

Hydrodynamic interactions in tandem flapping wing systems

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We review recent studies on the hydrodynamic interactions in tandem flapping wing systems, with a focus on the interplay between vortex dynamics and structural flexibility. We examine configurations including rigid wings with prescribed motion, spanwise-flexible wings, and chordwise-flexible wings. For rigid wings, three-dimensional effects are shown to reduce thrust, but they can be mitigated through aspect ratio tuning. Flexibility introduces fluid-structure resonance phenomena that enhance aerodynamic performance. In tandem systems, performance optimization requires careful coordination: forewings benefit from resonance, while hindwings perform best with slightly stiffer configurations to effectively capture upstream wake structures. Finally, in systems of self-propelled flexible flappers, hydrodynamic interactions give rise to coordinated motion and energy savings through flow-mediated feedback.

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I. INTRODUCTION

This article reviews our recent work on hydrodynamic interactions in systems with two flapping wings, either rigid or flexible, following a recent invited talk at the 2024 meeting of the Division of Fluid Dynamics of the American Physical Society [1].

The development of bioinspired flapping-wing systems has attracted significant attention in recent years, driven by their potential applications in micro air vehicles (MAVs) [2–4] and biomimetic underwater propulsion systems [5,6]. These systems offer advantages including enhanced maneuverability and the ability to operate efficiently at low Reynolds numbers, where conventional propulsion methods become less effective. The promise of achieving hovering capabilities, rapid directional changes, and energy-efficient locomotion has motivated extensive research into understanding the fundamental fluid dynamics underlying the flight of insects and birds [7–12] and the swimming of fish [13–19].

One particularly interesting feature of these biological systems is their ability to manipulate the vortical structures they generate during movement, actively exploiting them to enhance propulsion and maneuverability. This is especially evident in systems with paired wings, where the motion of one wing alters the flow environment experienced by the other. Such wing-wing interactions are not passive side effects but integral components to force generation and energy efficiency. From this perspective, self-induced flow manipulation is a key aspect of their locomotor performance.

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Among the most compelling biological models for multiwing configurations is the dragonfly, whose tandem wing arrangement reflects a long evolutionary history of functional adaptation shaped by natural selection [20]. Dragonflies demonstrate remarkable flight performance through the coordinated motion of their fore- and hindwings, which can operate either in-phase or with complex phase relationships depending on flight requirements [21,22]. Their exceptional hovering stability, rapid turning capability, and forward flight speeds reaching up to around 7 m/s result from complex aerodynamic interactions between fore- and hindwings. The forewing generates vortical structures that can either constructively or destructively interact with the hindwing, depending on the phase relationship and wing kinematics, which influences both lift production and the energetic cost of flight.

Similarly, schooling fish provide valuable insights into collective hydrodynamic interactions, where multiple oscillating bodies coordinate their motion and relative position to achieve enhanced propulsive efficiency [23–26]. Fish schools exploit wake structures generated by neighboring individuals, with followers positioning themselves to benefit from vortical flows while avoiding detrimental interactions. This collective behavior demonstrates how vortex-mediated interactions can lead to energy savings and improved swimming performance at the group level. This suggests that understanding multibody oscillatory systems extends beyond individual propulsors to encompass swarm robotics and coordinated underwater vehicle applications.

The diverse mechanical properties of the wings and fins of these biological systems underscore the critical need to understand vortex-mediated wing-wing interactions across different flexibility regimes. While rigid wing models provide fundamental insights into basic hydrodynamic mechanisms, biological wings and fins exhibit varying degrees of flexibility that can significantly alter flow patterns and force generation [27]. The interplay between structural dynamics and fluid dynamics in flexible wing systems introduces additional parameters that can be exploited for performance optimization, yet our understanding of how wing flexibility modulates hydrodynamic interactions remains incomplete. Elucidating these fluid-structure interactions is essential for advancing the design of bioinspired propulsion systems that can match or even exceed the performance of their biological counterparts.

The manuscript is organized as follows. In Sec. II, we provide some background by introducing the fundamental physical parameters and flow features relevant to flapping wings, including vortex structures, aerodynamic forces, and the effects of flexibility. Section III discusses rigid tandem wings undergoing prescribed motions, focusing on the transition from two- to three-dimensional configurations and the impact of the wing’s aspect ratio. In Sec. IV, we explore spanwise-flexible tandem wings, highlighting the role of fluid-structure resonance and wake capture. We shift in Sec. V to self-propelled chordwise-flexible flappers and examine how hydrodynamic interactions give rise to emergent coordination and energy savings. Conclusions are provided in Sec. VI.

II. BACKGROUND

A. Aerodynamic fundamentals

1. Key parameters

The aerodynamics of flapping wings are governed by several dimensionless parameters that characterize the relative importance of various physical phenomena [27,28]. The balance between inertial and viscous forces is determined by the Reynolds number, $Re = \rho U c / \mu$, where ρ is the fluid density, U is a characteristic velocity, c is a mean chord length, and μ is the dynamic viscosity. For most biological flapping wings and their biomimetic counterparts, Reynolds numbers typically range from 10 to 10^5 , placing them in a regime where viscous effects remain significant, and laminar-to-turbulent transition plays a crucial role in flow dynamics. In forward motion, the characteristic velocity is typically taken as the flight or swimming speed, denoted here as U_∞ . In contrast, for hovering insects or small birds, the characteristic velocity is usually defined based on the wing’s motion during a stroke. A common choice is the mean wingtip velocity, given by $2fb\phi_0$, where b is the wingspan, f is the flapping frequency, and ϕ_0 is the peak-to-peak amplitude of the flapping angle.

The Strouhal number, St , characterizes the ratio between flapping speed and forward speed. It is typically expressed as $St = fL_{\text{ref}}/U_{\infty}$, where f is the flapping frequency and L_{ref} is a characteristic length scale. Several definitions for L_{ref} are used in the literature, depending on the context; a common choice is the width of the wake, such as $L_{\text{ref}} = 2h_0$, with h_0 being the semiamplitude of the wing's motion. Biological swimmers and flyers consistently operate within a Strouhal number range of 0.2–0.4 [29]. This range corresponds to optimal propulsive efficiency [30], and several theories have been proposed to explain these observations. It has been argued that peak performance occurs when the kinematics result in maximum amplification of the shed vortices in the wake; this is known as wake resonance [30]. Other authors have argued that for aquatic animals, the ratio of the tail beat amplitude to the length of the body dictates the Strouhal number, since it requires a balance between thrust and drag [31]. Finally, using scaling arguments, Floryan *et al.* [32] argued that the Strouhal number for peak efficiency is largely determined by fluid drag on the fins and wings.

