expand on below: first, micropumps where surface reactions induce electric fields which generate electro-osmotic flow, the primary focus of this work; second, micropumps based on diffusio-osmotic mechanisms; and finally, when the reactions occur on a particle then the same physical mechanisms will lead to self-phoretic behavior instead of pumping.

Surfaces with nonuniform catalytic reactivity generate flow by multiple mechanisms [10]. One mechanism, electro-osmosis, refers to flows generated when reaction-induced electric fields, driving current through the fluid, act on surface electric double layers screening the surface charge. This mechanism has been studied using a silver disk plated on a gold substrate immersed in a hydrogen peroxide solution [1,11] as well as for interdigitated gold and platinum electrodes [12]. Ibele *et al.* showed that different fuels can generate flow by the same mechanism by developing a system using hydrazine fuels [13]. Farniya and coworkers developed a technique for inferring chemical reaction rates and ion impurities in electrocatalytic micropumps by fitting simulations to detailed experiments measuring proton concentrations with fluorescence microscopy and tracking particles with positive and negative surface charges to measure flow velocities and electric fields [14]. In other experiments, it has been shown that separating the electrodes on opposite sides of a membrane [15] and utilizing the high surface charge of doped silicon provide promising avenues for enhancing the flow rates generated by electrocatalytic micropumps [16]. Recent simulations by Esplandiu *et al.* analyzed important parameters in self-electro-osmotic pumps and suggested that the induced electric fields are dependent on ionic strength, not total conductivity [17]. However, their model assumed that only protons contribute to the net current and ignored bulk equilibrium reactions.

A second mechanism, diffusio-osmosis, refers to flows generated by the presence of species concentration gradients tangent to the surface. When the concentration gradients are caused by nonuniform reactivity of the surface itself, the flows they generate are called self-diffusio-osmotic. Such gradients may generate flows in multiple ways depending on whether the solution is an electrolyte [18] or not [19]. In electrolyte solutions, gradients of species concentrations generate diffusio-osmotic flows due to the dependence of the pressure difference across the Debye screening layer on solution conductivity. The surface pressure is proportional to the local conductivity, so concentration gradients along the surface generate pressure gradients and hence flow. Also, for electrolytes where the positive and negative species have different diffusivities, concentration gradients cause internal electric fields which act on the Debye layer to generate flow. Diffusio-osmotic flows have been used to develop microfluidic pumps [3]. Hong *et al.* utilized the photocatalytic sensitivity of titanium dioxide to generate a diffusio-osmotic micropump controllable by light [20]. Niu *et al.* showed that even in micromolar salt concentrations, an ion-exchange resin can be used to create a self-diffusio-osmotic micropump [21].

Although we are primarily concerned with electrocatalytic micropumps, much research into electrocatalytic conversion of chemical to mechanical energy has focused on reaction-induced propulsion of nanoswimmers, i.e., self-electrophoresis and self-diffusiophoresis. The same mechanisms which cause pumping on a fixed surface lead to swimming of a particle with varying surface reactivity. Such self-phoretic mechanisms for motion were originally investigated in the context of cell motion [22,23]. Early experiments demonstrating reaction-induced propulsion in nonliving systems utilized bimetallic particles in peroxide solutions [24–26]. Various proposed mechanisms explaining their motion were proposed, and of these self-electrophoresis was found to be dominant [12,27]. Velocities on the order $10\text{ }\mu\text{m/s}$ were reported. Nanomachines such as these have created much excitement due to their potential to revolutionize fields such as drug delivery and biological sensing [28–31].

A number of theoretical studies have been performed to better understand self-catalytic swimming. Golestanian showed how asymmetric reaction-induced concentration gradients result in the propulsion of particles by self-diffusiophoretic mechanisms [32]. Moran and Posner investigated the self-electrophoretic mechanism by performing direct numerical simulations of the bulk transport equations coupled initially to fixed surface fluxes [33] which they refined by considering surface

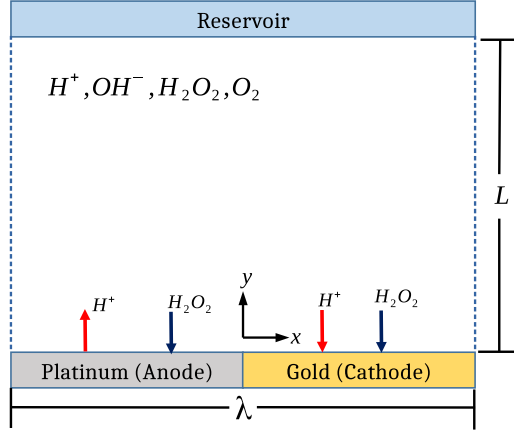


FIG. 1. Schematic showing the model problem. The electrocatalytic surface reaction pair consists of gold (Au) and platinum (Pt) at a periodic width of λ in contact with the electrolyte containing protons, hydroxide ions, hydrogen peroxide, and oxygen.

reaction kinetics [34]. They also analyzed the reduction in swimming speed with background electrolyte concentration [35]. However, their model predicted a quadratic relationship between swimming speed and peroxide concentration unlike the linear dependence observed in experiments. Yariv studied the problem using matched asymptotic expansions in the limit of thin electrical double layers and long-slender rods, predicting propulsion in experimentally observed direction [36]. Sabass and Seifert found that in the presence of background salt, the decrease in pH with increased peroxide concentration can alleviate diffusive limitations on proton transport [37]. Self-consistent nonlocal feedback theory, relating local fluxes to changes in the induced potential field, was developed to further improve understanding of electrocatalytic motors [38] and inspired our analysis. Further theoretical studies have found that the optimal location for surface reactivity is concentrated at the poles of the swimmer [39], a geometrical effect which may guide design of micropumps, and that bulk peroxide decomposition can significantly affect swimming speeds [40].

Here, we investigate the relation between surface reactivity patterns and the fluid flow they generate. For this, we analyzed the scaling relation of the induced potential (gradient) and velocity with surface reactivity. The system corresponds to a pattern of catalytic sites with distinct reaction rate constants (expressed by their corresponding second Damköhler numbers). We provide physical insight into how the different components of the induced potential lead to a balance of fluxes in the system. We also derive a simple model capturing the induced potential and flow velocity and compare this with direct numerical simulations of the governing transport equations.

II. MODEL PROBLEM

Many combinations of patterned metals and electrolytes lead to reactions which generate the asymmetric ion concentrations necessary to induce this type of flow [27]. In this work, we focus on electrocatalytic hydrogen peroxide oxidation at Pt next to its reduction on Au surfaces. The oxidation generates electrons and protons, while the reduction consumes protons and electrons. This asymmetric surface reaction pair generates and depletes protons near the surface, resulting in a reaction-induced charge distribution and electric field. The forces on the fluid caused by the induced electric field acting on the induced charge distribution give rise to an electro-osmotic flow which is driven by the electrocatalytic conversion.

Specifically, we model a two-dimensional domain of width λ and height L consisting of an electrolyte between a reactive surface and a reservoir as shown in Fig. 1. The domain is periodic in

the tangential x direction, and thus λ represents the period of a repeating patterned surface of which we simulate one period. This surface is modeled as completely flat, with a step change in surface reactivity at the intersection of the Au and Pt surface regions. The background electrolyte contains four species: protons H^+ , hydroxide ions OH^- , hydrogen peroxide H_2O_2 , and molecular oxygen O_2 . Peroxide oxidation at the platinum anode is given by



and reduction at the gold cathode is given by



This choice of reaction mechanism governing the decomposition of H_2O_2 has been successfully used in previous studies [17,33]. However, there remains uncertainty regarding the precise mechanism of peroxide decomposition at platinum and gold surfaces. Mechanisms other than the one used here have been proposed [41,42]. Our simulations and model could be modified to account for alternative mechanisms.

