

Smooth- and rough-wall boundary layer structure from high spatial range particle image velocimetry

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Two particle image velocimetry arrangements are used to make true spatial comparisons between smooth- and rough-wall boundary layers at high Reynolds numbers across a very wide range of streamwise scales. Together, the arrangements resolve scales ranging from motions on the order of the Kolmogorov microscale to those longer than twice the boundary layer thickness. The rough-wall experiments were obtained above a continuous sandpaper sheet, identical to that used by Squire *et al.* [*J. Fluid Mech.* **795**, 210 (2016)], and cover a range of friction and equivalent sand-grain roughness Reynolds numbers ($12\,000 \lesssim \delta^+ \lesssim 18\,000$, $62 \lesssim k_s^+ \lesssim 104$). The smooth-wall experiments comprise new and previously published data spanning $6500 \lesssim \delta^+ \lesssim 17\,000$. Flow statistics from all experiments show similar Reynolds number trends and behaviors to recent, well-resolved hot-wire anemometry measurements above the same rough surface. Comparisons, at matched δ^+ , between smooth- and rough-wall two-point correlation maps and two-point magnitude-squared coherence maps demonstrate that spatially the outer region of the boundary layer is the same between the two flows. This is apparently true even at wall-normal locations where the total (inner-normalized) energy differs between the smooth and rough wall. Generally, the present results provide strong support for Townsend's [*The Structure of Turbulent Shear Flow* (Cambridge University Press, Cambridge, 1956), Vol. 1] wall-similarity hypothesis in high Reynolds number fully rough boundary layer flows.

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I. INTRODUCTION

Turbulent boundary layers over rough walls are of considerable interest both practically and in their propensity to provide insight into the influence of the wall boundary condition on turbulence structure. The effect of surface roughness on the mean streamwise velocity profile has been extensively studied and is generally well accepted (see, for example, Raupach *et al.* [1]). However, there is ongoing uncertainty regarding how rough surfaces affect turbulent quantities and boundary layer structure.

One simple, yet persistent, question in rough-wall studies from the last several decades asks whether or not there is an outer region of the flow where the turbulence is universal regardless of wall condition. The notion that such a region could exist stems from Townsend's (1956) [2] Reynolds number similarity hypothesis, which predicts that in high Reynolds number wall bounded

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flows there is an outer “fully turbulent” region where the influence of the wall is reflected only in the velocity scale and length scale appropriate to this region (that is, the wall friction velocity, U_τ , and the boundary layer thickness, δ) [1]. In the context of rough-wall bounded flows, herein and within the literature Townsend’s postulation for outer region similarity is referred to as the wall-similarity hypothesis. A survey of the existing rough-wall literature reveals support for both possible outcomes: Perry and Abell [3], Acharya *et al.* [4], and Flack *et al.* [5], to name a few, conclude in favor of wall similarity, while Krogstad *et al.* [6], Keirsbulck *et al.* [7], Bhaganagar *et al.* [8], and others report outer region modification in rough-wall flows beyond that which can be described by the hypothesis.

In studies of rough-wall flows, a number of intrinsic experimental difficulties have contributed to slow progress and ongoing debate. The high turbulence intensities and large flow angles encountered, particularly near the wall, can induce errors that are difficult to diagnose and correct (for example in $\times$ -wire anemometry measurements: see Perry *et al.* [9] and Raupach *et al.* [10]). Estimation of the wall friction velocity, U_τ , is also particularly difficult in rough-wall flows. Many techniques, such as oil film interferometry, are not feasible. Two common approaches adapted from smooth-wall studies are the total shear stress method [6,11] and the modified Clauser method [12,13]. The former requires accurate measurements of the Reynolds shear stress profile, $\overline{uw}$, which, as noted above, are particularly challenging in rough-wall flows. The latter involves forcing the mean streamwise velocity to adhere to a predefined logarithmic law. While there is considerable evidence for a logarithmic region in rough-wall flows, there is no wholly vigorous specification for the onset and extent of this region. Furthermore, there is ambiguity in defining the origin of the wall normal location, which varies spatially. Typically, spatial averaging is assumed, for example, in most formulations of the rough-wall logarithmic law (see Clauser [14] and Hama [15]), but this still requires knowledge of the zero plane displacement, ϵ , which is difficult to accurately estimate for practical roughness geometries [16,17]. The above points are relevant, of course, to all U_τ estimates that rely on similarity over some region of rough-wall flows (e.g., Krogstad *et al.* [6], Tachie *et al.* [18]).

In assessments of wall similarity, these uncertainties, when not appropriately managed, can lead to questionable conclusions. Moreover, in such assessments it is crucial that the conditions proposed by Townsend are considered. Because the hypothesis is founded on an asymptotic approximation, the Reynolds number should be high (although it is not clear what “high” means: Schultz and Flack [13] observe outer layer similarity of first- and second-order turbulence statistics at friction Reynolds numbers as low as $\delta^+ \approx 840$, whereas the data of Squire *et al.* [19] suggest that $\delta^+ \gtrsim 14\,000$ is necessary for such behavior). Additionally, Townsend’s hypothesis is only posed outside of the region where wall perturbations directly affect the flow. This is particularly pertinent given the growing understanding that certain roughness arrangements can produce very large motions, even where the roughness height is very small. Volino *et al.* [20], for example, demonstrate that 2D transverse bars have a dynamic effect farther into the boundary layer than 3D staggered cubes when the bar height is only 1/7th of the cube height. Based on investigation of existing data above a range of roughness geometries, Jiménez [21] suggests that generally the relative roughness height, k/δ , must be smaller than approximately 1/40 for wall similarity to be observed. This suggestion is even more convincing when flows over roughnesses with large spanwise length scales (such as 2D transverse bars) are not considered.

A. Spatial structure in rough-wall flows

The majority of existing rough-wall experiments have investigated the flow using single-point, typically time-averaged, measurements. There are a few notable examples of studies that use simultaneous measurements at multiple locations to examine the spatial structure of rough-wall bounded flows. Grass [22] observe, using hydrogen bubble visualizations, violent ejections of fluid, which contribute strongly to the Reynolds shear stress, rising almost vertically from the interstices between roughness elements. Similar, though apparently weaker, events were also observed above a smooth wall. The rough-wall visualizations of Grass [22] also reveal an apparent reduction in the frequency and strength of long streamwise vortices that are very apparent near to the smooth-wall

boundary during ejection events, likened to the wall velocity streaks of Kline *et al.* [23], leading the author to note that the dominant instability modes associated with the ejection events may differ with boundary roughness conditions. These observations have motivated numerous smooth-rough wall comparisons of the contribution to the Reynolds shear stress from ejection-type (and sweep-type) events [6,24,25]. Grass *et al.* [26] performed hydrogen bubble visualizations above various roughnesses. Their interest was predominantly in the flow very near to the roughness elements: specifically, in the spanwise spacing of near-wall streamwise vortices, which was observed to be approximately proportional to the roughness height. They note, however, that even for their largest roughness, which generates intense disturbances in the separating flow around the roughness elements, the rough-wall flow is “remarkable” in its ability to organise itself over a very small vertical distance.

A large proportion of more recent rough-wall studies attempt to assess the large-scale spatial structure of the outer region in the context of a hairpin-packet paradigm. This is motivated by considerable evidence that streamwise aligned packets of hairpin-like vortices are a recurrent feature in the outer region of smooth-wall bounded flows [27–30], and that these vortex packets contribute significantly to momentum and energy transport [31–33]. Volino *et al.* [20,34,35] and Wu and Christensen [25,36] compare smooth- and rough-wall two-point correlations of various velocity and vorticity components. These studies all conclude that the outer region in rough-wall flow is *qualitatively* similar to that of smooth-wall flow, characterized by similarly inclined packets of hairpin vortices (note, the correlation fields of Nakagawa and Hanratty [37] and Flores and Jimenez [38] also indicate a similar result, although these authors do not interpret their results in the framework of hairpin packets). It is important to note, however, that *quantitative* smooth-rough differences are noted in most of the above studies. Volino *et al.* [34], for example, note clear reductions, relative to the smooth-wall case, in the characteristic lengths of rough-wall two-point correlations involving wall-normal velocity and spanwise vorticity fluctuations. Similarly, Wu and Christensen [36] observe a decrease in the streamwise extent of two-point correlations over a surface replicated from a damaged turbine blade. Above 2D transverse bars, Volino *et al.* [20,35] observe a significant increase in all examined spatial scales despite the aforementioned qualitative similarities.

Recent rough-wall measurements by Hong *et al.* [39] suggest that the outer region of rough-wall bounded turbulence may contain a signature of the roughness in scales of $O(k)$, where k is the roughness height. Evidence for this is provided primarily from examination of their spatial energy spectra of rough-wall streamwise and wall-normal velocity fluctuations, which show distinct bumps in scales associated with motions of length $k - 3k$. The apparent modifications to the small-scale energy are small and do not noticeably affect the respective Reynolds stress profiles; Hong *et al.* [39] report outer layer agreement between their rough-wall Reynolds stress profiles and those from previous smooth-wall measurements. Similar observations are made by Hackett *et al.* [40] in oceanic particle image velocimetry (PIV) data, with small spectral bumps appearing across all examined elevations at a scale similar to the bottom ripple height. Hong *et al.* [41] extend the work of Hong *et al.* [39], investigating subgrid-scale energy transfer, and the coherent structures primarily associated with this transfer, above the same rough wall (and using predominantly the same PIV data, though additional time-resolved PIV data are included). Relevant to the present discussion, the authors note that in addition to the apparent excess of energy in scales of $O(k)$, there is further indication of an outer layer roughness-associated effect in the instantaneous spacing of spatially-filtered vortices in both the streamwise–wall-normal and streamwise–spanwise planes, which appears to scale with the wavelength of the roughness. However, no comparison to smooth-wall flow is made. A new rough-wall turbulence model is proposed by Hong *et al.* [41] (distinct from the smooth-wall hairpin-packet model described above), whereby the dominant vortical structure is a “U shaped” (inverted hairpin) vortex generated by the stretching of spanwise vorticity in regions of high streamwise velocity between roughness elements. These vortices are lifted away from the wall as they propagate downstream leading to the inclined trains of vortices observed instantaneously by Hong *et al.* [41] (and in previous rough-wall studies, see the paragraph above). Hong *et al.* [41] note, however, that for randomly distributed and/or closely packed rough surfaces, the characteristics

and interaction frequency of the roughness-induced vortices may differ from their case of a regular roughness, resulting in less obvious, or nonexistent, outer layer roughness-scale influences. There is also evidence that the vorticity structure in rough-wall boundary layer flows is influenced by the relative magnitude of the roughness scale and the wall-normal location of the peak in the Reynolds shear stress; see Mehdi *et al.* [42] and Ebner *et al.* [43].

B. The present contribution

In the present work, we utilize recently obtained PIV data to make true spatial comparisons between smooth- and rough-wall flows across an unprecedented range of scales. Two PIV arrangements are employed. Together, these are able to fully resolve over three orders of magnitude of streamwise scales, ranging from the roughness-sized motions investigated by Hong and colleagues, to the large-scale streamwise-coherent motions described above. Comparisons between the smooth- and rough-wall are made across a friction Reynolds number range $6000 \lesssim \delta^+ \lesssim 18\,000$, where δ is the boundary layer thickness (determined here as the wall-normal location at which the local mean streamwise velocity is 99% of the free-stream velocity, U_∞) and the $+$ superscript denotes inner normalization. The rough-wall measurements are taken above a sandpaper roughness that is well characterized according to the criteria of Jiménez [21]. Specifically, the relative roughness height, δ/k is greater than 370 ($\delta/k_s > 170$, where k_s is Nikuradse’s (1933) [44] equivalent sand grain roughness) for all rough-wall measurements. This also reduces the positional error associated with difficulties in determining ϵ , which is bounded by the roughness height [1]. A floating element drag balance is used to obtain U_τ to within an accuracy $< 1\%$ for all rough-wall measurements presented here (see Baars *et al.* [45]). Throughout this paper, x , y , and z denote the streamwise, spanwise, and wall-normal directions, respectively, with $z = 0$ located at the roughness crest. Mean quantities are denoted using uppercase variables, and fluctuations are denoted using lowercase variables.

II. EXPERIMENTAL DETAILS

The PIV measurements were performed above a smooth and rough wall in the High Reynolds Number Boundary Layer Wind Tunnel at the University of Melbourne (see Nickels *et al.* [46,47] for details). For the rough-wall measurements, the bounding wall was covered with a single sheet of P36 grit sandpaper (total area: 54 m^2). This is a three-dimensional, globally homogeneous surface roughness, with a normally distributed local roughness height, h . Here the physical roughness height of the sandpaper surface is defined as $k = 6\sigma(h) = 0.902\text{ mm}$, and Nikuradse’s [44] equivalent sandgrain is determined to be $k_s = 1.96\text{ mm}$. Key parameters of the rough surface are provided in Table I and, in further detail, in Squire *et al.* [19].

Two streamwise–wall-normal PIV arrangements were employed above the smooth and rough wall. Both arrangements utilize eight 14 bit PCO 4000 PIV cameras (2672×4008 pixels), providing stitched images of the streamwise–wall-normal plane that are almost 90 MP in size. One arrangement [hereafter the large-field-of-view (LFOV) arrangement; see Sec. II A] was targeted at investigating structural features with reasonably large streamwise dimension. The other arrangement (hereafter the tower arrangement; see Sec. II B) had a narrower streamwise extent to provide well-resolved information on the wall-normal structure of the boundary layer. The center of the field of view of

TABLE I. Key sandpaper surface parameters. Note that h' is the surface deviation about the mean roughness height ($h' = h - \bar{h}$).

Parameter	Class	k	k_s	k_a	k_p
Formula	–	$\sqrt{h'^2}$	See [19]	$ \overline{h'} $	$h'_{\max} - h'_{\min}$
Value [mm]	P36	0.902	1.960	0.119	1.219

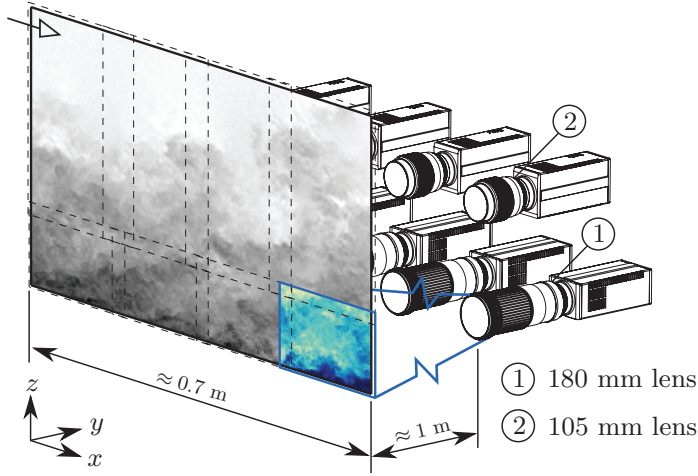


FIG. 1. Schematic of the large-field-of-view PIV arrangement (not to scale) including an example stitched instantaneous field. The region of color shows the field of view contributed by one example camera, and the dashed boxes show how each camera is incorporated into the total stitched field. Note that the wind tunnel walls are removed for clarity; all cameras are located outside of the wind tunnel test section. The open arrow indicates the direction of the flow.

both arrangements was located nominally 21.7 m downstream of the boundary layer trip. In all PIV experiments the flow was seeded with tracer particles (diameter, $d \approx 1 \mu\text{m}$), delivered by a Rosco aqueous-glycol-solution smoke generator. The polyamide particles were ingested into the wind tunnel upstream of honeycomb straighteners and screens used for flow conditioning.