Instead of the Strouhal number, sometimes the reduced frequency, $k = \pi fc/U_{\infty}$, is employed. It is used to quantify the unsteadiness of the flow and determine whether quasisteady or fully unsteady approaches are appropriate for analysis. At low reduced frequencies ($k < 0.1$), quasisteady models may suffice, while higher values require consideration of added mass effects, vortex dynamics, and history-dependent forces.

Other important parameters are the aspect ratio of the wing, $AR = b^2/S$, and the solid-to-fluid density ratio, $\tilde{\rho} = \rho_s/\rho$, where S is the wing's planform area and ρ_s is the solid density. The aspect ratio controls the importance of tip effects and the shape of the three-dimensional flow structures. The density ratio is used to estimate the effective inertia of the wing,

$$\Pi_0 = \tilde{\rho} \frac{h_s}{c} \left(\frac{fc}{U_{\infty}} \right)^2, \quad (1)$$

where h_s represents the thickness of the wing. If the wing is flexible, it is necessary to characterize also its structural properties. This is done by defining the effective stiffness

$$\Pi_1 = \frac{1}{12} \frac{E}{\rho U_{\infty}^2} \left(\frac{h_s}{c} \right)^3, \quad (2)$$

where E is the Young modulus of the wing's material.

2. Vortical structures

The complex three-dimensional vortex structures generated by flapping wings fundamentally determine force production and wake characteristics [12,33]. Understanding these vortical flows is essential for predicting performance and designing effective bioinspired systems.

When a flapping wing moves forward, the flow separates from the leading edge, wingtips, and trailing edge, shedding large-scale vortices into the wake: leading edge vortices (LEVs), trailing edge vortices (TEVs), and tip vortices (TiVs). Depending on the aspect ratio of the wing, all these vortices may be shed together forming a vortex ring.

The LEV represents perhaps the most critical flow feature in flapping wing aerodynamics [34,35]. During the downstroke, flow separation at the leading edge generates this coherent vortical structure, which is initially attached to the wing's upper surface and eventually is shed into the wake. The LEV creates a region of low pressure that significantly enhances lift generation beyond what quasisteady theory would predict. The stability and strength of the LEV depend on several factors, including the angle of attack, wing kinematics, and Reynolds number. At moderate Reynolds numbers typical of insect flight, the LEV can remain stable throughout much of the stroke cycle through a combination of spanwise flow and periodic shedding mechanisms. In forward flight, the LEV is shed periodically into the wake; depending on the particular motion and geometry of the wing, the development and breakup of the LEV presents complex patterns that are difficult to track [36].

The interaction between LEVs and TEVs often determines the instantaneous force production. The timing and strength of the TEV shedding are closely linked to wing acceleration and the distribution of surface vorticity over the wing, both contributing to the net aerodynamic force

[37,38]. TEVs have received comparatively less attention than LEVs, although in certain conditions they have been shown to contribute up to one-third of the total aerodynamic force on the wing [39]. Tip vortices arise from the pressure differences between the wing's upper and lower surfaces, creating spanwise flow that rolls up at the wing tips. In flapping flight, tip vortices interact with the oscillatory wake structure, creating complex three-dimensional flow patterns that can either enhance or diminish performance depending on the specific kinematics and geometry.

The interaction and evolution of these vortical structures create the characteristic wake patterns observed in flapping flight, ranging from the thrust-producing reverse von Kármán street to more complex three-dimensional vortex rings. The spatial and temporal organization of wake vortices serves as an indicator of the efficiency of momentum transfer and energy conversion in the flapping process, making vortex dynamics central to both the design and diagnosis of biological locomotion.

3. Aerodynamic forces

The aerodynamic force acting on heaving and pitching wings is commonly decomposed into two main components: added mass forces and circulatory forces [19,35]. This decomposition reflects a physical distinction. Added mass forces arise from the acceleration of fluid around the wing due to its unsteady motion [7]. When a body accelerates through a fluid, it must accelerate not only its own mass but also a volume of surrounding fluid, creating reaction forces proportional to the acceleration. These forces are particularly significant during rapid changes in motion direction or velocity, like the end-of-stroke transitions in flapping wing kinematics. On the other hand, circulatory forces are linked to the generation and evolution of vorticity (and circulation) in the flow. In the inviscid limit, they correspond to the bound circulation of the wing and the unsteady shedding of circulation into the wake. At the finite Reynolds numbers of interest here, circulatory forces are usually obtained by subtracting the added mass contribution from the total aerodynamic force [35], and they can be interpreted as the contributions to the total aerodynamic force from rotational flow elements (i.e., LEVs, TEVs, and tip vortices, plus the contribution of the surface vorticity around the wing) [37,38]. The circulatory forces depend on many parameters, including the wing's geometry and angle of attack, and the formation and shedding of LEV, TEV, and tip vortices. The relative importance of added mass and circulatory forces depends on several factors, such as the type of motion employed (pure heaving, pure pitching, or combined motions) and the Strouhal number, among others.

For purely heaving motions, thrust generation is primarily lift-based (circulatory), occurring when the foil moves perpendicular to the flow direction. As the wing oscillates vertically while moving forward, it creates a time-varying effective angle of attack due to the vector addition of the forward velocity and the instantaneous heaving velocity. This leads to the generation of circulation around the wing. The resulting lift force, when the foil is moving upward or downward, has a component in the forward direction that contributes to thrust. The efficiency of pure heaving depends critically on the Strouhal number, with optimal propulsive efficiency typically occurring in the range of $St = 0.2\text{--}0.4$, as discussed above.

Purely pitching motions present a more complex force generation mechanism. While added mass forces are indeed significant due to the rotational acceleration of the wing, circulatory forces also contribute substantially. As the wing pitches, it experiences varying angles of attack throughout the cycle, generating time-varying circulation and associated lift forces. The circulatory component becomes increasingly important at higher frequencies and larger pitch amplitudes. The thrust generation efficiency in pure pitching is generally lower than in pure heaving at comparable amplitudes, particularly at moderate Strouhal numbers.

When heaving and pitching motions are combined with appropriate phase relationships [40–42], a superior propulsive performance is achieved, compared to either motion alone. The interaction between these motions creates complex but beneficial fluid dynamic effects. The optimal phase difference between heave and pitch is approximately 90° , with pitch leading heave [43]. The pitch component effectively preangles the foil to align more favorably with the instantaneous flow

direction created by the heaving motion. This combination allows for greater heave velocities at the same effective angle of attack by adding the appropriate pitch component, effectively rotating more of the instantaneous lift vector into the thrust direction and increasing overall propulsive efficiency.