In this work, we only explicitly simulate the concentrations of the two ionic species, protons and hydroxide ions. We assume that the concentration of peroxide is constant over relevant timescales so its transport can be ignored. This assumption is justified with scaling arguments in Sec. II C. Transport of oxygen can be ignored because in the dilute limit oxygen only affects the system through the anode reactions. However, we assume that the anode reaction only proceeds in the forward direction in which oxygen does not participate [34].

A. Governing equations

This system is governed by the coupled Poisson-Nernst-Planck (PNP) equations for ion transport and unsteady Stokes equations of fluid motion along with appropriate surface and bulk reactions. The PNP equations relate species concentration change to advection, diffusion, reaction, and electromigration fluxes. Transport of ionic species is coupled through the Poisson equation for the electric potential. In dimensionless form, they are given by

$$\frac{\partial c_i}{\partial t} + \nabla \cdot (\mathbf{u} c_i) = D_i \nabla \cdot (\nabla c_i + z_i c_i \nabla \phi) + R_i, \quad (3)$$

$$-2\epsilon^2 \nabla^2 \phi = \sum_i z_i c_i, \quad (4)$$

where $c_i = \tilde{c}_i/c_0$ is the dimensionless concentration of species i which has valence z_i and diffusivity $D_i = \tilde{D}_i/D_0$, where the tilde ($\tilde{\cdot}$) represent dimensional quantities when there is a dimensionless counterpart. c_0 and D_0 are arbitrary reference values for concentration and diffusivity. In this study, we reference diffusivities to the diffusivity of hydroxide ions $D_0 = 5.273 \times 10^{-9} \text{ m}^2/\text{s}$ [34] and concentrations to the bulk concentrations of H^+ and OH^- in pure water, $c_0 = \sqrt{K_w} = 1 \times 10^{-7} \text{ M}$, where K_w is the water equilibrium constant. $\phi = \tilde{\phi}/V_T$ is the electrostatic potential normalized by the thermal voltage $V_T = RT/F \approx 25 \text{ mV}$, where R is the ideal gas constant, T is the temperature, and F is the Faraday constant. $\mathbf{u} = \tilde{\mathbf{u}}/(D_0/\lambda)$ is the fluid velocity normalized by the diffusion velocity. $t = \tilde{t}/t_0$ where $t_0 = \lambda^2/D_0$ is the diffusion time. The coefficient ϵ is the dimensionless Debye length $\epsilon = \lambda_D/\lambda$ where for our case of a symmetric binary electrolyte $\lambda_D = \sqrt{\epsilon V_T/2F c_0}$, where ϵ is the permittivity of water. $R_i = \tilde{R}_i/(c_0/t_0)$ is a bulk reaction term. For hydrogen peroxide and molecular oxygen, $R_i = 0$. For protons and hydroxide ions, $R_i = \text{Da}_b(K_w/c_0^2 - c_+ c_-)$ which enforces water self-ionization. $\text{Da}_b = k_b \lambda^2 c_0/D_0$ is a dimensionless bulk Damköhler number where k_b is the bulk water recombination reaction rate constant and c_+ and c_- are proton and hydroxide ion concentrations respectively. With this nondimensionalization, all lengths are normalized by λ and thus the dimensionless width is 1. We chose an aspect ratio of $L/\lambda = 4$, which is large enough that the reservoir boundary conditions do not affect the simulated flow near the electrodes.

The Stokes equations for momentum conservation along with continuity which enforces the incompressibility of water are

$$\frac{1}{Sc} \frac{\partial \mathbf{u}}{\partial t} = -\nabla p + \nabla^2 \mathbf{u} - \frac{\kappa}{2\epsilon^2} \rho_e \nabla \phi, \quad (5)$$

$$\nabla \cdot \mathbf{u} = 0. \quad (6)$$

Here we have included the electric body force term and ignored the nonlinear inertial term because it is small relative to the viscous and electric body force terms as verified in Sec. III A. $p = \tilde{p}/(\mu D_0/\lambda^2)$ is the pressure, and $\rho_e = \sum_i z_i c_i/c_0$ is the charge density. The Schmidt number $Sc = \mu/\rho D_0$, where μ is the dynamic viscosity and ρ is the density of water, and the electrohydrodynamic coupling constant $\kappa = \epsilon V_T^2/\mu D_0$. Note that we do not include any chemiosmotic forces (i.e., forces due to nonionic solute-wall interactions) because gradients in nonionic solutes (O_2 and H_2O_2) are expected to be very small. However, the diffusio-osmotic forces due to internal electric fields caused by salt concentration gradients with ions of different diffusivities are naturally included by this formulation.

B. Boundary conditions

The boundary condition for the Stokes equations at both the electrodes ($y = 0$) and reservoir ($y = 4$) is given by

$$\mathbf{u} = \mathbf{0} \quad (7)$$

due to the no-slip and no-penetration conditions at the solid surface, along with our assumption of a stationary reservoir.

The potential boundary condition at the electrodes is given by

$$\phi = 0, \quad (8)$$

thus representing the constant electric potential of a conductive metal surface which we have arbitrarily set to zero as a reference. Because we resolve the Debye layer in our simulation domain, the ζ potential does not directly enter this boundary condition. Because of the thinness of the Stern layer to the Debye length for the dilute conditions studied, we omit a Stern layer correction [17,34]. We have ignored other mechanisms which would induce a ζ potential [43] in order to focus on the reaction-induced potential as has been done previously [34]. Inclusion of a fixed ζ potential at each electrode would be a useful extension of the model. The potential boundary condition at the reservoir is

$$\frac{\partial \phi}{\partial y} = 0 \quad (9)$$

or zero normal electric field. As with the reservoir boundary condition for the momentum equations, this condition is valid for sufficiently large L/λ such that, in the absence of external electric fields, electric fields induced by the reactions decay before the reservoir. In other words, for this boundary condition to be valid, we expect that the potential at the reservoir should be approximately constant. The use of a Neumann boundary condition, instead of Dirichlet, at the reservoir allows the potential difference between the surface and reservoir to adjust as the simulation evolves until it reaches a steady state at which all fluxes are in balance as described in Sec. III A.

For species transport, anions do not participate in either of the surface reactions, so a no-flux boundary condition is enforced on the entire surface. This reduces to

$$\frac{\partial c_-}{\partial y} - c_- \frac{\partial \phi}{\partial y} = 0. \quad (10)$$

For cations, different boundary conditions are used for the platinum and gold portions of the surface. Both are derived by matching the combined electromigration and diffusion fluxes with the

surface reaction flux. The platinum ($-1/2 < x < 0$) and gold ($0 < x < 1/2$) boundary conditions respectively are

$$-D_+ \left(\frac{\partial c_+}{\partial y} + c_+ \frac{\partial \phi}{\partial y} \right) = D_+ \text{Da}_a, \quad (11)$$

$$-D_+ \left(\frac{\partial c_+}{\partial y} + c_+ \frac{\partial \phi}{\partial y} \right) = -D_+ \text{Da}_c c_+^2, \quad (12)$$

where $\text{Da}_a = k_{0, \text{anode}} \lambda \tilde{c}_{\text{H}_2\text{O}_2} / \tilde{D}_+ c_0$ is the anode Damköhler number and $\text{Da}_c = k_{r, \text{cathode}} \lambda \tilde{c}_{\text{H}_2\text{O}_2} c_0 / \tilde{D}_+$ is the cathode Damköhler number defining the dimensionless anode and cathode reaction rates. These Damköhler numbers represent ratios of reaction timescales to diffusive transport timescales, and hence by varying them we can consider regimes with different limiting mechanisms where reaction at one electrode versus the other controls the process. Both of these boundary conditions assume that the reactions proceed only in the forward direction and that the Stern layer potential drops are sufficiently small that reaction rate coefficients can be treated as constants.

