

Transient spray cooling: An analytic predictive approach

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This study examines transient spray cooling of a heated substrate under a single nozzle, a situation commonly encountered in spray cooling or quenching applications. The temperature range is such that the cooling process goes through the three cooling regimes in time—film boiling, transitional boiling, and nucleate boiling. The heat transfer associated with each drop impact is computed using validated theoretical expressions for each of the boiling regimes. These theoretical expressions and limits for delineating the three boiling regimes are taken from literature. The principle of superposition is then applied, using the local number flux of impacts on the surface, to compute the rate of heat transfer from the substrate. The simulation is performed over time as a full conjugate heat transfer problem, with the temperature decrease in the substrate being computed using a finite difference method. This predictive approach is validated using existing experimental results and then used to investigate the influence of various spray parameters on the cooling performance of a flat plate.

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I. INTRODUCTION

It is now well established that spray cooling can offer substantial performance advantages compared with conventional cooling technologies, especially for high-temperature, high-heat flux applications [1–3]. This is especially true when phase change is involved, i.e., with nucleate boiling, transitional boiling, or at critical heat flux (CHF) conditions. It is therefore particularly interesting to have a reliable predictive tool, accounting for the many parameters involved when designing a spray cooling system. There exists a very large body of literature characterizing spray cooling performance from which such predictive models can be derived. These studies consider for the most part steady-state cooling and express results in the form of Nusselt number or heat transfer correlations [4,5]. On the other hand, numerous applications involve transient spray cooling, i.e., cooling down of a substrate from a high temperature, whereby the cooling mechanism changes from film boiling, through transitional boiling to nucleate boiling and eventually to convection and evaporation. Examples of such applications include hot forging, steel quenching, or continuous casting [6,7]. For instance, it is well known that during the quenching of metal plates, structural transformations influence the mechanical material properties of the plate, e.g., microstructure, hardness, and residual stresses [8–10]. To predict such structural influences it is of utmost importance to consider phase transformation effects in modeling the heat transfer during quenching, as shown

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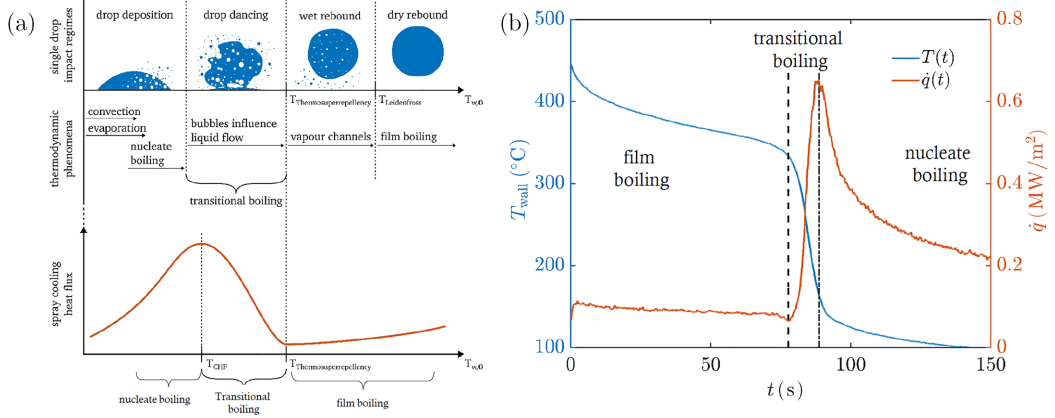


FIG. 1. Delineation of heat transfer regimes according to substrate temperature and time during spray cooling. (a) Drop behavior and heat flux for single drop impact as a function of substrate temperature; (b) exemplary transient temperature and heat flux values as a function of time. Adapted from [22].

by numerous authors [10–13]. Thus, it is important to compute simultaneously the temperature distribution in the substrate, i.e., as a fully conjugate heat transfer problem.

The present study introduces a predictive approach for transient spray cooling. This study differs substantially from many former studies in that it does not use a heat transfer coefficient (or the dimensionless Nusselt number) to quantify the heat transferred, but instead derives the heat transferred by summing over the ensemble of impacting drops on the substrate. This avoids the use of empirical correlations [14–16] and the corresponding uncertainty in applying them outside the range of temperature and heat transfer rates, or using different spray parameters, from which they were developed. Thus, instead of characterizing a spray using parameters such as nozzle orifice diameter, height or angle of nozzle above the substrate, mass flow rate (or pressure) through the nozzle, etc., this predictive approach relies on specifying local spray quantities at the substrate, i.e., drop number flux, impingement angle, and size and velocity distributions of the drops. Necessarily, the temperature distribution throughout the substrate must be simultaneously computed, since the surface temperature will be instrumental in determining what heat transfer mechanism each drop experiences.

This approach has the disadvantage that many of these quantities have not been explicitly measured in past experimental studies, especially their local values over the substrate, hindering verification. This situation is likely to change in future studies since local measurements of the spray are available using phase Doppler interferometry or other suitable techniques, and numerous methods have been developed for the measurement of local temperature and/or heat flux at the substrate [17,18]. The advantage of this approach is that once verified, it is equally applicable to steady-state spray cooling, cooling of complex geometries [19], or cooling with multiple sprays, which is advantageous in certain applications [6,20].

Fortunately, considerable progress has been made in identifying and theoretically modeling the heat transfer mechanisms for single drop impact under film boiling [21], transitional boiling [22], and nucleate boiling [23] conditions. Furthermore, the limits of these three regimes have now been clearly identified, enabling a simulation to recognize when to switch the heat transfer model while the substrate is cooling down [12,24,25].

Figure 1 summarizes the transient heat transfer process involved in spray cooling of a substrate [22]. Figure 1(a) indicates the different hydrodynamic and thermodynamic behavior of a single drop after impacting onto a heated surface. The three regimes are delineated by the substrate temperature. In spray cooling, however, the substrate temperature decreases with time and this is shown exemplary in Fig. 1(b). Note that the time period over which transitional boiling occurs

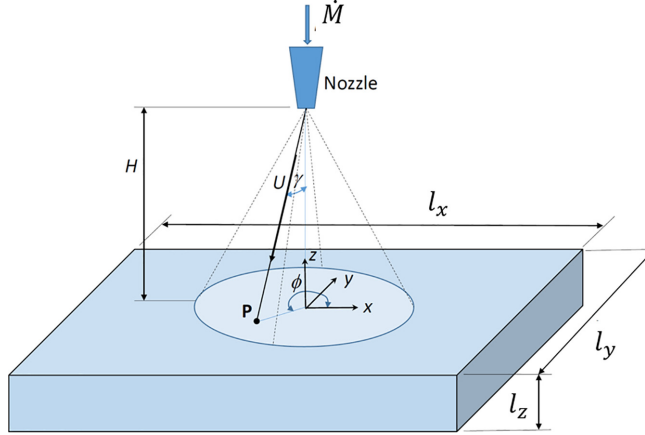


FIG. 2. Sketch of the geometric parameters defining the geometry of the substrate and spray cooling.

