

Statistics of velocity fluctuations of Geldart A particles in a circulating fluidized bed riser

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The statistics of fluctuating velocity components are studied in the riser of a closed-loop circulating fluidized bed with fluid catalytic cracking catalyst particles. Our analysis shows distinct similarities as well as deviations compared to existing theories and bench-scale experiments. The study confirms anisotropic and non-Maxwellian distribution of fluctuating velocity components. The velocity distribution functions (VDFs) corresponding to transverse fluctuations exhibit symmetry, and follow a stretched-exponential behavior up to three standard deviations. The form of the transverse VDF is largely determined by interparticle interactions. The tails become more overpopulated with an increase in particle loading. The observed deviations from the Gaussian distribution are represented using the leading order term in the Sonine expansion, which is commonly used to approximate the VDFs in kinetic theory for granular flows. The vertical fluctuating VDFs are asymmetric and the skewness shifts as the wall is approached. In comparison to transverse fluctuations, the vertical VDF is determined by the local hydrodynamics. This is an observation of particle velocity fluctuations in a large-scale system and their quantitative comparison with the Maxwell-Boltzmann statistics.

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I. INTRODUCTION

The interest in exploring the distribution of random particle velocity arises from its analogy with molecular velocity, albeit significant differences have been established in the past. Theoretical analyses including van Noije and Ernst [1], Ernst and Brito [2], and Barrat *et al.* [3] have proposed the velocity distribution function (VDF) to assume a stretched-exponential form $P(c) \sim e^{-\alpha|c|^\beta}$. The bench-scale experiments (on the scale of a laboratory work table) on vertically vibrated beds [4–8] have confirmed the existence of this behavior. The deviation from Maxwellian distribution has also been recorded in electrostatically driven granular media (Aranson and Olafsen [9] and Kohlstedt *et al.* [10]). According to the theories of Ben-Naim and Krapivsky [11] and Ben-Naim and Machta [12], the scaling in the VDF tails follows $P(c) \sim |c|^{-\gamma}$, also observed in the numerical simulations of Ben-Naim and Machta [12] and the experiments of Moka and Nott [13].

The knowledge of VDF is imperative in understanding the particle-scale dynamics and its influence on the macroscopic behavior of the system. VDF is representative of an energy cascade, and could be considered analogous to fast Fourier transform (FFT) spectra commonly used to characterize turbulent single-phase flows. Further, the constitutive relations for the Eulerian two-fluid model using the kinetic theory for granular flow (KTGF) are obtained from the moments of VDF. Finally, VDF measurements could provide an important database for validating the computational models for gas-solid flows. While developing the KTGF, it is common to approximate the VDF using a Sonine expansion about the reference state having a Gaussian distribution [14,15]. Deviations are present when the VDF tails become overpopulated [16,17]. Besides, the VDFs are assumed to be symmetric, which is not expected to be the case for a driven system having a preferred direction of mean motion.

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Bench-scale experiments of Moka and Nott [13] and the direct numerical simulation (DNS) results of Liu *et al.* [18] have confirmed the directional dependence of VDF on the nature of forcing.

It is important to check the validity of the proposed theoretical assumptions for a large-scale system being on the order of an industrial facility (based on dimensions and operating conditions). Particles are transported using a carrier phase in such applications, and it remains to be seen if additional perturbations are introduced to the Maxwell-Boltzmann statistics. In this Rapid Communication, we report the VDFs in a large-scale system. A significant feature is that velocity fluctuations are obtained when mean particle velocities are nonstationary [19], which is usually not the case for bench-scale experiments.

