

Unsteady flow structure and loading of a pitching low-aspect-ratio wing

Miguel R. Visbal*

Air Force Research Laboratory, Wright-Patterson Air Force Base, Ohio 45433, USA

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This study addresses the flow structure and unsteady loading arising over a pitching low-aspect-ratio rectangular wing under low-Reynolds-number conditions of interest in small unmanned aerial vehicle operation and gust interactions. Simulations are performed employing a high-fidelity computational approach capable of accurately capturing the complex unsteady transitional flows. The wing is pitched about its quarter-chord axis to a maximum incidence of 45° over time intervals ranging from four to 16 convective time scales. The Reynolds number based on the wing chord varied from 10^3 to 4×10^4 . For the highest pitch rate, good agreement between the computed three-dimensional (3D) flow structure and recent experimental measurements is demonstrated. The 3D dynamic stall process is characterized by the formation of an initially spanwise-oriented leading-edge vortex which evolves into an arch-type structure with legs anchored to the wing surface. The normal vorticity in the arch vortex legs establishes a low-pressure region and swirling pattern on the wing surface. A distinct characteristic of the arch vortex is its upstream propagation and persistence over the wing, postulated to be the result of the self-induced velocity of the vortex and its image underneath the plate. Increasing either pitch rate or Reynolds number promotes a more coherent arch vortex and circulation pattern, and delays the onset of stall to a higher angle of attack. Even for the lowest pitch rate considered, a significant increase in maximum lift is achieved relative to the static situation.

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I. INTRODUCTION

The process of unsteady separation and stall generated by large transient excursions in angle of attack, referred to in general as dynamic stall, represents a long-standing issue in maneuvering aircraft and rotorcraft applications [1–3]. In recent years, interest in dynamic stall at moderate Reynolds numbers has re-emerged driven by its importance to small unmanned air vehicle system operations [4].

Dynamic stall has been studied extensively [5–12] for effectively two-dimensional wing geometries represented by means of spanwise-periodic computations or wall-to-wall experimental configurations. These studies have provided a great deal of insight into the formation and transition of these dynamic stall vortices and on the highly unsteady loads elicited by their convection along the aerodynamic surface. In small-unmanned vehicle applications, wings may have a short span and therefore the induced three-dimensional nature of the unsteady separation process requires further elucidation. Although the literature on unsteady separation over three-dimensional configurations is not as numerous as its two-dimensional counterpart, several studies have provided valuable information on various aspects of the problem.

The nonlinear lift characteristics for stationary low-aspect-ratio planforms at low Reynolds numbers have been measured by Torres and Mueller [13]. Wind-tunnel smoke visualizations showing the structure of the combined leading-edge and tip vortex system over finite-aspect-ratio wings undergoing different transient maneuvers have been presented by Freymuth *et al.* [14–16]. Experimental investigations addressing the near-wake structure of oscillating low-aspect-ratio wings have been conducted by Von Ellenrieder *et al.* [17] and Buchholz and Smits [18]. Yilmaz and Rockwell [19] provided flow visualizations and PIV measurements above a plunging

*miguel.visbal@us.af.mil

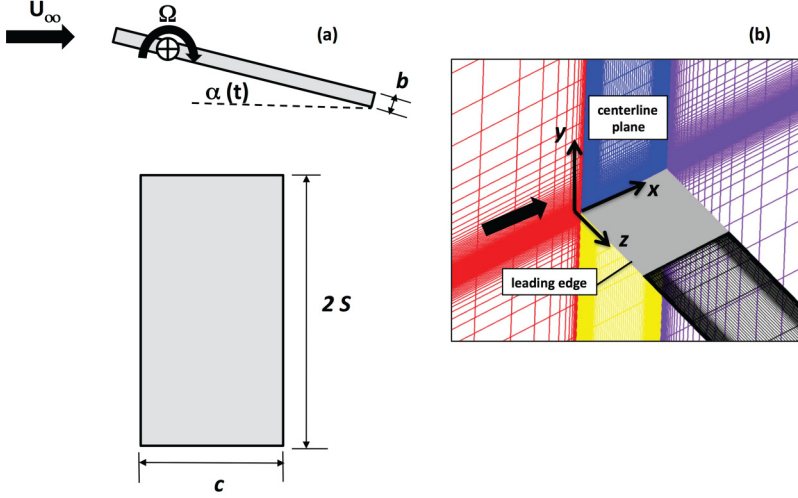


FIG. 1. (a) Pitching wing configuration and (b) overset grid system.

low-aspect-ratio rectangular wing under low-Reynolds-number conditions. Their investigation illustrated the existence of pronounced axial flows within the core of the developing leading-edge vortex (LEV) followed by the formation of large-scale patterns of streamwise-oriented vorticity near the wing centerline. Calderon *et al.* [20] studied the effect of small-amplitude plunging oscillations for low-aspect-ratio wings at poststall angles of attack, and determined optimal planform-dependent forcing frequencies for maximum lift production. Yilmaz and Rockwell [21] examined, by means of volumetric particle image velocimetry techniques, the flow structure generated over rectangular and elliptical wing planforms subjected to a rapid pitch-up maneuver. They found that despite differences in the initial separation process, both geometries exhibit a similar final flow state characterized by the presence of a large-scale swirl pattern on the wing. Calderon *et al.* [22] provided volumetric three-component velocimetry measurements for low-aspect-ratio rectangular and elliptical wings subjected to small amplitude plunging oscillations. The effects of forcing frequency and aspect ratio on the three-dimensional (3D) flow structure was described including the unusual formation of a tip vortex ring. Hartloper *et al.* [23] performed experiments for nominally two-dimensional and aspect-ratio-4 rectangular wings undergoing both slow and fast pitch-up motions. The interaction between the leading-edge and tip vortices as well as their formation time scales were examined.

A few related computational studies of maneuvering finite wings are also worth noting. Taira and Colonius [24] presented a detailed computational study of impulsively translating low-aspect-ratio wings at high incidence and for Reynolds numbers of 300 and 500. They also investigated the effect of simulated actuation in the poststall regime [25]. Visbal [26] performed high-fidelity computations for a heaving low-aspect-ratio wing and Reynolds numbers in the range $10^3 \leq Re_c \leq 2 \times 10^4$. The results were found to be in good agreement with flow visualizations and PIV measurements [19,27]. In Ref. [26], a model of three-dimensional dynamic stall for a low-aspect-ratio plunging wing was proposed which is characterized by the evolution of the leading-edge vortex into an arch vortex with legs normal to the wing surface. In accordance with Helmholtz law, this vortex is eventually shed as a ringlike structure following reconnection on the symmetry plane. However, it is not clear how universal this dynamic stall process and flow morphology are when considering various types of maneuvers, Reynolds numbers, and aspect ratio. This motivates the present study in which pitching maneuvers are examined.

High-fidelity simulations are employed to examine the unsteady flow structure and loading arising over a rectangular wing of aspect ratio 2 [Fig. 1(a)] subjected to high-amplitude transient pitching maneuvers. Although a high-frequency plunging was considered previously [26,27], the present

cases are more representative of the maneuvers and gust interactions encountered by a nonflapping small fixed-wing vehicle. The simulations also complement experimental studies by simultaneously providing the spatially resolved instantaneous flow structure along with surface loading. Furthermore, this work examines the universality of arch-vortex formation for a broader range of pitch rates and Reynolds number.

The finite wing is pitched about its quarter-chord axis to a maximum incidence of 45° with nominal nondimensional angular rates $0.05 \leq \Omega_o^+ = \dot{\alpha}c/U_\infty \leq 0.2$. The Reynolds number based on the wing chord is varied from 10^3 to 4×10^4 . The flow fields are computed employing an extensively validated high-fidelity large-eddy simulation approach [28,29]. Effects of grid resolution and comparison with available experiments [21] are presented for the highest pitch rate considered. In addition, the dynamic stall processes for a two-dimensional (i.e., infinite aspect ratio) and the finite wing are contrasted.

II. METHODOLOGY

A. Governing equations and numerical approach

For these maneuvering wing simulations, the governing equations solved are the unfiltered full compressible Navier-Stokes equations cast in strong conservative form after introducing a general time-dependent curvilinear coordinate transformation from physical to computational space [30–32]. These equations are augmented using the perfect gas relationship, a constant Prandtl number, Sutherland’s viscosity law, and Stokes’ hypothesis for the bulk viscosity coefficient. It should be noted that the governing equations correspond to the original *unfiltered* Navier-Stokes equations and are used without change in laminar, transitional, and turbulent regions of the flow. Unlike the standard large-eddy simulation (LES) approach, no additional subgrid stress and heat flux terms are appended. Instead, a high-order low-pass filter operator (described later) is applied to the conserved dependent variables during the solution of the standard Navier-Stokes equations. This highly discriminating filter selectively damps only the evolving poorly resolved high-frequency content of the solution [28,29]. This filtering regularization procedure provides an effective alternative to the use of standard subgrid-stress (SGS) models, and has been validated extensively for several canonical turbulent flows. A re-interpretation of this implicit LES (ILES) approach in the context of an approximate deconvolution model [33] has been presented by Mathew *et al.* [34]. As the grid resolution increases and/or Reynolds number decreases, this ILES approaches a direct numerical simulation (DNS).

All simulations are performed with the extensively validated high-order Navier-Stokes solver FDL3DI [35,36]. A finite-difference approach is employed to discretize the governing equations, and all spatial derivatives are obtained with high-order compact-differencing schemes [37]. In this investigation, a sixth-order scheme is used at interior points, whereas at boundary points higher-order one-sided formulas are invoked which retain the tridiagonal form of the scheme [35,36]. Typically, Neumann boundary conditions are implemented with third-order one-sided expressions.

For the case of a maneuvering wing, the grid is moved in a rigid fashion using the prescribed airfoil motion. To ensure that the geometric conservation law (GCL) is satisfied, the time metric terms are evaluated employing the procedure described in detail in Ref. [38].

In order to eliminate spurious components, a high-order low-pass spatial filtering technique [35,39] is incorporated. This filtering approach is based on templates proposed in Refs. [37,40] and with proper choice of coefficients provides up to tenth-order accuracy. Filter operators, along with representative filter transfer functions, can be found in Refs. [36,41]. The filter is applied to the conserved variables along each transformed coordinate direction. In the present simulations, an interior eighth-order filter with $\alpha_f = 0.4$ is prescribed. At near-boundary points, the filtering strategies described in Refs. [35,41] are used. For moderate Reynolds number transitional flows, the previous high-fidelity spatial algorithmic components provide a seamless DNS or ILES approach [28,29] in lieu of traditional SGS models. Finally, time marching is accomplished by incorporating an iterative, implicit approximately factored procedure [28,29].

B. Details of computations

Computations were carried out for the low-aspect-ratio plate wing geometry shown in Fig. 1(a) and considered in the experiments of Yilmaz and Rockwell [21]. The wing has an aspect ratio $2S/c = 2$, a uniform thickness $b/c = 0.016$, and all edges are squared off. Computations were performed on a stretched Cartesian mesh system consisting of five overlapped zones [Fig. 1(b)]. The baseline grid (denoted as grid 3) contained approximately 30.1 million points on the half-span configuration and had overall dimensions of $440 \times 469 \times 146$ in the streamwise, normal, and spanwise directions, respectively. The wing surface was discretized employing 245 and 100 points along the chord and half span, respectively. Clustering was provided near the plate in order to capture the near-field complex vortical structure generated by the pitching motion. The minimum normal spacing at the surface was $\Delta y/c = 1 \times 10^{-4}$.

Simulations were performed using the parallel high-order FDL3DI Navier-Stokes solver [42,43]. The baseline mesh was decomposed into a set of 256 overlapped subdomains or blocks which were assigned to individual processors. A five-point interblock overlap was employed in order to retain high-order numerical accuracy. Due to the simple topological structure of the mesh, all overlapping planes were coincident and high-order interpolation was not required.

The boundary conditions were prescribed as follows. Along the entire wing surface, a no-slip adiabatic condition was employed in conjunction with a zero normal pressure gradient. The surface velocity components (u_s, v_s, w_s) were determined from the imposed pitching motion described below. In order to improve spatial resolution with available computational resources, the flow was assumed to be symmetric about the wing centerline. This simplification was considered based on examination of the experiments of Yilmaz and Rockwell [21] which displayed a fairly symmetric overall flow structure during the transient time scales of interest. On the far field boundaries, located approximately 100 chords away from the wing, free-stream conditions were specified. It should be noted that prior to reaching this boundary, the grid is stretched rapidly. This stretching in conjunction with the low-pass spatial filter provides a buffer-type treatment shown previously [44] to be quite effective in reducing spurious reflections.