The phase relationship between heaving and pitching also fundamentally influences the formation of vortex structures in the wake, which directly correlates with propulsive efficiency. Properly phased combined motions are capable of generating reverse von Kármán vortex streets [44], where vortices rotate in the opposite direction compared to those behind a stationary bluff body. These reverse vortex streets produce jetlike wakes with a momentum surplus rather than a deficit, indicating net thrust production. The vortex strength and spacing can be controlled through the amplitude and frequency of the combined motions, allowing for optimization of the wake structure for maximum propulsive efficiency.

B. Wing-vortex interactions

Beyond the intrinsic vortex generation by isolated flapping wings, the interaction between external vortical disturbances and oscillating wings represents a fundamental aspect of flapping flight aerodynamics. These interactions occur in three contexts: (i) the encounter of wings with environmental turbulence or gusts [45–47]; (ii) the recapture by the wing of vortices that were shed during the previous stroke, as may occur during stroke reversal [48,49]; and (iii) the exploitation of upstream wake structures in tandem wing configurations [50–53]. These phenomena can be analyzed by examining how vortices modify the local flow field around the oscillating wing, thereby altering force production and wake development.

When a flapping wing encounters vortical disturbances, such as those arising from environmental turbulence or the wake of upstream objects, the interaction can significantly modify the wing's aerodynamic performance. The relative timing between the wing's oscillatory motion and the passage of external vortices determines whether the interaction is beneficial or detrimental. Constructive interactions occur when external vortices enhance the wing's bound circulation during the downstroke, leading to increased lift and thrust generation. Conversely, destructive interactions can reduce performance or introduce unsteady loading that challenges flight stability. The strength, size, and convection speed of the incoming vortical disturbances relative to the wing's natural shedding frequency create a complex parameter space that governs the interaction dynamics.

In tandem wing systems, the forewing's wake provides a controlled vortical environment for the hindwing, creating opportunities for enhanced performance through wake capture mechanisms. The LEVs and TEVs shed by the forewing encounter the hindwing at specific phases of its oscillatory cycle. When properly timed, the hindwing can capture beneficial vorticity from the forewing's wake, effectively preloading its bound circulation and reducing the energy required for force generation. Two-dimensional studies have revealed that optimal phase relationships between fore- and hindwings can lead to significant improvements in propulsive efficiency and thrust augmentation compared to isolated wing performance.

The success of vortex-wing interactions depends on the spatial and temporal matching between the incoming vortical structures and the wing's instantaneous kinematic state. Parameters such as interwing spacing, phase difference, and relative stroke amplitudes determine whether wake capture results in performance benefits or penalties. Additionally, the interaction process itself modifies the downstream wake structure, creating a coupled system where the performance of each wing element depends on the complex feedback between vortex generation, convection, and subsequent capture by downstream wings.

C. Flexibility effects

The introduction of structural flexibility fundamentally alters flapping wing aerodynamics by creating a coupled fluid-structure interaction system [27] where wing deformation actively participates in force generation and flow control. Unlike rigid wings, flexible wings can passively adapt their

shape in response to aerodynamic loading, potentially leading to enhanced performance through optimized local flow conditions and reduced power requirements.

Chordwise flexibility impacts aerodynamic force generation [54,55] through several interconnected mechanisms. Shape deformation adjusts the projected area normal to the trajectory, directly influencing drag and thrust components throughout the flapping cycle. The redistribution of thrust and lift occurs as the effective angle of attack changes along the chord due to bending-induced camber variations. In addition, the temporal phase relationships between prescribed wing motion and passive deformation can create beneficial coupling when the structural response amplifies desired aerodynamic effects while suppressing detrimental ones.

Spanwise flexibility introduces progressive deformation patterns from wing root to tip that create complex three-dimensional effects [56,57]. This flexibility generates varied phase shifts along the wingspan, with tip regions typically lagging behind root motion due to inertial and aerodynamic loading. The resulting nonuniform effective angle of attack distribution alters local flow velocities and acceleration patterns, which in turn modifies vortex formation and shedding dynamics along the span. Research indicates that properly tuned spanwise flexibility can enhance thrust generation in forward flight by optimizing the spanwise circulation distribution and reducing tip losses [56]. The wing's dynamic response is highly dependent on the ratio between flapping frequency and the wing's natural frequencies [58], with resonant conditions potentially leading to dramatic performance improvements or degradation.

Fluid-structure resonance represents a critical aspect of flexible wing design, occurring when the flapping frequency approaches the wing's structural natural frequencies. Near-resonant conditions can amplify wing deformations, leading to enhanced thrust production and improved propulsive efficiency when the phase relationship between forcing and response is favorable. This will be shown below in Sec. IV A. However, resonance can also result in excessive deformations, structural fatigue, or performance degradation if not properly controlled.

D. Computational methodology and notation

The studies that we review in the following sections have been all performed with TUCAN, an in-house code that has been developed in our group over the past 10 years. Here we summarize the main characteristics of the solver, while further details can be found in the corresponding references. The fluid solver [43,59] is based on a numerical solution to the Navier-Stokes equations for incompressible flow of a Newtonian fluid. We use a finite difference approach on a staggered grid configuration. Spatial derivatives are discretized using second-order central differences, and the temporal integration is handled through a three-stage semi-implicit Runge-Kutta method. The presence of arbitrarily shaped objects in the flow is modeled using the immersed boundary method proposed by Uhlmann [60].

When structural flexibility is primarily limited to a single spatial direction, flexible objects can often be approximated as a series of rigid segments connected by joints incorporating elastic elements, such as torsional springs [61]. This modeling strategy is well suited for configurations like spanwise-flexible wings (see Sef. IV) or chordwise-flexible flappers (see Sef. V). Hence, the solver used for the simulations presented in Secs. IV and V couples our in-house fluid solver TUCAN to a multibody solver (MBS) [62]. Details of the implementation of this coupling can be found in [63]. TUCAN has been extensively validated by comparisons to benchmark test cases in the literature and experimental data [39,59,63,64]. There is also a recent GPU implementation of TUCAN [65].

The aerodynamic force coefficients are defined as

$$C_{k,i} = \frac{2\mathbf{F}_i \cdot \mathbf{e}_k}{\rho U_\infty^2 S_i}, \quad (3)$$

where the subindex i denotes the i -wing, $\mathbf{F}_i$ is the total aerodynamic force, and $\mathbf{e}_k$ is the unitary vector parallel to the k -axis. If the force coefficient for a two-dimensional (2D) configuration is provided, a lower case is employed, $c_{k,i}$, and instead of the wing surface the chord length is

employed for nondimensionalization. In addition to force coefficients, the efficiency of locomotion is an important quantity of interest that can be measured in various ways. For an overview of the different approaches, see, for example, Section 2.5 of Zhang *et al.* [66]. In Secs. III and IV below, the performance of the wings is assessed by means of the individual propulsive efficiency of each wing, computed as

$$\eta_i = \frac{\bar{C}_{T,i}}{\bar{P}_i}, \quad (4)$$

where $\bar{C}_{T,i}$ is the average thrust coefficient (computed as the average of $-C_{x,i}$ over a cycle), and $\bar{P}_i$ is the averaged nondimensional power required to move the i -wing over a cycle. See the corresponding references [67–70] for the computation of this quantity in each case.