II. EXPERIMENT

The circulating fluidized bed (CFB) experiments were performed in an 8-in.-diam CFB riser at Particulate Solid Research, Inc. (PSRI) laboratories [19]. Fluid catalytic cracking (FCC) particles were used having a mean diameter of 81 μm , and a mean particle density of 1490 kg/m^3 . According to the classical Geldart classification [20], particles are grouped based on their physical properties. FCC particles belong to group A and are slightly cohesive. Mechanisms including electrostatics, capillary, and van der Waals forces enhance the rate of cluster formation [21], whose occurrence results in a stretched Gaussian velocity distribution, as explained in Sec. III. The fluidizing medium was air at atmospheric conditions. The description of the facility can be found in Gopalan and Shaffer [19,22]. The superficial gas velocity in the experiments considered is 18.3 m/s, while the solid's flux varies (49 $\text{kg}/\text{m}^2/\text{s}$ for PSRI1 and 390 $\text{kg}/\text{m}^2/\text{s}$ for PSRI2). The transverse and streamwise fluctuating velocity data reported in the current analysis were obtained at an elevation of 20 m (98 riser diameters) above the gas distributor at the following radial locations: 0 (center), 2, 3, 3.5, and 4 in. (wall).

The high-speed particle image velocimetry (HSPIV) borescope system used to acquire the data consists of the following components: (1) Vision Research Phantom v12.1 high-speed camera (1280 $\times$ 1024 pixels); (2) Vision Research Cinestation high-speed data recording system; (3) Nikon Micro-NIKKOR 105 mm lenses; (4) high-intensity light sources for high-speed video; and (5) a custom designed borescope from Gradient Lens Corporation [19,22]. The Vision Research v12.1 camera has a 12-bit grayscale resolution and has the capability of recording up to 6800 frames per second at full resolution. At a lowered resolution, the frame rate was between 21.5k and 37.5k frames per second, and the magnification was adjusted such that 1 pixel displacement would correspond to 16–26 μm . This provides adequate temporal resolution to measure very high particle velocities and the required spatial resolution to ensure accuracy.

The HSPIV technique developed at the National Energy Technology Laboratory, Department of Energy (NETL-DOE) was used to measure particle velocities in fluidized beds [21] and CFB risers [22]. Calculating local mean velocity is trivial when there is a sufficient number of particles in the field of view. However, the standard Reynolds decomposition technique is inadequate, especially in regions having a low particle count, and hence a local window averaging technique [19] is used. A temporal extension is provided to the field of view for accurate statistics. As the number of particles in the averaging window is increased, there is a transition from the undersampled regime to the nonstationary regime which represents an optimum window size. The results shown are confirmed to be insensitive to small variations in the averaging window.

III. RESULTS

The measurements from the CFB riser are obtained under dense upflow conditions [22], where the mean particle velocity is positive across any given cross section. The VDFs (Fig. 1) correspond to the histogram of normalized fluctuating velocity component,

$$c_x = \frac{v_x}{\sqrt{2v^2}}, \quad (1)$$

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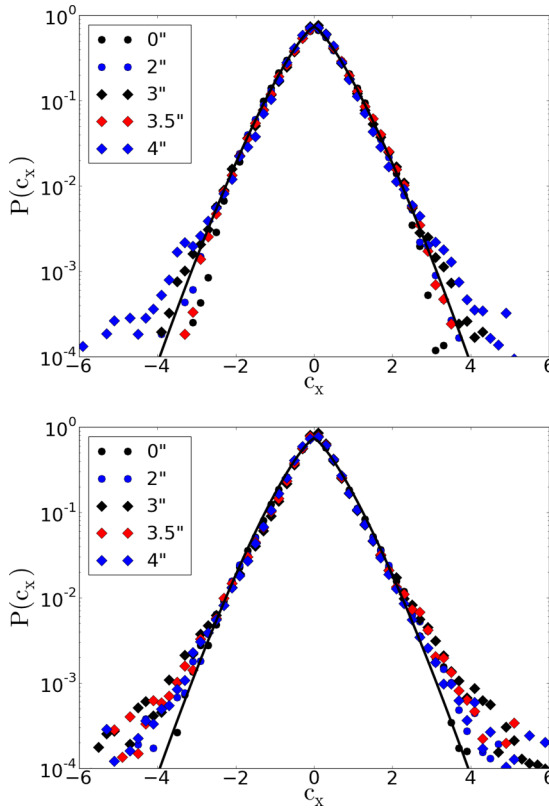


FIG. 1. Fluctuating horizontal VDF measured from the experiments (symbols) at different radial locations for PSRI1 (top) and PSRI2 (bottom). The solid line represents the stretched Gaussian distribution $P(c_x) \sim e^{-1.5|c_x|^{1.3}}$.