The reservoir boundary condition for both species is simply a Dirichlet condition at the reservoir concentration

$$c_+ = c_- = 1. \quad (13)$$

The left and right boundaries are treated as periodic boundaries.

C. Validity regime of two-species approximation

Our assumption that we can treat the concentration of H_2O_2 as a constant is valid so long as the timescale associated with changes in the concentration of H_2O_2 is much longer than dynamically relevant timescales of the system. To estimate this timescale, we start with the balance of reactive and diffusive fluxes at the anode surface. The scaling of these fluxes can be written in dimensionless form as

$$j_{\text{reac}} \sim \frac{\text{Da}_a D_+ c_{\text{H}_2\text{O}_2}}{c_{\text{H}_2\text{O}_2, 0}}, \quad (14)$$

$$j_{\text{diff}} \sim \frac{D_{\text{H}_2\text{O}_2} \Delta c_{\text{H}_2\text{O}_2}}{l}, \quad (15)$$

where l is a characteristic length scale over which the concentration of H_2O_2 changes by $\Delta c_{\text{H}_2\text{O}_2}$ and $c_{\text{H}_2\text{O}_2, 0}$ is its initial concentration. As a conservative choice for l , we choose the diffusion length $l \sim \sqrt{D_{\text{H}_2\text{O}_2} t}$. This choice is conservative because additional mixing from the induced flow would only shorten the length scale. Substituting this value for l into (15), equating the two fluxes, and solving for t gives a scaling for the time associated with a given change in H_2O_2 concentration

$$t_{\text{H}_2\text{O}_2} \sim \left(\frac{\Delta c_{\text{H}_2\text{O}_2}}{c_{\text{H}_2\text{O}_2}} \right)^2 \frac{D_{\text{H}_2\text{O}_2} c_{\text{H}_2\text{O}_2, 0}^2}{\text{Da}_a^2 D_+^2}. \quad (16)$$

In the cases investigated in this study, over dynamically relevant times, which will be at worst $t \sim 1$ (the diffusion time across the domain), $\Delta c_{\text{H}_2\text{O}_2} / c_{\text{H}_2\text{O}_2}$ is small. Therefore, in effectively unbounded domains such as the current study, it is justifiable to ignore changes in H_2O_2 concentration over intermediate timescales, which are long relative to system dynamics but short relative to timescales for depletion of H_2O_2 at the surface. For longer term behavior, one may adopt our solution as a quasisteady solution in which the bulk concentration of H_2O_2 is adjusted gradually with time. In this case, (16) would imply that the length scale of depletion of H_2O_2 would be much longer than λ while varying slowly in time. Another possibility is the presence of an effective reservoir H_2O_2

concentration, due to either bulk mixing or for nanomotors the motion of the swimmer itself, in which case a suitable H_2O_2 concentration may be assumed.

D. Computational framework

We solve the governing equations using a custom written OPENMP parallel C++ code described thoroughly and compared against a commercial solver in Ref. [44]. The code uses second-order staggered finite differences to discretize the governing equations where species concentrations, electric potential, and pressure fields are stored at cell centers while velocities and fluxes are stored at cell faces. The linear Poisson and Poisson-like equations for pressure, electric potential, and momentum are solved using a pseudospectral method where Fourier transforms are performed in the tangential x direction and leaving a tridiagonal system to be solved at each wave number in the electrode normal. Time integration is done with a semi-implicit method which iteratively solves the transport, momentum, and potential equations in a decoupled manner until the full implicit solution for the next time step is reached. The species transport equations are solved implicitly during each iteration only in the stiff electrode-normal direction. Verification of the accuracy and convergence of the code was performed using the method of manufactured solutions [45]. This code and its variants have been successfully used in previous studies for prediction of chaotic electroconvection [46,47], induced-charge electro-osmosis [48], and flow over patterned membranes [49]. Mesh convergence testing was performed to ensure that a sufficiently fine mesh was used to resolve the results shown.

III. RESULTS AND ANALYSIS

A number of dimensionless parameters are present in the above equations. We summarize these along with the values used in our simulations in Table I. In this work, most of the parameters are held constant while two parameters, the surface Damköhler numbers, are varied. For reference to a physical system, the value $\epsilon = 4 \times 10^{-3}$ is representative of a 0.25-mm pattern length λ and pure water reservoir concentration of $c_0 = 1 \times 10^{-7}$ M. Qualitative steady-state fields of concentration, charge density, electric potential and electric field lines, and velocity magnitude with flow streamlines for nominal Damköhler numbers of $\text{Da}_a = 1.48$ and $\text{Da}_c = 2.68$ are shown in Fig. 2. These Damköhler numbers come from rate constants $k_{o, \text{anode}} = 5.5 \times 10^{-9}$ m/s and $k_{r, \text{cathode}} = 1 \text{ m}^7 \text{ s}^{-1} \text{ mol}^{-2}$ [34] and peroxide concentration $c_{\text{H}_2\text{O}_2} = 1 \times 10^{-3}$ M.

Evident from the concentration field in Fig. 2(a) is that, outside of the double layer, the concentration of H^+ is equal to c_0 . Because of electroneutrality, the concentration of OH^- is also equal to c_0 . The combination of water equilibrium and electroneutrality prevent concentration gradients from forming in the bulk. This would change if other ionic species were present. Note the expanded y scale in Figs. 2(a) and 2(b).

TABLE I. Dimensionless governing parameters.

Parameter	Description	Formula	Value(s)
ϵ	Dimensionless Debye length	λ_D/λ	4×10^{-3}
Sc	Schmidt number	$\mu/\rho D_0$	1000
κ	Electrohydrodynamic coupling constant	$\epsilon V_T^2/\mu D_0$	.094
	domain aspect ratio	L/λ	4
D_+	H^+ diffusivity	$\tilde{D}_+/D_0$	1.7658
D_-	OH^- diffusivity	$\tilde{D}_-/D_0$	1
Da_b	Bulk Damköhler number	$k_b \lambda^2 c_0/D_0$	9.38×10^4
Da_a	Anode Damköhler number	$k_{o, \text{anode}} \lambda c_{\text{H}_2\text{O}_2}/D_+ c_0$	$2.5 \times 10^{-3} - 2.5 \times 10^2$
Da_c	Cathode Damköhler number	$k_{r, \text{cathode}} \lambda c_0 c_{\text{H}_2\text{O}_2}/D_+$	$2.5 \times 10^{-3} - 2.5 \times 10^2$