is relatively short, as also confirmed by Tenzer *et al.* [25]. Any simulation of the spray cooling must therefore use appropriate heat transfer models for each of the regimes and furthermore, the temperature at which the regime boundaries occur must be known in order to invoke a change in the model. The present simulation adopts an approach that has been implemented and validated in previous studies for spray cooling in the film boiling and nucleate boiling regimes [19,26]. In this approach, the spray is first specified in terms of mass flow rate at the nozzle, a spray cone angle, and a size and velocity distribution for the drops (Sec. II B). This allows a number flux of drops impacting at each point on the heated substrate to be computed. The heat transfer for each drop impact is then computed according to the appropriate regime model. A summation of all drops within a small portion of the substrate surface and over time is performed, yielding the local heat flux. This is the principle of superposition, which has been shown to be very reliable in past validation studies. In particular, this principle of superposition finds its physical justification in the high-speed video observations of [25], in which throughout all heat transfer regimes, the actual covered or wetted area on the surface was never observed to form a continuous, uninterrupted film. This is in stark contrast to pool boiling, which at all times has liquid resident above the surface, albeit sometimes above a vapor layer.

Allowance is made for the probability that at higher number fluxes, drops may interact on the surface, yielding a heat transfer less than the sum of the individual drops. This probability is considered in terms of Poisson statistics (Sec. II C) and a suggestion is made on how to account for this in the overall heat transfer balance. Parallel to consideration of the spray drop impacts, the temperature distribution within the substrate must be computed, since the surface temperature is instrumental in determining what heat transfer regime is to be used for each drop impact. Thus, a finite difference computation of the substrate is carried out for each new time step, updating the surface boundary condition according to the computed local heat flux. This computation is outlined in Sec. II A. The approach represents therefore a two-way coupling, i.e., a conjugate heat transfer problem.

II. GEOMETRY AND BOUNDARY CONDITIONS

The geometric situation considered in this study is sketched in Fig. 2, in which a single spray nozzle is directed normal to the substrate at distance H from the nozzle. A Cartesian coordinate system (x, y, z) is placed with its origin directly below the nozzle on the substrate and with the unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$. The substrate dimensions in the x , y , and z directions are given, respectively, as l_x , l_y , and l_z . The impingement angle of individual drops is a function of the impingement position on the surface ($\mathbf{P}$) and takes the value $\psi = \pi - \gamma$, where γ is the local angle at which the drop exits

the nozzle towards point $\mathbf{P}$ with a velocity U . Thus, the angles γ and ϕ specify the trajectory of the drop within the spray cone.

A. Substrate boundary conditions

The temperature throughout the substrate is computed using a finite difference method (FDM), requiring either a temperature or heat flux to be specified on all sides of the plate. To align the computations with validation experiments taken from literature, all sides and the bottom of the plate are considered to be adiabatic, i.e., $\dot{q} = 0$. Experimentally, this requires insulation, details of which will not be discussed further in the context of the present study.

The boundary condition on the substrate upper surface is specified separately for the area of the spray impingement cone and the remaining area. For the area within the spray cone, the heat flux computed according to the respective heat transfer model is used, as elaborated in Sec. III. For the area on the upper surface outside of the spray cone, the heat transfer coefficient is not as easily specified, since in any practical application the flow will be some mixture of radiation, natural convection, and induced convection; the latter being induced by both the spray and the natural convection. In terms of boundary conditions, this situation is extremely ill defined, presumably also depending to some extent on the far-field boundary conditions (e.g., size of room or containment). On the other hand, this is not expected to be an important factor, given the dominant role of the spray on the overall heat transfer. For the present purposes, a heat transfer coefficient is estimated with the help of the empirical expression [27] for the Nusselt number

$$\text{Nu} = \frac{hL'}{k} \approx 0.54\text{Ra}^{1/4}. \quad (1)$$

In the above expression, a length scale L' is required and here we use the ratio of the plate area to its perimeter, i.e., $L' = l_x l_y / 2(l_x + l_y)$ and k is the thermal conductivity of air. The Rayleigh number is given by

$$\text{Ra} = \frac{\beta \Delta T g L'^3}{\alpha \nu}, \quad (2)$$

whereby ν , α , and β , are the kinematic viscosity, thermal diffusivity, and thermal expansion coefficient of air, respectively. All material constants are evaluated at the film temperature, i.e., the mean of the surface temperature and the far field room temperature. Expression (1) for the Nusselt number represents a value typical of natural convection from a finite size, horizontal heated plate in an infinite surrounding, which can only be very approximate [28]. Some variation of this value is explored in Sec. IV to determine the sensitivity of the results to this value, and in particular to confirm the assumption that this factor is of only of second-order importance.

The substrate is assumed to be polished flat with a surface roughness which influences neither the hydrodynamics nor the thermodynamics of the drop impact. The thermal effusivity of the substrate expresses its ability to exchange thermal energy with its surroundings and is given by $e_w = \sqrt{k \rho_p c_p}$, where k is the substrate thermal conductivity, ρ_p its density, and c_p its specific heat capacity.

In the finite difference method implementation, a nonuniform discretization mesh is employed to achieve higher resolution in the spray region. The plate thickness l_z is discretized into 100 nodes. Within the spray cone the mesh size was $l_x/200$ and $l_y/200$; outside the spray cone $l_x/100$ and $l_y/100$ was used. Preliminary computations with varying numbers of nodes were conducted to ensure that the solution is independent of the mesh size.

The governing equation for each mesh volume can be written as

$$\frac{\partial T}{\partial t} = \alpha_w \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right). \quad (3)$$

This equation is solved for each time step, assuming constant boundary conditions over the time step. Once the temperature distribution throughout the substrate has been computed, this yields a

new surface temperature distribution; hence, a new local heat transfer arising from the spray used for the next time step. This solution procedure marches forward in time. The time step has been selected such that any shorter value would not significantly influence the result. The solution procedure uses a first-order forward difference scheme for the temporal derivative and a second-order central difference scheme for all spatial derivatives. The finite difference code has been validated using the theoretical solution of an infinite plate of finite thickness, whereby the upper surface is exposed to a prescribed temperature at $t = 0$ (the results of which are not shown here).

B. Spray parameters

The mass flow rate through the nozzle is given as $\dot{M}$ [kg/s] and is atomized into a spray cone with an opening (half) angle of $\gamma_{\max}$. The liquid mass is assumed to be uniformly atomized throughout the spray cone, thus the mass flow rate through a small area $dA_s = d\gamma d\phi$ on a spherical shell centered around the nozzle exit is given by

$$d\dot{M} = \dot{M} \frac{\sin \gamma d\gamma d\phi}{4\gamma_{\max}}. \quad (4)$$

The assumption of uniform atomization is only out of convenience; if in a practical case, the drop size and velocity distributions and the number flux varying over the spray angle are known, this information could easily be adopted into the solution procedure. Such information is normally acquired using, for instance, a phase Doppler instrument.