where v_x and v represent the fluctuation velocity, and the root mean square (rms) velocity in the transverse direction. The distributions exhibit non-Maxwellian behavior having overpopulated tails. The VDFs up to three standard deviations can be represented by

$$P(c_x) \sim e^{-\alpha|c_x|^\beta}, \quad (2)$$

where $\alpha = 1.5$, $\beta = 1.3$. Similar forms have been reported in Rouyer and Menon [6] and Reis *et al.* [8] based on measurements in a granular bed with vertical vibrations. The difference in the coefficient α and exponent β could be due to a combination of the following factors: difference in the hydrodynamics, difference in dimensionality of the problem, and difference in particle properties. It is interesting to note that the data points can be collapsed onto a single distribution when normalized using v , and the core structure of the VDF is preserved as we move from the center to the wall. The lack of external forcing in the transverse direction results in a symmetric distribution at any given radial location.

Following the theory of van Noije and Ernst [1], the VDF is obtained using the Sonine expansion about the Maxwell-Boltzmann distribution which follows

$$P(c_i) = P_{\text{MB}}(c_i) \left(1 + \sum_{p=1}^{\infty} a_p S_p(c_i^2) \right), \quad (3)$$

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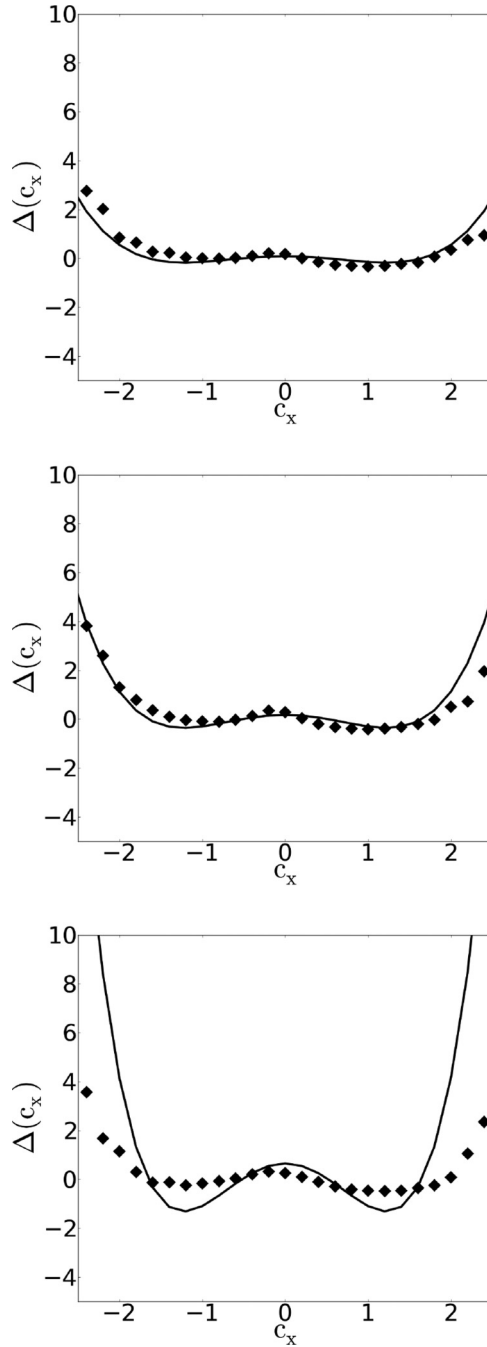


FIG. 2. Deviation from Maxwellian distribution for PSRI1 at $r = 0$ in. (top), $r = 2$ in. (middle), and $r = 4$ in. (bottom). Symbols represent the deviation calculated using the experimental data, and the solid line corresponds to the theoretical deviation from the Maxwell-Boltzmann distribution.