Pitching simulations were started from the corresponding computed static solutions at a zero-degree angle of attack. The wing is pitched about its quarter-chord axis [Fig. 1(a)] from zero incidence to a maximum angle of attack $\alpha_{\max} = 45^\circ$ and then held fixed. In order to avoid discontinuities in angular acceleration, the angle of attack is prescribed employing a modified version of the ramping function of Eldredge *et al.* [45]. The angle of attack α is given by

$$\alpha(t) = \frac{\Omega_0^+}{2a} \ln \left[\frac{\cosh[a(t - t_1)]}{\cosh[a(t - t_2)]} \right] + \frac{1}{2} \alpha_{\max}. \quad (1)$$

In this equation, $a = 11$ and $t_1 = 0.5$ control the smoothing of the angular acceleration and deceleration, $\Omega_0^+ = \dot{\alpha}c/U_\infty = \alpha_{\max}/(t_2 - t_1)$ is the nominal nondimensional pitch rate, and t_1 and t_2 define the nominal pitching normalized time interval. For future reference, the time elapsed after the nominal start of the pitch up maneuver is defined as $t^* = t - t_1$. Three different rotation rates are considered in this study corresponding to nominal pitch intervals $t_2 - t_1 = 16.0, 8.0$, and 4.0 with corresponding $\Omega_0^+ = 0.049, 0.098$, and 0.196 . For simplicity, these cases are designated as $\Omega_0^+ = 0.05, 0.1$, and 0.2 in the remainder of the paper. Finally, the instantaneous angular rate can be obtained by straightforward differentiation of Eq. (1). The variations of angle of attack an angular rate are shown in Fig. 2 for all pitch rates considered.

A very small computational nondimensional time step $\Delta t = 6.25 \times 10^{-5}$ was prescribed in order to provide sufficient temporal resolution of the transitional fine-scale flow evolution. This value of Δt corresponded to 64 000 time steps during the pitching interval $t_2 - t_1$ for the highest pitch rate considered. Finally, all computations were performed employing a low free-stream Mach number ($M_\infty = 0.1$) in order to effectively eliminate potential compressibility effects.

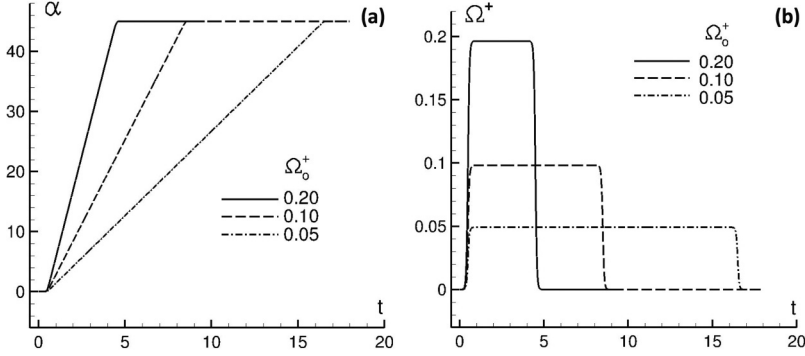


FIG. 2. Pitching maneuver: (a) angle of attack and (b) pitch rate.

C. Grid sensitivity study

In order to provide confidence on the results, the sensitivity of the computed solution to spatial resolution was examined for $\Omega_0^+ = 0.2$. To this end, two additional coarser grids were derived from the baseline 30.1-million mesh. These less resolved grids (denoted as grid 1 and grid 2) contained 15.1 and 7.6 million mesh points respectively. The chordwise and spanwise spacing on the wing varied by 62.5% between the coarsest and baseline mesh. The effect of spatial resolution on the instantaneous flow structure at $\alpha = 30^\circ$ is shown in Figs. 3(a) and 3(b). Although fine-scale features emerged with increased grid resolution, the overall flow structure remained unchanged. The effect of resolution on the computed lift history [Fig. 3(c)] is also found to be negligible. In addition, the instantaneous spanwise distributions of the sectional lift coefficient, displayed in Fig. 3(d) for two angles of attack during pitch-up, did not exhibit significant changes.

III. RESULTS

A. Stationary wing

In order to provide a reference for comparison and to highlight the effects induced by the pitching maneuvers, the flow structure over the stationary wing is briefly considered. Further details for the stationary cases can be found in Refs. [26,46]. Figures 4(a)–4(c) display the flow structure for $\alpha = 8^\circ$ at the baseline Reynolds number $Re_c = 10^4$. The instantaneous three-dimensional flow structure, depicted in terms of an isosurface of total pressure [Fig. 4(a)], is characterized by an initially laminar shear layer separating from the wing leading edge. This shear layer rolls up to form discrete vortical structures which eventually experience breakdown into fine-scale features further downstream. The convergence toward the symmetry plane of the edges of the vortex sheet emanating from the the leading edge is quite apparent [Fig. 4(a)]. As described in Refs. [26], there is a strong transverse flow towards the center plane and the flow is highly three dimensional even at this modest incidence. The instantaneous surface pressure and limiting streamlines on the wing half span along with contours of vorticity magnitude on the midplane ($z/c = 0$) are shown in Fig. 4(b). The instantaneous limiting streamlines display a swirling pattern encompassing a region of low pressure. This feature is observed more clearly in the time-averaged representation [Fig. 4(c)]. The surface topology is characterized by a pair of foci close to the wing front corners, as well as by a well-defined line of attachment. The highest suction is centered about the foci. This overall structure resembles the pattern observed, for instance, in the experiments of Ref. [47] albeit for different conditions ($Re_c = 1.14 \times 10^5$ and $\alpha = 7^\circ$). Separation from the wing tip also results in the formation of a weak tip vortex which induces a very narrow region of lower pressure on the wing surface [Fig. 4(b)]. The spanwise distribution of the time-averaged sectional lift coefficient, shown in Fig. 5, indicates that the direct contribution from the tip vortex to the lift is only minor, as most of the lift arises from the leading-edge separation zone.

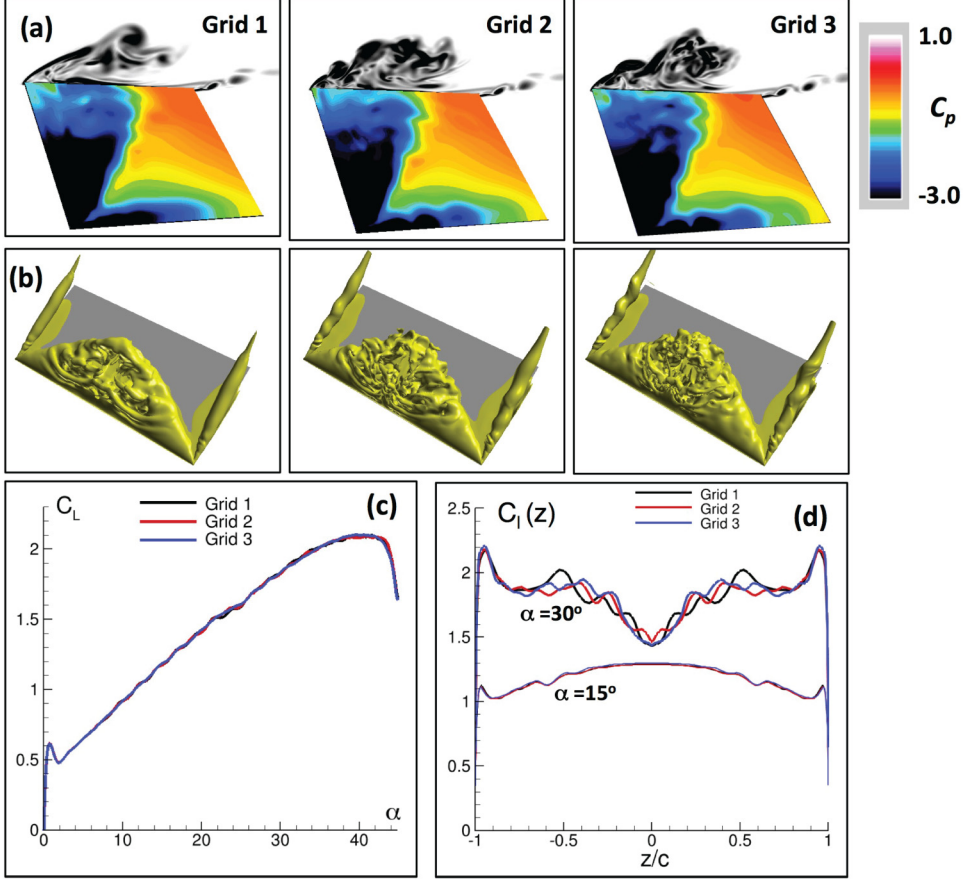


FIG. 3. Effect of grid resolution on unsteady flow structure and loading for pitching wing ($\Omega_o^+ = 0.2, Re_c = 10^4$): (a) instantaneous pressure contours on wing surface (only half span shown) and contours of vorticity magnitude on midplane ($z/c = 0$) at $\alpha = 30^\circ$, (b) isosurface of instantaneous total pressure, (c) lift coefficient histories, and (d) spanwise distributions of sectional lift.

As the angle of attack increases to $\alpha = 15^\circ$ [Figs. 4(d)–4(f)], a more prominent tip vortex emerges as well as a larger region of separation. Relative to $\alpha = 8^\circ$, the edges of the shear layer emanating from the wing leading edge exhibit less convergence towards the plane of symmetry. The surface topology displays a better established circulatory pattern, and an attachment line which in this case does not extend to the wing centerline. The spanwise distribution of the mean sectional lift (Fig. 5) indicates that even at this higher angle of attack, despite the signature of the tip vortex becoming more distinct, the dominant contribution to the aerodynamic lift is still associated with the 3D region of separation above the wing.

Results for the highest static angle of attack considered ($\alpha = 25^\circ$) are shown in Figs. 4(g)–4(i). At this higher incidence, the extent of separated flow normal to the wing increases significantly, as seen in the contours of vorticity magnitude on the symmetry plane [Fig. 4(h)]. The surface flow topology is characterized by foci which are placed further downstream. The line of attachment seen at lower incidence is no longer present. There is still a well-defined circulatory pattern which results in a weaker region of low pressure on the surface. The tip vortices move further away from the wing and as a result, the signature of the tip vortex on the surface pressure is no longer visible. Furthermore, the spanwise distribution of the sectional lift coefficient (Fig. 5) shows no peak associated with the tip vortices but rather a smooth distribution.

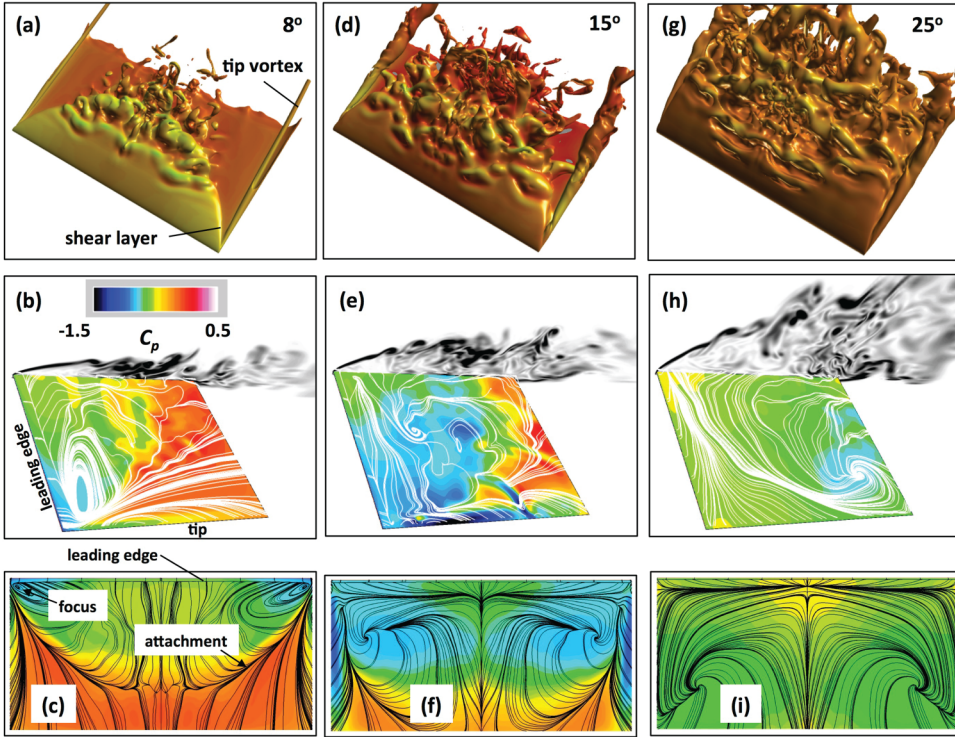


FIG. 4. Flow structure for stationary wing at $\alpha = 8^\circ$, 15° , and 25° and $Re_c = 10^4$: (a),(d),(g) isosurfaces of instantaneous total pressure; (b),(e),(h) instantaneous pressure contours on wing surface (only half span shown) and vorticity magnitude on midplane $z/c = 0$; (c),(f),(i) time-averaged limiting streamlines and pressure on wing surface.

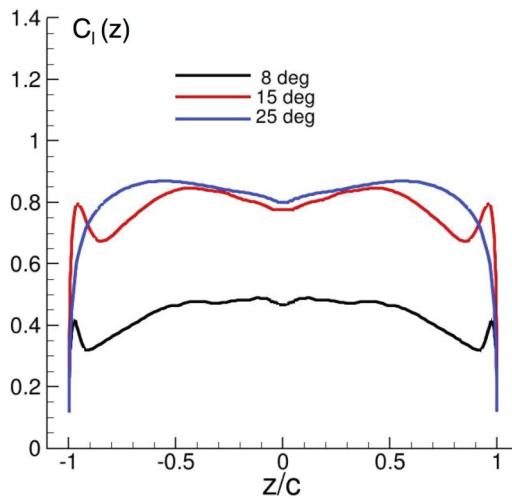


FIG. 5. Spanwise distributions of time-averaged sectional lift for static wing at $\alpha = 8^\circ$, 15° , and 25° , $Re_c = 10^4$.