Allowance is made for a drop size distribution and for purposes of illustration a log-normal probability density function was chosen

$$p(D) = \frac{1}{D\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln D - \mu)^2}{2\sigma^2}\right), \quad (5)$$

where μ and σ correspond to the expectation (mean) and the standard deviation of drop size. Other distributions may be more suitable for certain applications, but this distribution is frequently encountered in spray studies. Indeed, if an experimentally measured size distribution is available, this could also be incorporated into the analysis. Simplifications in subsequent expressions are possible by assuming all drops have the same velocity and/or the same size.

In all computations presented below (except for those shown in Sec. IV C), a monodispersed size distribution will be used. The expected difference in heat transfer between using a polydispersed spray and a monodispersed spray based on either the arithmetic mean or Sauter mean diameter has been studied in [26] and shown to be small at lower mass flow rates typical of the computations performed in the present study. In Sec. IV C, the explicit influence of the size distribution standard deviation is explored.

A nonzero correlation between size and velocity of drops in a spray is not unusual and for the present purposes the following linear relation is used:

$$U = a + bD, \quad (6)$$

where a and b are freely chosen constants expressing either a positive or negative correlation. For the applications motivating the present work, in which the spray is injected into quiescent surroundings, a positive correlation is appropriate, since larger drops maintain their velocity longer due to their higher inertia. However, negative correlations can be expected if a spray is injected into a fast-moving gas flow.

From these spray parameters it is necessary to compute the number flux of drops onto a given plate area for each drop size and velocity class in order to estimate the probability of drop-drop interactions (see Sec. II C). The number flux is computed knowing the mass flux normal to a given area on the plate and taking into account the size distribution. Since the desired flux is a vector with a direction in the unit normal to the plate, the effective impingement angle must be used to compute the number flux. Further details of similar computations can be found in [19].

C. Drop interaction

If the number flux of drops impacting the substrate increases, either through a higher mass flow rate or if multiple nozzles were directed onto the same surface, or if the residence time of drops on the substrate (before rebound or complete evaporation) is prolonged due to the decreasing temperature, then the probability of drop-drop interaction on the surface increases. This can decrease the rate of heat transfer when compared to a simple superposition (summation) over all individual drops. The probability of drop-drop interaction and a factor correcting for the resultant heat transfer was first introduced by Breitenbach *et al.* [12] and has been used in previous studies by the authors [19,26], therefore, the main working relations will only be briefly summarized here.

The basic assumption in this approach is that the heat transfer will scale with the actual area covered by the impinging drops. If drop-drop interaction occurs, then the actual covered area will be less than the sum of all drop areas without interaction. The ratio of the two is expressed as an effective wetted or coverage ratio, η_{cov} . To predict this ratio, a random (Poisson) distribution of drop impact positions and times is assumed. η_{cov} is then given as

$$\eta_{\text{cov}} = \frac{1 - e^{-\lambda}}{\lambda}, \quad (7)$$

where λ is the expectation of the cumulative wetted area of the impacting drops. This area can be estimated as

$$\lambda = \dot{n} \int_0^\infty p(D) \int_0^{t_{\text{cov}}} \pi r_c(t)^2 dt dD, \quad (8)$$

with the probability function of the drop diameter in a polydispersed spray $p(D)$. The number flux $\dot{n}$ [#/(m²s)] can be estimated from the values of the measured mass flux on the current elemental area and the mean drop diameter D_{10} , leading to the following expression for the expectation of the covered area:

$$\lambda = \frac{6\dot{m}}{\rho_l D_{10}^3} \int_0^{t_{\text{cov}}} r_c(t)^2 dt. \quad (9)$$

In these equations, t_{cov} refers to the contact or spreading time and $r_c(t)$ is the spreading radius of the the impacting droplet on the surface. This time will be specified in the following section for each of the different heat transfer regimes, whereby its designation is then t_c , t_e , or t_{in} , depending on the heat transfer regime. Furthermore, the radius of the drop spreading on the substrate with time, $r_c(t)$, will also depend on the heat transfer regime and is given in the following section either as a radius or diameter. Once a heat transfer for a given elemental area on the surface is computed, the value is modulated by the factor η_{cov} , representing the effective reduced area of the impacting drops. Note that the factor η_{cov} is always less than unity.

Note also that the underlying assumption made in using η_{cov} as a modulating factor of the heat transfer presently has no direct experimental validation. While there are several studies examining the hydrodynamics of multiple drop impacts [29,30], only very few include also the altered heat transfer [31–34]. For this reason, we expect that our predictive model will be less accurate at lower values of η_{cov} , when the probability of drop-drop interaction increases. Presently, we consider a value of $\eta_{\text{cov}} > 0.85$ to be acceptable. However, this knowledge deficit highlights the need for further experimental and/or numerical investigations of multiple drop impacts onto heated surfaces.

III. HEAT TRANSFER MODELS

The heat transfer for each of the three heat transfer regimes outlined in Fig. 1 is described by validated theoretical models taken from the literature. Whether all or only some of the models are invoked in the simulation, depends on the initial temperature of the substrate (e.g., a temperature above the Leidenfrost point), the cooling rate which is achieved under the given operating conditions, and the duration over which the cooling is activated. Irrespective of which regimes are

invoked, the approach is analogous in all cases. The heat removed from the surface by a single drop impact is expressed as

$$Q_{sd}(D) = \int_0^{t_c, t_e, t_{in}} \dot{q}(t, D) A(t, D, \psi) dt, \quad (10)$$

whereby the integration time interval is specified differently for the different regimes and the local impact angle, ψ , can influence both the area covered by a single drop and the number flux $\dot{n}$ over the elemental portion of the surface being examined. The heat removed per unit area per unit time (heat flux), taking into consideration the size distribution, would be

$$\dot{q} = \dot{n} \int_0^\infty Q_{sd}(D) p(D) dD. \quad (11)$$

This assumes that the velocity is uniquely determined by size. If this is not the case, then the joint size-velocity distribution $p(D, U)$ would have to be known and Eq. (11) would become a double integral. The heat flux specified in Eq. (11) is then applied as a boundary condition on the surface of the plate, locally evaluated according to the position within the spray cone. To account for diminished heat flux due to possible drop-drop interaction, this heat flux is multiplied by the factor η_{cov} , as outlined in the previous subsection.

It becomes evident that it is now necessary to know for each boiling regime and for a single drop impact the contact time (t_c, t_e, t_{in}), the time-dependent coverage area $A(t, D, \psi)$, and an analytic expression for the heat flux $\dot{q}(t, D)$. This information, taken from the respective literature, is summarized in the following subsections.