III. RIGID WINGS IN PRESCRIBED MOTION: FROM 2D TO 3D

In this section, we review two studies [67,68] that deal with the transition from 2D to 3D aerodynamics in tandem flapping wing systems. Many studies in the literature have used two-dimensional models to shed light on the basic mechanisms of tandem wing interactions [52,71,72]. However, relatively few investigations [73–75] have addressed the three-dimensional effects that inevitably arise in more realistic configurations. The work of Arranz *et al.* [67] examines how 3D effects influence the aerodynamic performance of rigid wings undergoing prescribed heaving and pitching motions in tandem configurations, with particular emphasis on the modification of vortex interactions and force generation mechanisms.

The simulations were performed at $Re = 1000$ and $St = 0.34$ considering two flat plates of equal aspect ratio 2 (denoted case AR22) and 4 (case AR44), and comparing to the 2D configuration. The prescribed motion of the wing, a combination of sinusoidal, heaving, and pitching motion about the midchord, was based on the optimal, 2D configuration proposed by Ortega-Casanova and Fernández-Feria [76]. These authors performed an optimization process that aimed to maximize the efficiency of the configuration. As discussed by Smits [19], high-efficiency motion for heaving and pitching airfoils occurs at low thrust, since increasing the thrust reduces the efficiency. Therefore, for high-efficiency kinematics, the formation of the LEV has to be avoided. As a consequence, in the proposed 2D optimal configuration [76], no LEV is formed on the forewing. During the flapping cycle, however, a TEV is shed from the forewing that promotes the formation of an induced LEV (iLEV) on the hindwing. The interaction of TEV and iLEV substantially influences the aerodynamic performance of the hindwing.

For illustration, Fig. 1 shows flow visualizations at three instants during the cycle, $t/T = 0.5$, 0.75 and 1, where T is the flapping period. Panels (a), (d), and (g) correspond to case AR44, and panels (b), (e), and (h) correspond to case AR22. The first instant, $t/T = 0.5$, corresponds to the end of the forewing downstroke, and the TEV and iLEV are clearly visible in panel (a). For both aspect ratios, the TEV and the iLEV tilt upwards and break near the tips of the hindwing. However, for AR44, there is a region at the midspan where both vortices resemble two-dimensional vortices. This breakdown observed near the wing tips progresses towards the midsection, as observed at later time instants in panels (d), (e), (g), and (h). The progression of the breakdown seems to occur at the same rate for both aspect ratios. As a consequence, the dipole in the midspan section of the AR22 case breaks down earlier than in the AR44 case. Finally, near the wing tips, the TEV and the iLEV merge with the tip vortices of the hindwing and the forewing, forming a Y-branching bifurcation. The interactions among the iLEV, the TEV, and the tip vortices depend on the aspect ratio of both wings.

Figure 2 shows the aerodynamic force coefficients ($C_{x,i}$ and $C_{z,i}$) for both aspect ratios, together with the forces of the 2D reference case. Qualitatively, the aerodynamic forces of the finite wings are similar to those of the 2D case. Both the fore- and the hindwing produce thrust (i.e., negative horizontal force) during most of the cycle. Figure 2(a) shows two thrust peaks for the forewing at mid-downstroke and mid-upstroke; these peaks are also present for the hindwing, Fig. 2(c), but

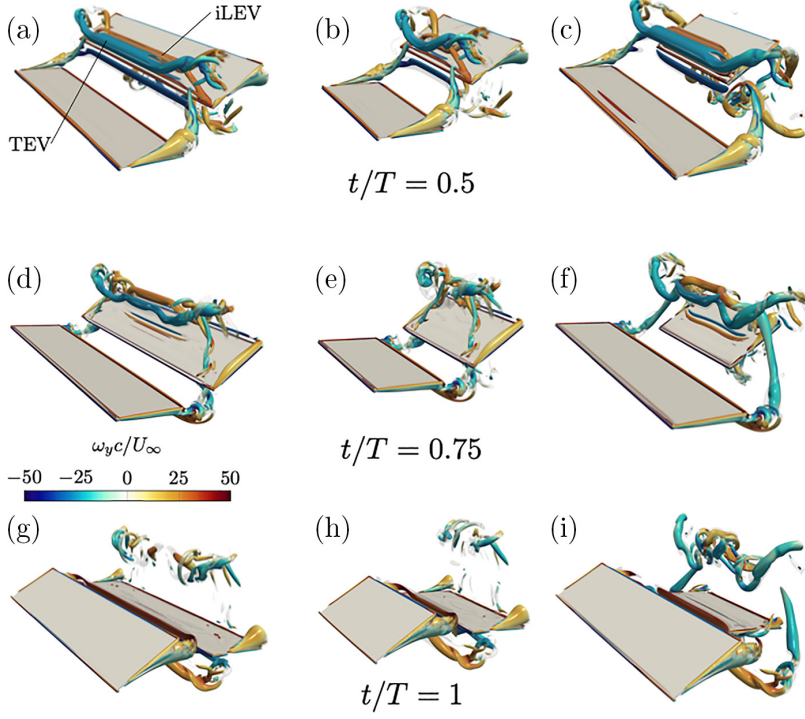


FIG. 1. Flow visualization at various time instants: (a)–(c) end of the forewing downstroke; (d)–(f) forewing mid-upstroke; and (g)–(i) end of the forewing upstroke. (a), (d), (g) Case AR44. (b), (e), (h) Case AR22. (c), (f), (i) Case AR42. Semitransparent isosurface corresponds to $Q = 3\Omega_0^2$, with $\Omega_0 = 2\pi f$; isosurface colored with the spanwise vorticity, ω_y , corresponds to $Q = 6\Omega_0^2$. Adapted from Jurado *et al.* [68].

they are less intense. On the other hand, the vertical force for both wings is mostly positive during the downstroke and negative during the upstroke [Figs. 2(b) and 2(d)]. These forces yield a zero mean lift, contrary to the nonzero mean lift observed in the 2D case. This was attributed to the fact that in the 2D case the wake was deflected, while in 3D this phenomenon did not occur. A similar observation had been reported in the literature [77] on single plunging foils and wings at similar reduced frequencies.