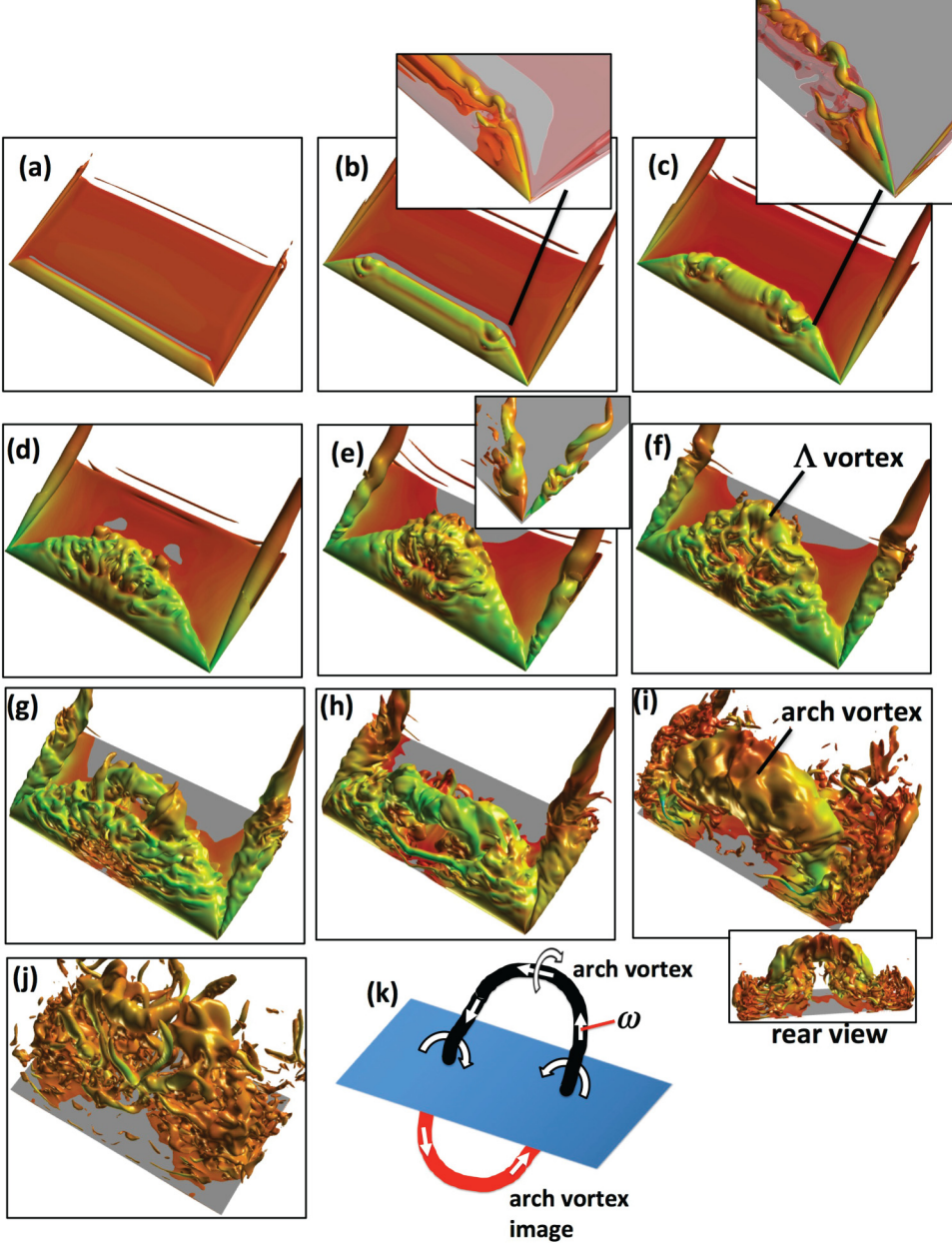


FIG. 6. Instantaneous three-dimensional flow structure for pitching wing ($\Omega_o^+ = 0.2$, $Re_c = 10^4$) depicted by an isosurface of total pressure: (a) 8° , $t^* = 0.71$, (b) 15° , $t^* = 1.33$ (inset shows structure near corner), (c) 20° , $t^* = 1.78$, (d) 25° , $t^* = 2.22$, (e) 30° , $t^* = 2.67$ (inset shows tip vortex breakdown), (f) 35° , $t^* = 3.11$, (g) 40° , $t^* = 3.55$, (h) 45° , $t^* = 4.0$, (i) 45° , $t^* = 6.3$ (inset shows rear view), (j) 45° , $t^* = 8.5$, and (k) schematic of arch vortex and its image below the plate.

B. Flow structure for $\Omega_o^+ = 0.2$

The three-dimensional instantaneous flow structure for the highest pitch rate ($\Omega_o^+ = 0.2$) is shown in Fig. 6 at selected points during the maneuver employing an isosurface of constant total pressure. The corresponding pressure on the wing semispan as well as contours of vorticity magnitude on the

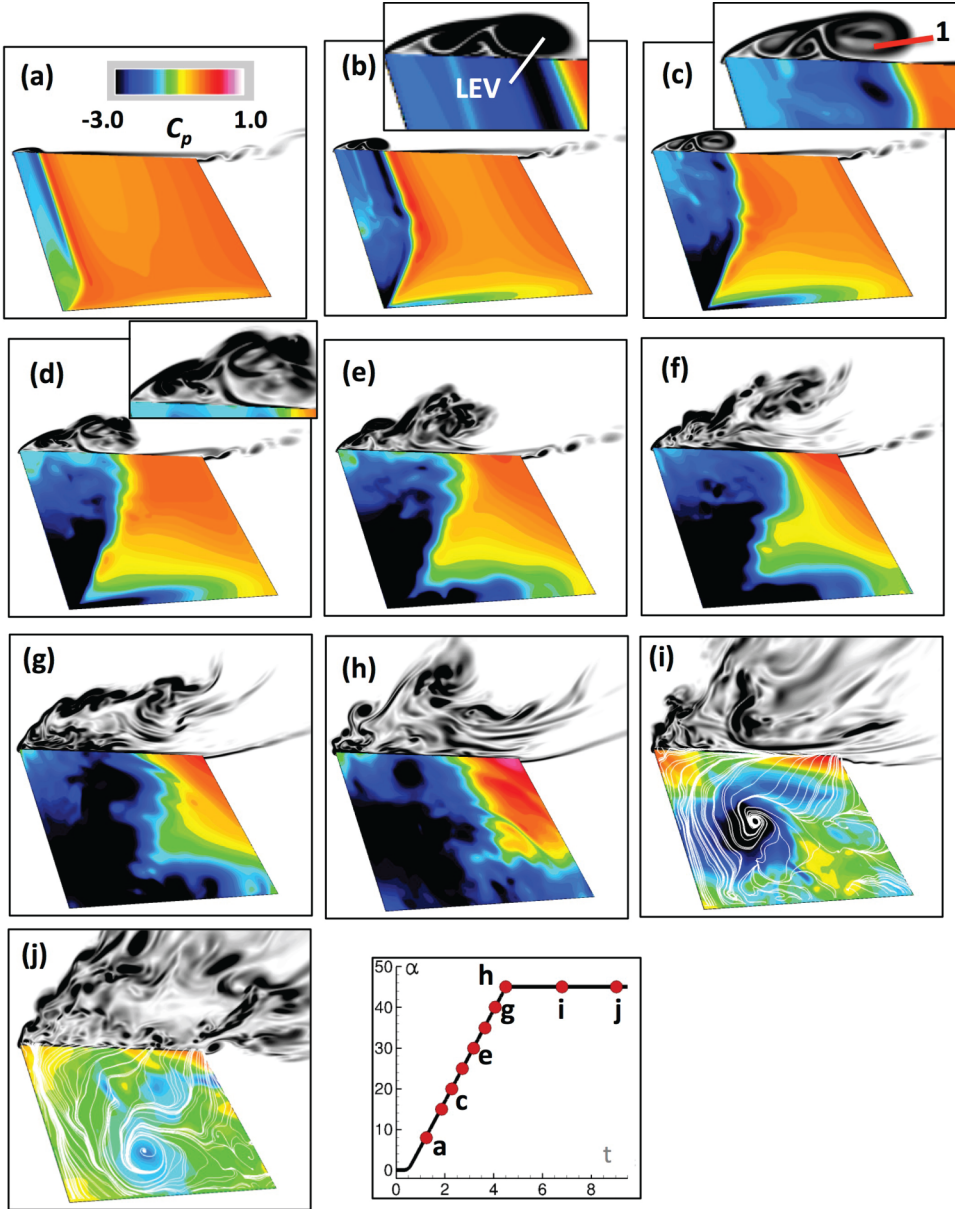


FIG. 7. Instantaneous pressure contours on wing surface (only half span shown) and contours of vorticity magnitude on midplane $z/c = 0$ for $\Omega_o^+ = 0.2$, $Re_c = 10^4$: (a) 8° , $t^* = 0.71$, (b) 15° , $t^* = 1.33$ (inset shows structure of LEV), (c) 20° , $t^* = 1.78$, (d) 25° , $t^* = 2.22$, (e) 30° , $t^* = 2.67$, (f) 35° , $t^* = 3.11$, (g) 40° , $t^* = 3.55$, (h) 45° , $t^* = 4.0$, (i) 45° , $t^* = 6.3$, and (j) 45° , $t^* = 8.5$.

midplane plane are shown in Fig. 7. Following the onset of the pitching motion, one can observe at $\alpha = 8^\circ$ a nascent leading-edge vortex which is pinned at the wing front corners and which appears initially to be fairly uniform across a significant portion of the span [Fig. 6(a)]. However, examination of the surface pressure [Fig. 7(a)] indicates that three-dimensional effects are present, with higher levels of suction found near the wing midplane, consistent with the finite span of the wing. The emergence of three dimensionality is more apparent in the transverse cross section of the

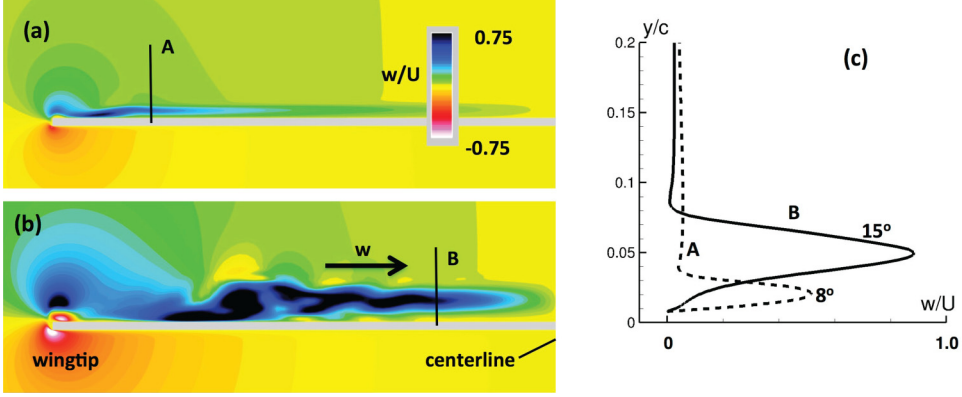


FIG. 8. Spanwise-oriented velocity on a transverse plane through the LEV core over pitching wing with $\Omega_o^+ = 0.2$, $Re_c = 10^4$: (a) $\alpha = 8^\circ$, $x/c = 0.1$, (b) $\alpha = 15^\circ$, $x/c = 0.2$, and (c) spanwise velocity profiles at stations shown in (a) and (b).

vortex depicted in Fig. 8(a) using contours of the spanwise velocity component. One can observe a developing cross-flow current in the core of the leading edge vortex which propagates toward the wing centerline. The w -velocity profile, shown in Fig. 8(c), indicates a maximum axial velocity magnitude of approximately $0.5U_\infty$. This axial vortex flow is common to the structure found in previous experimental and computational studies for a heaving wing [19,27], as well as in the wake of oscillating airfoils [48] and wall jets [49].

By the time the wing has reached $\alpha = 15^\circ$, the flow field exhibits pronounced three-dimensional effects in the leading-edge vortex. The straight portion of the LEV parallel to the wing front edge is reduced in spanwise extent [Fig. 6(b)] as the corner separated region propagates toward the centerline [Fig. 7(b)]. The axial flow within the vortex core has penetrated close to the wing symmetry plane [Fig. 8(b)] and the maximum cross-flow magnitude is of the order of the free-stream velocity ($w_{\max} \approx 0.9U_\infty$). Examination of the vortical structure on the symmetry plane [Fig. 7(b)] indicates that the LEV is initially laminar and similar in appearance to that found in 2D dynamic stall [8,10]. The process of vortex compression associated with the reduction of velocity as the axial current approaches the wing center plane results in the ringlike structure of the LEV denoted as “1” in Fig. 7(c). Nonetheless, at this stage, the LEV still appears to be laminar. It is also of interest to note that by $\alpha = 20^\circ$ the strongest suction on the surface begins to shift from the wing center to underneath the corner shear layer [Fig. 7(b)]. The tip vortex (to be discussed in more detail in Sec. III C 1) also increases in strength. The internal structure of the corner shear layer is displayed in the insets of Figs. 6(b) and 6(c) using a lower value for the total pressure isosurface. The LEV exhibits a spiral near the corner with sense of winding opposite to that of the primary vortex circulation.