where the normalized Maxwell-Boltzmann distribution is given by

$$P_{\text{MB}}(c_i) = \frac{e^{-c_i^2}}{\sqrt{\pi}}. \quad (4)$$

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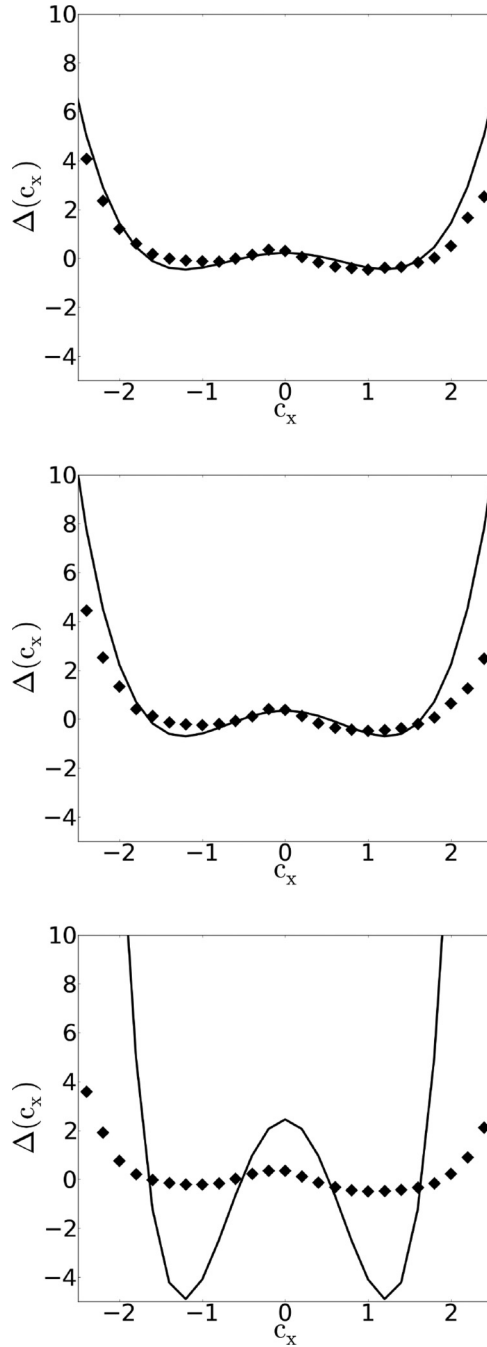


FIG. 3. Deviation from Maxwellian distribution for PSRI2 at $r = 0$ in. (top), $r = 2$ in. (middle), and $r = 4$ in. (bottom). Symbols represent the deviation calculated using the experimental data, and the solid line corresponds to the theoretical deviation from the Maxwell-Boltzmann distribution.

The second-order Sonine polynomial is given by

$$S_2(c_i^2) = \frac{1}{2}c_i^4 - \frac{3}{2}c_i^2 + \frac{3}{8}, \quad (5)$$

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and a_2 , which is the first nontrivial coefficient in the expansion, is given by

$$a_2 = \frac{4}{3} (\langle c_i^4 \rangle - 3 \langle c_i^2 \rangle^2). \quad (6)$$

The deviation from the Maxwell-Boltzmann distribution is representative of kurtosis of the VDF given by

$$\Delta(c_i) = a_2 S_2(c_i^2). \quad (7)$$

Work pertaining to measuring this deviation using experiments is reported in Reis *et al.* [8], and we follow a similar approach for the most part. The coefficient a_2 was used as a fitting parameter by Reis *et al.* [8], which was attributed to sampling noise at high c_i , and was found to be fairly constant for all the conditions considered. The deviation in the current analysis is obtained after evaluating a_2 from the data given the fact that the decomposition technique [19] is proven to yield accurate statistics over a wide range of operating conditions. The measurements are in very good agreement with the theoretical deviation away from the wall but deviate as the near-wall region is approached (Figs. 2 and 3).