A. Large-field-of-view PIV configuration

The LFOV configuration consists of two horizontal rows of four cameras, at two magnifications as shown in Fig. 1. The four cameras on the top row were equipped with Sigma AF 105 mm f/2.8 EX DG macro lenses, providing a lower magnification than the bottom four cameras, which were fitted with Tamron SP AF 180 mm macro lenses. The smooth-wall LFOV measurements were previously published by de Silva *et al.* [48], de Silva *et al.* [49], and Chauhan *et al.* [50], with experimental details provided therein. For the rough-wall measurements, the light sheet was introduced from the top of the tunnel through a 6 mm thick glass plate. The particles were illuminated using a Spectra Physics “Quanta-Ray” PIV 400 mJ/pulse Nd:YAG double-pulse laser. Two plano-convex cylindrical lenses reduced the spanwise thickness of the coincident beams to approximately 0.5 mm (determined using burn tests). The narrow beams were subsequently spread into a sheet using three plano-concave cylindrical lenses (of -25 mm, -40 mm, and -50 mm focal length) arranged in series. The separation in time between releasing each energised laser cavity was chosen to obtain a maximum particle displacement of approximately 16 pixels.

LFOV PIV measurements were obtained at three free-stream velocities above the smooth wall ($U_\infty = 10$ m/s, 20 m/s and 30 m/s), and at two free-stream velocities above the rough wall ($U_\infty = 12.3$ m/s and 20 m/s). These were chosen to enable two comparisons between rough- and smooth-wall datasets at approximately matched δ^+ ($\delta^+ \approx 12\,000$ and $18\,000$), and at approximately matched Re_x ($\text{Re}_x \approx 1.6 \times 10^7$ and 2.9×10^7). All datasets capture a streamwise extent, L_x , of at least 2δ and were processed to have approximately the same final interrogation window size in viscous wall (plus) units ($w_{I_x}^+ \times w_{I_z}^+ \approx 80 \times 80$) to enable comparisons at similar spatial resolution. The exception to the latter is the smooth-wall data at 30 m/s, which required an unfeasibly small interrogation window size in pixels to obtain this spatial resolution, resulting in a high percentage of spurious

TABLE II. Details of the LFOV PIV measurements.

Wall	Boundary layer parameters						PIV parameters			
	U_∞	δ^+	Re_x	k_s^+	U_τ	δ	Field of view ($L_x \times L_z$)		Final window size ($w_{Ix} \times w_{Iz}$)	
–	(m/s)	–	($\times 10^{-7}$)	–	(m/s)	(mm)	(mm)	(number of δ)	(pixels) ^a	(plus units)
Smooth	10.1	6460	1.420	–	0.34	296	726×388	2.5×1.3	64×64	83.9×83.9
Smooth	20.0	12 200	2.823	–	0.64	295	576×395	1.9×1.3	32×32	79.3×79.3
Smooth	29.7	16 970	4.180	–	0.92	284	731×394	2.6×1.4	32×32	155.0×155.0
Rough	12.3	11 600	1.724	62	0.49	367	722×447	2.0×1.2	48×48	77.5×77.5
Rough	20.3	19 640	2.856	104	0.82	370	729×450	2.0×1.2	32×32	86.7×86.7

^aThe window size in pixels used for the top row of cameras was 75% of this value, such that the spatial resolution in wall units is matched across the field of view.

vectors and substantial small-scale noise, and hence was processed with $w_{Ix}^+ \times w_{Iz}^+ \approx 155 \times 155$. Note that for all LFOV measurements, the images from the top four cameras were processed using an interrogation window size in pixels that was 25% smaller than that of the bottom cameras in order to obtain matched spatial resolution across the entire field of view. Key experimental parameters from the LFOV PIV measurements are given in Table II.

B. Tower PIV configuration

In the tower configuration, presented in Fig. 2, the cameras are orientated with the larger sensor dimension in the wall-normal direction of the measurement plane. The cameras are staggered vertically, such that the combined field of view is a narrow vertical column that spans the entire boundary layer height. Due to the size constraints of the cameras, every second camera in the imaging array was located on the opposite side of the wind tunnel working section. For the smooth-wall measurements, all eight cameras were equipped with a Tamron SP AF 180 mm macro photography lens, a Sigma APO EX DG 2× teleconverter, and a 109 mm bellows (note

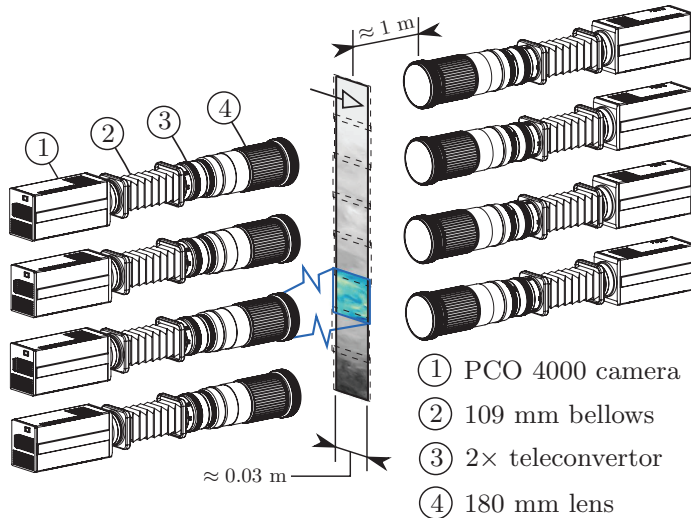


FIG. 2. As in Fig. 1 but for the tower PIV arrangement.

TABLE III. Details of the tower PIV measurements.

Wall	Boundary layer parameters						PIV parameters			
	U_∞	δ^+	Re_x	k_s^+	U_τ	δ	Field of view ($L_x \times L_z$)		Final window size ($w_{Ix} \times w_{Iz}$)	
–	(m/s)	–	($\times 10^{-7}$)	–	(m/s)	(mm)	(mm)	(plus units)	(pixels)	(plus units)
Smooth	10.3	6560	1.473	–	0.34	291	33×406	740×9168	24×24 48×48	7.6×7.6 15.3×15.3
Smooth	20.5	12 310	2.916	–	0.65	290	33×406	1388×17 247	24×24 32×32	14.4×14.4 19.2×19.2
Smooth	29.6	16 940	4.182	–	0.91	288	33×406	1946×23 940	24×24	20.0×20.0
Rough	12.4	12 140	1.781	64	0.49	372	26×543	854×17 715	32×32 ^a	15.2×15.2 ^a
Rough	20.4	18 460	2.862	104	0.82	347	27×543	1425×28 835	24×24 ^b	18.6×18.6 ^b

^aThe top camera in this measurement was processed using a 64×64 pixel interrogation window size, giving 100.2×100.2 plus units.

^bThe top camera in this measurement was processed using a 64×64 pixel interrogation window size, giving 162.7×162.7 plus units.

that long-range microscopes, such as the Infinity-K2 [51] do not utilize the full extent of the large format PCO 4000 sensors, and are therefore not well suited to these measurements). One effect of the rough wall, however, is to increase the boundary layer thickness, δ , relative to that observed above a smooth wall at matched free-stream velocity, U_∞ . Thus, to ensure that the full wall-normal extent of the boundary layer was observed during the rough-wall measurements, the top camera was equipped only with a Tamron SP AF 180 mm macro lens, generating a larger field of view (and poorer spatial resolution) than the lower cameras. Both the smooth- and rough-wall tower PIV measurements utilised a laser and optical path identical to that used in the LFOV configuration, except that the narrowed beams were spread into a sheet using only a single plano-concave cylindrical lens of focal length –25 mm. For the smooth-wall measurements, the maximum in-plane displacement of the particles was approximately 16 pixels. However, during the rough-wall measurements, it was infeasible to optimize the particle displacement for all cameras using a single laser. Thus, the time between each laser pulse was chosen to provide a maximum particle displacement in the lower seven cameras of approximately 18 pixels. The corresponding maximum particle displacement in camera eight was equal to approximately five pixels.

Datasets were obtained using the tower PIV arrangement under the same wind tunnel conditions as for the LFOV arrangement (see Table III). Again, the data were processed to enable approximately matched spatial resolution comparisons between smooth- and rough-wall flows at similar δ^+ and at similar Re_x . However, the spatial resolutions of the tower measurements are significantly better than those obtained using the LFOV arrangement.

C. Vector evaluation

Both PIV arrangements were calibrated using targets consisting of an array of precisely located dots, equispaced in the streamwise and wall-normal directions (by 5 mm and 2.5 mm for the LFOV and tower PIV measurements, respectively). For the tower PIV measurements, a glass calibration target was employed to allow imaging from both sides of the wind tunnel. The accuracy of PIV measurements is typically influenced by perspective errors, and by distortions caused by optical components. This is particularly true in high-resolution measurements, such as the tower PIV measurements, where a large number of lenses are required in series to provide the desired magnification. Because of this, the use of a dewarping pixel-to-real conversion process, rather than the application of a single scaling factor, is crucial. In the present measurements, a third-order

polynomial mapping was used to convert all pixels in a particular PIV image to a corresponding position in real space. The use of higher-order polynomial mapping functions was observed to have negligible effect on the resulting vector fields. Due to significant reflection of the laser sheet from the sandpaper, the lowest wall-normal extent of the field of view is approximately 3 mm from the crest of the local roughness height. This location was determined directly from the grid locations on the calibration target. Above the smooth wall, the wall-normal location of the field of view was determined as half of the distance between the lowest line of calibration dots and their reflection in the glass wall.

Prior to displacement evaluation, each raw PIV image was histogram filtered, and the average image for each camera was subtracted. Vector evaluation was performed using a fast-Fourier-transform (FFT)-based cross-correlation algorithm with multigrid [52], window deformation [53–55], and second-order correlation for spurious correction [56]. Window deformation was not utilized when processing the top camera of the rough-wall tower PIV arrangement, since this camera (which images a larger region than the lower seven cameras) mostly images the free stream. Two multigrid passes were used in all measurements, with each dimension of the initial interrogation window corresponding to twice that of the final pass, and the vector spacing equal to half of the final interrogation window size. The velocity fields from each camera were stitched such that the overlap region of the stitched field for each camera pair was equal to the overlap region from the camera in the pair that contains the lowest number of spurious vectors in the overlapping region.

D. Boundary layer parameters

The wall friction velocity, U_τ , for all rough-wall PIV measurements was determined directly using a floating element drag balance, located between $x = 19.5$ m and $x = 22.5$ m downstream of the beginning of the test section. These measurements provide estimates for U_τ across the present range of free-stream velocities with an estimated uncertainty of less than 1%. Baars *et al.* [45] provide a description of the rough-wall drag balance measurements, and an analysis of the corresponding errors. For the smooth-wall measurements, U_τ was determined using the composite fit of Chauhan *et al.* [57]. Although this approach also provides an estimate of the boundary layer thickness, for consistency with previous studies we use $\delta = \delta_{99}$, the wall-normal location at which the mean streamwise velocity is 99% of U_∞ . The free-stream velocity was determined for all PIV datasets using a free-stream Pitot-static tube that was sampled periodically throughout each measurement.

In Fig. 3 the dependence of the roughness function, ΔU^+ , on the inner normalized equivalent sandgrain roughness is presented. The solid black line in the figure shows the curve determined by Squire *et al.* [19] using hot-wire anemometry (HWA) above the same rough surface. The measurements taken at the higher roughness Reynolds number ($k_s^+ = 104$) appear to be in the fully rough regime and lie close to the fully rough asymptote of Nikuradse [44] (dashed black line). The data obtained with $k_s^+ \approx 64$, however, are apparently transitionally rough, although these data are close to exhibiting fully rough characteristics. The roughness function in Fig. 3 was determined for the present rough-wall data by minimizing the least squares error between the inner-normalized mean streamwise velocity and the rough-wall logarithmic law:

$$U^+ = \frac{1}{\kappa} \log(z + \epsilon)^+ + A - \Delta U^+ \quad (1)$$

on the inertial subdomain with $\kappa = 0.39$ and $A = 4.3$ [58]. (Note that the logarithmic region is herein taken to coincide with the inertial subdomain.) The zero plane displacement, ϵ , accounts for the fact that the roughness displaces the entire flow away from the wall. Here we assume $\epsilon = k/2$ and note that any sensible choice of ϵ does not appreciably change the results presented herein (for more details see Squire *et al.* [19]). To estimate the wall normal location of the start of the rough-wall inertial subdomain, z_I , we follow Mehdi *et al.* [42], who, guided by the balance of terms in the mean momentum equation, provide convincing evidence that the onset of leading order inertial dynamics in rough-wall flows scales with the peak in the Reynolds shear stress, z_m . This is contrary to the

classically assumed k or δ scaling. Squire *et al.* [19] and Morrill-Winter *et al.* [59] adopt the Mehdi *et al.* [42] scaling, using the empirical formulation for z_m provided therein, and show that for the present rough surface an inertial subdomain spanning $2.5 \times z_m < z < 0.19\delta_{99}$ is consistent with the classical features of the logarithmic region. Hence, these same bounds are employed in the present study, where, from Mehdi *et al.* [42], for the present roughness regime [$z_m/k_s > O(1)$]:

$$z_m = 0.89(\nu/U_\tau)^{0.36} k_s^0 \delta_{99}^{0.64}. \quad (2)$$

For the smooth-wall data, we adopt the log law bounds and constants of Marusic *et al.* [58]: $3\sqrt{\delta\nu/U_\tau} < z < 0.19\delta$, $\kappa = 0.39$ and $A = 4.3$, respectively, which are also consistent with the analysis of the mean momentum equation over smooth walls Klewicki *et al.* [60]. Note that for the outer bound Marusic *et al.* [58] actually employ $z = 0.15\delta_c$, where δ_c is computed using the composite fit of Chauhan *et al.* [57]. Thus, the coefficient has been adapted here for use with the present definition of δ .