Regarding the effect of the aspect ratio on the forces, Figs. 2(a) and 2(b) show that, on the forewing, the larger aspect ratio leads to higher forces during the mid-upstroke and mid-downstroke. On the hindwing [Figs. 2(c) and 2(d)], the temporal evolution of the force coefficients presents more oscillations due to the interaction with the wake of the forewing, like in the 2D configuration [76]. Despite this, a qualitatively similar increase in the peak forces of the hindwing with aspect ratio was also observed.

In fact, the average forces over the half-cycle of each wing decrease roughly 10% from AR44 to AR22. This is reported in Table I, which gathers the thrust coefficient averaged over a cycle ($\overline{C}_{T,i}$), the lift coefficient averaged over half a cycle ($\overline{C}_{z,i}$), and the propulsive efficiencies of each wing (η_i). For comparison, we also report in the table data for the 2D tandem system and for 3D isolated wings of aspect ratio 2 and 4 with the same motion as the forewing. If the mean thrust of each wing is compared to its two-dimensional counterpart, a reduction of approximately 21% and 11% is obtained for the forewing of AR22 and AR44, respectively. The data for the isolated wings (AR4 and AR2) show that the thrust generated by the forewing is not very much affected by the presence of the hindwing, in agreement with other studies [78,79]. This suggests that the thrust reduction in the forewing from 2D to 3D is mainly due to wingtip effects, and not due to wing-wing interactions.

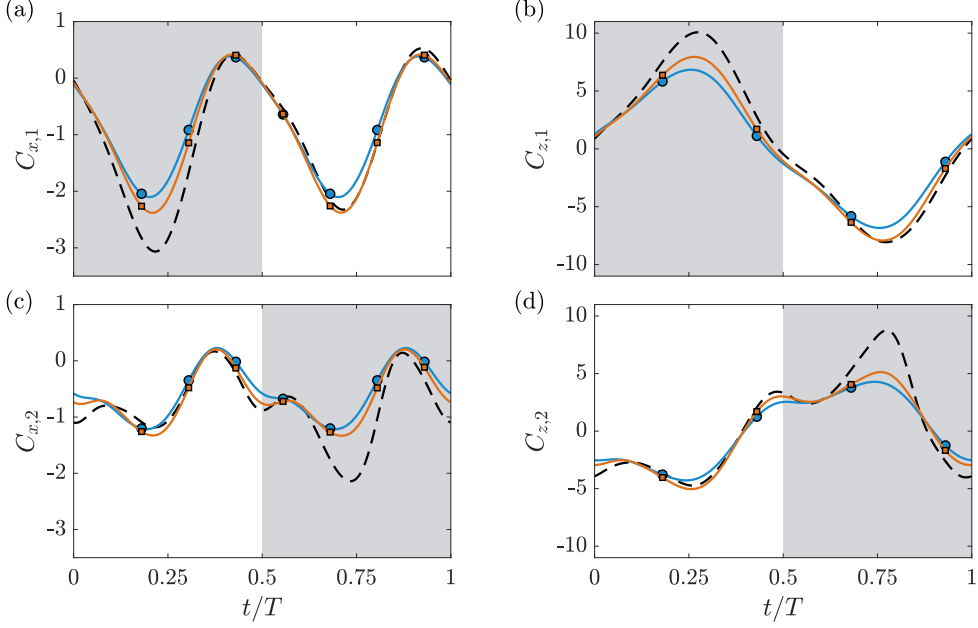


FIG. 2. Temporal evolution of the force coefficients. (a), (b) Forewing; (c), (d) hindwing. (a) and (c) Horizontal force coefficient; (b) and (d) vertical force coefficient. Black dashed line, 2D reference case; blue line with circular symbols, AR22; orange line with square symbols, AR44. In all plots, the shaded region corresponds to the downstroke of each wing. Taken from Arranz *et al.* [67].

For the hindwing, the thrust reduction from 2D to 3D is larger, from 29% for case AR22 to 20% for case AR44. Consequently, three-dimensional vortical mechanisms, which are not present in the 2D case (i.e., wingtip vortices and vortex breakdown), influence the hindwing's thrust generation. Indeed, the thrust reduction of the hindwing with respect to the forewing ($1 - \bar{C}_{T,2}/\bar{C}_{T,1}$) is very similar for both aspect ratios (30% and 28% for AR22 and AR44, respectively), and higher than for the two-dimensional case, which is close to 20%. Despite the detriment in thrust generation, the propulsive efficiency of the hindwing (η_2) is very similar to the propulsive efficiency of the forewing (η_1) and only slightly lower than the individual propulsive efficiencies of the 2D case. Consequently,

TABLE I. Mean force coefficients and propulsive efficiencies for each wing. The thrust coefficient is averaged over a full cycle, and the lift coefficient is averaged over half a cycle. Data include tandem wing systems of various aspect ratios, isolated wings of aspect ratio 2 and 4, and 2D tandem system. The latter exhibited the phenomenon of wake deflection, tainting the values of $\bar{C}_{z,i}$ (which are not provided in the table). Adapted from Arranz *et al.* [67] and Jurado *et al.* [68].

Case	$\bar{C}_{T,1}$	$\bar{C}_{T,2}$	$\bar{C}_{z,1}$	$\bar{C}_{z,2}$	η_1	η_2
AR44	0.91	0.65	4.39	2.24	0.23	0.20
AR22	0.81	0.57	3.83	2.05	0.23	0.21
2D tandem	1.02	0.81			0.24	0.23
AR4 isolated	0.93		4.58		0.22	
AR2 isolated	0.81		3.95		0.22	
AR43	0.93	0.63	4.47	1.96	0.23	0.20
AR42	0.93	0.62	4.52	2.01	0.23	0.19
AR32	0.89	0.59	4.24	1.84	0.23	0.20

the propulsive efficiency of the whole system, η , is virtually the same, and not much dependent on the aspect ratio, a result that is consistent with previous data for isolated wings [77,80].

The observation that the thrust coefficient of the hindwing decreases significantly due to the influence of vortex interactions generated at the wing tips motivated the followup study by Jurado *et al.* [68]. To decrease the influence of the wingtip vortices of the forewing on the hindwing, it was proposed to reduce the aspect ratio of the hindwing compared to the aspect ratio of the forewing. Three additional simulations were performed with aspect ratios of the tandem system, 4 and 3 (case AR43), 4 and 2 (case AR42), and 3 and 2 (case AR32); see Table I for the force coefficients and efficiencies.