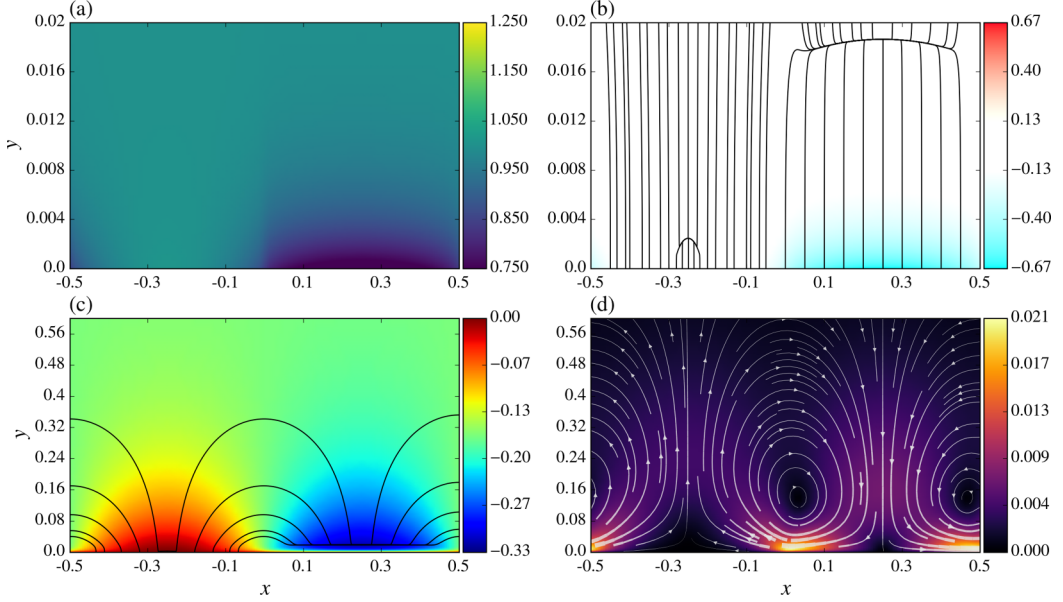


FIG. 2. Field quantities for $Da_a = 1.48$, $Da_c = 2.68$. (a) H^+ concentration field. (b) ρ_e field with electric field lines. (c) ϕ field with electric field lines. (d) Velocity magnitude field with streamlines.

The charge density field along with electric field lines are shown in Fig. 2(b). Notable from the electric field lines are a small region near the anode around $y = .002$ and a larger region above the cathode around $y = .018$ where field lines intersect. These correspond to regions where bulk charge is present, causing the electric field to not be divergence free. However, the charge at these locations is too small to be seen because we have chosen the color scale based on the largest charge density magnitude in the domain. Similar features have been observed previously [34]. These locations where field lines converge result from a superposition of field lines in the bulk which point from the anode to the cathode transporting the current and field lines associated with the double layer whose direction is determined by the local ζ potential. When these types of field lines point in opposite directions, then there can be a location at which the direction of the overall electric field switches and hence field lines intersect. In this case, the ζ potential is positive over most of the surface with field lines pointing away from the surface and necessitating the change in direction above the cathode. However, there is a small region near the center of the anode where the ζ potential switches sign, leading to field lines near the surface pointing toward the anode which must change direction.

The electric field lines in the bulk can be seen more clearly in Fig. 2(c) along with the associated electric potential. These electric field lines act on the charged Debye layer to generate the flow field shown in Fig. 2(d) consisting of a pair of counter-rotating vortices.

One quantity of interest is the induced potential and particularly the induced potential difference between the surface and far field which we label ϕ_∞ . Figure 3 shows induced electric potential and associated electric field lines for different values of the anode Damköhler number. The color scales in the plots have been set to emphasize variations about ϕ_∞ for each case and do not necessarily contain the zero potential of the surface. One consequence of this choice is the potential field within the double layer saturates in Figs. 3(a) and 3(c), leading to a sharp change in the color. As the ratio of Damköhler numbers changes, the sign of the induced net electric potential changes, and two patterns in the potential field are evident. First, there is a one-dimensional (1D) component associated with the net induced potential drop strongly visible within the double layer very near the surface and varying only in the y direction. There is also evident a component of the potential associated with

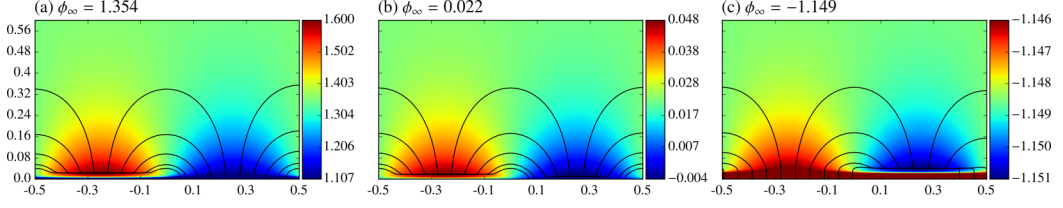


FIG. 3. Electrostatic potential fields with electric field lines plotted for several values of the Damköhler numbers: (a) $Da_a = 2.5$, $Da_c = .25$, (b) $Da_a = .25$, $Da_c = .25$, and (c) $Da_a = .025$, $Da_c = .25$.

the outer nearly semicircular electric field lines, which does not appear to be significantly affected by the changes in Da_a . The superposition of these two components determines the local ζ potential.

A. An approximate model

In this section, we develop an approximate model, shown schematically in Fig. 4, to understand the results of our numerical simulations. Our model utilizes balances of fluxes along with the approximation that current travels along semicircular electric field lines outside of the double layer. We also take inspiration from asymptotic modeling, e.g., Refs. [9,50], in which an “inner” solution within the thin charged double layer is matched with an “outer” solution valid in the electroneutral bulk. This means that our model requires $\epsilon \ll 1$. We calculate several important quantities which we compare against the results of our numerical simulations. One quantity is the induced potential difference between the surface and bulk $\phi_\infty = \phi(y \rightarrow \infty)$. A second is the reaction-induced ζ potential $\zeta(x) = -[\phi_\infty + \phi_{\text{tan}}(x)]$, where $\phi_{\text{tan}}(x)$ describes the variation along the electrode surface just outside of the double layer, analogous to the matching location between asymptotic inner and outer solutions, of the electric potential and hence the tangential electric field responsible for inducing a slip velocity is $E_{\text{tan}} = \frac{d\zeta}{dx} = -\frac{d\phi_{\text{tan}}}{dx}$. These allow us to use the Helmholtz-Smoluchowski equation to calculate a third quantity, the slip velocity u_{slip} .