E. Spatial resolution

By utilizing the two experimental arrangements described above, the present measurements enable the structure of the rough-wall turbulent boundary layer to be scrutinized across a uniquely large range of scales. This is demonstrated in Fig. 4(a), which shows the inner-normalized premultiplied energy spectrogram of the streamwise velocity fluctuations above the smooth wall at $U_\infty = 20$ m/s. The blue and red regions show the range of streamwise scales, λ_x , and wall-normal locations, z , captured by the LFOV and tower PIV arrangements, respectively, with the region of overlap shown in purple. Combined, the two PIV arrangements cover streamwise scales that span three orders of magnitude, ranging from scales longer than twice the boundary layer thickness, to scales on the order of the Kolmogorov microscale. To demonstrate the latter point, Fig. 4(b) shows the inner normalized Kolmogorov length scale, η^+ , as a function of wall-normal location for the smooth- and rough-wall boundary layer parameters examined here. The Kolmogorov scale is estimated using previously published HWA data from the Melbourne wind tunnel (all with $l^+ < 26$: smooth-wall data from Hutchins *et al.* [61]; rough-wall data from Squire *et al.* [19]), assuming local isotropy such that the turbulent dissipation rate, $\epsilon \approx 15\nu(\partial u/\partial x)^2$. Also plotted in Fig. 4(b) is the ratio of the PIV interrogation window size to the Kolmogorov microscale, l/η , for all of the tower PIV measurements. Note that only the smallest interrogation window size employed in each measurement (see Table III)

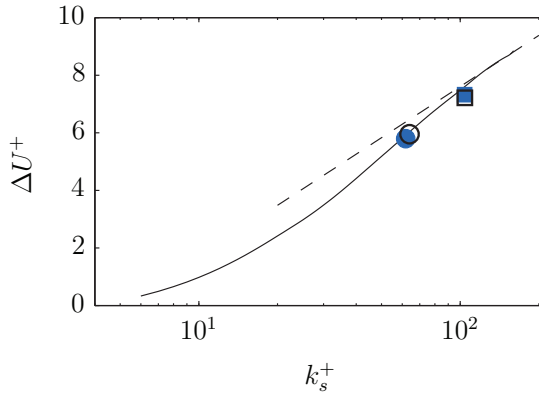


FIG. 3. The roughness function, ΔU^+ , as a function of the inner-normalized equivalent sand-grain roughness height, k_s^+ . Rough-wall data at $U_\infty = 12.3$ and 20 m/s are shown for LFOV (● and ■, respectively) and tower (○ and □, respectively) arrangements. The solid black line shows the curve determined by Squire *et al.* [19] using hot-wire anemometry above the same rough surface, and the dashed black line shows the fully rough asymptote of Nikuradse [44].

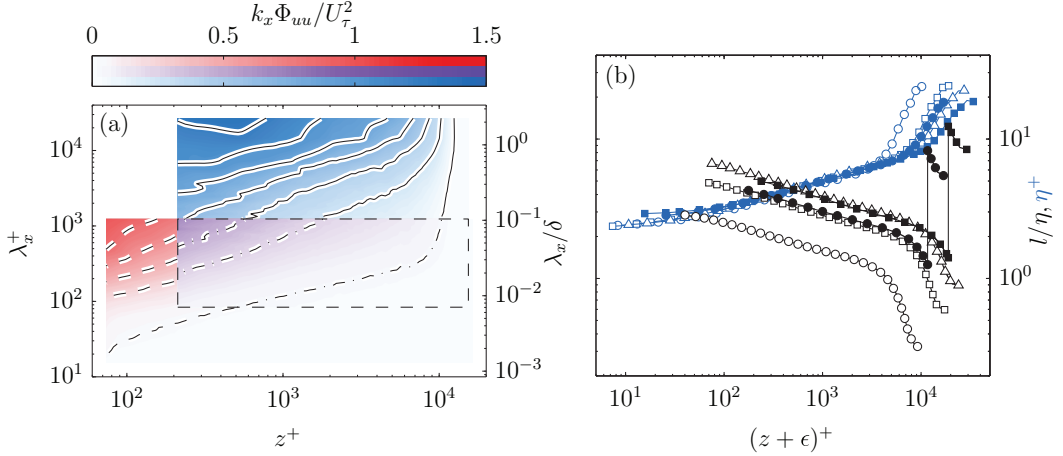


FIG. 4. Resolvable scales using the present PIV measurements. (a) The one-sided inner-normalized premultiplied energy spectrogram of the streamwise velocity fluctuations above the smooth wall at $U_\infty = 20$ m/s. The blue and red regions show the range of streamwise scales and wall-normal locations captured by the LFOV and tower PIV arrangements, respectively. The region of overlap between the LFOV and tower PIV spectrograms is denoted with a dashed box and colored in purple. Contour lines are shown for the LFOV (—), tower (---), and overlap (— · —) regions for $k_x \phi_{uu} / U_\tau^2 = 0.04 \rightarrow 1.24$ with a spacing of 0.2. (b) The inner-normalized Kolmogorov microscale (η^+) from HWA measurements (blue), and the number of Kolmogorov scales within an interrogation window (l/η) for the tower PIV measurements (black). Note that only results obtained using the smallest interrogation window size employed in each tower measurement are shown (see Table III). The $\circ$, $\square$, and $\triangle$ symbols show smooth-wall hot-wire data at $\delta^+ = 6120$, 11 830 and 15 210 [61]; the $\bullet$ and $\blacksquare$ symbols show rough-wall hot-wire data at $\delta^+ = 12\,300$ with $k_s^+ = 63$ and $\delta^+ = 20\,160$ with $k_s^+ = 103$ [19]. Similarly, the $\circ$, $\square$, and $\triangle$ symbols show smooth-wall tower PIV data at $\delta^+ = 6560$, 12 310, and 16 940; and the $\bullet$ and $\blacksquare$ symbols show rough-wall tower PIV data at $\delta^+ = 12\,140$ with $k_s^+ = 64$ and $\delta^+ = 18\,460$ with $k_s^+ = 104$.

is represented in Fig. 4(b). It is evident in Fig. 4(b) that the tower PIV arrangement is capable of resolving scales on the order of the Kolmogorov microscale ($l/\eta \lesssim 10$) across all smooth- and rough-wall boundary layer parameters examined.

III. TURBULENCE STATISTICS

All statistics are computed by ensemble averaging across independent realisations (PIV vector fields), followed by line averaging in the streamwise direction. The friction Reynolds number increase across the field of view is estimated to be less than 0.2% for the tower measurements and 4% for the LFOV measurements, meaning that this approach to averaging produces approximately local estimates of statistics. Note that except where clearly stated, only the dataset with the highest spatial resolution is plotted for each speed and wall condition throughout this section (see Table III). Additionally, filled symbols are used exclusively throughout to denote rough-wall data, while open symbols denote smooth-wall data. In all figures, profiles have been downsampled (usually to approximately 30 points) for clarity.

A. Reynolds number trends and validation

1. Tower PIV statistics

First- and second-order turbulence statistics obtained using the tower PIV arrangement are presented in Fig. 5. The first row shows smooth-wall data at $U_\infty \approx 10$ m/s, 20 m/s, and 30 m/s, and

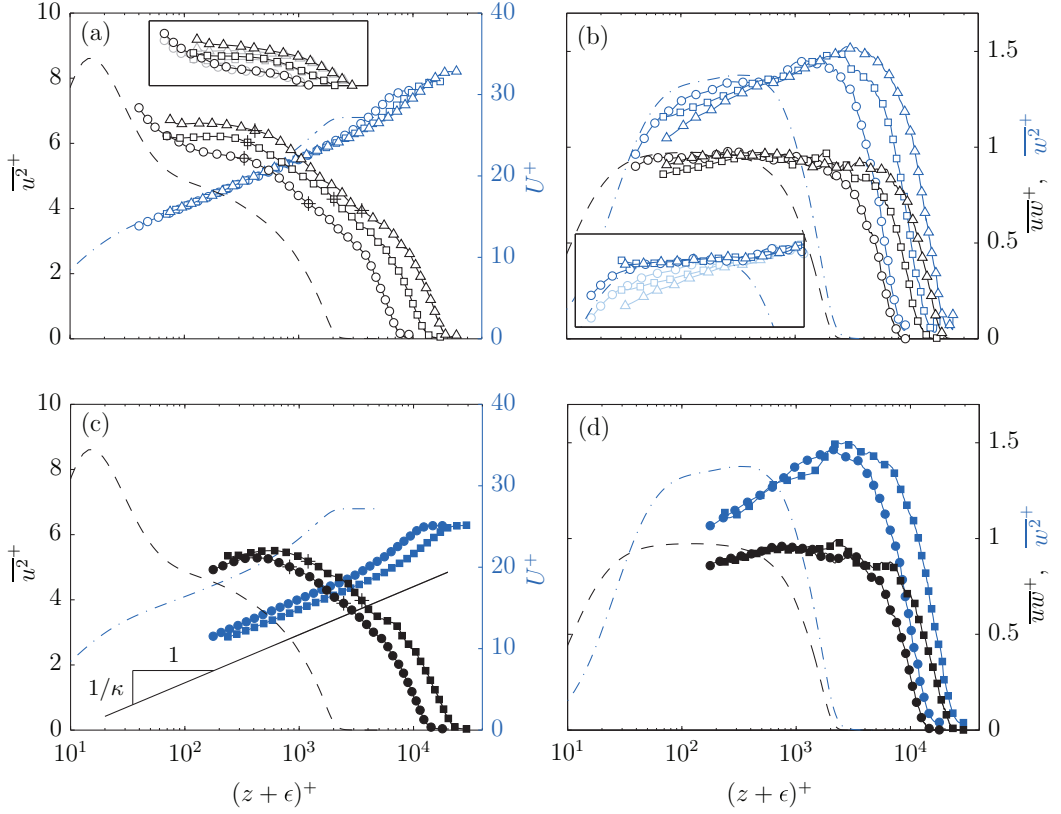


FIG. 5. Smooth-wall (a and b) and rough-wall (c and d) first- and second-order turbulent statistics obtained using the tower PIV arrangement. Statistics are presented for the smooth wall at $\delta^+ = 6560$ ($\circ$), 12 310 ($\square$), and 16 940 ($\triangle$), and for the rough wall at $\delta^+ = 12\,140$ with $k_s^+ = 64$ ($\bullet$) and $\delta^+ = 18\,460$ with $k_s^+ = 104$ ($\blacksquare$). In (a) and (c), blue symbols show the inner-normalized mean streamwise velocity, U^+ , while in (b) and (d) these symbols show the variance of the wall-normal velocity fluctuations, w^{2+} . Symbols with crosses overlayed in the $\overline{u^2}^+$ profiles in panels (a) and (c) indicate the approximate location of the beginning and end of the logarithmic region according to the definitions in Sec. II D. Smooth-wall statistics from the DNS study of Sillero *et al.* [62] are also included for comparison: U^+ ($-\cdot-\cdot-$) and $\overline{u^2}^+$ ($- - -$) in panels (a) and (c), and $\overline{w^2}^+$ ($- \cdot - \cdot -$) and $\overline{uw}^+$ ($- - -$) in panels (b) and (d). Note that $\epsilon = 0$ for the smooth-wall data. The insets in panels (a) and (b) show $\overline{u^2}^+$ and $\overline{w^2}^+$, respectively, corrected for spatial attenuation affects using the approach of Ref. [63] (faded symbols show the original profiles, solid symbols show the corrected profiles).

the second row shows rough-wall data at $U_\infty \approx 12.3$ m/s and 20 m/s. Profiles from smooth-wall DNS at $\delta^+ \approx 2000$ (Sillero *et al.* [62]) are shown in all plots for comparison. The smooth-wall mean streamwise velocity profiles [Fig. 5(a)] compare well and exhibit the expected Reynolds number behavior in the log and wake regions. Similarly, both of the rough-wall mean velocity profiles in Fig. 5(c) show a convincing log-linear region, shifted vertically downwards by ΔU^+ due to the roughness-induced increase in wallward momentum flux [1]. The slope of the rough-wall logarithmic region is very similar to that in the smooth-wall flow. Recall that U_τ for the rough-wall profiles was determined directly using drag balance data, i.e., no κ value is assumed in Fig. 5(c), so the similarity in slope between the smooth and rough wall is an empirical result. This observation has been previously made from HWA measurements above the same rough surface [19].

The Reynolds stress profiles also show self-consistent Reynolds number trends, which agree with previous studies (e.g., Hutchins *et al.* [61], Morrill-Winter *et al.* [64] for smooth-wall flows, and

Schultz and Flack [13], Squire *et al.* [19] for rough-wall flows). Both wall conditions exhibit a region of log linearity in $\overline{u^2}^+$ across the inertial subdomain, as predicted by Townsend's [65] attached eddy hypothesis (note that symbols with crosses overlayed in Figs. 5(a) and 5(c) indicate the approximate start and end of the inertial subdomain according to the definitions in Sec. II D). Previous evidence for such a region was provided by Marusic *et al.* [58] in flows bounded by smooth walls, and by Squire *et al.* [19] in rough-wall flows. Inside the inertial subdomain, the magnitude of the rough-wall streamwise variance appears to decrease towards the wall, whereas the smooth-wall data indicate an increase towards a near-wall peak (as is observed in the DNS and is well documented in smooth-wall flows). We lack the near-wall data to determine whether such a peak is present in either rough-wall case. However, the observed attenuation of near-wall energy in fully rough flows is well documented, resulting from roughness-generated disturbances to the near-wall cycle [13,22]. Squire *et al.* [19] note that the degree of near-wall attenuation is tied to the roughness strength, here quantified by k_s^+ , which is consistent with the near-wall merging of the different Reynolds number profiles in Fig. 5(c). Interestingly, the rough-wall $\overline{w^2}^+$ profiles also indicate near-wall attenuation relative to the smooth wall. Direct comparisons of smooth- and rough-wall statistics are made later in the paper.