A. Film boiling

The validated model for film boiling regime comes from the work of Breitenbach *et al.* [21] and makes the following assumptions:

(i) The dominant period of heat transfer is the initial spreading phase of the drop. During this period, the driving temperature gradient is the largest and the vapor layer between the substrate and the drop is the thinnest.

(ii) A correction factor of the order unity (Γ) will be introduced to account for the heat flux during the later phases of the impact, which is also considered in Breitenbach *et al.* [21].

(iii) The time of this initial spreading phase is approximated using $t_{in} = D/U$, whereby U is understood as being the normal component of the impact velocity.

The analysis of [21] leads to a heat flux from the surface to the drop with normal impact given as

$$\dot{q}_f = \frac{2g_2 e_w (T_w - T_{sat})}{\sqrt{\pi} (g_1 + 2g_2) \sqrt{t}}, \quad (12)$$

with

$$g_1 = \sqrt{(g_2 - g_3)^2 + \frac{4g_2^2}{\sqrt{\pi}}} - g_2 - g_3, \quad (13a)$$

$$g_2 = \frac{\sqrt{5} (T_{sat} - T_D) e_f}{\sqrt{\pi} (T_w - T_{sat}) e_w}, \quad (13b)$$

$$g_3 = \frac{\sqrt{\pi} k_v \rho_f L}{2(T_D - T_{sat}) e_w^2}, \quad (13c)$$

where $e_f = \sqrt{k_f \rho_f c_{pf}}$ is the thermal effusivity of the liquid/fluid, using the fluid thermal conductivity k_f , the fluid density ρ_f , and the specific heat capacity c_{pf} ; k_v is the thermal conductivity of the vapor; T_{sat} is the saturation temperature of the liquid; L is the latent heat of vaporization; T_w is the surface (wall) temperature; and T_D is the temperature of the drop before impact. We might note that Eq. (13)

simply defines some consolidated quantities used in Eq. (12), but that they have no direct physical meaning.

The spreading area of the drop $A(t, D, \psi)$ during the time t_{in} is taken from Rioboo *et al.* [35] and uses the expression for diameter given by

$$D'(t, D, \psi) = 2.8D\sqrt{tU \cos \psi / D}, \quad (14)$$

whereby we can write $A(t, D, \psi)$ as

$$A(t, D, \psi) = \pi D'^2(t, D, \psi)/4. \quad (15)$$

These values for t_{in} , $\dot{q}_f$ and $A(t, D, \psi)$ are used in Eq. (10), whereby the total heat removed by a single drop is then multiplied by the dimensionless correction factor Γ , accounting for the heat flux during the later stages of drop spreading and retraction. In the present study, this factor takes the value $\Gamma = 3.2$, as used in Breitenbach *et al.* [21].

B. Transitional boiling

Transitional boiling occurs during spray cooling when the substrate temperature decreases to the thermosuperrepellency temperature, also known as the Leidenfrost point [36]. At this temperature, the heat flux takes a minimum. Subsequently, the heat flux increases and the temperature decreases rapidly. As indicated in Fig. 1, the duration of the transitional boiling regime is relatively short, nevertheless, the heat transfer rate becomes very high, thus this regime must be accounted for. The heat transfer occurring in this regime is described in Schmidt *et al.* [24] and Schmidt *et al.* [22]. The hydrodynamics and thermodynamics of transitional boiling are complex and involve both direct contact of the drop liquid with the substrate, but also rebound of a significant portion of the liquid. At the first instance of contact, strong nucleate boiling occurs, followed by a large volume of the liquid rebounding from the surface, caused by vapor percolation. The time until the liquid detaches is described by the theoretical percolation time t_ϵ . The heat removed by a single drop can be computed by integrating the heat flux from the substrate over the wetted area of the drop and the percolation time since most of the drop liquid detaches after t_ϵ .

Schmidt *et al.* [22] derive the percolation time to be

$$t_\epsilon = \frac{1}{k_\epsilon} \left[\frac{(1 - \epsilon_c) \rho_l D L^*}{e_w \Delta T_w \text{Re}^{2/5}} \right]^2, \quad (16)$$

with k_ϵ being an empirical factor of the order unity. The time of percolation t_ϵ describes the beginning of the drop dancing, when most of the liquid is only connected to a thin layer of liquid on the substrate by small separated wetted areas or liquid bridges. As soon as the percolation time t_ϵ reaches the time of the drop natural oscillations $t_\sigma = \sqrt{\rho_l D^3 / \sigma}$, percolation leads to a drop rebound [37]. In Eq. (16), the quantity ϵ_c represents the relative wetted area at the percolation threshold and is given by [24]

$$\epsilon_c = \exp(\lambda_c) \approx 0.32. \quad (17)$$

The quantity λ_c is the Poisson parameter expressing the randomness for the expected number of bubbles covering a particular point on the surface during transitional boiling. It will not be discussed further but is given by percolation theory. The heat flux during this percolation time is given by [23,38]

$$\dot{q}_t = \frac{e_w \Delta T_w}{\sqrt{\pi t}}, \quad (18)$$

with the thermal effusivity of the wall e_w , the wall superheat $\Delta T_w = T_w - T_{\text{sat}}$, and time t . The heat removed by one drop, Q_{sd} , is computed using Eq. (10), which requires the covered area $A(t, D, \psi)$. This area can be computed from the equivalent contact radius, $r_c(t)$, expressed as a

parabolic function in time, reaching a known maximum radius given by [39]

$$r_{\max} = D(0.44\text{Re}^{1/5} - 0.20\text{Re}^{2/5}\text{We}^{-1/2}), \quad (19)$$

and taking the form

$$r_c(t) = \begin{cases} r_{\max} \sqrt{1 - \left(1 - \frac{t}{t_{\max}}\right)^2}, & \text{if } t < t_v \\ r_{\max} \sqrt{1 - \left(1 - \frac{t_v}{t_{\max}}\right)^2} = \text{const}, & \text{if } t \geq t_v. \end{cases} \quad (20)$$

In these equations, the Reynolds number and Weber number are given by $\text{Re} = UD/\nu_l$ and $\text{We} = \rho_l DU^2/\sigma$, whereby ν_l and σ are evaluated at the drop temperature. The time $t_{\max}$ is equal to half of the drop oscillation time $t_{\max} = t_\sigma/2$ with t_σ being the typical time of drop natural oscillations

$$t_\sigma = \frac{D\text{We}^{1/2}}{U}. \quad (21)$$

The viscous time scale t_v depends on the impact parameters and is given by

$$t_v = \frac{D\text{Re}^{1/5}}{U}, \quad (22)$$

again using the normal impact velocity for U in both expressions.

C. Nucleate boiling

The model used for heat transfer from a single drop in the nucleate boiling regime is taken from Breitenbach *et al.* [23] and makes the following assumptions:

(i) The entire drop mass is evaporated on the substrate, i.e., no rebound or splash. As such, the total heat transfer is not a function of drop velocity, size, or surface temperature (although the heat transfer rate is a function of these parameters).