By $\alpha = 25^\circ$, the emergence of fine-scale structures is apparent due to the onset of transitional effects [Fig. 6(d)]. This process appears to be enhanced by the strong axial currents within the vortex core which have reached the wing center plane. At this stage, the LEV structure on the symmetry plane [Fig. 7(d)] is less coherent and no longer laminar. With further increase in incidence, the leading-edge vortex system evolves into a Λ shape [Figs. 6(e) and 6(f)] as the vortex lifts away from the plate near the symmetry plane while remaining pinned at the wing front corners. The wing surface pressure displays a region of strong suction underneath the separated region [Figs. 7(d)–7(f)] with the lowest values near the corner where the vortex is in close proximity to the wing.

Eventually the LEV detaches from the edges forming an arch-type vortex with legs normal to the wing surface [Figs. 6(g)–6(i)]. This structure is similar to that found computationally by Visbal [26] for a heaving wing and confirmed by experimental observations [27]. However, in the present case the arch vortex is less compact and its legs are located further outboard.

Under the legs of the arch vortex a concentric region of low pressure emerges on the wing surface and a large-scale swirling pattern is established [Fig. 7(i)]. Animations of the flow field revealed that this region of low pressure (corresponding to the legs of the arch vortex) propagates towards the wing leading edge before it eventually moves downstream and outboard. In contrast with the previous high-frequency plunging study [26,27], in the present situation, a distinct property of the arch vortex is its very long residence time over the wing which in turn induces a strong swirling pattern on the surface. The frames in Figs. 6(j) and 7(j) correspond to an elapsed time of 4.5 convective time scales after the wing has reached its maximum angle of attack. Even at this stage, a weakened arch vortex is still observed over the wing and a region of circulation and low pressure persists on the wing upper surface with implications for loading, as discussed below. The upstream propagation and persistence of the arch vortex is attributable, based on inviscid arguments, to the self-induced velocity generated by its interaction with its image on the plate. This is depicted in the schematic of Fig. 6(k). The arch vortex and its image underneath the plate form a loop or vortex ring which has a self-induced velocity directed upstream. This effect tends to resist the downstream convection of the vortex by the free stream and explains in part the upstream propagation and persistence exhibited by the arch vortex. This represents an interesting finding associated with the three-dimensional unsteady separation process for a low-aspect-ratio wing which contrasts with the 2D counterpart wherein the dynamic stall vortex is readily shed following its detachment from the leading edge [6,8,10]. Further discussion of the comparison between 2D and 3D dynamic stall is provided in Sec. III E.

Comparison with experiment

As noted earlier, the highest pitch rate ($\Omega_o^+ = 0.2$) was selected to allow comparison with recent experiments by Yilmaz and Rockwell [21]. The experimental and computed flow structures are compared in Fig. 9 at two points during the pitching maneuver corresponding to angles of attack prior to and following the formation of the arch vortex. The experimental frames [Figs. 9(a) and 9(e)] show an isosurface of the Q criterion [50] ($Q = 5$) for the phased-averaged flow. These figures clearly display the Λ -type vortical structure and the subsequent formation of the arch-type vortex described earlier. The corresponding instantaneous iso- Q representations for the computed flow field are presented in Figs. 9(b) and 9(f). The computational isosurfaces exhibit fine-scale features not seen in the experiments. This is attributed primarily to the much finer spatial resolution provided by the computational mesh relative to the experimental representation. Differences also arise in part from the phased-averaging employed in the experiments. Unfortunately, a meaningful phase-averaged representation is not possible in the transient computations unless small perturbations (occurring naturally in the experimental setup) are added for different realizations, a process which becomes prohibitive in terms of computational resources.

In order to provide a more direct comparison between experiment and computation, the computed flow field was postprocessed using the PIV-like reduction procedure described in Garmann *et al.* [51]. In this approach, the solution is filtered down to the spatial resolution used in the experimental measurements, which includes the size of the PIV interrogation window, as well as the distance between sampling planes for the volume reconstruction [21]. The Q isosurfaces derived from the filtered computational results are shown in Figs. 9(c) and 9(g). On similar spatial resolutions, the good agreement between the computed and experimental three-dimensional flow structures is now apparent. Both the computation and the experimental measurements display the same key flow elements, including the Λ -shaped, arch and tip vortical structures. The last row in Fig. 9 displays the computed flow structure in terms of an isosurface of stagnation pressure on the highly resolved mesh. One can observe that this variable (used in previous and most subsequent figures) inherently filters out fine-scale features and provides a clearer depiction of the dominant flow elements. The previous favorable comparison between computation and experiment in conjunction with the grid resolution study provide confidence on the unsteady 3D separation process delineated above.

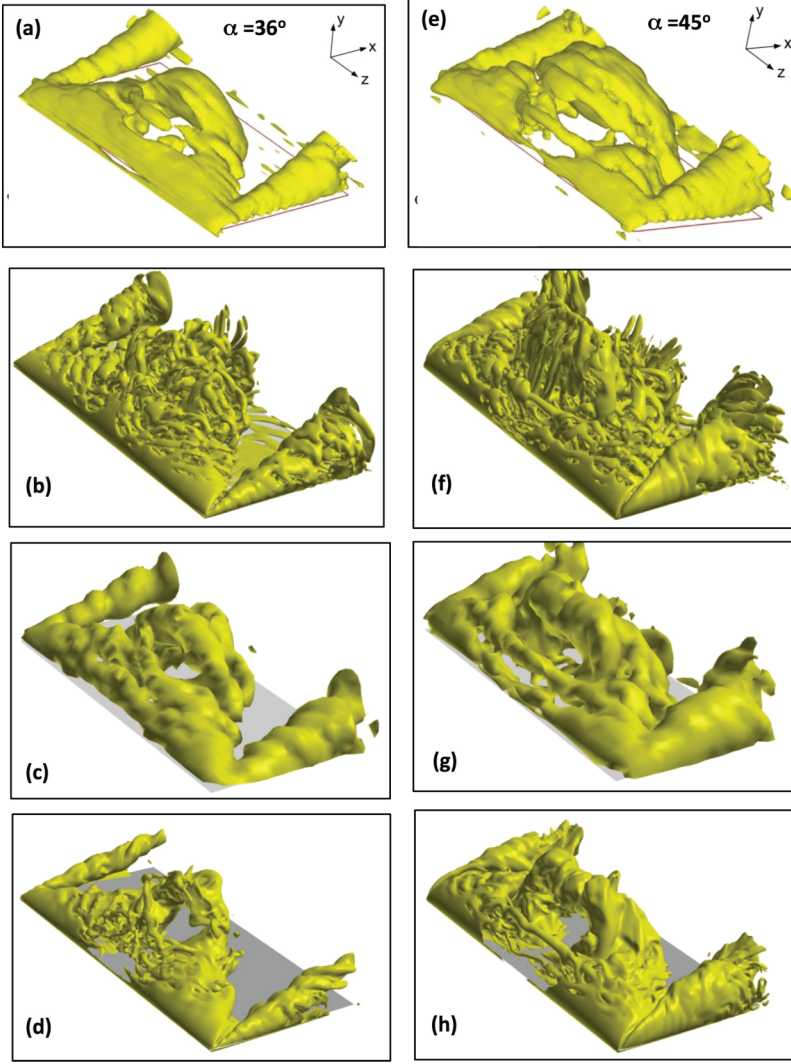


FIG. 9. Comparison of computed and experimental [21] three-dimensional instantaneous flow structure for $Re_c = 10^4$ and $\Omega_o^+ = 0.2$. (a),(e) experimental Q -criterion isosurface, (b),(f) computed Q -criterion isosurface, (c),(g) computed Q -criterion isosurface for filtered computational data, and (d),(h) computed total pressure isosurface.

C. Effect of pitch rate

Results for the lower pitch rates $\Omega_o^+ = 0.1$ and 0.05 are shown in Figs. 10–13. Comparison of all pitch rates indicates that the unsteady 3D separation process is characterized by the same sequence of events starting with the leading-edge vortex formation and culminating with its evolution into an arch-type vortical structure whose legs generate an accompanying swirling motion and a region of low pressure on the wing surface. There are, however, significant effects of pitch rate insofar as the timing and strength of these features. With increasing pitch rate several trends can be observed: (i) the growth of the leading-edge separation region, the onset of transition, as well as the emergence of the arch vortex take place at higher incidence, (ii) the initial LEV vortex and arch vortex become

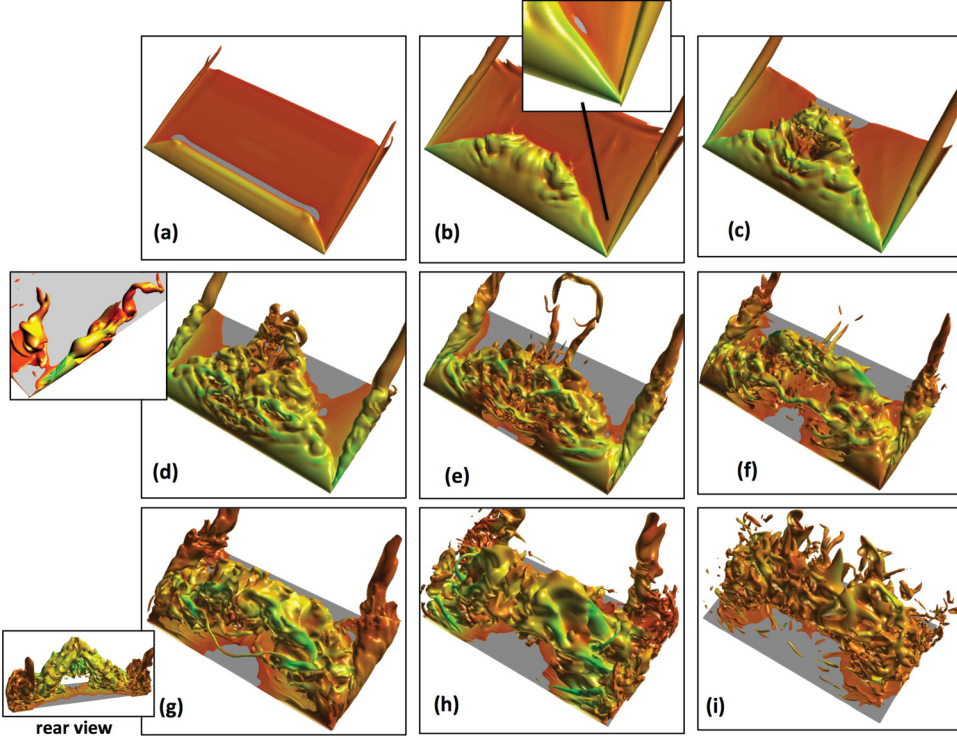


FIG. 10. Instantaneous three-dimensional flow structure for pitching wing ($\Omega_o^+ = 0.1$, $Re_c = 10^4$) depicted by an isosurface of total pressure: (a) 8° , $t^* = 1.42$, (b) 15° , $t^* = 2.67$ (inset shows structure near corner), (c) 20° , $t^* = 3.55$, (d) 25° , $t^* = 4.44$ (inset shows tip vortex breakdown), (e) 30° , $t^* = 5.33$, (f) 35° , $t^* = 6.22$, (g) 40° , $t^* = 7.11$ (inset shows rear view), (h) 45° , $t^* = 8.5$, and (i) 45° , $t^* = 11.5$.

more compact, and (iii) lower values of pressure and a stronger swirling pattern are observed on the wing surface.

The case with $\Omega_o^+ = 0.1$ illustrates more clearly the dynamics of the arch vortex with its accompanying low-pressure region and circulatory pattern on the wing. As the arch-vortex fully emerges between $\alpha = 30^\circ$ and 40° [Figs. 10(e)–10(g)], the surface pressure [Figs. 11(e)–11(g)] evolves into a more coherent concentric distribution with progressively lower values of minimum pressure. This process is due to the establishment of a circulatory pattern on the wing surface associated with the normal vorticity of the legs of the arch vortex. The approximate center of this low-pressure region propagates upstream and gets close to the wing leading edge at the instant shown in Fig. 11(h) before subsequently moving downstream [Fig. 11(i)]. This upstream movement of the arch vortex against the prevailing flow is again consistent with the self-induced velocity of the vortex and its image system described earlier in reference to Fig. 6(k). The long residence time of the arch vortex and swirling motion above the wing is also observed for $\Omega_o^+ = 0.1$. The last frames in Figs. 10 and 11 show that both features are still well defined even after a time interval of 3.5 convective time scales has elapsed following cessation of the pitch-up motion.