As was previously observed by Morrill-Winter *et al.* [64] and Morrill-Winter *et al.* [59] in smooth- and rough-wall flows, respectively, the location and magnitude of the peak in $\overline{w^2}^+$ appears to increase with increasing friction Reynolds number. Particularly for the smooth-wall profiles in Fig. 5(b), $\overline{w^2}^+$ is increasingly attenuated near to the wall as Reynolds number is increased. This is likely to be the result of spatial attenuation, since the spatial resolutions of the compared datasets are not exactly matched. The streamwise scale of the energy containing wall-normal fluctuations is significantly smaller than that of the streamwise fluctuations across the entire boundary layer, which explains why spatial attenuation is less obvious in $\overline{u^2}^+$ than in $\overline{w^2}^+$. Recently Lee *et al.* [63] proposed an approach for validating underresolved PIV statistics in wall-bounded turbulence that predicts missing energy for a given interrogation window size and laser sheet thickness from box filtered smooth-wall DNS fields. This approach assumes that the contribution to the total energy from the small scales is approximately fixed with Reynolds number for u , v , and w . The inset in Fig. 5(b) shows the smooth-wall $\overline{w^2}^+$ profiles, corrected using the approach of Lee *et al.* [63] using the interrogation window sizes given in Table III and a laser sheet thickness of 0.5 mm. The correction has negligible effect external to $(z + \epsilon)^+ \approx 1000$, so data are presented only in the range $30 < (z + \epsilon)^+ < 2000$. All data show improved agreement with the DNS, indicating that the near-wall differences observed between the uncorrected $\overline{w^2}^+$ profiles in Fig. 5(b) are most likely due to the differences in spatial resolution. The same correction is also applied to the $\overline{u^2}^+$ profiles in Fig. 5(a), and its effect is shown in the inset of this figure. As above, data are plotted only in the range $30 < (z + \epsilon)^+ < 2000$. As expected, the correction has less influence on $\overline{u^2}^+$ and does not affect the data trends discussed above. Of course, the scheme proposed by Lee *et al.* [63] is only applicable in smooth-wall flow measurements and cannot be applied to the rough-wall data in Fig. 5.

The smooth- and rough-wall Reynolds shear stress profiles in Figs. 5(b) and 5(d) show similar behavior, plateauing at approximately unity inside of $z/\delta \approx 0.3$. All profiles exhibit a decrease in intensity near to the wall, to below that of the smooth-wall DNS profiles. Such behavior was not observed by Morrill-Winter *et al.* [64] above smooth walls, who showed near-wall agreement with the Sillero *et al.* [62] DNS profile up to $\delta^+ = 12\,509$. The length of each sensing element in wall units of the probe used by Morrill-Winter *et al.* [64] was generally smaller than the interrogation window sizes presented here, suggesting the that the near-wall attenuation of $\overline{uw}^+$ in Figs. 5(b) and 5(d) may be a result of spatial attenuation (hot-wire anemometry and PIV measurements also apply different filters). Unfortunately, however, the correction scheme proposed by Lee *et al.* [63] is posed only for u , v , and w turbulence intensity profiles and so cannot be used to validate this suggestion. A common approach to determine U_τ in smooth- and rough-wall flows is to equate the total stress (viscous stress and Reynolds shear stress) and wall shear stress in the inner part of the boundary

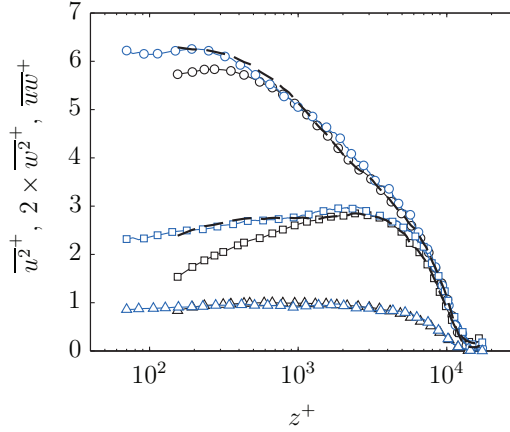


FIG. 6. Comparison of Reynolds stress profiles obtained from the tower (blue symbols) and LFOV (black symbols) PIV arrangements above the smooth wall at 20 m/s. $\overline{u^2}^+$, $\overline{w^2}^+$, and $\overline{uw}^+$ are denoted using $\circ$, $\square$ and $\triangle$, respectively. The black dashed lines show the predicted LFOV statistics obtained when the missing energy due to the difference in spatial resolution between the LFOV and the tower measurement is estimated using the scheme of Ref. [63].

layer (e.g., Refs. [11,25,66,67]). Except for very close to the wall, the viscous stress term is typically very small and is often neglected. Applying such an approach to the data in Fig. 5 produces estimates for U_τ that agree with the present drag balance determined values to within $\pm 3\%$. (The peak in each $\overline{uw}^+$ profile is determined here as the peak in a second order polynomial fit to the data across the range $400 < (z + \epsilon)^+ < 2000$, and the viscous stress component is computed from $\nu \partial U / \partial z$ at the wall-normal location of this peak.) It is noted, however, that some studies above regular roughnesses report significant modifications to the $\overline{uw}^+$ profile where the mean velocity profile, and hence its gradient, appear relatively unaffected [6,68]. Therefore, aside from the obvious dependence of the total stress technique on the quality of the uw data and on the spatial resolution of the dataset (uw is a difficult quantity to measure), it is possible that the veracity of this technique in rough-wall flows depends on the nature of the roughness.

2. LFOV PIV statistics

All turbulence statistics presented thus far were obtained using the tower PIV arrangement due to its superior spatial resolution compared to the LFOV arrangement. In Fig. 6 the second order velocity statistics obtained using the LFOV arrangement above the smooth wall at 20 m/s are compared to those obtained from the tower arrangement under the same conditions. Near the wall, both $\overline{u^2}^+$ and $\overline{w^2}^+$ from the LFOV exhibit attenuation relative to the respective statistic from the tower. Again, the scheme of Lee *et al.* [63] is employed to investigate whether the attenuation observed is consistent with spatial resolution differences between the two measurements. Here, however, we use the scheme to predict the missing energy due to the difference in spatial resolution between the LFOV and tower measurement. The corrected LFOV statistics are shown as thick dashed lines in Fig. 6 and show good agreement with the tower statistics. Recall that Lee *et al.* [63] do not investigate the effect of PIV spatial resolution on $\overline{uw}^+$. However, consistent with the observations in Fig. 6, it is expected that the level of near-wall attenuation observed in $\overline{uw}^+$ will always be less than that observed in $\overline{w^2}^+$, since the scales of motions that contribute to the former are larger near the wall. For brevity, comparisons are not presented between every tower and LFOV dataset, but we note that similar consistency between the datasets to that observed in Fig. 6 is observed across all free-stream velocities and wall conditions.

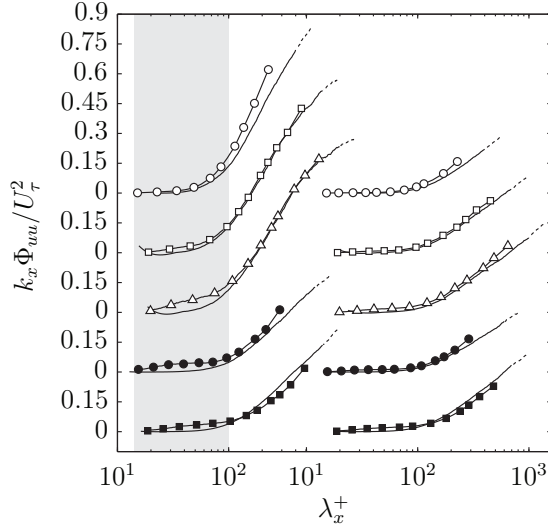


FIG. 7. Comparison of inner-normalized premultiplied energy spectrograms of the streamwise velocity fluctuations, $k_x \Phi_{uu}/U_\tau^2$, obtained using the tower PIV arrangement (symbols) and using HWA (lines) at approximately matched δ^+ . Details of each comparison are given in Table IV. The comparisons in row (a) are given at the lowest wall-normal location of each PIV measurement (from top to bottom: $z^+ = 63, 80, 84, 175, 260$). Those in row (b) are given at $z^+ = 500$. The gray regions indicate the approximate range of wavelengths, λ_x^+ , where noise is present in the PIV.

3. PIV noise

In addition to attenuation due to spatial resolution effects, PIV measurements are also potentially subject to measurement noise that can cause artificial amplification of turbulence statistics [69]. It is therefore possible to obtain sensible looking estimates of turbulent fluctuations that are in fact spatially under-resolved and contaminated by measurement noise. In Fig. 7 inner-normalized premultiplied energy spectra of the streamwise velocity fluctuations, $k_x \Phi_{uu}/U_\tau^2$, obtained from the tower PIV arrangement are compared to those from previously published HWA measurements at approximately matched δ^+ . Details of the compared datasets, including their exact Reynolds numbers and spatial resolutions, are given in Table IV. Note that in both rough-wall comparisons, the compared PIV and HWA measurements are obtained above identical roughnesses, and are at

TABLE IV. Summary of the PIV-HWA comparisons in Fig. 7. The “Type” indicates which velocity components are resolved by the hot-wire measurement: “—” means that only u fluctuations are measured, and “ $\times$ ” means that both u and w fluctuations are measured simultaneously. The wire length in each HWA measurement is denoted using l . H09, T14, S16, and M16 refer to Hutchins *et al.* [61], Talluru *et al.* [71], Squire *et al.* [19], and Morrill-Winter *et al.* [59], respectively.

Wall	PIV dataset			HWA dataset			
	Symb.	δ^+	$w_{l,x,z}^+$	δ^+	l^+	Type	Ref.
Smooth	○	6560	7.6	6120	22.9	—	H09
Smooth	□	12 310	14.3	10 940	20.1	$\times$	T14
Smooth	△	16 940	19.8	15 200	21.6	—	H09
Rough	●	12 140	15.3	12 080	16.0	$\times$	S16, M16
Rough	■	18 460	18.7	20 160	26.1	—	S16

matched k_s^+ . In Fig. 7, the comparisons in row (a) are at the lowest inner-normalized wall-normal location of the respective PIV measurement, while those in row (b) are given at $z^+ = 500$. For each of the PIV measurements, the spectra are computed for each realisation and then ensemble averaged across all realizations. Because the streamwise extent of the field of view is small, there are nonperiodic end effects in the resulting spectra, which infect the entire range of measurable wavelengths. In an attempt to minimize these effects, a Hamming window is applied. The use of Hamming window, however, imposes spurious energy at scales in the range of window length (i.e., the length of the field of view). Therefore, the energy at the largest three wavelengths are not plotted in Fig. 7. We note that it is possible that the influence of the Hamming window on the spectra extends to even smaller scales. For the HWA spectra, we use the local mean streamwise velocity to convert, using Taylor's [70] frozen turbulence hypothesis, between time and approximate wavelength. All HWA spectra are truncated at $\lambda_s^+ \approx 2000$ to facilitate comparison to the PIV spectra.

When comparing the PIV and HWA spectra in Fig. 7, it is critical to note that the spatially attenuating effects of PIV, single-wire HWA, and multiwire HWA are all different. The obvious differences, for example, observed between the smooth-wall comparisons at $\delta^+ \approx 6400$ probably indicate only that the inherent attuation of the PIV ($w_{I_x}^+ \times w_{I_z}^+ \times w_{I_y}^+ = 7.6 \times 7.6 \times 11.4$) is less than for the single-wire hot wire ($I^+ = 22.9$). However, all PIV spectra exhibit a range of scales over which the small-scale energy exhibits significant amplification relative to the associated HWA spectrum (i.e., for $\lambda_x^+ \lesssim 100$; see the gray-shaded regions in Fig. 7). The amplification of the PIV spectra does not appear to be a result of differences in spatial attenuation between PIV and HWA measurements, but is instead due to noise in the PIV measurements (as demonstrated below). The contribution to the total energy from noise apparently decreases with increasing wall-normal position and is much less apparent in the spectral comparisons at $z^+ = 500$. Similar observations (not presented here for brevity) are made in the spectra of the wall-normal fluctuations, where PIV measurement noise is also apparent for $\lambda_x^+ \lesssim 100$.

The characteristics and causes of PIV noise have been extensively studied. Foucaut *et al.* [72] demonstrate that PIV noise is well described by white noise that has undergone the same transfer function as the raw PIV images during processing. This transfer function is well described by a box filtering process (e.g., Lee *et al.* [63]), which in Fourier space represents a multiplication by a squared cardinal sine (sin_c) function. Thus, Foucaut *et al.* [72] suggest that a given 1D spectra, Φ , can be described using

$$\Phi_{\text{PIV}}(\lambda) = [\Phi_{\text{flow}}(\lambda) + \Phi_{\text{noise}}(\lambda)] \left[\text{sin}_c \left(\frac{\pi w_I}{\lambda} \right) \right]^2, \quad (3)$$

where Φ_{flow} and Φ_{PIV} are the true and measured spectra, respectively, Φ_{noise} is due to measurement noise, and w_I is the interrogation window size. Foucaut *et al.* [72] show that superimposing white noise onto HWA spectra in this manner produces very good estimates of PIV spectra. There are, however, two assumptions made in constructing Eq. (3): first, that no noise is introduced *during* processing of the PIV images, and, second, that the 1D slice of the 3D sin_c function given in Eq. (3) sufficiently describes the integrated effect of box filtering on the 1D spectra. The latter assumption is certainly questionable. Atkinson *et al.* [69], for example, compute the true 1D transfer function from boundary layer DNS data and show that this is attenuated below the 1D sin_c function in Eq. (3). Nonetheless, Eq. (3) is still used in numerous studies to estimate the effect of PIV noise where estimates of the “noiseless” spectra, Φ_{flow} , are available (see, for example, Foucaut *et al.* [72], Herpin *et al.* [73]). Here we employ Eq. (3), approximating Φ_{flow} using spectra from (spatially attenuated) HWA measurements obtained under matched conditions to the PIV measurements (see Table IV), and assuming that the measurement noise, Φ_{noise} , is white. Our aim is to demonstrate only that the small-scale humps observed in Fig. 7 are consistent with PIV measurement noise. Estimates of the contribution to the total energy from this noise are subject to all of the assumptions mentioned above.