We start with a balance which helps explain the behavior of ϕ_∞ . At steady state, the total reactive currents at the anode and cathode must balance. At the anode, the reaction rate in our model is determined solely by the constant concentration of H_2O_2 . This means that the system must adjust the reaction rate at the cathode to conserve current. The only way that this adjustment can occur is

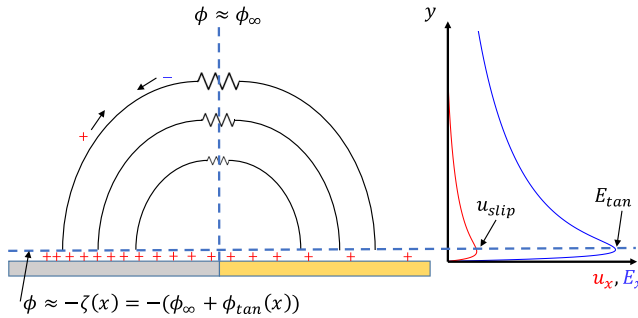


FIG. 4. Schematic showing conceptually our approximate model and how we evaluate it against the numerical simulations. The ζ potential just outside the double layer here shown as positively charged. $\zeta(x)$ is calculated as a sum of two terms, the far-field potential ϕ_∞ and a component which varies along the surface $\phi_{\text{tan}}(x)$. Current in the bulk travels along semicircular lines which act as resistors. The model-predicted slip velocity u_{slip} and tangential electric field E_{tan} are compared against the maximum x velocity and x electric field along the line $x = 0$.

by changing the surface H^+ concentration to an appropriate average value via an induced potential ϕ_∞ . We label this estimate of the induced potential $\phi_{\infty,0}$ because we will later calculate ϕ_∞ in a more complete way, accounting for transport between anode and cathode, and $\phi_{\infty,0}$ will show up as the first term in that equation. For now, however, we will consider only the ability of a spatially uniform induced ζ potential to lead to a balance of anode and cathode reaction rates. When c_+ obeys a Boltzmann distribution within the double layer, valid for small currents, and is equal to the reservoir concentration outside of the double layer, then

$$j_{Pt}^+ = D_+ Da_a, \quad (17)$$

$$j_{Au}^+ = (e^{\phi_{\infty,0}})^2 D_+ Da_c, \quad (18)$$

where j_i^+ is the magnitude of the local reactive surface flux. Because we have (temporarily) assumed a spatially uniform induced potential and the anode and cathode are of equal size, integrating the reactive flux over each surface amounts to setting these fluxes equal. Doing this and solving for $\phi_{\infty,0}$ gives

$$\phi_{\infty,0} = 1/2 \ln(Da_a/Da_c). \quad (19)$$

Physically, the dependence of $\phi_{\infty,0}$ on the ratio of Damköhler numbers can be interpreted as the potential adjusting to either increase or decrease the surface concentration of H^+ in such a way as to balance the reaction rates at the electrodes. This leads to a dependence on the ratio of reaction rate constants and the bulk H^+ concentration, but not on the H_2O_2 concentration.

In order to understand the tangential component of the electric field, we look at the transport of ions from anode to cathode. In the absence of significant bulk charge, as is evident in Fig. 2(b), current is carried predominantly in an Ohmic manner. Equating scaling of electromigration current and the cathodic current gives

$$E_{\tan} \sim Da_a. \quad (20)$$

In addition to governing transport of cations from anode to cathode, this component of the electric field will also affect the average ζ potential of the cathode. In other words, even if $Da_a = Da_c$ and thus $\phi_{\infty,0} = 0$, the fact that the potential at the anode outside of the double layer must be higher than the potential at the cathode to drive transport from one to the other means that ϕ_∞ must be induced to equalize the average reaction rates. This contribution to the net potential will scale as the electric field multiplied by the length scale over which it acts.

We estimate the coefficients associated with this scaling and account for the quadratic dependence of the reaction rate on the H^+ concentration at the cathode by starting with the equation for bulk charge transport

$$\nabla \cdot (\sigma \mathbf{E}) = 0, \quad (21)$$

where $\sigma = D_+ c_+ + D_- c_-$ is the conductivity and $\mathbf{E}$ is the electric field. This equation, a formulation of Ohm's law, is asymptotically valid in the electroneutral bulk for small ϵ . We then, using a heuristic model based on Ramos's work [8] on alternating current electro-osmosis, assume that the outer electric field lines are semicircular. This gives a bulk current of

$$i = \sigma E \quad (22)$$

$$= (D_+ + D_-) \frac{\phi_{\tan}(-x) - \phi_{\tan}(x)}{\pi x}, \quad (23)$$

where E is magnitude of the electric field along the field line starting at $-x$ just outside the electric double layer and ending at x and i the current density along it. We have also assumed $c_+ = c_- = 1$ outside the double layer. The value of E for each field line comes from the difference between the potential at the ends of the field line divided by its length. Equation (23) is valid in the range $0 < x < 1/4$ where current lines travel from the anode to the right to the cathode without passing

through the periodic boundary. We restrict our analysis to this region for simplicity and without loss of generality because the problem is symmetric about the center of each electrode. We will use (17) along with (23) to predict an i which is constant along the surface. While this is not generally true at the cathode where i depends on c_+ and therefore is a function x , this model provides a useful approximation

$$\phi_{\tan}(x) = \frac{-\text{Da}_a \pi x}{2(1 + D_-/D_+)}. \quad (24)$$

Now that we have an estimate for the tangential variation of the electric potential, we can calculate an estimate of ϕ_∞ which accounts for it. Previously, to find $\phi_{\infty,0}$, we assumed that the potential drop across the double layer, and hence the surface concentration and reaction flux were independent of x . We now relax this assumption and include the combined potential difference across the double layer

$$\zeta(x) = \phi(x, y = 0) - \phi(x, 1 \gg y \gg \epsilon) \quad (25)$$

$$= -(\phi_\infty + \phi_{\tan}). \quad (26)$$

This definition of ζ (25) comes from the local ζ potential representing the potential difference between the surface and a location far from the thin double layer based on inner scales $y \gg \epsilon$, but still close to the surface relative to outer scales $1 \gg y$. In other words, the potential difference between the surface and matching location between inner and outer solutions. The negative sign in (26) comes from our choice to reference potentials to the surface as given by the boundary condition of ϕ in (8), meaning that $\phi(x, y = 0) = 0$.

At steady state, the surface reactions constitute the only species transport into or out of the domain. Setting the total flux at the cathode equal to that at the anode by integrating over the surface gives

$$\int_0^{1/4} j^+(x, y = 0) dx = - \int_{-1/4}^0 j^+(x, y = 0) dx, \quad (27)$$

$$\int_0^{1/4} (e^{-\zeta})^2 D_+ \text{Da}_c dx = \frac{\lambda}{4} D_+ \text{Da}_a, \quad (28)$$

where we have restricted our integral to be over the half of the cathode where (24) gives an estimate for the contribution of transport to the potential difference across the double layer. In (28), we have again assumed that the concentration of c_+ is uniform and equal to the reservoir concentration outside of the double layer and Boltzmann distributed within it. Because we have included only H^+ and OH^- in the model, the combination of bulk water equilibrium and electroneutrality will prevent bulk concentration gradients. However, when additional species are present, pH gradients will arise [14], and the variation in c_+ outside of the double layer will need to be accounted for. The domain is symmetric about the center of the anode and cathode, so the current through the other half of the cathode ($1/4 < x < 1/2$) will come from the other half of the anode ($-1/2 < x < -1/4$). Solving for ϕ_∞ gives

$$\phi_\infty = \phi_{\infty,0} + \frac{1}{2} \ln \left(\frac{\beta \text{Da}_a}{1 - \exp(-\beta \text{Da}_a)} \right), \quad (29)$$

where $\beta = \pi/[4(1 + D_-/D_+)]$. In order to evaluate these models and scalings, we performed simulations over a broad range of anode and cathode Damköhler numbers. Figure 5 shows a comparison between ϕ_∞ from those simulations and the results of our model.