Flow visualizations of case AR42 are shown in panels (c), (f), and (i) of Fig. 1. The structure of the iLEV is quasi-2D over the whole span of the hindwing [panel (c)], significantly different from the iLEV of the case AR22 [panel (b)]. This indicates that, in general, the structure of the iLEV depends on the spanwise length of the TEV, which is dictated by the aspect ratio of the forewing only. Furthermore, the fact that the iLEV is quasi-2D leads to an increased thrust production on the hindwing by 8% compared to configurations with equal aspect ratios (see Table I). The data in the table show that despite the increased thrust, the propulsive efficiency of the hindwing is not increased. It was shown [68] that this is because the power required to sustain pitching motions increases proportionally with thrust gains, indicating a tradeoff between force production and energy expenditure. These findings highlight how forewing geometry influences wake interactions in tandem systems, indicating that using different aspect ratios for the wings can reduce 3D performance losses without compromising efficiency.

These results highlight a general trend: three-dimensional effects systematically reduce aerodynamic performance in tandem flapping systems, particularly for the hindwing, due to vortex breakdown and tip vortices. However, performance penalties can be mitigated through geometric tuning, such as adjusting the aspect ratio asymmetrically. This represents a simple strategy for mitigating the loss of efficiency in 3D tandem arrangements.

IV. SPANWISE-FLEXIBLE WINGS IN TANDEM

The results from Arranz *et al.* [67] and Jurado *et al.* [68] discussed in the previous section illustrate how aspect ratio selection affects the performance of flapping systems. While rigid wings with prescribed kinematics benefit from quasi-2D vortex preservation, flexible wings may further enhance performance by dynamically adjusting spanwise loading. This is the subject of this section, where we review the work of Martínez-Muriel *et al.* [70] for tandem systems of spanwise-flexible wings in forward flight.

In this investigation, a heaving and pitching motion is imposed on the midspan section of each of the wings, while the rest of the wing deforms passively. Unlike the previous section, where propulsive efficiency was the primary focus of the optimization procedure [76], we concentrate here on aerodynamic force generation. To this end, the base motion is selected to promote the formation of an LEV on each wing without relying on interaction with the TEV. The optimization procedure used to define this base motion is detailed in Ref. [81]. Note that the presence of LEVs in both fore- and hindwings results in stronger wing-wing interactions, and consequently more opportunities to tune the wings' flexibility to optimize the system's performance.

The simulations were performed at a Reynolds number $Re = 1000$ and a Strouhal number $St = 2h_0f/U_\infty = 0.38$, based on twice the semiamplitude of the midspan section's heaving motion, h_0 . Wings of aspect ratio 2 and 4 were considered. The material properties of the wing were varied in order to study the effect of flexibility; this was done by varying the effective stiffness of the wings in a broad range, including also the rigid wing case, $\Pi_1 \rightarrow \infty$, for reference. The effective inertia was kept constant to a value that is relevant to insect and bird flight, $\Pi_0 \approx 0.1$. The wings were modeled using a lumped-torsional flexibility model, as sketched in Fig. 3. The stiffness of the torsional springs was adjusted to match the natural frequencies in vacuum of a corresponding Euler-Bernoulli beam [61].

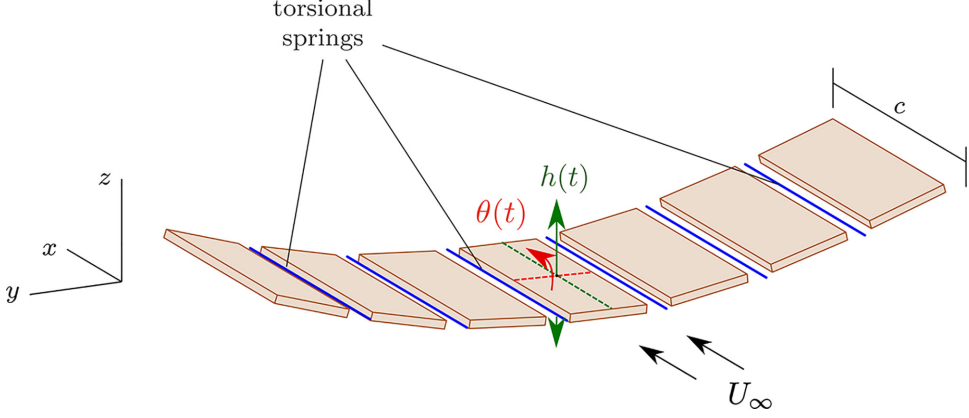


FIG. 3. Sketch of the multibody model to qualitatively describe the spanwise flexibility of a wing, where the system of bodies is connected via torsional springs. Adapted from Martínez-Muriel *et al.* [69].

To analyze the results, it was shown by Kang *et al.* [57] that it is possible to incorporate the effect of the aspect ratio by adding a factor to the effective inertia, namely $\Pi_1(2/AR)^4$. It is also possible to define the natural frequency of the structure in the fluid, $\omega_{n,f}$, by considering the additional moment of inertia of the wing due to added mass [61,82]. This allows characterizing the wing's flexibility as $\omega_{n,f}/\omega$, where $\omega = 2\pi/f$ is the angular frequency of the imposed motion on the wing. Indeed, it can be shown [69] that there is a linear relationship between $\log(\omega_{n,f}/\omega)$ and $\log(\Pi_1(1/AR)^4)$ when the effective inertia and the wing's geometry remain constant. Since both quantities are equivalent, and given the straightforward physical interpretation of $\omega_{n,f}/\omega$, in the following we employ this ratio to characterize the wing's flexibility.

The objective of the study was to investigate the effects of spanwise flexibility in a tandem wing configuration, but first it is necessary to examine the performance of a corresponding, isolated, spanwise-flexible wing as a reference case, as reported by Martínez-Muriel *et al.* [69].

A. Isolated wing

A total of 13 simulations were performed, 5 for $AR = 4$ and 8 for $AR = 2$ (for the complete list, see Table 2 of Martínez-Muriel *et al.* [69]). We will discuss here mainly the $AR = 4$ cases: case IsolR, with $\omega_{n,f}/\omega \rightarrow \infty$ (rigid); and cases IsolF1-4, with $\omega_{n,f}/\omega = 2.22, 1.11, 0.74$, and 0.44 . Case IsolF2 is the case closer to the resonance, with $\omega_{n,f}/\omega = 1.11$. Note that the imposed motion was symmetric during the upstroke and the downstroke, as for the rigid wings in the previous section, resulting in a time-averaged lift of zero.

The time evolution of thrust and lift coefficients of the isolated wings (Fig. 4) revealed a strong influence of wing flexibility, with nonmonotonic changes in their peak values as flexibility increases—initially enhancing thrust (cases IsolR, IsolF1, and IsolF2), followed by a sharp drop in more flexible cases (IsolF3 and IsolF4). Additionally, the timing of peak force generation shifts with flexibility, occurring earlier in stiffer wings and later in more flexible ones, a trend observed for both C_T and C_L . The effect of wing flexibility is more clearly visible when analyzing the time-averaged thrust coefficient as a function of the wing flexibility, as shown in Fig. 5(a). In this figure, the data for the two aspect ratios considered in Ref. [69] are included. The data follow a single trend, indicating that the rescaling with the aspect ratio proposed by Kang *et al.* [57] and $\omega_{n,f}/\omega$ are both able to characterize unambiguously the wing flexibility. Figure 5(b) shows the corresponding propulsive efficiencies as a function of wing flexibility, showing the same sharp decrease in performance at around $\omega_{n,f}/\omega \approx 1$ as the mean thrust coefficient.