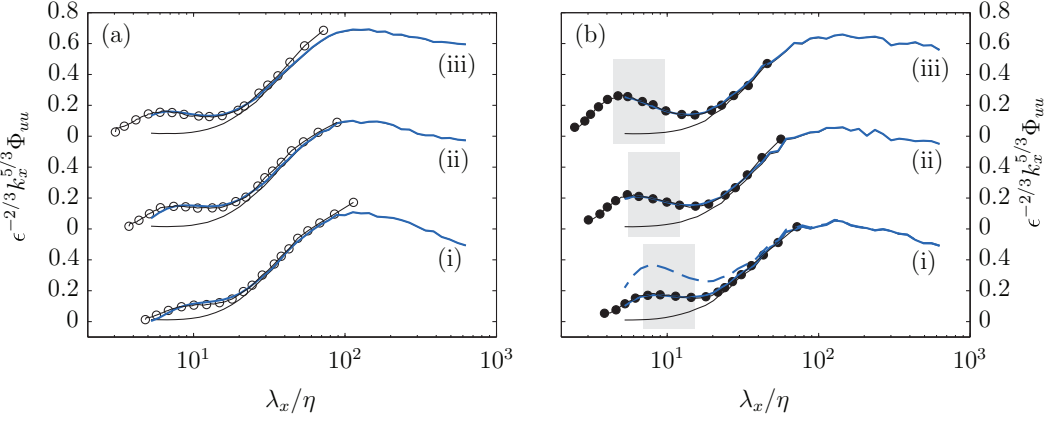


FIG. 8. Comparison of the small-scale compensated energy spectra of the streamwise velocity fluctuations for (a), the smooth, and (b), the rough, wall from HWA (thin black lines) and tower PIV (symbols) measurements at $\delta^+ \approx 12\,000$. For each wall condition, spectra are shown at $(z + \epsilon) = 0.03, 0.1$, and 0.3 [profiles (i), (ii), and (iii), respectively]. The thick blue lines show an estimate of the PIV spectra obtained using Eq. (3) with Φ_{flow} approximated using the HWA spectra, and Φ_{noise} (assumed white) determined to optimize the collapse between the estimated and measured PIV spectra. The thick dashed blue line shows the PIV spectra estimated in this manner when Φ_{noise} is chosen to contribute $\sim 3\%$ to the streamwise turbulence intensity at that location. In panel (b), the gray regions show $k < \lambda_x < 2k$.

Figure 8 compares the compensated energy spectra of the streamwise velocity fluctuations from HWA and tower PIV measurements across three wall-normal locations. This energy scaling accentuates high-wave-number, small-scale, energy, enabling further scrutiny of the apparent noise observed in Fig. 7. Comparisons are shown for the smooth and rough wall at $\delta^+ \approx 12\,000$ [Figs. 8(a) and 8(b), respectively]. In both comparisons the HWA measurement employed uses a multiwire probe which resolves u and w fluctuations simultaneously and occupies a similar volume (in wall units) to the interrogation volume of the respective PIV measurement (see Table IV). Also shown for each spectral comparison in Fig. 8 is an estimate of the PIV spectra (thick blue lines), obtained using the HWA spectra and Eq. (3), where Φ_{noise} was determined to maximize collapse between the estimated and measured PIV spectra. At all wall-normal locations, the estimated and measured PIV spectra show excellent agreement, indicating that the small-scale humps observed in the measured PIV spectra in Figs. 7 and 8 result from measurement noise, and that this noise is approximately white. Similar conclusions are drawn from the spectra of the wall-normal fluctuations and from the cospectra of the streamwise and wall-normal fluctuations. As mentioned above, the magnitude of Φ_{noise} is unlikely to accurately represent the true magnitude of the PIV measurement noise. It is worth noting, however, that the maximum noise contribution determined by comparing the HWA and PIV spectra (Φ_{uu} , Φ_{ww} and Φ_{uw}) from all available measurements in Table IV is equal to $\Phi_{\text{noise}} \approx 1.45 \times 10^{-6} \text{ m}^3 \text{ s}^{-2}$ ($\delta \Phi_{\text{noise}}^+ \approx 2.26 \times 10^{-6}$). Integrating this energy across all resolvable wave numbers leads to an estimated contribution from noisy energy that is only 1.5% of the local variance. Additionally, the contribution from noise to the Reynolds shear stress predicted using the above approach is consistently less than that obtained for $\overline{uw}$ and $\overline{ww}$, implying that the noise on u is uncorrelated with the noise on w . Thus, the observed PIV measurement noise will have negligible effect on most common tools for investigating spatial structure, for example, two-point correlations and conditional averages, since the noise is uncorrelated locally (i.e., between velocity components) and spatially [72].

There is an important observation to be made from Fig. 8. Hong *et al.* [39] examine rough-wall compensated energy spectra obtained using PIV and note a distinct peak throughout the boundary layer in scales of $k - 3k$ which they associate with a dynamic influence of their rough wall. In

Fig. 8(b), we observe a similar roughness-scale peak in the compensated spectra obtained from the rough-wall tower PIV measurements at all examined wall-normal locations [the gray-shaded regions in Fig. 8(b) show, for each spectral comparison, $k < \lambda_x < 2k$]. However, this peak is generated by noise in the PIV measurement, rather than by a physical roughness influence on the flow: the peak is consistent with expected spectral behavior of measurement noise, is also present in the smooth-wall PIV measurements, and is not observed in the HWA spectra. The latter observation then suggests that the small-scale differences between the rough- and smooth-wall PIV spectra in Fig. 8 indicate only that the rough-wall data are slightly noisier than the smooth-wall data. This will be discussed further in the following section.

The magnitude of the noise-associated small-scale energy peak clearly depends on the noise level of a given PIV measurement. It is worth noting, however, that sizable artificial spectral peaks can be generated from noise levels that contribute very little to the total energy. The thick dashed blue line in Fig. 8(bi), for example, shows Eq. (3) with Φ_{noise} chosen to contribute only $\sim 3\%$ to the streamwise velocity variance at that location (again, using the HWA spectra to estimate Φ_{flow} , and assuming that Φ_{noise} is white). Such a level of noise is expected in most PIV measurements [69]. As a cautionary anecdote, consider a hypothetical rough-wall PIV measurement that aims to resolve the full wall-normal extent wall layer and does so using a modern camera sensor array with approximately 3000 pixels in this dimension. Assuming an interrogation window size of 32×32 pixels for this measurement yields a smallest resolvable length approximately equal to 0.01δ . As demonstrated in Fig. 8, PIV measurement noise manifests as a peak in estimates of premultiplied energy spectra, occurring, for 50% overlapped interrogation windows, at approximately twice the smallest resolvable length. Therefore, any noise in the hypothetical measurement described above would generate spectral humps in scales $k - 5k$ when the roughness being examined has $50 < \delta/k < 250$. This range encompasses most laboratory roughness measurements including those of Hong and colleagues.

The previous section has illustrated that the present PIV data are self-consistent and exhibit Reynolds number trends that, for the most part, agree with existing literature. The tower PIV measurements, in particular, contain measurement noise. However, this noise appears to be uncorrelated and contributes very little to the total energy. We therefore turn our attention now to comparing the statistical and spatial structure of smooth- and rough-wall boundary layers. As previously mentioned (and demonstrated below), the present data enables comparison at matched Re_x and at matched δ^+ across a very wide range of streamwise scales. The former are interesting from a practical standpoint since they demonstrate the direct affect of roughness on the intensity of turbulent fluctuations in developing flows, for example on the hull of a ship. The latter attempts to isolate from the comparisons inherent trends in the rough- and smooth-wall data with δ^+ . Similar matched δ^+ comparisons were made by Squire *et al.* [19] and Morrill-Winter *et al.* [59] using HWA data, with more in-depth discussions and demonstrations of the rationale provided therein.

B. Smooth- and rough-wall comparisons

In Figs. 9(a) and 9(b), smooth- and rough-wall second order turbulence statistics obtained using the tower arrangement are compared at $\text{Re}_x \approx 1.6 \times 10^7$ and 2.9×10^7 , respectively. For the rough-wall profiles in Fig. 9(a), $k_s^+ = 64$, while in Fig. 9(b), $k_s^+ = 104$. In both comparisons, the interrogation window size in wall units is approximately matched (see Table III), and all statistics are normalized by the free-stream velocity, U_∞ . Recall that all PIV measurements are taken in the wind tunnel at the same streamwise location. Therefore, matching smooth- and rough-wall Re_x implies matching (approximately) U_∞ . When comparing inner-normalized smooth- and rough-wall turbulence statistics, it is easy to lose sight of the extent to which surface roughness can modify the turbulent boundary layer. Predictably, the introduction of wall roughness causes significant increases in the local wall drag and in the thickness of the boundary layer; for the profiles in Fig. 9(a) the percentage increase in these quantities relative to the smooth wall at matched Re_x are 44% and 28%, respectively. Additionally, all measured turbulent fluctuations are significantly intensified in the outer region relative to the free-stream velocity. The percentage increase, relative to smooth-wall flow,

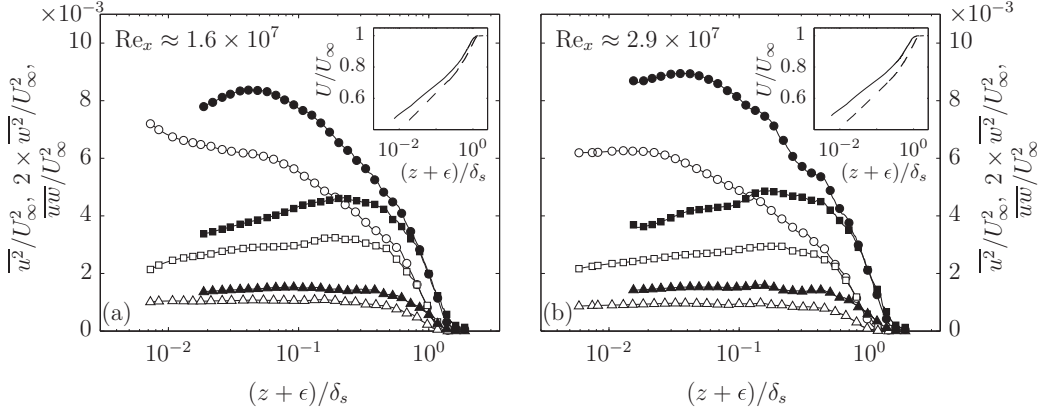


FIG. 9. Comparison of smooth- and rough-wall second-order turbulent statistics at matched Reynolds number, Re_x [$Re_x \approx 1.6 \times 10^7$ (a) and $Re_x \approx 2.9 \times 10^7$ (b)]. The $\circ$ symbols show u^2/U_∞^2 , the $\square$ symbols show $2 \times w^2/U_\infty^2$, and the $\triangle$ symbols show $\overline{uw}/U_\infty^2$. Empty symbols show smooth-wall data, and filled symbols show rough-wall data. The insets in each panel show the mean streamwise velocity for the smooth-wall (—) and rough-wall (---) flows.

increases with k_s^+ , such that larger differences are observed between the smooth- and rough-wall Reynolds stress profiles in Fig. 9(b) than in Fig. 9(a). Increasing k_s^+ also causes a reduction, relative to the smooth-wall flow, in the mean streamwise velocity due to the associated increase in momentum absorption by the rough wall [smooth- and rough-wall mean streamwise velocity profiles are shown in the insets of Figs. 9(a) and 9(b)]. Thus, in the outer layer the roughness generates a significant increase in the intensity of streamwise fluctuations relative to their local mean, and this effect increases with k_s^+ . For example, at $k_s^+ = 64$ the rough wall increases $\sqrt{u^2}/U$ at $z/\delta = 0.1$ by $\sim 22\%$ relative to the smooth wall at matched Re_x , whereas at $k_s^+ = 104$ the percentage increase at the same location is $\sim 37\%$.

Figure 10 compares smooth- and rough-wall second order turbulence statistics at matched δ^+ and matched spatial resolution [$\delta^+ \approx 12\,000$ in (a) and $18\,000$ in (b)]. Generally, all smooth- and rough-wall statistics show good outer region agreement, suggesting Townsend’s wall-similarity hypothesis provides sensible approximations of $\overline{u^2}$, $\overline{w^2}$, and $\overline{uw}$ for the present sand-grain roughness at large but finite friction Reynolds number. Numerous rough-wall studies have attempted to quantify the wall-normal extent of the effect of the roughness, i.e., the edge of the roughness sublayer. Raupach *et al.* [1], for example, compile a wide range of extant data and note that direct roughness influences are typically contained within $2k$ to $5k$ of the wall. Similar estimates for the extent of the roughness sublayer have been observed/assumed in many studies since (e.g., Flack *et al.* [5], Wu and Christensen [25], Schultz and Flack [66]). In Fig. 10, however, the edge of the roughness sublayer does not appear to scale in any straightforward manner on roughness height (the dashed black lines in each figure show the location of $5k_s^+$). Instead, as was also observed by Squire *et al.* [19], similarity of second order statistics appears to be observed only where the rough-wall flow is inertial. It is worth noting here that Townsend’s wall-similarity hypothesis is posed external to “the flow patterns set up by individual roughness elements” and only over a region that is “fully turbulent and independent of the magnitude of the fluid viscosity.” With the classical supposition that the roughness height is the smallest relevant scale in rough-wall flows, it is logical to conclude that the roughness layer should scale on k . However, Mehdi *et al.* [42], and subsequently Squire *et al.* [19] and Morrill-Winter *et al.* [59], provide evidence that ν/U_τ is an important parameter in rough-wall flows. If this is the case, then in certain flows Townsend’s criteria for wall similarity may only be satisfied in the region where viscosity is negligible; i.e., in the inertial region of the flow.

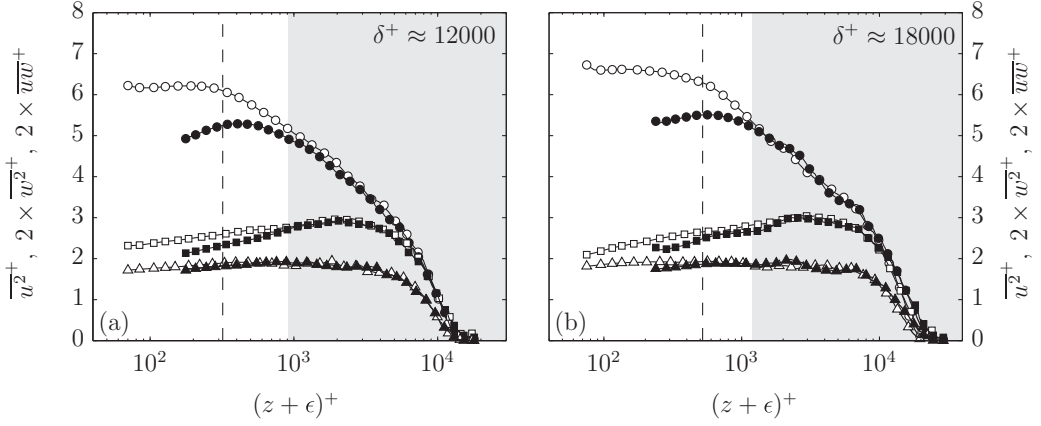


FIG. 10. Comparison of smooth- and rough-wall second-order turbulent statistics at matched friction Reynolds number, δ^+ [$\delta^+ \approx 12\,000$ (a) and $\delta^+ \approx 18\,000$ (b)]. The roughness Reynolds number in (a) and (b) is $k_s^+ = 64$ and 104 , respectively. The $\circ$ symbols show $\overline{u^2}^+$, the $\square$ symbols show $2 \times \overline{w^2}^+$, and the $\triangle$ symbols show $2 \times \overline{uw}^+$. Empty symbols show smooth-wall data, while filled symbols show rough-wall data. The dashed black line shows the wall-normal location of $5k_s^+$, and the shaded region shows the region over which the rough-wall flow is inertial (i.e., $z > z_l$) according to the definitions in Sec. II D.