1. Tip vortex evolution

Another important flow element studied in both experiments [20,21,23] and computations [24,27] of maneuvering finite wings is the tip vortex. For a rectangular planform with a sharp corner, the leading-edge and tip vortices emerge as distinct structures, as shown in the inset of Fig. 10(b). The vorticity in the leading-edge separated region is oriented predominantly in the spanwise direction

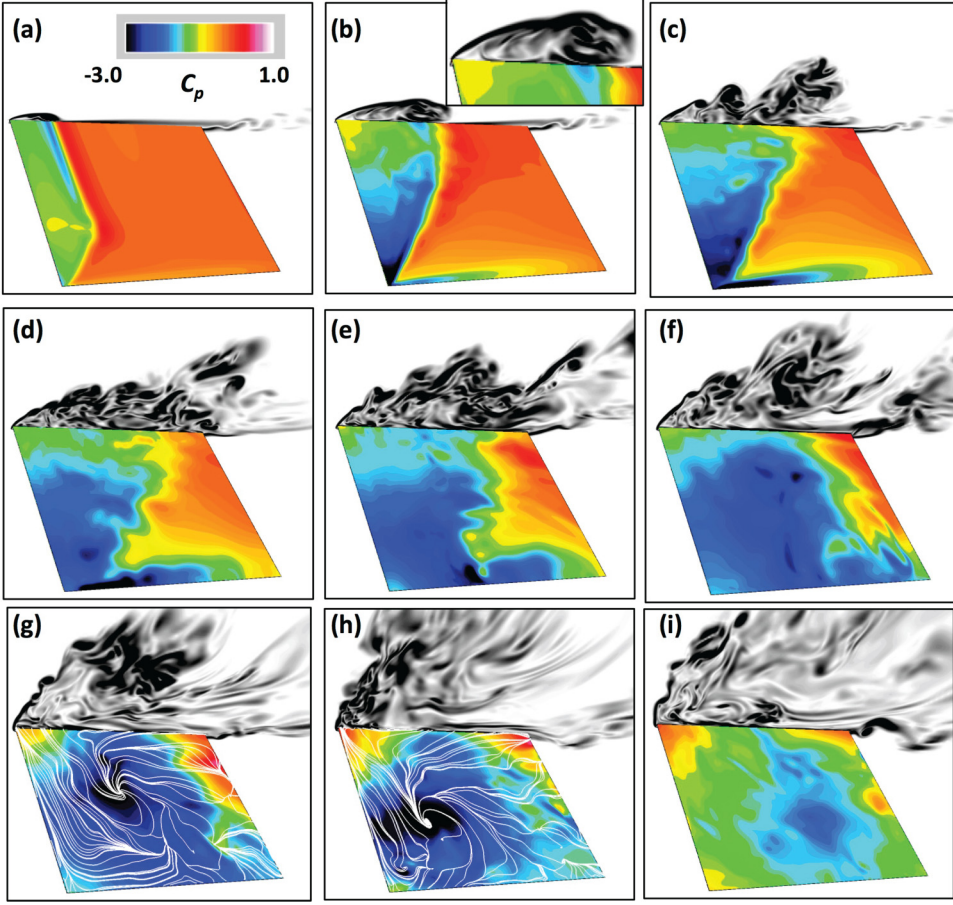


FIG. 11. Instantaneous pressure contours on wing surface (only half span shown) and contours of vorticity magnitude on midplane $z/c = 0$ for $\Omega_o^+ = 0.1$, $Re_c = 10^4$: (a) 8° , $t^* = 1.42$, (b) 15° , $t^* = 2.67$, (c) 20° , $t^* = 3.55$, (d) 25° , $t^* = 4.44$, (e) 30° , $t^* = 5.33$, (f) 35° , $t^* = 6.22$, (g) 40° , $t^* = 7.11$, (h) 45° , $t^* = 8.5$, and (i) 45° , $t^* = 11.5$.

whereas the tip vortex is directed in the streamwise direction. As such, the dynamics of the tip vortex should exhibit some similarities with vortex development on maneuvering highly swept delta wings.

The overall evolution of the tip vortex can be observed for all pitch rates in the corresponding isosurface plots of Figs. 6, 10, and 12. At low angles of attack, while the tip vortex is still close to the wing, its signature on the surface pressure is also discernible [e.g., Figs. 11(b)–11(d)]. The region of suction induced by the tip vortex is considerably smaller than that associated with the main leading-edge separation zone and hence it has a limited direct effect on the total aerodynamic lift, as discussed in the next section. Beyond a certain angle of attack, the tip vortex exhibits breakdown as shown in the insets of Figs. 6(e), 10(d), and 12(c) employing an isosurface of lower total pressure which highlights the vortex core. With increasing pitch rate, the onset of vortex breakdown takes place at a higher incidence. For instance, for $\Omega_o^+ = 0.05$, breakdown is well established by $\alpha = 20^\circ$ [Fig. 12(c)]. This breakdown is characterized by a bubble-type bursting with a spiral tail which winds in a sense opposite to the prevailing tip vortex swirl. This structure is similar to that observed during the onset of vortex breakdown above a pitching delta wing [52,53]. At the highest pitch rate, vortex breakdown emerges at $\alpha = 30^\circ$ [Fig. 6(e)]. Its shape is more conical in nature and exhibits the features of a spiral-type breakdown. Examination of flow animations indicated that although the

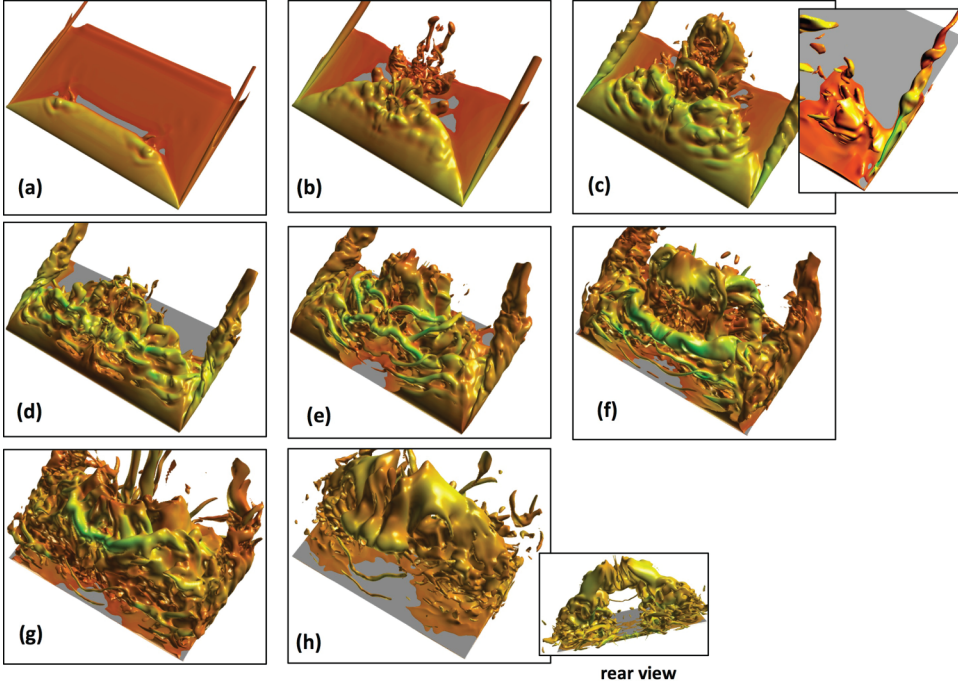


FIG. 12. Instantaneous three-dimensional flow structure for pitching wing ($\Omega_o^+ = 0.05$, $\text{Re}_c = 10^4$) depicted by an isosurface of total pressure: (a) 8° , $t^* = 2.84$, (b) 15° , $t^* = 5.33$, (c) 20° , $t^* = 7.11$ (inset shows tip vortex breakdown), (d) 25° , $t^* = 8.89$, (e) 30° , $t^* = 10.67$, (f) 35° , $t^* = 12.44$, (g) 40° , $t^* = 14.22$, and (h) 45° , $t^* = 17.5$ (inset shows rear view).

onset of bursting occurs at higher α for increasing Ω_o^+ , once established, it propagates upstream with a speed which increases with pitch rate. The features seen here for the tip vortex breakdown as a function of Ω_o^+ namely, the onset angle, the type of breakdown, and its propagation speed are qualitatively consistent with observations over pitching delta wings [54,55].

The cross-sectional structure of the tip vortex in the transverse plane at midchord ($x/c = 0.5$) is examined in more detail for all pitch rates. Figure 14 shows the flow over the outboard portion of the wing using contours of vorticity magnitude at $\alpha = 8^\circ$. For $\Omega_o^+ = 0.2$, the tip vortex is observed to be more compact and closer to the wing relative to the static and lower pitch-rate cases. This effect is consistent with the behavior observed on a delta wing following a sudden change in incidence. As shown in the experiments of Lambourne *et al.* [56], a time interval of the order of one convective time is required for the delta wing vortex to adjust to its new condition. At $\alpha = 8^\circ$, the time elapsed for the highest pitch rate is only $t^* = 0.7$ which is not quite sufficient for the tip vortex to develop. In contrast, for $\Omega_o^+ = 0.05, 0.1$, $t^* = 1.4, 2.8$, which allows for the tip vortex to approach the static case, assuming the pitch-rate independence described next.

Following its initial formation and prior to the onset of breakdown, the effect of pitch rate on the tip vortex structure at a given angle is not pronounced. For instance, at $\alpha = 15^\circ$ (Fig. 15), the tip vortex structure is similar for all pitch rates including the static case (i.e., $\Omega_o^+ = 0.0$). This contrasts with the evolution of the leading-edge separation region which lags significantly with increasing pitch rate. A more quantitative comparison is shown in Figs. 15(e) and 15(f) in terms of the axial and swirl velocity components in the vortex core. While the axial velocity profiles exhibit some differences in the vortex viscous core, the swirl velocities are in remarkable agreement with each other despite the significant variation in pitch rate. The fact that the swirl profiles are so congruent, even without the need to scale with local vortex size, supports the argument that the formation time

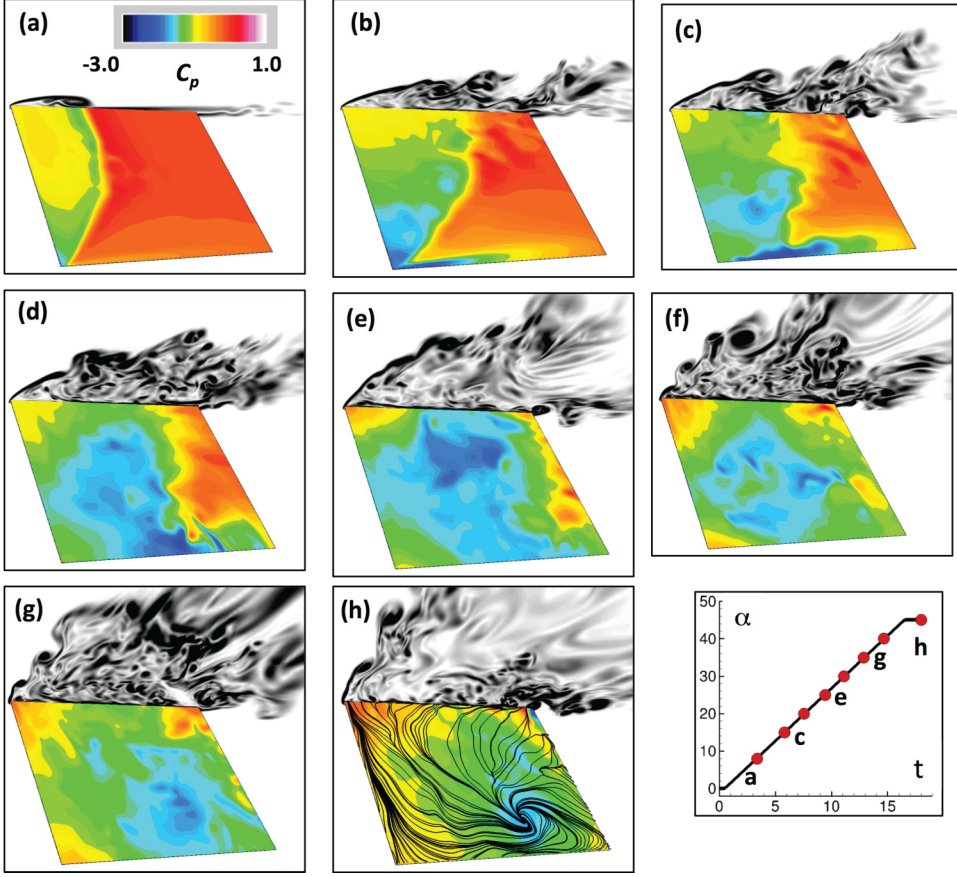


FIG. 13. Instantaneous pressure contours on wing surface (only half span shown) and contours of vorticity magnitude on midplane $z/c = 0$ for $\Omega_o^+ = 0.05$, $Re_c = 10^4$: (a) 8° , $t^* = 2.84$, (b) 15° , $t^* = 5.33$, (c) 20° , $t^* = 7.11$, (d) 25° , $t^* = 8.89$, (e) 30° , $t^* = 10.67$, (f) 35° , $t^* = 12.44$, (g) 40° , $t^* = 14.22$, and (h) 45° , $t^* = 17.5$.

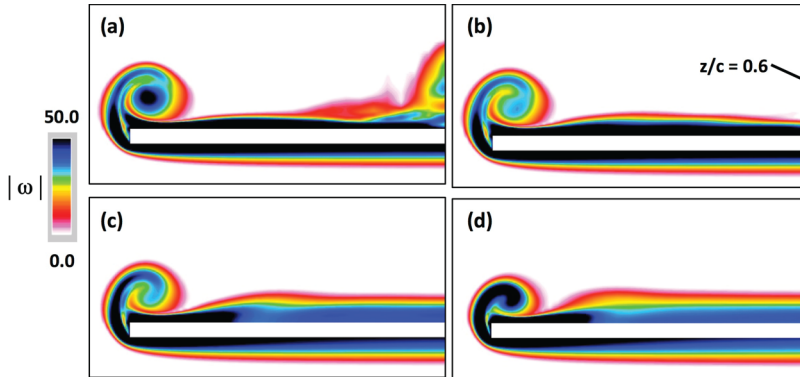


FIG. 14. Effect of pitch rate on wing tip vortex development at $\alpha = 8^\circ$ depicted using contours of vorticity magnitude on transverse plane at midchord ($x/c = 0.5$): (a) static, $\Omega_o^+ = 0.0$, (b) $\Omega_o^+ = 0.05$, $t^* = 2.84$, (c) $\Omega_o^+ = 0.1$, $t^* = 1.42$, and (d) $\Omega_o^+ = 0.2$, $t^* = 0.71$.