(ii) The wetted area/footprint of the evaporating drop is approximately constant throughout the entire contact time. This area is a function of the initial drop diameter, i.e., $A = \kappa\pi D^2$, where κ is an empirical but highly universal parameter expressing the maximum spreading diameter to the original drop diameter and is set at $\kappa = D_{\max}/D = 1.6$. Note that the area is therefore neither a function of time nor of impact velocity.

Given these assumptions, the contact time (evaporation time) is computed as

$$t_c = \pi \left(\frac{\rho_l L^* D}{12\kappa e_w \Delta T_w} \right)^2, \quad (23)$$

where ΔT_w is defined as the difference between the substrate (wall) temperature T_w and the saturation temperature T_{sat} of the liquid and $L^* = L + \Delta H_0$ is the sum of latent heat of evaporation L and the enthalpy difference ΔH_0 between the initial drop and saturated liquid (sensible heat).

The heat transfer rate to the drop in the nucleate boiling regime is then given by [23]

$$\dot{q}_n = \frac{e_w(T_w - T_{\text{sat}})}{\sqrt{\pi} \sqrt{t}}. \quad (24)$$

D. Bounding limits of each regime

Throughout the entire spray cooling simulation, it is necessary to determine at each position on the substrate and at each time, which heat transfer model is to be invoked; hence, the limits between film boiling and transitional boiling and between transitional boiling and nucleate boiling must be determined.

Film boiling $\rightarrow$ *Transitional boiling*. The condition defining this boundary is the Leidenfrost point (LEP), which corresponds to the minimum heat flux. Reaching the LEP for a heated substrate when impacted by a spray is not determined by a single factor; rather, it depends on a combination

of the fluid and substrate properties. Understanding how these properties influence the LEP requires consideration of two time scales related to the spraying fluid: the surface tension time scale t_σ and the viscous time scale t_ν . Schmidt *et al.* [24] demonstrated that two distinct conditions exist under which superheat can be determined:

$$\Delta T_{\text{LEP}} = \begin{cases} 0.48 \frac{D^{1/4} L^* \rho_l^{3/4} \sigma^{1/4}}{e_w \text{Re}^{2/5}} & \text{if } \text{We} < 2.5 \text{Re}^{2/5} \\ 1.92 \frac{\rho_l \sqrt{\nu_l} L^*}{e_w} & \text{if } \text{We} > 2.5 \text{Re}^{2/5}, \end{cases} \quad (25)$$

where the first expression is applicable for single drop impacts, whereas the second expression is more suitable for spray cooling. An interesting observation from the above equations is that droplet parameters like diameter D or velocity U become significant only when the condition $\text{We} < 2.5 \text{Re}^{2/5}$ is met. This indicates that the viscous time scale t_ν is considerably larger than the surface tension time scale t_σ . Conversely, when $\text{We} > 2.5 \text{Re}^{2/5}$, only the latent heat L^* of the fluid and kinematic viscosity ν influence the Leidenfrost point. In both cases, surface effusivity e_w appears in the denominator, leading to an inverse relationship with superheat. The film boiling model must therefore be used if and as long as the local substrate temperature exceeds this superheat.

Transitional boiling $\rightarrow$ Nucleate boiling. From Secs. III B and III C, it is evident that the actual heat transfer rate is the same in both transitional and nucleate boiling. These regimes differ only in the contact time, i.e., in transitional boiling a portion of the liquid detaches from the surface at some contact time, thus less fluid from each drop is vaporized. The boundary between the two regimes can therefore be taken as the condition when the contact time of transitional boiling becomes approximately equal to that of nucleate boiling. Explicitly, this can be expressed by examining at every time instant and at every point on the substrate the magnitude of contact times given in Eqs. (16) and (23), i.e., $t_\epsilon \approx t_c$:

$$\frac{1}{k_\epsilon} \left[\frac{(1 - \epsilon_c) \rho_l D L^*}{e_w \Delta T_{w1} \text{Re}^{2/5}} \right]^2 \approx \pi \left(\frac{\rho_l D L^*}{12 \kappa e_w \Delta T_{w2}} \right)^2. \quad (26)$$

Note that in the above expression, ΔT_{w1} and ΔT_{w2} are different. ΔT_{w1} is denoted as $(T_{\text{nb}} - T_{\text{sat}})$ where T_{nb} is the temperature of the substrate at the end of the transition boiling, whereas ΔT_{w2} is seen as $(T_{\text{LEP}} - T_{\text{sat}})$. Further details about this condition can be found in [22,24].

IV. COMPUTATIONAL RESULTS

A. Validation of the methodology

This section addresses the validation of the approach proposed in Sec. III. For this computation, we rely on the experimental work of Tenzer *et al.* [25]. For performing the computation, it is essential to evaluate the exact input conditions; however, in [25] there is no specific information regarding the angle of impact of the spray onto the substrate. By examining figures [Figs. 2(a)–2(c) from [25]], we find that the heated substrate is almost entirely covered by the spray. Thus, the heat flux of only approximately 20% of the surface must be computed according to Eqs. (1) and (2). Within this portion of the substrate, the heat flux varied between a maximum of 6% and 14% of the mean heat flux under the spray cone for nucleate and film boiling, respectively. Therefore, this represents between 1.2% and 2.8% of the total heat removed from the plate for the nucleate and film boiling. Hence, the assumption that this portion of the heat transfer is negligible and that Eqs. (1) and (2) are adequate for the present case appears to be valid. These assumptions may have to be revisited if a larger portion of the plate is not wetted by the spray, or if extenuating boundary conditions exist, for example, a strong convective flow over the substrate.

The mass flux $\dot{m}$ distribution across the entire sprayed surface for a particular nozzle pressure is nearly uniform, with only slight variations at the edges. Additionally, the droplet diameter D and velocity U distributions over the spray radius show minimal deviation from the center to the periphery. Thus, we can use average and uniform values of the mass flux $\dot{m}$, droplet diameter D , and

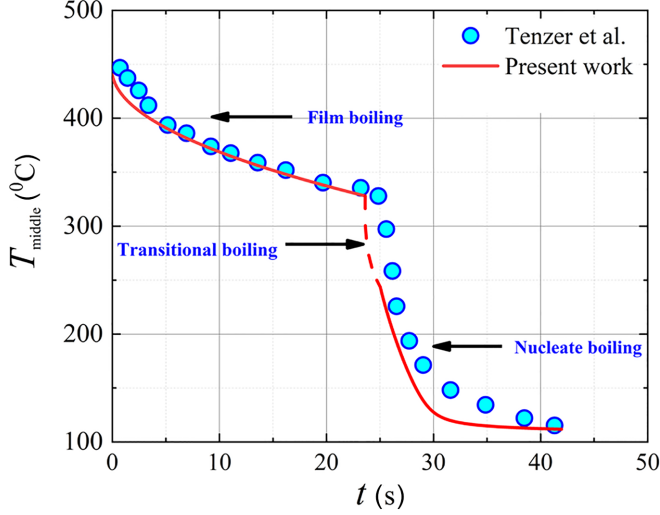


FIG. 3. Transient behavior of temperature at the center of the heated surface using the parameters $D_0 = 50 \mu\text{m}$, $U_0 = 13 \text{ m/s}$, $T_{w0} = 450 \text{ }^\circ\text{C}$, and mass flow rate $\dot{M} = 0.1 \text{ kg/s}$.

impact velocity U as input conditions. The effusivity takes the value $e_w = 8432 [\text{J/m}^2\text{K}\sqrt{\text{s}}]$ using $k = 18 [\text{J/msK}]$, $\rho_p = 7900 [\text{kg/m}^3]$, and $c_p = 500 [\text{J/Km}^3]$.