The recent studies of Squire *et al.* [19] and Morrill-Winter *et al.* [59] above the present sandpaper surface demonstrated attenuation of rough-wall $\overline{u^2}^+$ and $\overline{w^2}^+$ profiles (relative to smooth-wall measurements at matched δ^+) in the transitionally rough regime. The degree of attenuation was shown to be tied to the magnitude of k_s^+ , with increased attenuation at low k_s^+ and outer region similarity only apparent in the fully rough regime. It is therefore important to emphasize that the rough-wall data presented in Fig. 10 are close to fully rough (see Fig. 3) in a regime where the roughness-associated attenuation of $\overline{u^2}^+$ and $\overline{w^2}^+$ observed by Squire *et al.* [19] and Morrill-Winter *et al.* [59] is of the same order as the experimental uncertainty of the present measurements. Here we lack data over the range of k_s^+ that enabled Squire *et al.* [19] and Morrill-Winter *et al.* [59] to draw conclusions regarding the apparent k_s^+ trend.

The above statistical comparisons do not address the instantaneous spatial structure of rough- and smooth-wall boundary layers. In smooth-wall flows, there have been considerable advancements in recent years in the development of structural descriptions (or models) that prescribe statistical observations. Spatial data, for example, from PIV and DNS, have been instrumental in providing empirical evidence in this regard. Above rough walls, the majority of such studies investigate whether prevalent smooth-wall turbulence models are consistent with rough-wall observations. Generally, most studies note quantitative similarities between smooth- and rough-wall boundary layer structure in the outer region. However, there is still considerable debate regarding the extent (if at all) to which a rough bounding wall can modify the outer layer spatial structure of a wall layer. In the following section, we qualitatively compare the spatial structure of smooth- and rough-wall boundary layers at $\delta^+ \approx 12\,000$ using the present PIV measurements.

IV. SPATIAL CORRELATIONS AND COHERENCE

A common approach for investigating the spatial structure of turbulent flows is to plot the two-point correlation of various turbulent quantities, relative to a reference location, z_r :

$$R_{qr}(z_r + \Delta z) = \frac{\overline{q(x, z_r)r(x + \Delta x, z_r + \Delta z)}}{\sigma_q(z_r)\sigma_r(z_r + \Delta z)}. \quad (4)$$

Here q and r are the turbulent quantities of interest, separated in the streamwise and wall-normal directions by Δx and Δz , respectively, and σ_q and σ_r are the standard deviations of q and r at z_r and $z_r + \Delta z$, respectively. In Fig. 11 the two-point correlations R_{uu} (column a), R_{ww} (column b), and R_{uw} (column c) are shown for, from top to bottom, $z_r/\delta \approx 0.5, 0.3, 0.05$, and 0.02 , for the smooth- and rough-wall flows at $\delta^+ \approx 12\,000$. Note that the lower two reference points are within the logarithmic region of the flow. The two-point correlation map is computed for each LFOV PIV realization, and then ensemble averaged over all realizations to obtain the plots in Fig. 11. Also shown for $z_r/\delta \approx 0.3, 0.05$ is a slice through the ensemble-averaged correlation map at $z + \epsilon = z_r$; i.e., the auto-correlations of the streamwise and wall-normal velocity fluctuations (first and second columns, respectively), and the cross-correlation of the streamwise and wall-normal velocity fluctuations (third column). Data from smooth- and rough-wall HWA measurements, also at $\delta^+ \approx 12\,000$, are included in these plots for comparison.

Before scrutinizing the correlations in Fig. 11, it is important to note that examination of contours of the correlation can artificially amplify small differences between the smooth and rough walls, since at large separations from the correlation point, the two-point correlation is a slowly decaying function. Therefore, a small difference in the magnitude between the smooth- and rough-wall correlation at large Δx and/or Δz , can result in low-level contours that are significantly separated spatially. Additionally, while any measurement noise in the PIV measurements is expected to be random and uncorrelated, such noise can still influence the present correlations by artificially increasing the standard deviation of the measured velocity components. The normalization, in Eq. (4), by the standard deviations of each correlated signal also negates many differences observed in the magnitude of the energy between the smooth and rough walls. This is particularly important given the recent study by Squire *et al.* [19] that shows clear near-wall differences, above the present roughness, between the smooth- and rough-wall large-scale inner-normalized energy of the streamwise velocity fluctuations. (Note that similar differences are also observed in the spectra obtained using the present PIV measurements.) With these points in mind, all two-point correlation maps in Fig. 11 generally show good agreement between the smooth- and rough-wall flows. Contours of R_{uu} indicate that a prevalent structure associated with streamwise velocity fluctuations in both flows is a large, forward inclined structure, which, at least in the logarithmic region, has a strong correlation with the near-wall flow. This characteristic shape has been interpreted as a structural imprint of streamwise aligned packets of hairpin vortices, which appear to be a dominant and robust feature in smooth-wall [29] and rough-wall [34] flows. Christensen and Wu [74], for example, demonstrate that the inclination angle of R_{uu} is comparable to those observed in instantaneous snapshots of vortex packets [28,75]. Qualitatively, the approximate inclination angle of R_{uu} in Fig. 11 is similar between the smooth- and rough-wall flows, increasingly slightly with wall-normal position. Generally, these observations are consistent with recent smooth- and rough-wall studies [34,36,74]. It is worth noting, however, that Krogstad and Antonia [76] report a significant increase, relative to a smooth-wall flow, in the inclination of R_{uu} above a mesh roughness similar to that investigated by Volino *et al.* [34]. Referring to the center and right-hand columns of Fig. 11, the spatial extent of R_{ww} and R_{uw} is significantly less than that of R_{uu} . Yet, again, qualitatively, the smooth and rough walls indicate strong similarities in both shape and magnitude. Comparisons of the auto- and cross-correlations at $z + \epsilon = z_r$ of course demonstrate similar qualitative agreement between smooth- and rough-wall structure across the full spatial extent captured by the PIV measurements and also generally agree with the respective predictions from the HWA measurements.

The correlations in Fig. 11 are dominated by large-scale coherent motions, since only motions with length of $O(\Delta x)$ or larger can contribute significantly to the correlation at Δx (similarly for the z direction). Details of small-scale coherence of two given quantities are contained within their correlation close to the zero-lag point of the correlation, but this information is superimposed with that of all coherent scales larger than the scale of interest. In Fig. 11(v), for example, the smooth- and rough-wall two-point correlation maps in Fig. 11(iv) are reproduced, but using fields of velocity fluctuations that have been high-pass filtered at the length of the LFOV (i.e., only streamwise scales smaller than $\lambda_x \approx 2\delta$ are included). Note that the same contour levels are plotted in Figs. 11(iv)

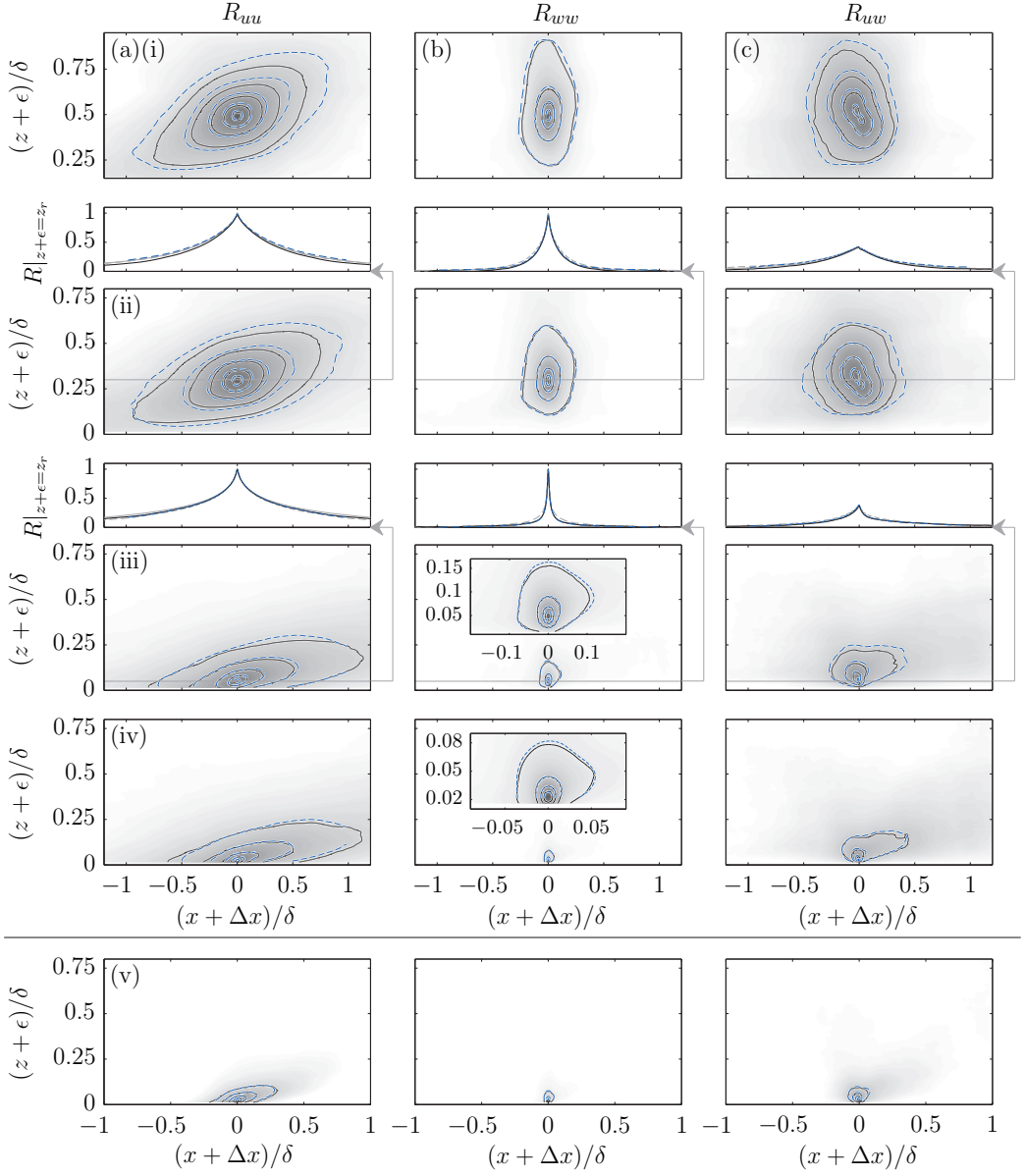


FIG. 11. Two-point correlations, R_{uu} (first column), R_{wu} (second column), and R_{uw} (third column) from the smooth-wall (—) and rough-wall (---) LFOV PIV measurement at $\delta^+ \approx 12\,000$. Correlations are given for reference locations at, from top to bottom: $z_r/\delta \approx 0.5, 0.3, 0.05$, and 0.02 . In row (v), the correlation maps in row (iv) are reproduced using fields that have been high-pass filtered at the streamwise length of the LFOV (i.e., $\lambda_x \lesssim 2\delta$). Contour levels are shown across the ranges: $R_{uu} = 0.25 \rightarrow 0.85$ with a spacing of 0.25 ; $R_{wu} = 0.14 \rightarrow 0.74$ with a spacing of 0.2 ; and $R_{uw} = -0.15 \rightarrow -0.39$ with a spacing of -0.08 . Background gray-scale shading shows filled contours of the respective smooth-wall correlation map. At $z_r/\delta \approx 0.3$, and 0.05 [rows (ii) and (iii)], a wall-parallel slice through each correlation map at $z + \epsilon = z_r$ is also provided (indicated by the gray arrows), and HWA data at $\delta^+ \approx 12\,000$ is also included in these plots for comparison (smooth wall: —, rough wall: ---). Insets show magnified views of the plots in which they are inset.

and 11(v). Comparison of the unfiltered and filtered correlation maps demonstrates that the unfiltered correlations are strongly influenced by structures with streamwise extents larger than that which can be fully resolved by the PIV. This is especially true for correlations involving the streamwise velocity fluctuations, which are regularly coherent over large streamwise extents (e.g., Hutchins and Marusic [77], Mejia-Alvarez *et al.* [78]). We therefore seek a quantitative means for assessing the mean structure of the boundary layer across all scales. This is particularly pertinent in light of studies, such as Hong *et al.* [39] and Hackett *et al.* [40], which report an imprint of the roughness in the small-scale turbulence across the entire rough-wall layer.

The magnitude-squared coherence, hereafter referred to as the coherence, between signals q and r , γ_{qr}^2 , is defined as

$$\gamma_{qr}^2(\lambda) = \frac{|\Phi_{qr}(\lambda)|^2}{\Phi_{qq}(\lambda)\Phi_{rr}(\lambda)}, \quad (5)$$

where $\Phi_{qr}(\lambda)$ is the cross-spectral density of q and r , and $\Phi_{qq}(\lambda)$ and $\Phi_{rr}(\lambda)$ are the auto-spectral densities of q and r , respectively. The coherence function estimates the extent to which r may be predicted by a linear time invariant function of q ; specifically, the proportion of energy in r at wavelength λ which can be explained by such a function. The coherence is bounded between values of 0 and 1. In these regards, the coherence provides a measure of correlation as a function of wavelength, enabling comparison of the structure of smooth- and rough-wall flows across all (resolvable) energetic scales. In Fig. 12 we present the streamwise coherence of smooth- and rough-wall streamwise velocity fluctuations, $\gamma_{uu}^2(\lambda_x)$, for three reference locations, $z_r/\delta \approx 0.02$, 0.08, and 0.3 using the LFOV PIV at $\delta^+ \approx 12\,000$. Here $\gamma_{uu}^2(\lambda_x)$, is defined from

$$\gamma_{uu}^2(\lambda_x, z_r + \Delta z) = \frac{|\Phi_{u(z_r)u(z_r+\Delta z)}(\lambda_x)|^2}{\Phi_{u(z_r)u(z_r)}(\lambda_x)\Phi_{u(z_r+\Delta z)u(z_r+\Delta z)}(\lambda_x)} \quad (6)$$

and is presented in row (ii) of Fig. 12, with row (i) showing, for each z_r , the estimation of the zeroth wave number ($\lambda_x = \infty$) coherence. The latter represents to some extent (discussed below) the coherence of all streamwise motions larger than the extent of the PIV measurement. Note that for clarity in Fig. 12, the coherence is shown only for $\lambda_x/\delta > 0.06$; the small-scale coherence is investigated later in the paper using the tower PIV datasets. Also shown in Fig. 12 is a representation of the phase of the cross-spectral density term in Eq. (6). However, the phase is expressed in Fig. 12(iii) as an inclination angle, θ_i , relative to the reference location. For a structure with a given length, λ_x , the schematic at the bottom of Fig. 12 demonstrates how θ_i is defined from the estimated phase. Note that θ_i is defined to be positive both above and below the reference location to simplify interpretation.