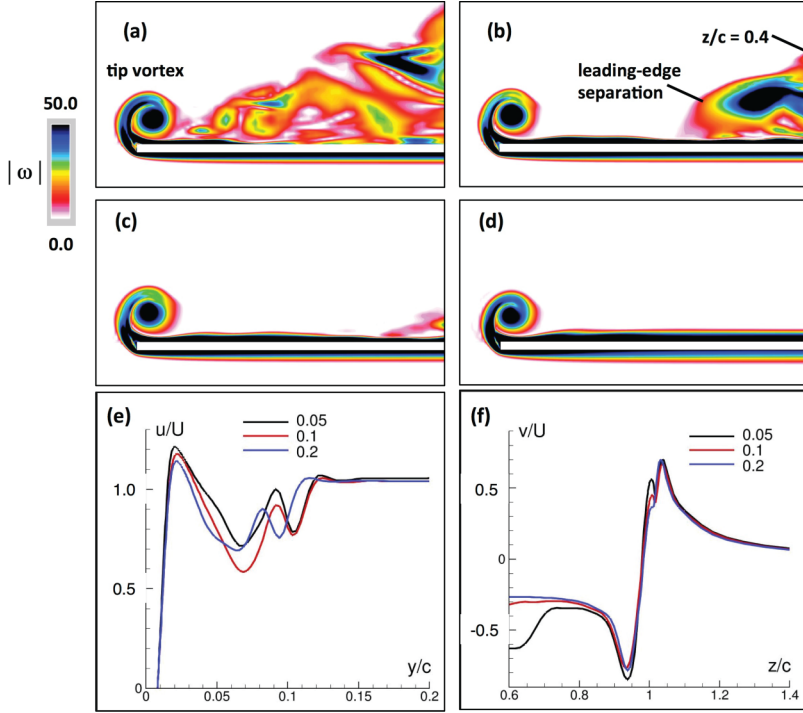


FIG. 15. Effect of pitch rate on wing tip vortex development at $\alpha = 15^\circ$ depicted using contours of vorticity magnitude on transverse plane at midchord ($x/c = 0.5$): (a) static, $\Omega_o^+ = 0.0$, (b) $\Omega_o^+ = 0.05$, $t^* = 5.33$, (c) $\Omega_o^+ = 0.1$, $t^* = 2.67$, (d) $\Omega_o^+ = 0.2$, $t^* = 1.33$, (e) corresponding axial velocity profiles at a z -constant line through tip vortex center, and (f) swirl velocity along y -constant line through tip vortex center.

scale of the predominantly streamwise-oriented tip vortex plays a dominant role. The tip vortex is therefore seen to evolve somewhat independently of the leading-edge vortex and bears resemblance to vortex formation from a swept edge, as found in delta wings. A similar good correlation of the tip vortex as function of incidence for slow and fast pitching maneuvers of a low-aspect-ratio wing has been noted in the experiments of Ref. [23].

The aforementioned independence of the tip vortex from pitch rate at a given incidence is however lost once vortex breakdown emerges over the wing. For instance, by $\alpha = 20^\circ$ (Fig. 16), significant differences are apparent as a function of pitch rate. For $\Omega_o^+ = 0.05$, vortex breakdown has considerably changed the structure of the tip vortex core which at this stage exhibits a significant region of reversed axial flow [Fig. 16(d)]. For the intermediate pitch rate, incipient vortex breakdown is observed whereas at the highest Ω_o^+ breakdown is still absent at this angle of attack.

2. Aerodynamic loading

The effect of pitch rate on the wing aerodynamic loading is considered next. The aerodynamic coefficient histories are presented in Fig. 17. Several trends are apparent in the lift and pitching moment coefficient histories as a function of pitch rate. In all cases, there is a noncirculatory spike in the lift and pitching moment induced by the angular acceleration [Figs. 17(a) and 17(c)]. The peak in lift ΔC_{L1} displays a linear variation as a function of pitch rate, as shown in Fig. 17(d). Following this initial surge, the lift distribution becomes fairly linear with a slope close to the theoretical [57] value of π (in radians) but shifted by an offset ΔC_{L2} [as defined in the inset of Fig. 17(d)]. Figure 17(d) also displays the quantity denoted as ΔC_M which corresponds to the magnitude of the approximately constant negative value of pitching moment established after the end of angular acceleration. For

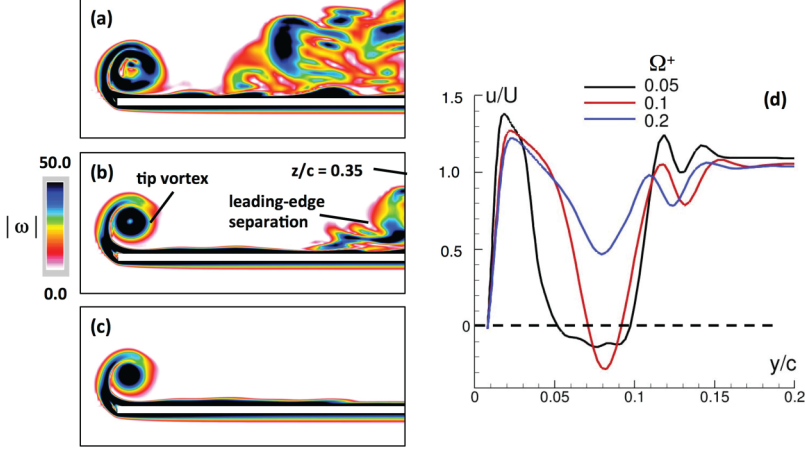


FIG. 16. Effect of pitch rate on wing tip vortex development at $\alpha = 20^\circ$ depicted using contours of vorticity magnitude on transverse plane at midchord ($x/c = 0.5$): (a) $\Omega_o^+ = 0.05$, $t^* = 7.11$, (b) $\Omega_o^+ = 0.1$, $t^* = 3.55$, (c) $\Omega_o^+ = 0.2$, $t^* = 1.78$; (d) corresponding axial velocity profiles along a z -constant line through tip vortex center.

convenience, this value is selected at $\alpha = 4^\circ$ at which point a constant angular velocity prevails even for the highest Ω_o^+ considered. The offset values ΔC_{L2} and ΔC_M also exhibit an effectively linear dependence with pitch rate. The offsets in lift and moment coefficients observed here after the wing reaches a constant angular rate are consistent with a rotation-induced chordwise camber [55,58,59].

The overall history of the lift coefficient [Fig. 17(a)] indicates that both the maximum value of C_L and its corresponding angular position ($\alpha_{CL_{\max}}$) increase with pitch rate. The values of $C_{L_{\max}}$ and $\alpha_{CL_{\max}}$ for $\Omega_o^+ = 0.05$, 0.1, and 0.2 are approximately $(1.5, 31.3^\circ)$, $(1.89, 38.4^\circ)$, and $(2.1, 40.6^\circ)$ respectively. Even for the smallest pitch rate, there is a significant increase in maximum lift which is almost twice the maximum static value [Fig. 17(a)]. The moment coefficient histories [Fig. 17(c)] show that after a fairly flat initial distribution a rapid drop in C_M ensues at an angle which increases with Ω_o^+ .

Figure 17(e) displays the chordwise position of the center of lift X_{CL} as a function of angle of attack. This parameter was computed for only a few available flow fields after the computations had been performed and therefore the distributions are rather coarse. Nonetheless, a few trends can be observed. For the static case, X_{CL} moves downstream with increasing incidence as found experimentally [13]. For the lowest pitch rate, the center of lift is upstream of the static situation ($8^\circ \leq \alpha \leq 25^\circ$) as it would be expected from lag effects in the separation process. This is not the case at low angle of attack at the higher pitch rates for which X_{CL} increases with Ω_o^+ . This effect is due to the higher pressures on the wing lower surface induced by rotation. At higher angles of attack ($\alpha \geq 25^\circ$), as the separated flow and its low pressure region have evolved further, the center of lift is found to be positioned consistently further upstream with increasing pitch rate in accordance with lag effects. An interesting feature for all pitching cases is the local dip in X_{CL} observed in the distributions (for instance, at $\alpha = 40^\circ$ for $\Omega_o^+ = 0.1$ and $\alpha = 30^\circ$ for $\Omega_o^+ = 0.05$). This feature correlates with the previously noted temporary movement toward the wing leading edge and subsequent downstream convection of the arch vortex and its corresponding swirling region of low pressure.

Selected sectional lift distributions for the intermediate pitch rate $\Omega_o^+ = 0.1$ are shown in Fig. 17(f). The distributions for the other two pitch rates were found to be qualitatively similar and are not included. Prior to the emergence of the arch vortex (see distributions at $\alpha = 8^\circ, 15^\circ, 25^\circ$), the sectional lift displays a mild variation across a significant portion of the span, similar to the static situation (Fig. 5). At $\alpha = 25^\circ$, several undulations begin to emerge near midspan and well-defined peaks [with the highest value of $C_l(z)$] are induced by the tip vortices. There is a transformation in

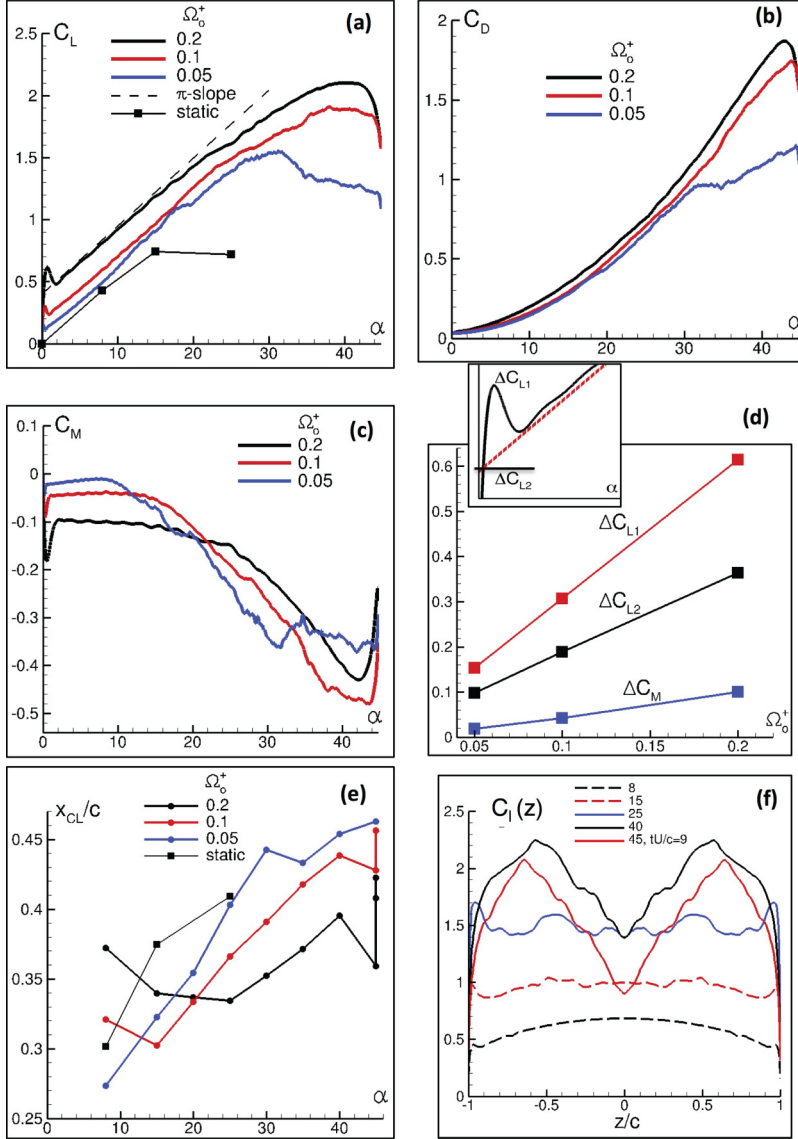


FIG. 17. Effect of nondimensional pitch rate on wing aerodynamic loading for $Re_c = 10^4$: (a)–(c) lift, drag, and quarter-chord pitching moment coefficient histories, (d) offsets in C_L and C_M due to wing rotation, (e) chordwise position of center of lift, and (f) spanwise distribution of sectional lift for $\Omega_o^+ = 0.1$.

the character of the sectional lift distribution following the appearance of the arch vortex ($\alpha = 40^\circ$). The sectional lift exhibits a strong broad peak on each side of the wing associated with the legs of the arch vortex and the induced swirling pattern over the wing. No signature of the tip vortex is apparent at this stage of the flow. The distributions at $\alpha = 40^\circ, 45^\circ$ also show the outboard movement of the center of the circulatory pattern as lift stall ensues.