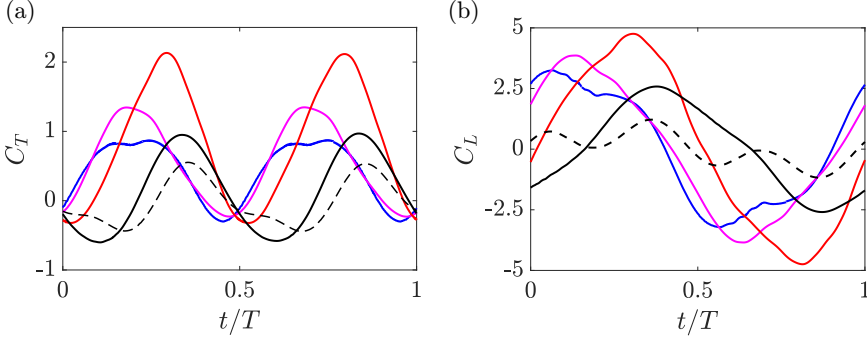


FIG. 4. Temporal evolution of (a) thrust coefficient, C_T , and (b) lift coefficient, C_L , of $AR = 4$ wings. Line colors correspond to —: IsolR. —: IsolF1. —: IsolF2. —: IsolF3. — —: IsolF4. Taken from Martínez-Muriel *et al.* [69].

Figure 5 clearly shows that there is an optimal aerodynamic performance of the wing that occurs when the imposed frequency of oscillation approaches the first natural frequency of the structure in the fluid, $\omega_{n,f}/\omega \approx 1$. For that case, the time-averaged thrust is maximum, increasing by a factor of 2 with respect to the rigid case. The associated increase in propulsive efficiency is milder, about 3–6 % in absolute terms, since the increase in thrust production is also linked to an increase in the required power to maintain the wing motion (not shown here). With increasing flexibility beyond the optimal case, $\omega_{n,f}/\omega < 1$, the aerodynamic performance drops significantly, both in terms of thrust production and propulsive efficiency.

To characterize the structural response, Fig. 6 shows the time evolution of midchord deflection from root to tip for increasing wing flexibility. In all cases, the wing bends in its first mode, with tip-to-root deflections increasing with flexibility. For low flexibility [IsolF1, panel (a)], deviations from rigid motion are small, forming a mildly diverging envelope. As flexibility increases [optimal case, IsolF2, panel (b)], the diverging pattern becomes more pronounced, indicating greater heaving amplitude along the span. However, beyond the optimal case, the deflection lines form a convergent-divergent pattern, and the spanwise heaving amplitude at the tip decreases despite larger overall deflections, leading to lower aerodynamic performance [IsolF3, panel (c)].

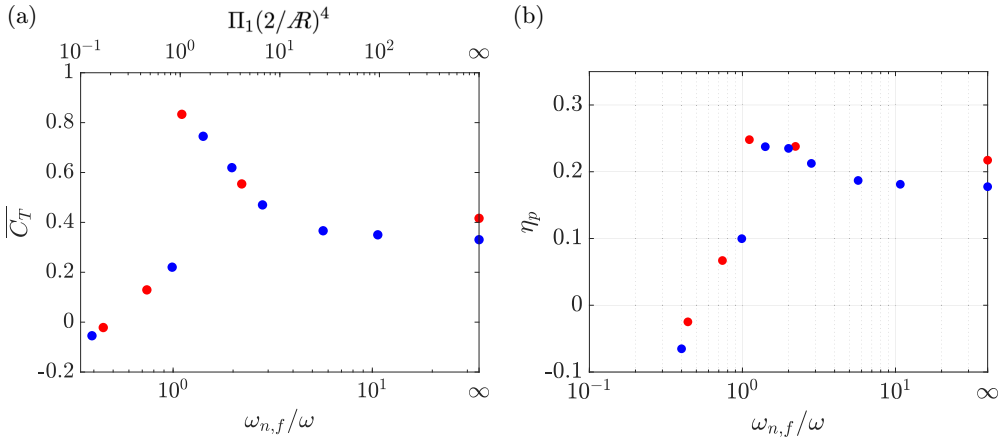


FIG. 5. (a) Mean thrust coefficient $\overline{C_T}$ and (b) propulsive efficiency η_p . Blue symbols, $AR = 2$. Red symbols, $AR = 4$. Adapted from Martínez-Muriel *et al.* [69].

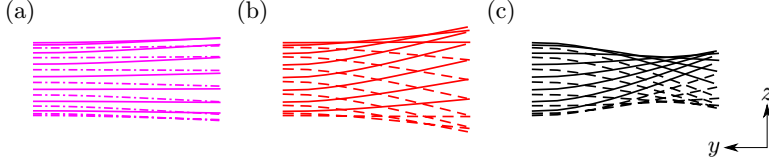


FIG. 6. Deflection of the midchord line of the wing during the cycle. Only half the wing is shown from root (left) to tip (right). Solid (dashed) lines correspond to the downstroke (upstroke). The following cases with increasing flexibility are shown: (a) IsolF1 (—). (b) IsolF2 (optimal) (—). (c) IsolF3 (—). Taken from Martínez-Muriel *et al.* [69].

The flow structure around the wing is significantly affected by flexibility. For brevity, we show an instantaneous snapshot at the end of the downstroke for three cases in Fig. 7, and we refer to the Supplemental Material of Martínez-Muriel *et al.* [69] for animations. At the start of the downstroke (not shown), all wings begin to develop an LEV. In the rigid case, the LEV extends uniformly along the span, while in the optimal case it is strongest near the midspan and weaker toward the tips. For the suboptimal case, the LEV appears only in the central region. As the wing continues moving downward, the LEV intensifies and is advected downstream. At the end of the downstroke, Fig. 7 shows that the LEV structures differ markedly: the rigid wing shows a quasi-2D, span-aligned LEV; the optimal wing shows an inclined LEV that has advected further at midspan; and in the suboptimal wing, the central LEV has broken down, with new vortices forming near the tips. Two key observations arise: in the suboptimal case, out-of-phase tip motion disrupts LEV coherence, reducing spanwise aerodynamic loading; in the optimal case, LEV development is delayed compared to the rigid wing, as indicated by the persistence of the previously shed LEV [tagged LEV2 in Fig. 7(b)], which is only clearly visible in this case.