The smooth- and rough-wall coherence in Fig. 12 agree well for all reference locations considered. Even for $\lambda_x = \infty$ [see Fig. 12(i)] the estimated smooth- and rough-wall coherence show remarkable collapse. This is particularly interesting for $z_r/\delta \approx 0.02$ given that the spectrograms of Squire *et al.* [19] indicated a difference at this location in the inner-normalized energy of these approximate scales. Of course, it is not possible to determine with the present measurements whether it is the same large streamwise scales that contribute to the smooth-rough differences and agreement, respectively, in the inner-normalized energy spectra and coherence. However, given that the HWA measurements of Squire *et al.* [19] predict that the majority of energy-containing motions larger than the streamwise extent of the present PIV exhibit smooth-rough differences, it seems likely that this is the case. The implication then is that the large-scale inner-normalized energy differences observed by Squire *et al.* [19] do not result from rough-wall influences on boundary layer structure, which may indicate that U_τ is not the appropriate scaling parameter for these large-scale streamwise motions. Certainly, the local turbulence levels associated with these motions are very different between the smooth- and rough-wall flows; see Sec. III B. The notion that large-scale structures in the log region may not scale exclusively on U_τ is consistent with previous smooth-wall studies (see, for example, Hutchins and Marusic [77]) and requires further study. In the scales that are fully resolved by the PIV,

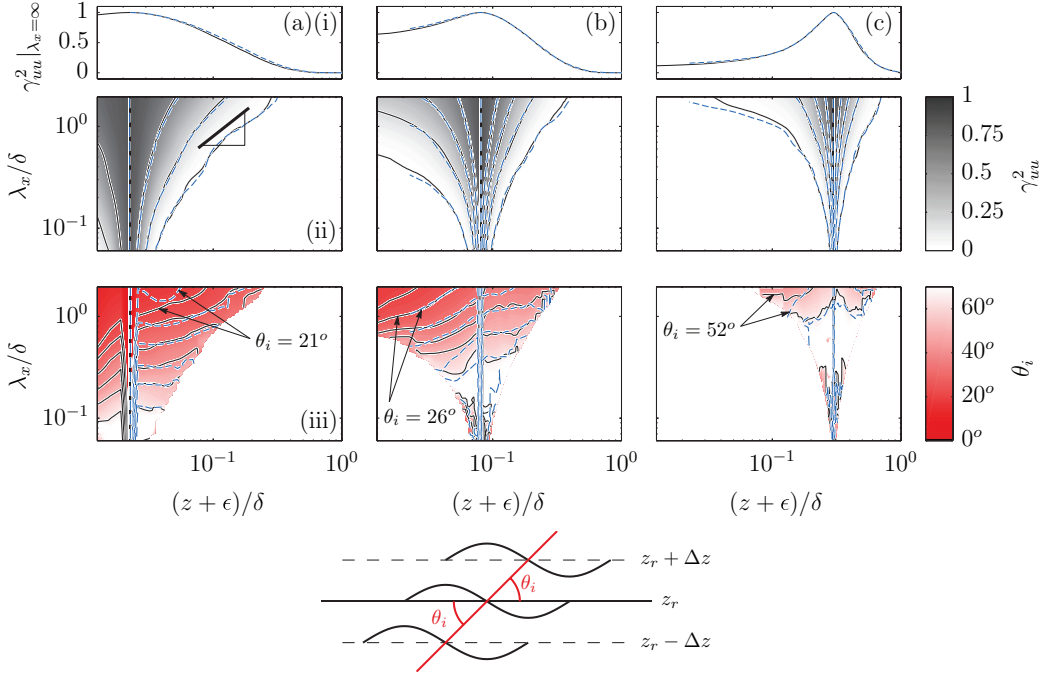


FIG. 12. The magnitude-squared coherence, γ_{uu}^2 [rows (i) and (ii)] and inclination angle, θ_i [row (iii)] for the smooth wall (—) and rough wall (— — —) at $\delta^+ \approx 12\,000$, relative to reference locations $z_r/\delta \approx 0.02$, 0.08 , and 0.3 [columns (a), (b), and (c), respectively]. θ_i is only plotted where the corresponding coherence is greater than 0.02 . Row (i) shows the coherence computed for $\lambda_x = \infty$, which represents, nontrivially, the coherence of all streamwise scales larger than the streamwise extent of the PIV measurements. The schematic at the bottom of the figure shows how θ_i is defined from the phase of numerator of Eq. (6). Contour levels are shown at $\gamma_{uu}^2 = 0.01, 0.2, 0.5, 0.8$, and 1 ; and $\theta_i = 0^\circ, 21^\circ, 26^\circ, 34^\circ, 42^\circ, 52^\circ$, and 65° . A reference contour level is provided in each plot of θ_i . Background shading shows filled contours of the respective smooth-wall quantity.

Fig. 12 shows that larger streamwise motions are coherent over a larger wall-normal extent, indicating that coherent motions that are longer are also generally taller. Relative to a reference location near to the beginning of the logarithmic region [$z_r/\delta \approx 0.02$: Fig. 12(aii)], the streamwise scale and wall-normal extent of motions within the logarithmic region are approximately linearly related; the thick black line in this plot spans the rough-wall logarithmic region and has a slope of one. This observation is consistent with distance from the wall scaling, and specifically the attached eddy hypothesis of Townsend [65] as later developed by Perry and Chong [79] and others. Farther into the layer, the large scales appear to maintain a relatively strong correlation with those near the wall. Even for a reference point located in the wake region [$z_r/\delta \approx 0.3$: Fig. 12(cii)], a nonzero coherence is estimated for $\lambda_x = \infty$ at the lowest wall-normal location for both the smooth- and rough-wall measurements.

The inclination angle, θ_i , also shows good agreement between the smooth- and rough-wall flows. This angle can be interpreted in the same way as the inclination of R_{uu} , but here the angle is presented as a function of resolved streamwise scale, λ_x . Below and within the logarithmic region, coherent streamwise structures with scales on the order of the extent of the LFOV measurements are inclined at $\sim 20^\circ$ in both the smooth- and rough-wall flows. Generally, smaller structures are inclined at steeper angles across all reference locations examined. At a given scale, the inclination angle apparently increases with wall-normal position. This behavior was also observed in the R_{uu}

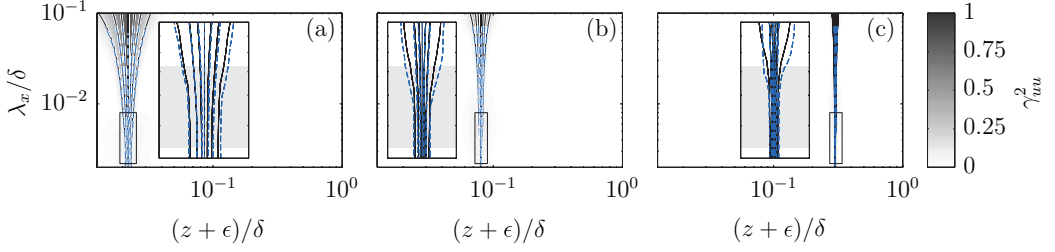


FIG. 13. The small-scale magnitude-squared coherence, γ_{uu}^2 , for the smooth wall (—) and rough wall (---) obtained from tower PIV data at $\delta^+ \approx 12\,000$, relative to reference locations $z_r/\delta \approx 0.02, 0.08$, and 0.3 [columns (a), (b), and (c), respectively]. Contour levels are shown at $\gamma_{uu}^2 = 0.2, 0.5, 0.8$, and 1 . Background shading shows filled contours of the respective smooth-wall quantity. In each magnified inset, the gray-shaded region spans $k < \lambda_x < 2k$.

correlation maps in Fig. 11. Recall, however, that the contours of R_{uu} in Fig. 11 are dominated by coherent motions longer than the streamwise extent of the present PIV measurements. Here we observe an apparent increase in inclination angle with z for all examined scales. We note that although the present measurements do not allow θ_i to be resolved for scales larger than the length of PIV field, extrapolation of the present smooth- and rough-wall trends would indicate that the very large energy containing streamwise motions are, on average, inclined at approximately 10° – 15° near to the wall, and increase in inclination with z . This is similar to previous smooth-wall estimates of the average inclination of hairpin vortex packets [28,29,80].

The small-scale coherence, γ_{uu}^2 , is investigated in Fig. 13 using smooth- and rough-wall tower PIV data at $\delta^+ \approx 12\,000$ and approximately matched spatial resolution. As in Fig. 12, the coherence is presented relative to reference points at $z_r/\delta \approx 0.02, 0.08$, and 0.3 [in (a), (b), and (c), respectively], but here the trends observed in row (ii) of Fig. 12 are extended down to wavelengths of $O(\eta)$. Note that for all scales fully resolved by the tower PIV measurements, $\theta_i \approx 90^\circ$, and is therefore not presented. The insets in Figs. 13(a), 13(b), and 13(c) show magnified views of the coherence for streamwise scales with length similar to the roughness height k ; the gray-shaded region in each inset spans $k < \lambda_x < 2k$. Recall that it is in these approximate scales that Hong *et al.* [39] report a roughness signature across their entire channel flow. Here, however, we observe general agreement between the smooth- and rough-wall coherence of scales with streamwise extent $O(k)$. The smooth-rough agreement in Fig. 13 also suggests that the small-scale differences observed between the rough- and smooth-wall compensated spectra obtained using the tower PIV arrangement (see Fig. 8) result from differences between the two datasets in the level of measurement noise, which, presumably, is spatially uncorrelated.

For brevity, small- and large-scale $\gamma_{ww}^2, \gamma_{uw}^2$ and their associated inclination are not presented here. But we note a similar level of agreement between smooth- and rough-wall flows to that observed in Figs. 12 and 13.

V. CONCLUSIONS

A set of PIV measurements has been used to empirically compare the structure of high Reynolds number rough-wall boundary layers across a uniquely large range of streamwise scales. This scale range was achieved using two eight camera PIV arrangements: one is capable of capturing streamwise motions of up to 2δ in length, the other can resolve scales on the order of the Kolmogorov microscale. Together the two arrangements can resolve over three orders of magnitude of streamwise scales across the full wall-normal extent of the boundary layer. The rough-wall measurements were obtained above a well characterized sandpaper surface in the transitional and fully rough regimes. A floating element drag balance was used to determine U_τ directly to within 1% error for all rough-wall measurements. Complementary smooth-wall measurements, obtained using the same PIV arrangements, facilitate

direct comparison between rough- and smooth-wall flows at matched δ^+ ($\delta^+ \approx 12\,000$ and $18\,000$), and matched Re_x ($\text{Re}_x \approx 1.6 \times 10^6$ and 2.9×10^6). Data were processed to enable comparisons at matched spatial resolution for each case.

All smooth- and rough-wall measurements show Reynolds number trends that are consistent with previous studies. For the smooth-wall data, the correction scheme of Ref. [63] is used to demonstrate a high level of consistency between profiles taken at different Reynolds numbers when the effects of spatial attenuation are accounted for. The same scheme indicates that statistics obtained from both PIV arrangements under identical flow conditions are also self-consistent. The rough-wall profiles show on the inertial domain regions of logarithmic dependence of both the mean streamwise velocity and the variance of the streamwise velocity fluctuations, consistent with predictions from Townsend's attached eddy hypothesis [65]. Comparisons of smooth- and rough-wall second order statistics at matched δ^+ agree well with the observations of Squire *et al.* [19] and Morrill-Winter *et al.* [59] above the same rough surface. These results demonstrate that at sufficiently high k_s^+ the edge of the roughness sublayer coincides approximately with the onset of inertial dynamics. This follows logically from Townsend's [2] original Reynolds number similarity hypothesis, given that ν/U_τ is an important parameter in rough-wall flows.

Similar to the findings of Hong *et al.* [39] and others, the power spectral density of the streamwise (and wall-normal; not shown) velocity fluctuations above the rough wall show a distinct peak throughout the boundary layer for streamwise motions approximately $k - 3k$ long. However, it is demonstrated that this peak results from measurement noise, rather than from roughness-associated dynamical influences as suggested by Hong *et al.* [39]: the peak is consistent with the expected spectral behavior of measurement noise, is also present in the smooth-wall PIV measurements, and is not observed in spectra obtained using HWA under matched flow conditions. The measurement noise is shown to be small (contributing, at worst, only $\sim 1.5\%$ to the local turbulence intensity) and uncorrelated.

The two-point magnitude squared coherence is used to compare the spatial structure of the smooth- and rough-wall flow at $\delta^+ \approx 12\,000$. Contrary to two-point correlation functions that are dominated by the largest motions present in the flow, the coherence provides a measure of correlation as a function of wavelength. Generally, the two-point coherence of streamwise velocity fluctuations (and the phase of the two-point cross spectral density, expressed here as a structure inclination angle) evidences strong similarities between smooth- and rough-wall structure across all resolvable scales, even near to the wall where the total (inner-normalized) energy of the streamwise fluctuations differs between the two flows. Squire *et al.* [19] demonstrated that much of this near-wall difference resides in streamwise motions longer than $\sim \delta$. However, the coherence suggests that structurally these motions are similar in shape and extent between smooth- and rough-wall flows, indicating that the observations of Squire *et al.* [19] result simply from roughness-induced reductions in the magnitude of the large-scale inner-normalized energy, rather than from modifications to the spatial structure of the boundary layer. This is consistent with previous suggestions that U_τ might not be the appropriate scaling parameter for very large-scale motions.

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- [1] M. R. Raupach, R. A. Antonia, and S. Rajagopalan, Rough-wall turbulent boundary layers, *Appl. Mech. Rev.* **44**, 1 (1991).