In Fig. 5(a) cases, where one of the Damköhler numbers is fixed and the other varied, very good agreement is observed over the entire range studied. Experimentally varying this ratio would be difficult because it depends on the ratio of reaction rate coefficients; however, some variation may be possible by choosing different electrode materials. Figure 5(b) shows cases where the ratio of the Damköhler numbers is fixed and their magnitude is varied together, representative of varying the

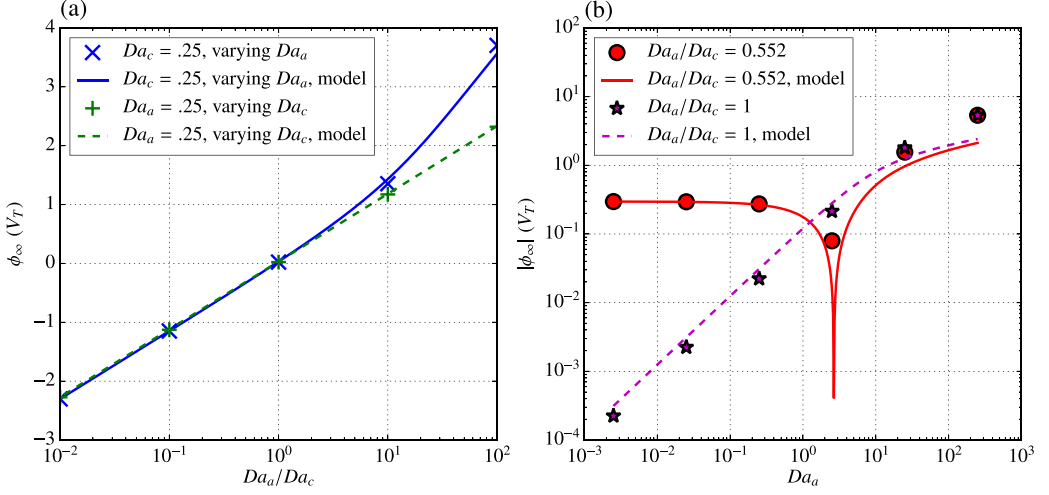


FIG. 5. Scaling of ϕ_∞ and comparison of simulation with model (29). (a) Scaling with Damköhler ratio showing the predicted logarithmic scaling when Da_a is small. (b) Scaling with Da_a for two fixed values of Da_a/Da_c .

concentration of H_2O_2 . Over most of the parameter space, good agreement is observed except when Da_a becomes very large $O(100)$ and the model assumptions used to determine the component of ϕ_∞ caused by tangential transport break down. Additionally, at $Da_a = 2.5$ and a ratio of $Da_a/Da_c = .552$, some error is apparent because ϕ_∞ changes sign near this point, making the result on the log scale highly sensitive to the exact Damköhler number at which this sign change occurs and appear as a sharp feature. Both of these plots demonstrate that despite the many assumptions in the model, it is able to accurately capture not merely the scaling, but the induced net potential in a broad parameter space.

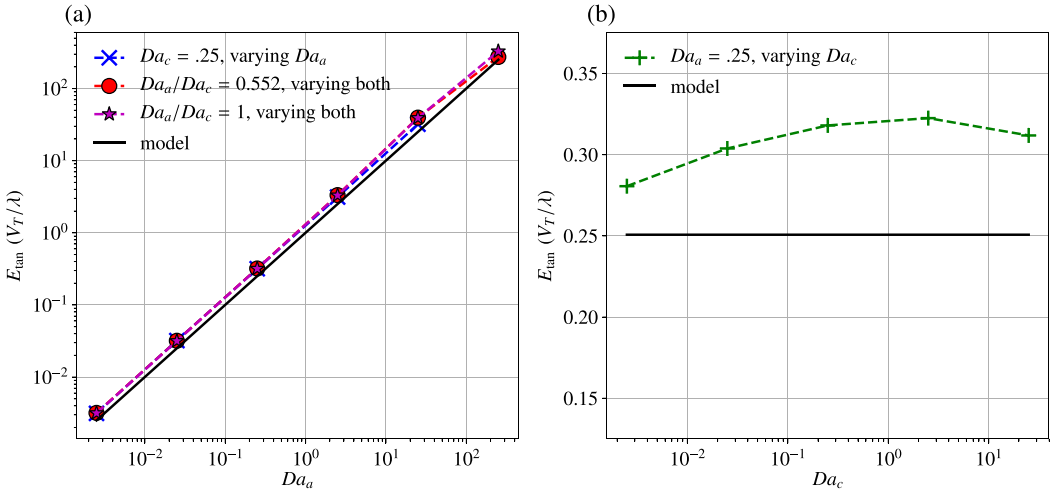


FIG. 6. Scaling of tangential electric field with Damköhler numbers. Panel (a) shows that as predicted E_{tan} scales linearly with Da_a . Panel (b) shows the independence of E_{tan} from Da_c .

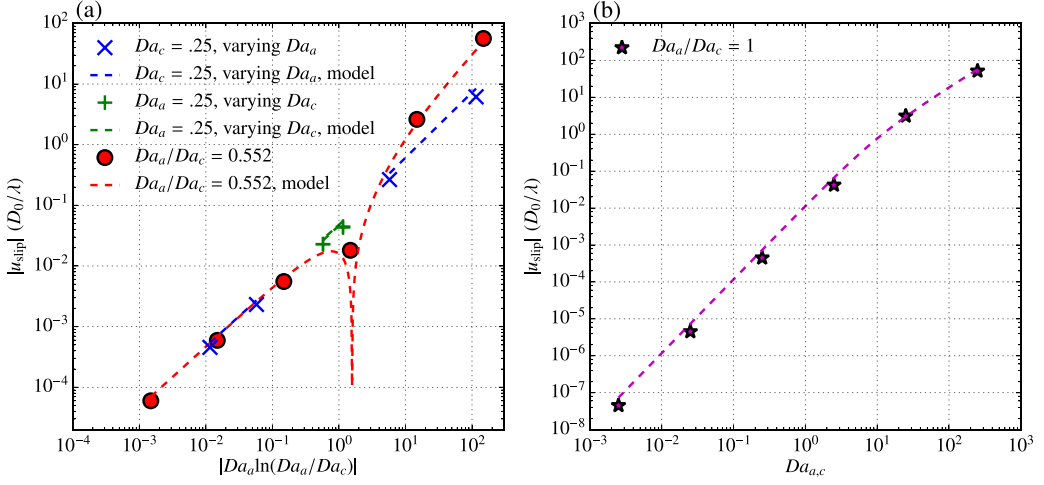


FIG. 7. Comparison of maximum tangential velocity predicted by our model (dashed lines) with simulations (symbols). (a) Comparison for $Da_a/Da_c \neq 1$. (b) Comparison for $Da_a/Da_c = 1$.

To evaluate our predicted linear scaling of the tangential electric field with Da_a , Fig. 6 shows a comparison of our model

$$E_{\text{tan}} = -\frac{d\phi_{\text{tan}}}{dx} \quad (30)$$

$$= 2\beta Da_a, \quad (31)$$

with E_{tan} from the simulations defined as $\max(E_x(x=0, y))$. We again see that the scaling is quite accurate over the entire range simulated with the predicted linear behavior when Da_a is varied and almost no change when Da_c is varied while Da_a is held constant. Additionally, our simple model is able to estimate the magnitude of the tangential electric field to within an $O(1)$ constant over the entire range.