In validating the code the heat flux is computed over the entire substrate; however, we chose to make a comparison of the heat flux and substrate temperature only at the center of the substrate, where drop impingement is normal to the surface, and where measurement data from the experiment are available. Figure 3 compares our simulation results of the substrate surface temperature in time with the experimental data from [25]. Our model accurately captures the regimes associated with film and transitional boiling. In the nucleate boiling zone, the numerical model reaches saturation ($T_w = 100 \text{ }^\circ\text{C}$) somewhat earlier than observed in the experiment. This discrepancy could be due to uncertainties of the exact experimental conditions (especially $\gamma_{\max}$) or less reliable estimation of the heat transfer at lower substrate temperatures, where drop-drop interactions become increasingly significant. Furthermore, Eq. (24), which expresses the heat flux in the nucleate boiling regime, is estimated on the assumption that the wall temperature at the interface is close to T_w , which is valid if the temperature difference is high. However, when the temperature approaches T_w , the error in the model becomes notable.

B. Example computations

In this section, we use the validated code to investigate the transient cooling behavior under varying spray conditions. In our geometry the lengths l_x and l_y are set to 0.5 m. The spray impinges onto the heated surface from a distance $H = 0.3 \text{ m}$, with a maximum spray cone angle of $\gamma_{\max} = 27^\circ$. The geometry corresponds to that described in Fig. 2, and the transient temperature change ($T_i(t, x = 0, y = 0)$) on the substrate upper surface and directly beneath the spray is computed, whereby the substrate initially has the uniform temperature T_{w0} , lying within the film boiling regime. The spray is characterized by a mean droplet diameter D_0 and an impact velocity U_0 . With the maximum cone angle of 27° ($\gamma_{\max}$), the spray impact area becomes πR_{FP}^2 , where R_{FP} is the footprint radius of the spray given by $R_{\text{FP}} = H \tan \gamma_{\max}$. The area over which the heat flux must be computed according to Eqs. (1) and (2) is therefore 70%. Thus, the portion of heat removed outside of the spray cone is expected to reach values of 4% to 10% of the total. Despite the uncertainty of this computed heat flux, we expect the investigated parametric trends to be valid since this portion of the heat flux is not expected to change significantly with substrate temperature. The working fluid used

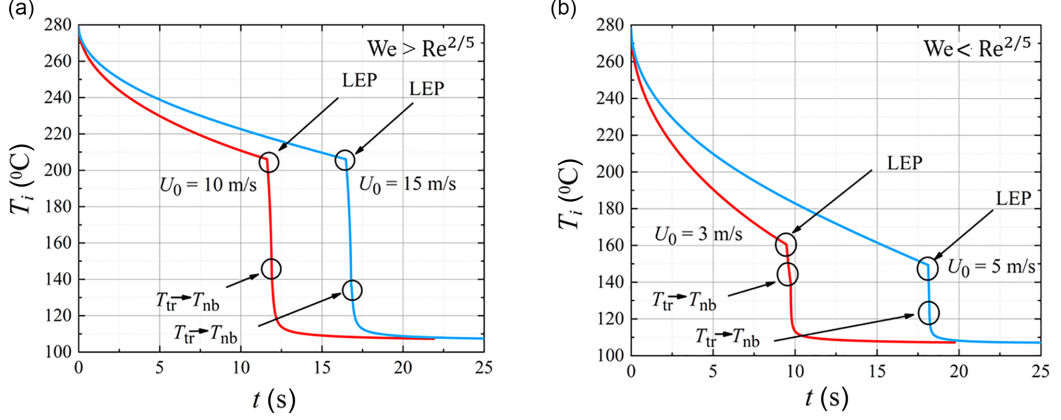


FIG. 4. Computation performed for spray cooling of an aluminum flat plate. (a) Variation of the surface temperature ($T_i(t, 0, 0)$) with cooling time (t) for (a) high velocity impact of $U_0 = 10$ m/s (red line) and $U_0 = 15$ m/s (blue line) and for (b) low velocity impact of $U_0 = 3$ m/s (red line) and $U_0 = 5$ m/s (blue line). Parameters used are $D_0 = 30$ μ m, $T_{w0} = 280$ °C, $H = 30$ cm, $\gamma_{\max} = 27^\circ$, and $\dot{M} = 0.1$ kg/s.

was water, with a latent heat of vaporization of 2.275×10^6 J/kg/K. The heated surface, assumed to be flat, was made of aluminum and had a roughness comparable to that reported by Breitenbach *et al.* [23]. Its thermal effusivity was 2.236×10^4 Ws^{0.5}/(m²K).

The first result, shown in Fig. 4, illustrates the effect of droplet impact velocity U_0 on the substrate temperature (measured at the center of the top surface) over the cooling time t . As expressed in Eq. (25), the bound between film boiling and transitional boiling exhibits a Weber/Reynolds number dependency; hence, we expect a difference in heat transfer between low and high spray velocities. Therefore, the high and low spray velocity results are shown separately in Figs. 4(a) and 4(b), both using the same droplet diameter ($D_0 = 20$ μ m) and both showing the temperature evolution for two different velocities.

Several noteworthy observations can be made from these diagrams. The first diagram [Fig. 4(a)] shows that the velocity has no effect on the Leidenfrost point, as dictated by Eq. (25) for the condition $We < 2.5Re^{2/5}$. However, the temperature and time at which the cooling shifts from transitional boiling to nucleate boiling ($T_{tr} \rightarrow T_{nb}$) are both affected. As the impact velocity increases, the temperature at which transitional boiling reverts to nucleate boiling is lower. This follows directly from Eq. (26), equating t_ϵ with t_c . A higher velocity increases the Reynolds number and for the same value of t_ϵ the Δt_{W1} must be lower, thus T_{nb} must be lower. Nevertheless, as droplet impact velocity increases, the total cooling time increases, primarily due to the longer period required to reach the Leidenfrost point. This result is initially counterintuitive, but can be explained by the following. In the film boiling regime, a higher velocity increases the spreading diameter proportional to $\sqrt{U}$; hence, the area [Eqs. (14) and (15)], which would accelerate the heat transfer. On the other hand, the contact time in the film boiling regime is given by D/U , thus decreasing with increasing velocity. The net effect is a lower rate of heat transfer at higher velocities. If we now also consider Eq. (12), prescribing a $1/\sqrt{t}$ dependency of heat flux, then we can expect a change of velocity from 10 m/s to 15 m/s as shown in Fig. 4(a), a difference of cooling time in the order of 1.5, which approximately corresponds to the difference between 12.5 s and 19 s. Small deviations will occur in this ratio due to the differences in the value of the coverage ratio (η_{cov}), taking lower values for higher impact velocities (larger probability of drop-drop interactions on the substrate).