- [2] A. A. Townsend, *The Structure of Turbulent Shear Flow* (Cambridge University Press, Cambridge, 1956), Vol. 1.
- [3] A. E. Perry and C. J. Abell, Asymptotic similarity of turbulence structures in smooth-and rough-walled pipes, *J. Fluid Mech.* **79**, 785 (1977).
- [4] M. Acharya, J. Bornstein, and M. P. Escudier, Turbulent boundary layers on rough surfaces, *Exp. Fluids* **4**, 33 (1986).
- [5] K. A. Flack, M. P. Schultz, and T. A. Shapiro, Experimental support for Townsend's Reynolds number similarity hypothesis on rough walls, *Phys. Fluids* **17**, 035102 (2005).
- [6] P.-Å. Krogstad, R. A. Antonia, and L. W. B. Browne, Comparison between rough-and smooth-wall turbulent boundary layers, *J. Fluid Mech.* **245**, 599 (1992).
- [7] L. Keirsbulck, L. Labraga, A. Mazouz, and C. Tournier, Surface roughness effects on turbulent boundary layer structures, *J. Fluids Eng.* **124**, 127 (2002).
- [8] K. Bhaganagar, J. Kim, and G. Coleman, Effect of roughness on wall-bounded turbulence, *Flow Turbul. Combust.* **72**, 463 (2004).
- [9] A. E. Perry, K. L. Lim, and S. M. Henbest, An experimental study of the turbulence structure in smooth-and rough-wall boundary layers, *J. Fluid Mech.* **177**, 437 (1987).
- [10] M. R. Raupach, A. S. Thom, and I. Edwards, A wind-tunnel study of turbulent flow close to regularly arrayed rough surfaces, *Bound. Lay. Meteorol.* **18**, 373 (1980).
- [11] P. M. Ligrani and R. J. Moffat, Structure of transitionally rough and fully rough turbulent boundary layers, *J. Fluid Mech.* **162**, 69 (1986).
- [12] A. E. Perry and J. D. Li, Experimental support for the attached-eddy hypothesis in zero-pressure-gradient turbulent boundary layers, *J. Fluid Mech.* **218**, 405 (1990).
- [13] M. P. Schultz and K. A. Flack, The rough-wall turbulent boundary layer from the hydraulically smooth to the fully rough regime, *J. Fluid Mech.* **580**, 381 (2007).
- [14] F. H. Clauser, The turbulent boundary layer, *Advan. Appl. Mech.* **4**, 1 (1956).
- [15] F. R. Hama, *Boundary-Layer Characteristics for Smooth and Rough Surfaces* (SNAME, New York, 1954).
- [16] A. S. Thom, Momentum absorption by vegetation, *Quart. J. R. Meteorol. Soc.* **97**, 414 (1971).
- [17] P. S. Jackson, On the displacement height in the logarithmic velocity profile, *J. Fluid Mech.* **111**, 15 (1981).
- [18] M. F. Tachie, D. J. Bergstrom, and R. Balachandar, Rough wall turbulent boundary layers in shallow open channel flow, *J. Fluids Eng.* **122**, 533 (2000).
- [19] D. T. Squire, C. Morrill-Winter, N. Hutchins, M. P. Schultz, J. C. Klewicki, and I. Marusic, Comparison of turbulent boundary layers over smooth and rough surfaces up to high Reynolds numbers, *J. Fluid Mech.* **795**, 210 (2016).
- [20] R. J. Volino, M. P. Schultz, and K. A. Flack, Turbulence structure in boundary layers over periodic two- and three-dimensional roughness, *J. Fluid Mech.* **676**, 172 (2011).
- [21] J. Jiménez, Turbulent flows over rough walls, *Annu. Rev. Fluid Mech.* **36**, 173 (2004).
- [22] A. J. Grass, Structural features of turbulent flow over smooth and rough boundaries, *J. Fluid Mech.* **50**, 233 (1971).
- [23] S. J. Kline, W. C. Reynolds, F. A. Schraub, and P. W. Runstadler, The structure of turbulent boundary layers, *J. Fluid Mech.* **30**, 741 (1967).
- [24] M. P. Schultz and K. A. Flack, Outer layer similarity in fully rough turbulent boundary layers, *Exp. Fluids* **38**, 328 (2005).
- [25] Y. Wu and K. T. Christensen, Outer-layer similarity in the presence of a practical rough-wall topography, *Phys. Fluids* **19**, 085108 (2007).
- [26] A. J. Grass, R. J. Stuart, and M. Mansour-Tehrani, Vortical structures and coherent motion in turbulent flow over smooth and rough boundaries, *Phil. Trans. R. Soc. A* **336**, 35 (1991).
- [27] M. R. Head and P. Bandyopadhyay, New aspects of turbulent boundary-layer structure, *J. Fluid Mech.* **107**, 297 (1981).
- [28] J. Zhou, R. J. Adrian, S. Balachandar, and T. M. Kendall, Mechanisms for generating coherent packets of hairpin vortices in channel flow, *J. Fluid Mech.* **387**, 353 (1999).
- [29] R. J. Adrian, C. D. Meinhart, and C. D. Tomkins, Vortex organization in the outer region of the turbulent boundary layer, *J. Fluid Mech.* **422**, 1 (2000).

- [30] C. J. Delo, R. M. Kelso, and A. J. Smits, Three-dimensional structure of a low-Reynolds-number turbulent boundary layer, *J. Fluid Mech.* **512**, 47 (2004).
- [31] B. Ganapathisubramani, E. K. Longmire, and I. Marusic, Characteristics of vortex packets in turbulent boundary layers, *J. Fluid Mech.* **478**, 35 (2003).
- [32] M. Guala, S. E. Hommema, and R. J. Adrian, Large-scale and very-large-scale motions in turbulent pipe flow, *J. Fluid Mech.* **554**, 521 (2006).
- [33] B. J. Balakumar and R. J. Adrian, Large-and very-large-scale motions in channel and boundary-layer flows, *Phil. Trans. R. Soc. A* **365**, 665 (2007).
- [34] R. J. Volino, M. P. Schultz, and K. A. Flack, Turbulence structure in rough-and smooth-wall boundary layers, *J. Fluid Mech.* **592**, 263 (2007).
- [35] R. J. Volino, M. P. Schultz, and K. A. Flack, Turbulence structure in a boundary layer with two-dimensional roughness, *J. Fluid Mech.* **635**, 75 (2009).
- [36] Y. Wu and K. T. Christensen, Spatial structure of a turbulent boundary layer with irregular surface roughness, *J. Fluid Mech.* **655**, 380 (2010).
- [37] S. Nakagawa and T. J. Hanratty, Particle image velocimetry measurements of flow over a wavy wall, *Phys. Fluids* **13**, 3504 (2001).
- [38] O. Flores and J. Jimenez, Effect of wall-boundary disturbances on turbulent channel flows, *J. Fluid Mech.* **566**, 357 (2006).
- [39] J. Hong, J. Katz, and M. P. Schultz, Near-wall turbulence statistics and flow structures over three-dimensional roughness in a turbulent channel flow, *J. Fluid Mech.* **667**, 1 (2011).
- [40] E. E. Hackett, L. Luznik, A. R. Nayak, J. Katz, and T. R. Osborn, Field measurements of turbulence at an unstable interface between current and wave bottom boundary layers, *J. Geo. Res.: Oceans* **116**, C02022 (2011).
- [41] J. Hong, J. Katz, C. Meneveau, and M. P. Schultz, Coherent structures and associated subgrid-scale energy transfer in a rough-wall turbulent channel flow, *J. Fluid Mech.* **712**, 92 (2012).
- [42] F. Mehdi, J. C. Klewicki, and C. M. White, Mean force structure and its scaling in rough-wall turbulent boundary layers, *J. Fluid Mech.* **731**, 682 (2013).
- [43] R. L. Ebner, F. Mehdi, and J. C. Klewicki, Shared dynamical features of smooth-and rough-wall boundary-layer turbulence, *J. Fluid Mech.* **792**, 435 (2016).
- [44] J. Nikuradse, Laws of flow in rough pipes, NASA Tech. Memo (1950).
- [45] W. J. Baars, D. T. Squire, K. M. Talluru, M. R. Abbassi, N. Hutchins, and I. Marusic, Wall-drag measurements of smooth- and rough-wall turbulent boundary layers using a floating element, *Exp. Fluids* **57**, 1 (2016).
- [46] T. B. Nickels, I. Marusic, S. Hafez, and M. S. Chong, Evidence of the k -1 Law in a High-Reynolds-Number Turbulent Boundary Layer, *Phys. Rev. Lett.* **95**, 074501 (2005).
- [47] T. B. Nickels, I. Marusic, S. Hafez, N. Hutchins, and M. S. Chong, Some predictions of the attached eddy model for a high Reynolds number boundary layer, *Phil. Trans. R. Soc. Lond. A* **365**, 807 (2007).
- [48] C. M. de Silva, J. Philip, K. Chauhan, C. Meneveau, and I. Marusic, Multiscale Geometry and Scaling of the Turbulent-Nonturbulent Interface in High Reynolds Number Boundary Layers, *Phys. Rev. Lett.* **111**, 044501 (2013).
- [49] C. M. de Silva, E. P. Gnanamanickam, C. Atkinson, N. A. Buchmann, N. Hutchins, J. Soria, and I. Marusic, High spatial range velocity measurements in a high Reynolds number turbulent boundary layer, *Phys. Fluids* **26**, 025117 (2014).
- [50] K. Chauhan, J. Philip, C. M. de Silva, N. Hutchins, and I. Marusic, The turbulent/non-turbulent interface and entrainment in a boundary layer, *J. Fluid Mech.* **742**, 119 (2014).
- [51] C. J. Kähler, S. Scharnowski, and C. Cierpka, High resolution velocity profile measurements in turbulent boundary layers, in *16th Int. Symp. Appl. Laser Tech. Fluid Mech.* (Springer-Verlag, Portugal, 2012), p. 9.
- [52] C. Willert, Stereoscopic digital particle image velocimetry for application in wind tunnel flows, *Meas. Sci. Tech.* **8**, 1465 (1997).
- [53] H. T. Huang, H. E. Fiedler, and J. J. Wang, Limitation and improvement of PIV. II: Particle image distortion, a novel technique, *Exp. Fluids* **15**, 263 (1993).

- [54] K. Jambunathan, X. Y. Ju, B. N. Dobbins, and S. Ashforth-Frost, An improved cross correlation technique for particle image velocimetry, *Meas. Sci. Tech.* **6**, 507 (1995).
- [55] F. Scarano, Iterative image deformation methods in PIV, *Meas. Sci. Tech.* **13**, R1 (2002).
- [56] D. P. Hart, PIV error correction, *Exp. Fluids* **29**, 13 (2000).
- [57] K. A. Chauhan, P. A. Monkewitz, and H. M. Nagib, Criteria for assessing experiments in zero pressure gradient boundary layers, *Fluid Dyn. Res.* **41**, 021404 (2009).
- [58] I. Marusic, J. P. Monty, M. Hultmark, and A. J. Smits, On the logarithmic region in wall turbulence, *J. Fluid Mech.* **716**, R3 (2013).
- [59] C. Morrill-Winter, D. T. Squire, J. C. Klewicki, N. Hutchins, M. P. Schultz, and I. Marusic, Turbulent stress behaviours in boundary layers over sandpaper roughness (unpublished).
- [60] J. C. Klewicki, P. Fife, and T. Wei, On the logarithmic mean profile, *J. Fluid Mech.* **638**, 73 (2009).
- [61] N. Hutchins, T. B. Nickels, I. Marusic, and M. S. Chong, Hot-wire spatial resolution issues in wall-bounded turbulence, *J. Fluid Mech.* **635**, 103 (2009).
- [62] J. A. Sillero, J. Jiménez, and R. D. Moser, One-point statistics for turbulent wall-bounded flows at Reynolds numbers up to $\delta^+ \approx 2000$, *Phys. Fluids* **25**, 105102 (2013).
- [63] J. H. Lee, Kevin, J. P. Monty, and N. Hutchins, Validating under-resolved turbulence intensities for PIV experiments in canonical wall-bounded turbulence, *Exp. Fluids* **57**, 129 (2016).
- [64] C. Morrill-Winter, J. Klewicki, R. Baidya, and I. Marusic, Temporally optimized spanwise vorticity sensor measurements in turbulent boundary layers, *Exp. Fluids* **56**, 1 (2015).
- [65] A. A. Townsend, *The Structure of Turbulent Shear Flow* (Cambridge University Press, Cambridge, 1976), Vol. 2.
- [66] M. P. Schultz and K. A. Flack, Turbulent boundary layers over surfaces smoothed by sanding, *J. Fluids Eng.* **125**, 863 (2003).
- [67] I. P. Castro, Rough-wall boundary layers: Mean flow universality, *J. Fluid Mech.* **585**, 469 (2007).
- [68] P.-Å. Krogstad and R. A. Antonia, Surface roughness effects in turbulent boundary layers, *Exp. Fluids* **27**, 450 (1999).
- [69] C. Atkinson, N. A. Buchmann, O. Amili, and J. Soria, On the appropriate filtering of PIV measurements of turbulent shear flows, *Exp. Fluids* **55**, 1 (2014).
- [70] G. I. Taylor, The spectrum of turbulence, *Proc. R. Soc. Lond. A* **164**, 476 (1933).
- [71] K. M. Talluru, R. Baidya, N. Hutchins, and I. Marusic, Amplitude modulation of all three velocity components in turbulent boundary layers, *J. Fluid Mech.* **746**, R1 (2014).
- [72] J. Foucaut, J. Carlier, and M. Stanislas, PIV optimization for the study of turbulent flow using spectral analysis, *Meas. Sci. Tech.* **15**, 1046 (2004).
- [73] S. Herpin, C. Y. Wong, M. Stanislas, and J. Soria, Stereoscopic PIV measurements of a turbulent boundary layer with a large spatial dynamic range, *Exp. Fluids* **45**, 745 (2008).
- [74] K. T. Christensen and Y. Wu, Characteristics of vortex organization in the outer layer of wall turbulence, in *Proceedings of Fourth International Symposium on Turbulence and Shear Flow Phenomena* (Begell House, USA, 2005), Vol. 3, pp. 1025–1030.
- [75] K. T. Christensen and R. J. Adrian, Statistical evidence of hairpin vortex packets in wall turbulence, *J. Fluid Mech.* **431**, 433 (2001).
- [76] P.-Å. Krogstad and R. A. Antonia, Structure of turbulent boundary layers on smooth and rough walls, *J. Fluid Mech.* **277**, 1 (1994).
- [77] N. Hutchins and I. Marusic, Evidence of very long meandering features in the logarithmic region of turbulent boundary layers, *J. Fluid Mech.* **579**, 1 (2007).
- [78] R. Mejia-Alvarez, Y. Wu, and K. T. Christensen, Observations of meandering superstructures in the roughness sublayer of a turbulent boundary layer, *Int. J. Heat Fluid Flow* **48**, 43 (2014).
- [79] A. E. Perry and M. S. Chong, On the mechanism of wall turbulence, *J. Fluid Mech.* **119**, 173 (1982).
- [80] C. D. Tomkins and R. J. Adrian, Spanwise structure and scale growth in turbulent boundary layers, *J. Fluid Mech.* **490**, 37 (2003).