Our model for the electric field (31) predicts that it should depend on the diffusivities of both species. In contrast, previous studies [17] found that it should depend only on the ionic strength and H^+ diffusivity. They argue that the conductivity should not be important because only H^+ ions carry net current. However, when bulk water equilibrium reactions are accounted for, current in the bulk can be carried by OH^- ions which are generated by water splitting near the cathode and recombine with H^+ ions near the anode, leading to a dependence of the electric field on their diffusivity.

The velocity of the flow induced by the micropump provides a useful measure of the ability of the reaction-induced convection to provide additional transport on top of diffusion. From a balance of the electric body force and diffusive terms in the momentum equations, the velocity should scale as the product of ζ and E_{tan} . For our model, we use the Helmholtz-Smoluchowski formula for electro-osmotic flow past a flat charged surface, $u_{\text{slip}} = -\varepsilon E \zeta / \mu$, valid for slip associated with thin equilibrium double layers. Corrections for quasi- and nonequilibrium double layers, due to passage of current, also exist and could be included [51]. In our dimensionless formulation, this gives

$$u_{\text{slip}}(x) = -\kappa E_{\text{tan}} \zeta(x) \quad (32)$$

$$= \kappa \beta Da_a \left[\ln \frac{Da_a}{Da_c} + \ln \frac{\beta Da_a}{1 - \exp(-\beta Da_a)} - 2\beta Da_a x \right]. \quad (33)$$

We evaluate this predicted slip velocity by comparing the predicted velocity at $x=0$ with the slip velocity from our simulations $\max(u_x(x=0, y))$ in Fig. 7. When $\phi_{\infty,0}$ is dominant, our model predicts that the velocity should be proportional to the quantity $Da_a \ln(Da_a/Da_c)$. From Fig. 7(a),

we see that the velocity does indeed scale with this quantity and that our model predicts the maximum simulated velocity to within an $O(1)$ in all cases. In Fig. 7(b), a comparison is shown for the case where $\text{Da}_a/\text{Da}_c = 1$ and hence $\phi_{\infty,0} = 0$. Again, for this case agreement to within an $O(1)$ constant is seen over many orders of magnitude change in the Damköhler number. Visually sharp features in Fig. 7(a) correspond to changes in the sign of the velocity, similar to those observed in Fig. 5. We can also see from this figure that our earlier assumption that the nonlinear inertial term $\mathbf{u} \cdot \nabla \mathbf{u}/\text{Sc}$ is small relative to the other terms in the momentum is reasonable by comparing the inertial and viscous terms. The largest velocities here are $u \sim O(10^2)$, with the smallest length scale $l \sim \epsilon \sim O(10^{-3})$. This means that the inertial term, which scales as $(1/\text{Sc})u^2/l$, can be at most $O(10^4)$ while the viscous term, which scales as u/l^2 , will be $O(10^8)$, a difference of four orders of magnitude.

IV. DISCUSSION AND CONCLUSIONS

In this work, we have modeled the reaction-induced electroconvection generated by the electrochemical decomposition of hydrogen peroxide on platinum and gold surfaces. Based on observations from our direct numerical simulations, we have developed a simple model without any tuning parameters which is able to predict both the induced electric potentials and flow velocity magnitudes to within a factor of ≈ 2 over a broad range of Damköhler numbers. This model can be used to aid in experimental design of systems using this reaction-induced flow to provide additional near-surface mixing. It can also be modified to provide estimations for systems which involve other electrochemical reactions.

Although the developed model agrees well with simulations over the parameter space investigated, one important question which remains to be addressed in future studies is the effect of additional charged species on the system. Our model and simulations assume ideal water while in experimental systems there will always be other ionic species due to either contaminants or their addition to induce other electrochemical reactions. The combination of the ideal water model and bulk electroneutrality means that significant bulk pH and salt concentration gradients cannot develop in our simulations and that double layers must remain in equilibrium.

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- [1] T. R. Kline, W. F. Paxton, Yang Wang, D. Velegol, T. E. Mallouk, and A. Sen, Catalytic micropumps: Microscopic convective fluid flow and pattern formation, *J. Am. Chem. Soc.* **127**, 17150 (2005).
 - [2] W. F. Paxton, S. Sundararajan, T. E. Mallouk, and A. Sen, Chemical locomotion, *Angew. Chem., Int. Ed.* **45**, 5420 (2006).
 - [3] C. Zhou, H. Zhang, Z. Li, and W. Wang, Chemistry pumps: A review of chemically powered micropumps, *Lab Chip* **16**, 1797 (2016).
 - [4] T. M. Squires, R. J. Messinger, and S. R. Manalis, Making it stick: Convection, reaction, and diffusion in surface-based biosensors, *Nat. Biotechnol.* **26**, 417 (2008).
 - [5] M. Z. Bazant and T. M. Squires, Induced-Charge Electrokinetic Phenomena: Theory and Microfluidic Applications, *Phys. Rev. Lett.* **92**, 066101 (2004).
 - [6] T. M. Squires and M. Z. Bazant, Induced-charge electro-osmosis, *J. Fluid Mech.* **509**, 217 (2004).