The second diagram, [Fig. 4(b), $We < Re^{2/5}$], also reveals an interesting observation, namely that at the lower velocity, the film boiling transitions directly into nucleate boiling, with no observable transitional boiling. The lower Reynolds number leads to viscous effects dominating over surface tension effects in determining the hydrodynamics of the drop impact. This may seem

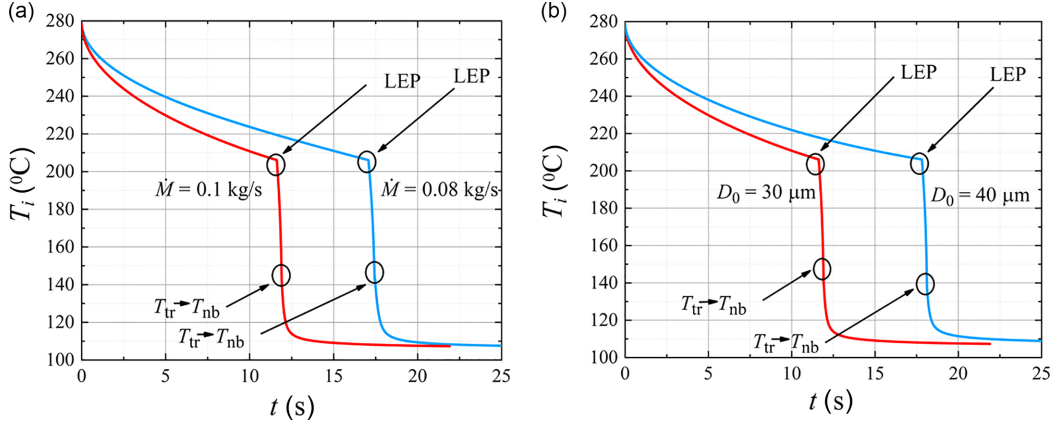


FIG. 5. Computation performed for spray cooling of an aluminum flat plate. Variation of the surface temperature ($T_i(t, 0, 0)$) with cooling time (t) for (a) two input mass flow rates $\dot{M} = 0.08$ kg/s (blue line) and 0.1 kg/s (red line) and for a mean droplet diameter ($D = 30$ μm and $H = 30$ cm, $\gamma_{\text{max}} = 27^\circ$). (b) For the same conditions as (a) with two different mean droplet diameters and a mass flow rate of $\dot{M} = 0.1$ kg/s.

advantageous from a cooling rate perspective, as the absence of the transitional boiling zone reduces the overall cooling time. However, the abrupt temperature drop is dramatic and may have detrimental consequences on the subsequent material properties after cooling.

The next results, shown in Fig. 5, report the surface temperature evolution in time ($T_i(t, 0, 0)$) for different mass flow rates $\dot{M}$ and for different drop mean diameters. In Fig. 5(a), it is evident that increasing the mass flow rate $\dot{M}$ from the nozzle leads to faster cooling, assuming the droplet diameter D and velocity U remain constant. This outcome is expected, as a higher mass flow rate corresponds to an increased droplet number flux $\dot{n}$ interacting with the heated surface. Since the heat flux $\dot{q}$ is directly proportional to $\dot{n}$ [see Eq. (11)], the rate of heat removal per time step Δt is higher for larger mass flow rates compared to lower ones. One notable observation from this diagram is that changes in mass flow rate do not influence the critical temperatures associated with the transitions from film to transition boiling, or from transition to nucleate boiling. This can be explained using Eqs. (25) and (26), as neither of these equations depends on the input mass flow rate. This is expected since these transition zones are governed solely by the combined effects of the surface and fluid properties.

Figure 5(b) presents cooling curves for different mean drop diameters while keeping the mass flow rate $\dot{M}$, the velocity U and H and γ unchanged. As expected, the cooling is faster with smaller droplets, primarily because of the earlier transition from film to transitional boiling. The explanation is simply that for a constant mass flow rate, the number flux increases with D^3 whereas the spread diameter increases only with D^2 . Indeed, this net linear dependence on D is seen in this figure, since the ratio $40/30$ corresponds to the time ratio of reaching the Leidenfrost point, i.e., $17/12$. Again, smaller deviations from this ratio can be expected due to the higher level of drop-drop interactions with larger drops.

A further result has been computed to illustrate the flexibility of this computational approach to handle geometric variations and in particular a variation often found in spray cooling literature. In Fig. 6, the substrate surface temperature is shown as a function of time for various positions of the spray above the substrate ($H = 0.25, 0.3, 0.35$ m), leaving all other parameters constant. Changing the spray nozzle height above the substrate changes the circular area over which the spray cone impacts the plate, the radius given by $H \tan \gamma$. For the maximum spray cone angle of $\gamma = 27^\circ$ this leads to a dimensionless diameter of $2r/l_x = 0.5, 0.6, 0.7$. Thus, in all cases, the spray does not extend beyond the limits of the substrate. Much more important, however, is the increase in number flux for smaller values of H . For a given mass flux through the nozzle, the local number flux at the

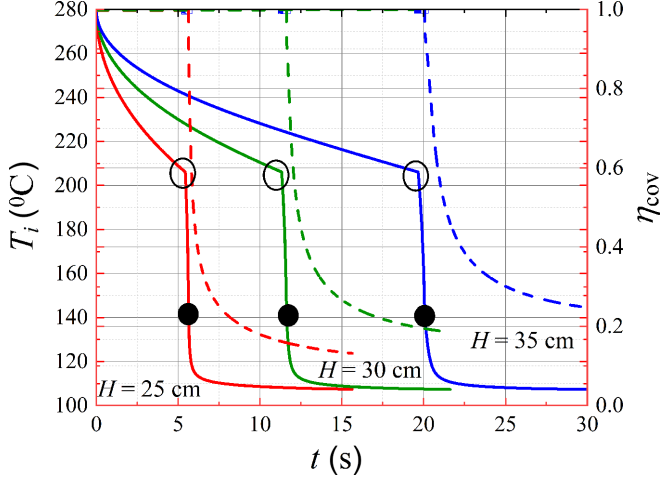


FIG. 6. Transient behavior of substrate surface temperature at the center of the substrate ($T_i(t, 0, 0)$) with various nozzle-to-surface distances (H) using the parameters $D_0 = 30 \mu\text{m}$, $U_0 = 10 \text{ m/s}$, $T_{w0} = 280^\circ\text{C}$, and mass flow rate $\dot{M} = 0.1 \text{ kg/s}$. Solid lines are substrate surface temperature; dashed lines are coverage ratio, η_{cov} . Open circles mark boundaries between film and transitional boiling. Filled circles mark boundaries between transitional and nucleate boiling.