D. Effect of Reynolds number

The effect of Reynolds number on the flow structure and loading generated by the pitching wing was investigated for the lowest pitch rate ($\Omega_o^+ = 0.05$) by considering two additional values of Re_c

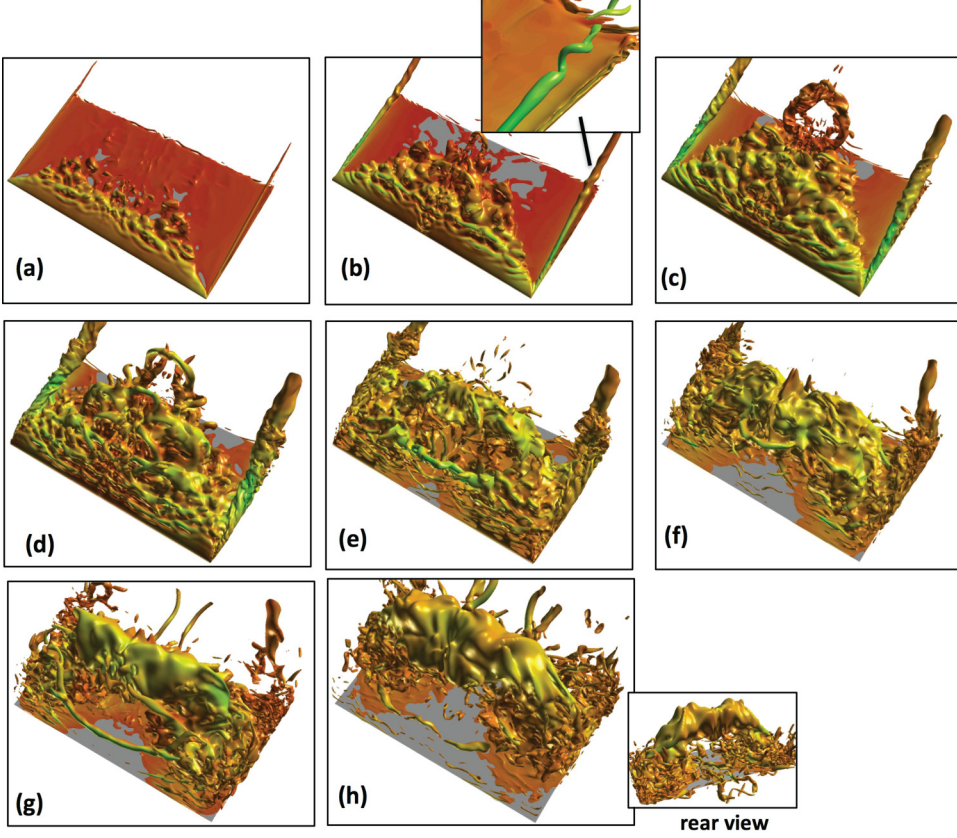


FIG. 18. Instantaneous three-dimensional flow structure for pitching wing ($\Omega_o^+ = 0.05$, $Re_c = 4 \times 10^4$) depicted by an isosurface of total pressure: (a) 8° , $t^* = 2.84$, (b) 15° , $t^* = 5.33$ (inset shows tip vortex spiral breakdown), (c) 20° , $t^* = 7.11$, (d) 25° , $t^* = 8.89$, (e) 30° , $t^* = 10.67$, (f) 35° , $t^* = 12.44$, (g) 40° , $t^* = 14.22$, and (h) 45° , $t^* = 17.5$ (inset shows rear view).

(10^3 and 4×10^4). The instantaneous flow structure for $Re_c = 4 \times 10^4$ is shown in Figs. 18 and 19. As seen previously for $Re_c = 10^4$, the flow evolution is again characterized by the transformation of the LEV into an arch-type structure with its accompanying swirling pattern over the wing surface. The arch vortex appears to be more compact, as can be observed by comparison of Figs. 12(h) and 18(h). The higher Reynolds number case exhibits earlier transition of the leading-edge vortex, finer scale features, as well as premature spiral breakdown of the tip vortex [inset of Fig. 18(b)]. In addition, lower pressures are also observed on the wing surface with increased Re_c [e.g., Figs. 13(g) and 19(g)].

The case of $Re_c = 10^3$ is displayed in Figs. 20 and 21. At this very low value of Reynolds number, one can observe throughout the entire pitch-up the lack of fine-scale flow features associated with transition. At the earlier stages of the flow evolution [$\alpha = 20^\circ, 25^\circ$, Figs. 20(c) and 20(d)], the structure is characterized by a large-scale leading edge vortex and by the formation of hairpinlike vortices near the wing trailing edge. These features display a broad similarity with the investigation of Taira and Colonius [24] for an impulsively translating wing at $Re_c = 300$ and $\alpha = 30^\circ$. From $\alpha = 30^\circ$ onwards [Figs. 20(f)–20(h)], the emergence of the arch-type vortex is apparent. The surface pressure contours and limiting streamlines in Fig. 21 show again the induced circulatory motion with much weaker suction relative to the higher Reynolds number cases.

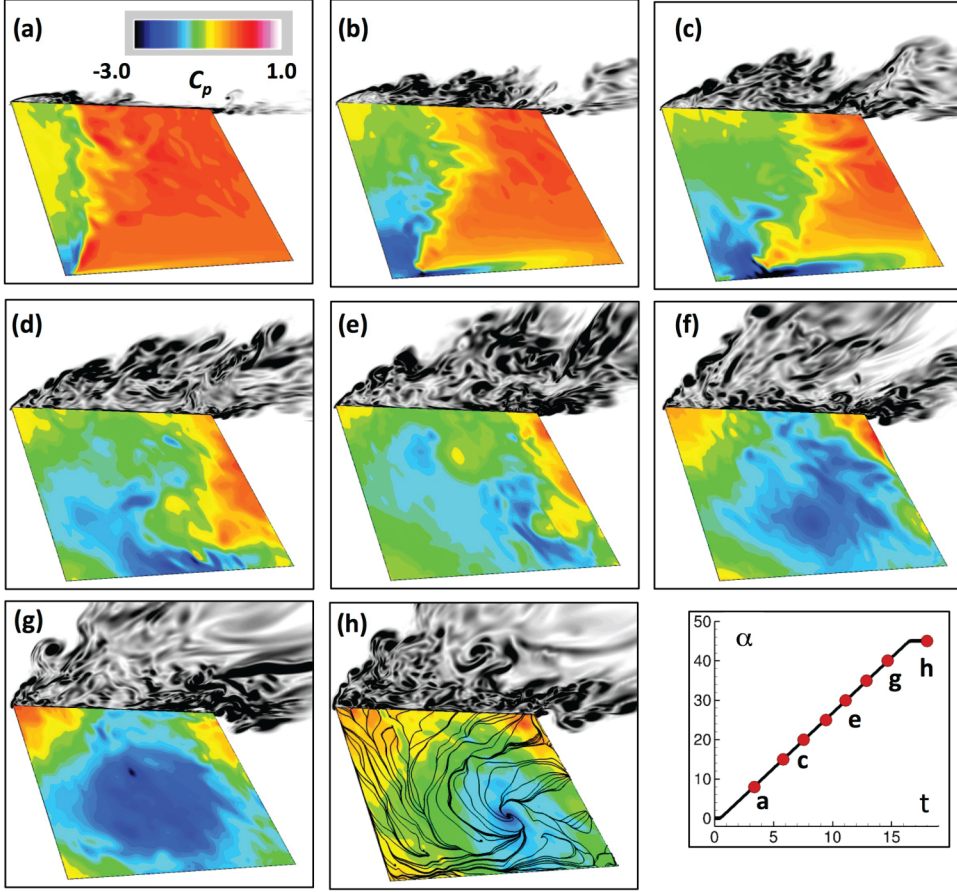


FIG. 19. Instantaneous pressure contours on wing surface (only half span shown) and contours of vorticity magnitude on midplane $z/c = 0$ for $\Omega_o^+ = 0.05$, $Re_c = 4 \times 10^4$: (a) 8° , $t^* = 2.84$, (b) 15° , $t^* = 5.33$, (c) 20° , $t^* = 7.11$, (d) 25° , $t^* = 8.89$, (e) 30° , $t^* = 10.67$, (f) 35° , $t^* = 12.44$, (g) 40° , $t^* = 14.22$, and (h) 45° , $t^* = 17.5$.

The histories of the aerodynamic lift, shown in Fig. 22(a), exhibit significant quantitative effects of the Reynolds number. As the Reynolds number increases, there is an increase in maximum lift and a delay in lift stall. Nonetheless, the lift distributions for the two highest Reynolds numbers are in reasonable agreement prior to stall. The C_L distribution for $Re_c = 10^3$ is however quite different, with a lower lift slope and a much smoother character. The pitching moment coefficient histories [Fig. 22(b)] exhibit a more pronounced drop with increasing Re_c . Again, the C_M distributions for $Re_c = 10^4$ and $Re_c = 4 \times 10^4$ are in good agreement up to $\alpha \approx 30^\circ$.

E. Comparison of 2D and 3D dynamic stall process for a pitching wing

In order to contrast the fundamental differences between the dynamic stall process for a 2D and a low-aspect-ratio wing, computations were also performed for an effectively infinite-aspect-ratio plate, emulated by means of spanwise periodic conditions on a prescribed spanwidth ($s/c = 0.4$). The sectional geometry and computational grid were the same as in the previously described 3D cases. Only the intermediate values of pitch rate $\Omega^+ = 0.1$ and Reynolds number ($Re_c = 10^4$) were considered for the 2D pitching configuration. The corresponding flow field evolution is presented in Fig. 23. The process of 2D dynamic stall is characterized by the formation of an initially laminar

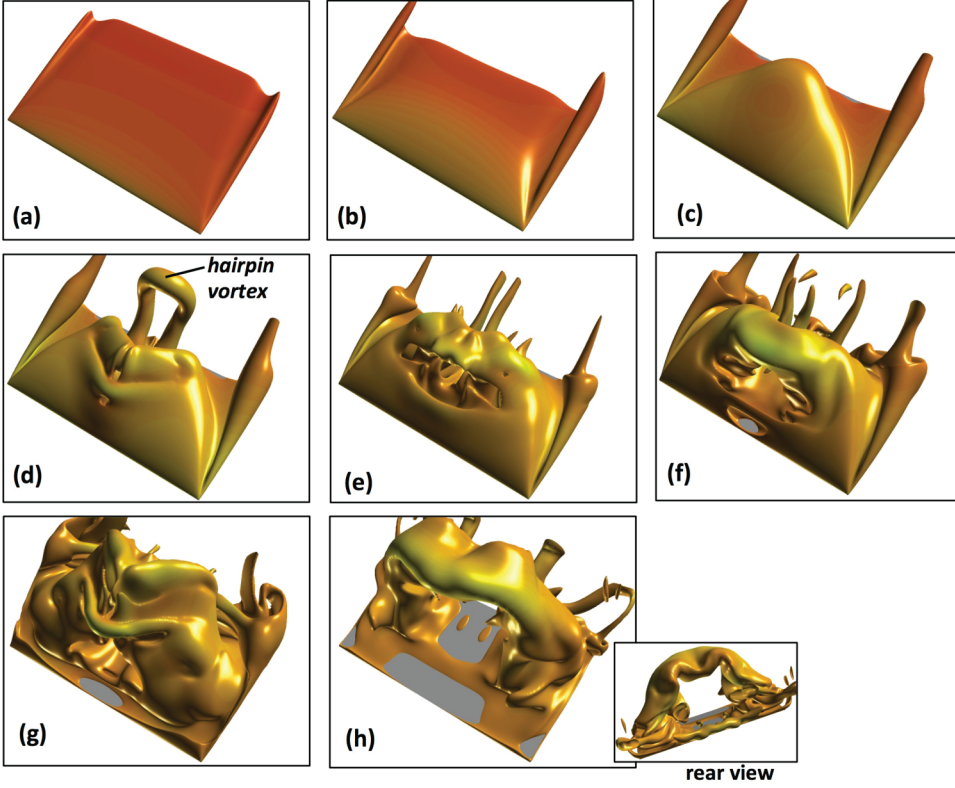


FIG. 20. Instantaneous three-dimensional flow structure for pitching wing ($\Omega_o^+ = 0.05$, $Re_c = 10^3$) depicted by an isosurface of total pressure: (a) 8° , $t^* = 2.84$, (b) 15° , $t^* = 5.33$, (c) 20° , $t^* = 7.11$, (d) 25° , $t^* = 8.89$, (e) 30° , $t^* = 10.67$, (f) 35° , $t^* = 12.44$, (g) 40° , $t^* = 14.22$, and (h) 45° , $t^* = 17.5$ (inset shows rear view).

leading-edge vortex system which subsequently undergoes transition due to the onset of spanwise instabilities despite the modest value of Reynolds number. The turbulent LEV convects along the wing, and as it nears the trailing edge, a trailing-edge vortex of opposite rotation emerges. Eventually both vortices are shed into the wake as the plate reaches its maximum angle of attack. The surface pressure contours clearly show that these vortices induce regions of low pressure which extend, with only small secondary variations, across the entire span. This flow evolution is similar to that found during the downstroke of a heaving 2D wing, described in detail in Ref. [10].