- [7] A. Ramos, H. Morgan, N. G. Green, and A. Castellanos, Ac electrokinetics: A review of forces in microelectrode structures, *J. Phys. D* **31**, 2338 (1998).
- [8] A. Ramos, H. Morgan, N. G. Green, and A. Castellanos, AC electric-field-induced fluid flow in microelectrodes, *J. Colloid Interface Sci.* **217**, 420 (1999).
- [9] A. Ramos, A. González, A. Castellanos, N. G. Green, and H. Morgan, Pumping of liquids with ac voltages applied to asymmetric pairs of microelectrodes, *Phys. Rev. E* **67**, 056302 (2003).
- [10] J. L. Anderson, Colloid transport by interfacial forces, *Annu. Rev. Fluid Mech.* **21**, 61 (1989).
- [11] T. R. Kline, J. Iwata, P. E. Lammert, T. E. Mallouk, A. Sen, and D. Velegol, Catalytically driven colloidal patterning and transport, *J. Phys. Chem. B* **110**, 24513 (2006).
- [12] W. F. Paxton, P. T. Baker, T. R. Kline, Yang Wang, T. E. Mallouk, and A. Sen, Catalytically induced electrokinetics for motors and micropumps, *J. Am. Chem. Soc.* **128**, 14881 (2006).
- [13] M. E. Ibele, Yang Wang, T. R. Kline, T. E. Mallouk, and A. Sen, Hydrazine fuels for bimetallic catalytic microfluidic pumping, *J. Am. Chem. Soc.* **129**, 7762 (2007).
- [14] A. Afshar Farniya, M. J. Esplandiu, D. Reguera, and A. Bachtold, Imaging the proton concentration and mapping the spatial distribution of the electric field of catalytic micropumps, *Phys. Rev. Lett.* **111**, 168301 (2013).
- [15] I. K. Jun and H. Hess, A biomimetic, self-pumping membrane, *Adv. Mater.* **22**, 4823 (2010).
- [16] M. J. Esplandiu, A. A. Farniya, and A. Bachtold, Silicon-based chemical motors: An efficient pump for triggering and guiding fluid motion using visible light, *ACS Nano* **9**, 11234 (2015).
- [17] M. J. Esplandiu, A. A. Farniya, and D. Reguera, Key parameters controlling the performance of catalytic motors, *J. Chem. Phys.* **144**, 124702 (2016).
- [18] J. L. Anderson, M. E. Lowell, and D. C. Prieve, Motion of a particle generated by chemical gradients, part 1: Non-electrolytes, *J. Fluid Mech.* **117**, 107 (1982).
- [19] D. C. Prieve, J. L. Anderson, J. P. Ebel, and M. E. Lowell, Motion of a particle generated by chemical gradients, part 2: Electrolytes, *J. Fluid Mech.* **148**, 247 (1984).
- [20] Y. Hong, M. Diaz, U. M. Córdova-Figueroa, and A. Sen, Light-driven titanium-dioxide-based reversible microfireworks and micromotor/micropump systems, *Adv. Funct. Mater.* **20**, 1568 (2010).
- [21] R. Niu, P. Kreissl, A. T. Brown, G. Rempfer, D. Botin, C. Holm, T. Palberg, J. de Graaf, and A. Sen, Microfluidic pumping by micromolar salt concentrations, *Soft Matter* **13**, 1505 (2017).
- [22] P. Mitchell, Self-electrophoretic locomotion in microorganisms: Bacterial flagella as giant ionophores, *FEBS Lett.* **28**, 1 (1972).
- [23] P. E. Lammert, J. Prost, and R. Bruinsma, Ion drive for vesicles and cells, *J. Theor. Biol.* **178**, 387 (1996).
- [24] S. Fournier-Bidoz, A. C. Arsenault, I. Manners, and G. A. Ozin, Synthetic self-propelled nanorotors. *Chem. Commun.* **0**, 441 (2005).
- [25] W. F. Paxton, A. Sen, and T. E. Mallouk, Motility of catalytic nanoparticles through self-generated forces, *Chem. Eur. J.* **11**, 6462 (2005).
- [26] P. M. Wheat, N. A. Marine, J. L. Moran, and J. D. Posner, Rapid fabrication of bimetallic spherical motors, *Langmuir* **26**, 13052 (2010).
- [27] Yang Wang, R. M. Hernandez, D. J. Bartlett, J. M. Bingham, T. R. Kline, A. Sen, and T. E. Mallouk, Bipolar electrochemical mechanism for the propulsion of catalytic nanomotors in hydrogen peroxide solutions, *Langmuir* **22**, 10451 (2006).
- [28] J. Wang, Can man-made nanomachines compete with nature biomotors?, *ACS Nano* **3**, 4 (2009).
- [29] J. Wu, S. Balasubramanian, D. Kagan, K. M. Manesh, S. Campuzano, and J. Wang, Motion-based DNA detection using catalytic nanomotors, *Nat. Commun.* **1**, 36 (2010).
- [30] S. J. Ebbens, J. R. Howse, S. Balasubramanian, S. Sarayasamitsathit, K. M. Manesh, G.-U. Flechsig, J. Wang, J. Wang, and V. H. Crespi, In pursuit of propulsion at the nanoscale, *Soft Matter* **6**, 726 (2010).
- [31] S. Sengupta, M. E. Ibele, and A. Sen, Fantastic voyage: Designing self-powered nanorobots, *Angew. Chem., Int. Ed.* **51**, 8434 (2012).
- [32] R. Golestanian, T. B. Liverpool, and A. Ajdari, Propulsion of a Molecular Machine by Asymmetric Distribution of Reaction Products, *Phys. Rev. Lett.* **94**, 220801 (2005).
- [33] J. L. Moran, P. M. Wheat, and J. D. Posner, Locomotion of electrocatalytic nanomotors due to reaction induced charge autoelectrophoresis, *Phys. Rev. E* **81**, 065302 (2010).

- [34] J. L. Moran and J. D. Posner, Electrokinetic locomotion due to reaction-induced charge auto-electrophoresis, *J. Fluid Mech.* **680**, 31 (2011).
- [35] J. L. Moran and J. D. Posner, Role of solution conductivity in reaction induced charge auto-electrophoresis, *Phys. Fluids* **26**, 042001 (2014).
- [36] E. Yariv, Electrokinetic self-propulsion by inhomogeneous surface kinetics, *Proc. R. Soc. London, Ser. A* **467**, 1645 (2011).
- [37] B. Sabass and U. Seifert, Nonlinear, electrocatalytic swimming in the presence of salt, *J. Chem. Phys.* **136**, 214507 (2012).
- [38] A. Nourhani, V. H. Crespi, and P. E. Lammert, Self-consistent nonlocal feedback theory for electrocatalytic swimmers with heterogeneous surface chemical kinetics, *Phys. Rev. E* **91**, 062303 (2015).
- [39] P. Kreissl, C. Holm, and J. De Graaf, The efficiency of self-phoretic propulsion mechanisms with surface reaction heterogeneity, *J. Chem. Phys.* **144**, 204902 (2016).
- [40] A. T. Brown, W. C. K. Poon, C. Holm, J. de Graaf, Q. He, D. Velegol, R. Golestanian, S. J. Ebbens, and T. Mallouk, Ionic screening and dissociation are crucial for understanding chemical self-propulsion in polar solvents, *Soft Matter* **13**, 1200 (2017).
- [41] S. B. Hall, E. A. Khudaish, and A. L. Hart, Electrochemical oxidation of hydrogen peroxide at platinum electrodes, part 1: An adsorption-controlled mechanism, *Electrochim. Acta* **43**, 579 (1998).
- [42] S. B. Hall, E. A. Khudaish, and A. L. Hart, Electrochemical oxidation of hydrogen peroxide at platinum electrodes, part II: Effect of potential, *Electrochim. Acta* **43**, 2015 (1998).
- [43] J. F. L. Duval, G. K. Huijs, W. F. Threels, J. Lyklema, and H. P. van Leeuwen, Faradaic depolarization in the electrokinetics of the metal electrolyte solution interface, *J. Colloid Interface Sci.* **260**, 95 (2003).
- [44] E. Karatay, C. L. Druzgalski, and A. Mani, Simulation of chaotic electrokinetic transport: Performance of commercial software versus custom-built direct numerical simulation codes, *J. Colloid Interface Sci.* **446**, 67 (2015).
- [45] K. Salari and P. Knupp, Code verification by the method of manufactured solutions, Technical Report No. SAND2000-144, Sandia National Laboratory, Albuquerque, NM, and Sandia National Laboratory, Livermore, CA, June 2000.
- [46] C. L. Druzgalski, M. B. Andersen, and A. Mani, Direct numerical simulation of electroconvective instability and hydrodynamic chaos near an ion-selective surface, *Phys. Fluids* **25**, 110804 (2013).
- [47] C. Druzgalski and A. Mani, Statistical analysis of electroconvection near an ion-selective membrane in the highly chaotic regime, *Phys. Rev. Fluids* **1**, 073601 (2016).
- [48] S. M. Davidson, M. B. Andersen, and A. Mani, Chaotic Induced-Charge Electro-Osmosis, *Phys. Rev. Lett.* **112**, 128302 (2014).
- [49] S. M. Davidson, M. Wessling, and A. Mani, On the dynamical regimes of pattern-accelerated electroconvection, *Sci. Rep.* **6**, 22505 (2016).
- [50] M. Z. Bazant, K. Thornton, and A. Ajdari, Diffuse-charge dynamics in electrochemical systems, *Phys. Rev. E* **70**, 021506 (2004).
- [51] I. Rubinstein and B. Zaltzman, Electro-osmotic slip of the second kind and instability in concentration polarization at electrodialysis membranes, *Math. Models Methods Appl. Sci.* **11**, 263 (2001).