The VDFs of normalized streamwise fluctuations $c_y = \frac{v_y}{\sqrt{2v^2}}$ are asymmetric (Fig. 4) at all radial locations. Away from the wall, the histograms lie close to a Gaussian distribution having a noticeable skewness. The inertia of the carrier phase generates a preference for upward velocities resulting in an overpopulated tail in the positive direction. The hydrodynamics change near the wall, and there is a preferential alignment along the direction of gravity. The skewness shifts and the population in the negative end of the tail increases. The shape of the VDF corresponding to vertical fluctuations is

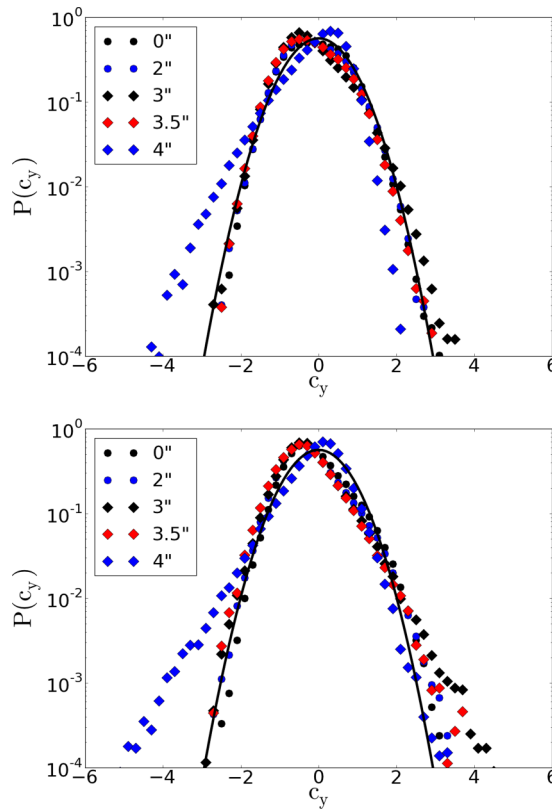


FIG. 4. Fluctuating vertical VDF measured from the experiments (symbols) at different radial locations for PSRI1 (top) and PSRI2 (bottom). The solid line represents the Gaussian distribution $P(c_y) \sim e^{-|c_y|^2}$.

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largely determined by the hydrodynamics, unlike the horizontal VDF, where the particle interactions have a dominant effect.

The results are analogous to the findings of Rouyer *et al.* [23], who observed stretched exponential behavior and anisotropy in VDFs, and attributed this to the Lévy flight of particles. In CFBs, clusters and streamers provide extended trajectories for particles. The interfacial shear exerted by gas on such high-density configurations results in excursions leading to a Lévy flightlike behavior. This deviation from the random walk process is more prominent for PSRI2 compared to PSRI1 where the particle loading is higher. Tracking individual particle trajectories will provide further details regarding the distributions and development of anisotropy. However, this would require a larger field of view representative of mesoscale structures in the system. The streamers could be as long as 1 m for the conditions reported in the current study [24], which makes it challenging to track individual particles having a mean diameter of 81 μm .

A detailed investigation of the VDF is required to ensure accurate modeling of the energy cascade process which influences the macroscopic behavior of the system. Further, the velocity fluctuation statistics could provide invaluable benchmark data for the successful development of computational techniques including discrete element modeling (DEM) and particle-in-cell methodologies.

IV. CONCLUSION

The large-scale CFB experiments using Geldart A (FCC catalyst) particles confirm the existence of nonequilibrium stationary VDFs in forced systems. There is an evident anisotropic behavior in the presence of a unidirectional driving mechanism. The transverse VDF is symmetric, and the shape is determined by the particle interactions. The overpopulation in the tails have been confirmed and a comparison is made with the leading order term of the Sonine expansion. The Lévy flight of particles becomes significant with increasing clustering frequency and wall proximity. The streamwise VDF exhibits asymmetry, and its shape is largely determined by interfacial drag and gravity. The skewness in the VDF shifts as we move from the center to the wall, where the mean particle velocity is less compared to the bulk region. In contrast to the previous bench-scale experiments and simulations, the results are obtained for particle-laden flows having an unsteady mean component in a large-scale system. It is interesting to note the collapse of data given the complexity of the system being analyzed. Our work validates some of the assumptions of the KTGF in the presence of a carrier phase. Further, we highlight the need for corrections due to the presence of nonisotropic forcing and wall boundaries.

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