A comparison of the lift and pitching moment coefficient histories for the 2D and finite-aspect-ratio plates is shown in Fig. 24. The lift and pitching moment for the 2D wing display similar offsets as discussed previously for the finite plate due to the rotation-induced apparent camber effect [58,59]. In addition, the 2D wing displays a distinct depression in the lift slope relative to its corresponding theoretical value of 2π (in radians), also plotted for comparison in Fig. 24(a). This effect previously noted in 2D dynamic stall experiments and computations [58,59] is associated with the lag in the establishment of circulation around the airfoil (i.e., Wagner-type effect) at a given angle of attack. Comparison of the aerodynamic lift indicates that a much smoother lift history is present for the 3D wing. In addition, the finite-aspect ratio promotes further delay of stall from $\alpha_{C_{L_{\max}}} \approx 28^\circ$ to $\alpha_{C_{L_{\max}}} \approx 38.5^\circ$. It is interesting to note that this difference in stall angle of attack ($\Delta\alpha_{C_{L_{\max}}} \approx 10^\circ$) is of the same order as the theoretical [57] downwash induced at the wing centerline by the finite-aspect ratio, $\epsilon = C_L/\pi AR$ (assuming C_L of order 1). Regarding the C_M history, there is also a delay in moment stall with the finite-aspect-ratio wing, and the elimination of the dual peak seen in the 2D

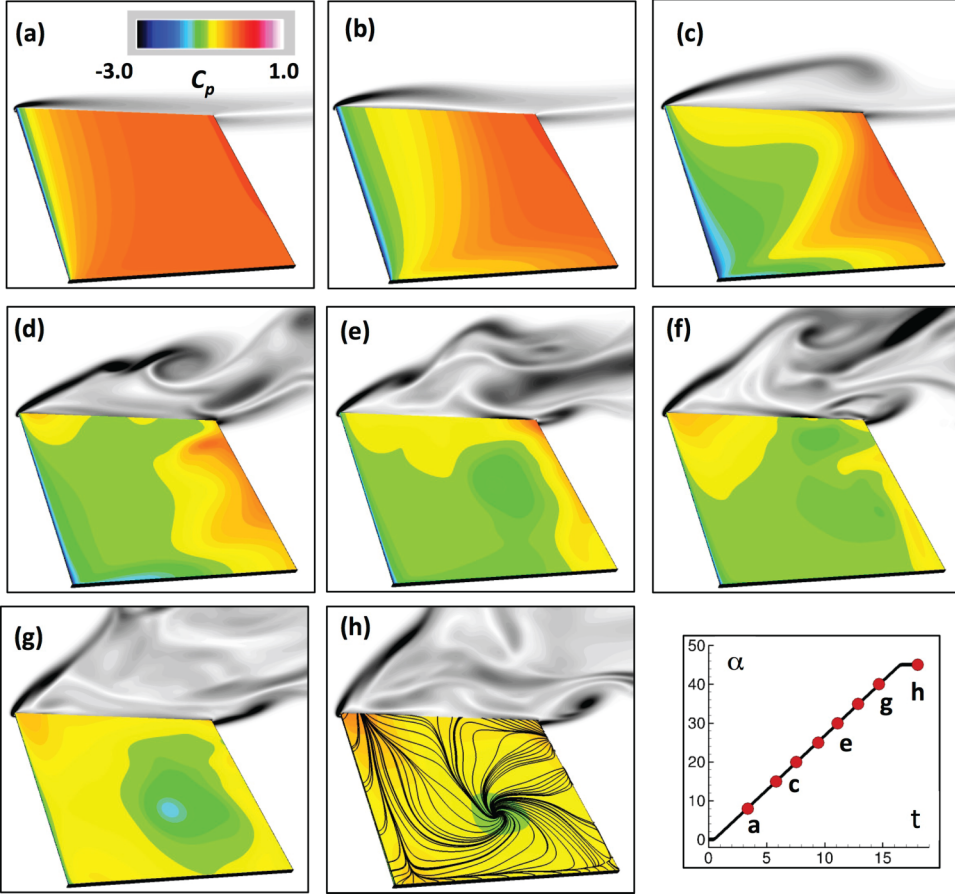


FIG. 21. Instantaneous pressure contours on wing surface (only half span shown) and contours of vorticity magnitude on midplane $z/c = 0$ for $\Omega_o^+ = 0.05$, $Re_c = 10^3$: (a) 8° , $t^* = 2.84$, (b) 15° , $t^* = 5.33$, (c) 20° , $t^* = 7.11$, (d) 25° , $t^* = 8.89$, (e) 30° , $t^* = 10.67$, (f) 35° , $t^* = 12.44$, (g) 40° , $t^* = 14.22$, and (h) 45° , $t^* = 17.5$.

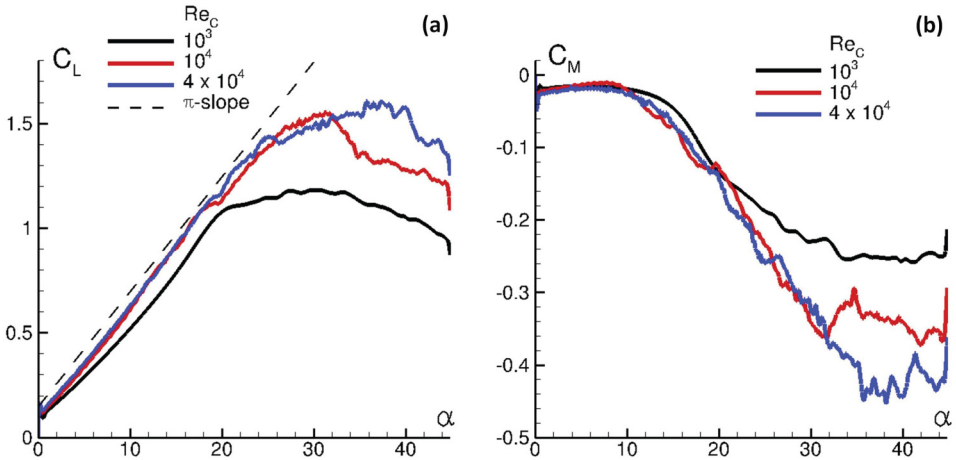


FIG. 22. Effect of Reynolds number on (a) lift and (b) pitching moment coefficients, $\Omega_o^+ = 0.05$.

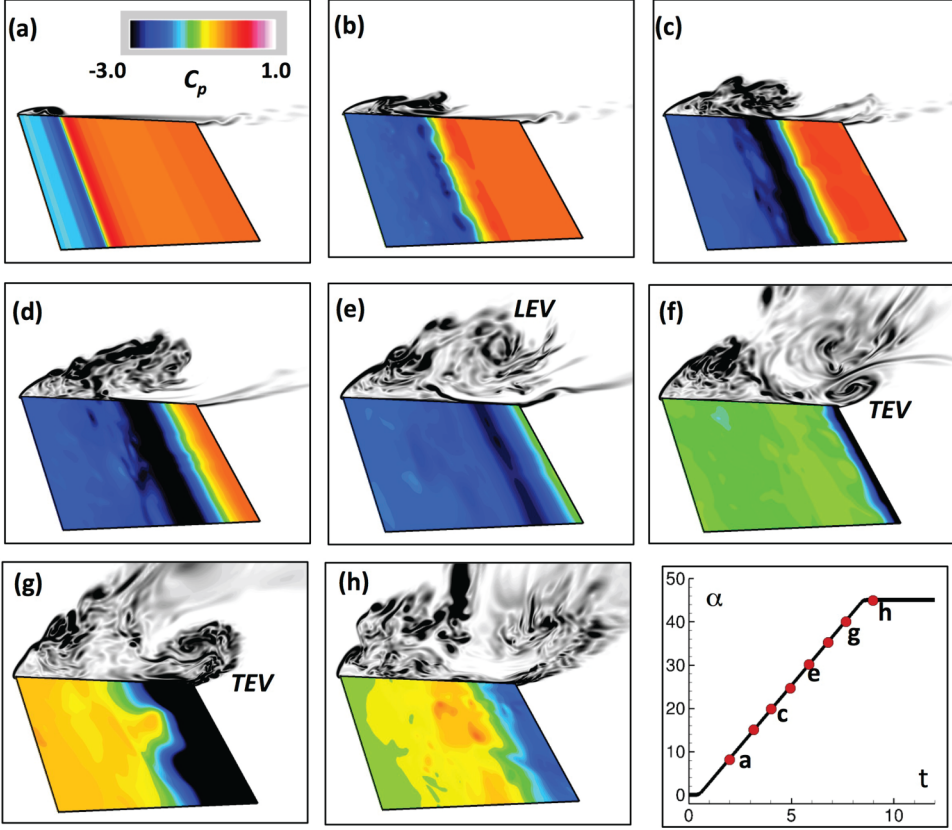


FIG. 23. Instantaneous pressure contours on wing surface and contours of vorticity magnitude on transverse plane ($z/c = 0$) for pitching 2D (infinite-aspect-ratio) wing, $\Omega_o^+ = 0.1$, $Re_c = 10^4$: (a) 8° , $t^* = 1.42$, (b) 15° , $t^* = 2.67$, (c) 20° , $t^* = 3.55$, (d) 25° , $t^* = 4.44$, (e) 30° , $t^* = 5.33$, (f) 35° , $t^* = 6.22$, (g) 40° , $t^* = 7.11$, and (h) 45° , $t^* = 8.5$.

case. This latter feature, as explained for a 2D plunging wing in Ref. [10], is due to the strong trailing-edge vortex which is absent in the 3D wing.

The previous comparison of the flow structure and aerodynamic loading for the finite-aspect-ratio and 2D plates clearly highlights the fundamental differences between 2D and 3D dynamic stall

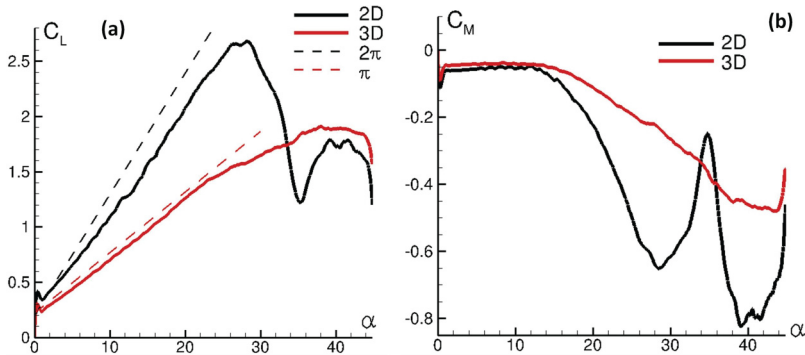


FIG. 24. Comparison of (a) lift and (b) pitching moment coefficient histories for pitching 2D and low-aspect-ratio wings, $\Omega_o^+ = 0.1$, $Re_c = 10^4$.

phenomena. The latter being characterized by the formation of an arch-type vortex and a strong swirl pattern over the wing which persists for longer times. In the 2D case, the spanwise-oriented leading- and trailing-edge vortices are shed much more readily inducing an earlier stall.

IV. CONCLUSIONS

This study addresses the flow structure and unsteady loading arising over a rectangular wing of aspect ratio 2 subjected to high-amplitude pitching maneuvers under low-Reynolds-number conditions of interest in small unmanned aerial vehicle operation and gust interactions. The flow fields are simulated employing an extensively validated high-fidelity computational approach capable of accurately capturing the complex unsteady transitional flows generated under such extreme maneuvers. The wing is pitched about its quarter-chord axis to a maximum incidence of 45° over nominal time intervals ranging from four to 16 convective time scales yielding maximum nondimensional angular rates $0.05 \leq \Omega_o^+ = \dot{\alpha}c/U_o \leq 0.2$. The Reynolds number based on the wing chord varied from 10^3 to 4×10^4 .

The flow field evolution is found to start with the formation of a spanwise-oriented leading-edge vortex which is initially pinned at the front edges of the plate. This vortex evolves at later times into the arch-type structure previously found for a heaving wing [26]. The arch vortex is here less compact and since the wing remains at high angle of attack, it has the time to establish a well-defined circulatory pattern over the wing surface. This swirling motion extends over most of the wing semispan (on each side of the wing) and encompasses a region of low pressure. The arch vortex exhibits a long residence time over the wing and persists for several convective time scales after cessation of the pitch-up motion. Following its formation, the arch vortex is observed to propagate upstream towards the leading edge of the wing. This interesting upstream propagation against the prevailing flow is postulated to be the result of the self-induced velocity of the combined vortex and its image system. In addition to the arch vortex, the flow is characterized by the emergence tip vortices at the wing tips. As the angle of attack increases, the tip vortices strengthen and eventually exhibit the onset of vortex breakdown in a manner similar to that found in the leading-edge vortex development over pitching delta wings [52,53,55].

Despite quantitative differences, the aforementioned morphology of 3D dynamic stall is found to be universal over the range of pitch rates and Reynolds numbers investigated. Several trends are apparent with increasing Reynolds number or pitch rate. These include (i) a more compact arch vortex system, (ii) a stronger circulatory pattern and lower pressures above the wing surface, (iii) increased maximum lift, and (iv) delay of stall. Even for the lowest pitch rate considered, a significant increase in maximum lift is achieved relative to the static situation. Transitional effects are observed to emerge in all cases with the exception of the lowest Reynolds number ($Re_c = 10^3$).

For $\Omega_o^+ = 0.2$, good agreement is found between the computed three-dimensional flow structure and the recent experimental measurements of Yilmaz and Rockwell [21] provided that the higher-resolution computational results are filtered down to the coarser resolution used in the experimental reconstruction. The experiments and computations also exhibit broad similarities in terms of the final arch-vortex formation and induced swirling pattern.

The present pitching wing simulations in conjunction with previous heaving-wing computations [26], as well as the available experimental observations [19,21,60] provide a consistent model of three-dimensional dynamic stall for a low-aspect-ratio wing.

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