substrate increases as the nozzle height decreases, thus accelerating the cooling. On the other hand, the higher number flux leads to increased drop-drop interactions and η_{cov} decreases from unity (no interactions) much more rapidly at lower H , as also shown in this figure. Note that the temperatures of the Leidenfrost point and the change from transitional to nucleate boiling are not significantly affected by this change in local number flux, only their times of occurrence change. The insight gained is not that the height of the nozzle above the substrate is the deciding parameter, but the local number flux, which could also be changed by varying the mass flow rate in the nozzle, albeit, possibly also influencing the drop sizes and velocities. Even if the nozzle height was increased such that part of the spray cone would not impact the plate, this would be accounted for in the present approach, since automatically the number flux over the impacting zone would be less.

C. Influence of diameter-velocity correlation

The influence of a possible size-velocity correlation on the heat removal rate from a flat plate is investigated, using the simplified linear relation given in Eq. (6). As being representative of many spray conditions, we consider a spray with droplet diameters ranging from $20 \mu\text{m}$ ($D_{\min}$) to $200 \mu\text{m}$ ($D_{\max}$) and velocities ranging from 20 m/s ($U_{\min}$) to 50 m/s ($U_{\max}$) with mean values of $110 \mu\text{m}$ and 30 m/s , respectively. The constants a and b are chosen to fit the condition

$$D = \begin{cases} D_{\max}, & \text{if } U = U_{\max} \\ D_{\min}, & \text{if } U = U_{\min} \end{cases}, \quad (27)$$

for a positive size-velocity correlation or

$$D = \begin{cases} D_{\max}, & \text{if } U = U_{\min} \\ D_{\min}, & \text{if } U = U_{\max} \end{cases}, \quad (28)$$

for a negative size-velocity correlation.

Figure 7 indicates that the heat transfer decreases as the width of the size distribution increases (σ), for both positive and negative size-velocity correlations and in both film and transitional boiling. Note that the impact velocity does not enter the heat transfer rate for nucleate boiling. The negative

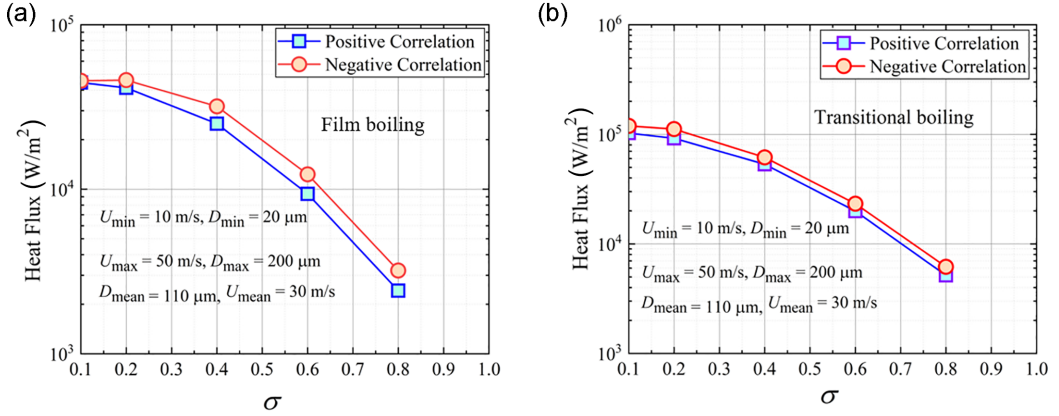


FIG. 7. Effect of positive and negative correlations on heat flux for spray cooling performed on an Al plate for (a) film boiling ($T_w = 300$ °C) and (b) transition boiling ($T_w = 180$ °C) for a range of spray droplet diameters ($D = 20$ μm – 200 μm) and velocity ($U = 10$ – 50 m/s) with a mean droplet diameter ($D_{\text{mean}} = 110$ μm) and mean velocity ($U_{\text{mean}} = 30$ m/s). Input conditions are: $H = 0.3$ m, $\gamma_{\text{max}} = 27^\circ$, and $\dot{M} = 0.1$ kg/s.

correlation exhibits a higher heat transfer rate for both transitional and film boiling. Thus, small fast droplets promote heat transfer more than large, slow droplets. This difference is attributed primarily to the conditions of film boiling. Whereas the higher velocity decreases heat transfer [see Fig. 4(a)], the smaller diameter improves the heat transfer twofold (see Fig. 5); once due to the higher number flux and second due to the lower probability of drop-drop interaction on the wall. The logarithmic scale of this plot indicates that the difference can reach up to 50% between the positive and negative size-velocity correlation. Note that many applications of spray cooling correspond to a positive size-velocity correlation, i.e., injection into quiescent ambient. On the other hand, the correlation is often very weak (small value of b) for steady sprays, since after an initial transient period, the spray produces an entrained gas flow of high velocity, equalizing the velocity of all drops. Nevertheless, a monodispersed size distribution at low values remains most desirable in terms of heat transfer.

V. CONCLUSIONS

The aim of this study was to introduce a physics-based approach to evaluate transient spray cooling throughout the regimes film, transitional, and nucleate boiling, whereby the approach is equally capable of handling steady-state cooling. Very little empirical data is used in this approach, the main inputs being the constants $\kappa = D_{\text{max}}/D = 1.6$ to account for the spread diameter of the drop during nucleate boiling and $\Gamma = 3.2$, giving the ratio of heat transferred over the entire period of drop impact in the film boiling regime compared to that transferred during the initial spreading to a maximum diameter. Both values are taken from documented validation experiments and lead to excellent validation in the present study.

The main interest of this approach is that the defining input parameters are directly the drop size, velocity, and number flux distributions in the spray cone. This data should be available at a downstream distance commensurate with the substrate position, avoiding any significant changes during the drop trajectories between the nozzle and the impact, e.g., evaporation, deceleration or acceleration, drop-drop coalescence, etc. With this input data, the cooling of arbitrary substrate geometries can be computed, provided that the thermodynamic boundary conditions of the substrate itself are well-defined and known. In this study, an adiabatic insulated flat plate has been used, but this was only to allow comparison with experimental data. Furthermore, with this approach arbitrary drop size distributions and size-velocity correlations can be implemented, as can variations of these distributions throughout the spray cone.

The parametric study has underlined the fact that mass flow rate is a dominant parameter in determining the heat transfer, but that smaller drops are in general preferable given a mass flow rate. This is because of the increased number of drops and lower probability of drop-drop interaction upon impact.

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