In summary, for isolated spanwise-flexible wings, fluid-structural resonance results in an enhanced aerodynamic performance. This is due to two reasons. First, the increased amplitude of motion of the outboard wing sections leads to larger effective angles of attack. Second, the motion of the outboard wing sections is delayed with respect to the motion of the midspan section of the wing. This results in a delayed development of the leading edge vortex, which, together with the larger effective angles of attack, explains the larger aerodynamic load in the outboard sections of the optimal wing. This beneficial fluid-structure interaction holds while the bending

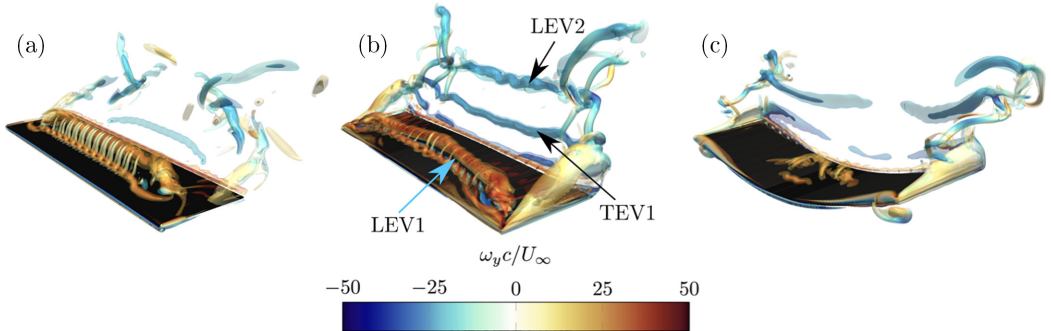


FIG. 7. Flow visualization at the end of the downstroke using isosurfaces of the second invariant of the velocity gradient tensor, Q , for cases with increasing flexibility. (a) IsolR (rigid). (b) IsolF2 (optimal). (c) IsolF3. Transparent isosurfaces correspond to $Q = 6\omega^2$, where $\omega = 2\pi f$. Filled isosurfaces correspond to $Q = 15\omega^2$. Isosurfaces are colored with the spanwise vorticity ω_y . The arrows in panel (b) highlight the LEV and TEV formed during the downstroke of the wing (LEV1 and TEV1), and the LEV formed during the previous upstroke of the wing (LEV2). Adapted from Martínez-Muriel *et al.* [69]

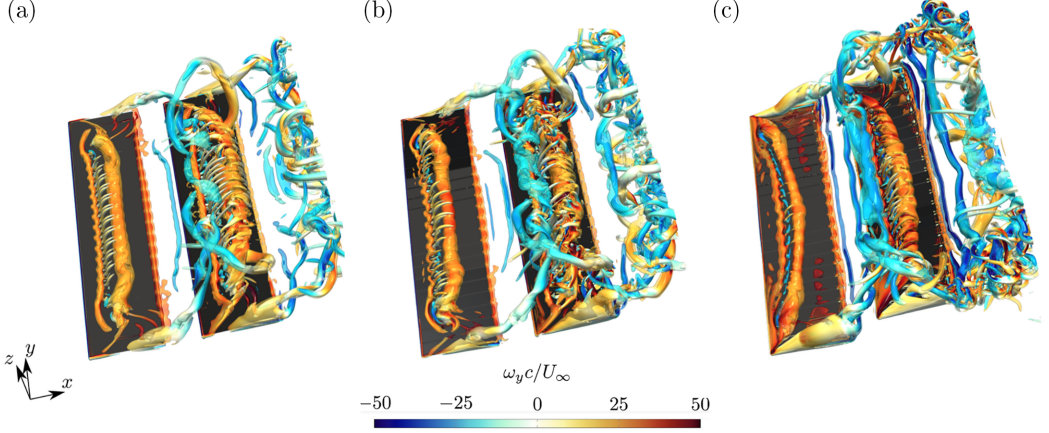


FIG. 8. Flow visualization at the end of the downstroke of the forewing, using the same style as in Fig. 7, for cases (a) TandR (rigid), (b) TandF1, and (c) TandF2. Adapted from Martínez-Muriel *et al.* [70].

deformation and the pitching motion of the wing are synchronized. Indeed, the outboard wing sections of the suboptimal cases are out-of-phase, leading to drag generation during the first half of each stroke. Overall, the coupling among deformation, force generation, and wing orientation (i.e., pitching angle) in the results is reminiscent of the aerodynamic tailoring arguments of Ramanarivo *et al.* [83] for chordwise-flexible wings, although there are fundamental differences between both configurations.

A key message from the results presented in this subsection is that fluid-structure resonance represents a general mechanism for enhancing aerodynamic performance in spanwise-flexible wings. Optimal thrust is achieved when the flapping frequency matches the structure's natural frequency in fluid, a principle that may guide the design of efficient flapping-wing systems.

B. Tandem wings

Once the isolated wings have been discussed for reference, we turn our attention to tandem systems as studied by Martínez-Muriel *et al.* [70]. A total of 12 simulations were performed, 4 for $AR = 4$ and 8 for $AR = 2$ (for the complete list, see Table 2 of Ref. [70]). As for the isolated wings, we will discuss here mainly the $AR = 4$ cases: Case TandR, with $\omega_{n,f}/\omega \rightarrow \infty$ (rigid); and cases TandF1-3, with $\omega_{n,f}/\omega = 2.22, 1.11$ and 0.74 . As before, case TandF2 is the case closer to the resonance, with $\omega_{n,f}/\omega = 1.11$.

Recall that in 2D tandem wings, the basic mechanism at play is wake capture, where the hindwing's aerodynamic performance is enhanced through interaction with the forewing's wake. For isolated wings, the previous section showed that the effect of the wing flexibility on its aerodynamic performance arises from a combination of unsteady aerodynamic mechanisms (LEV, TEV), fluid-structure resonance, and aerodynamic tailoring. Accordingly, key questions for tandem wings are as follows: (i) How do fluid-structure resonance and aerodynamic tailoring contribute in a tandem system? (ii) To what extent do hydrodynamic interactions between the forewing wake and the hindwing enhance performance? (iii) How do these interactions influence the aeroelastic response of the hindwing relative to the forewing?

The complex vortical structures generated by the tandem wings are illustrated in Fig. 8, while corresponding animations can be found in the Supplemental Material of Ref. [70]. The visualizations correspond to the end of the downstroke of the forewing, and they may be compared to those of the isolated wing (Fig. 7). We focus first on the forewing: In the rigid wing case [Fig. 8(a)], the LEV runs spanwise along the wing with a relatively uniform alignment. In contrast, for the

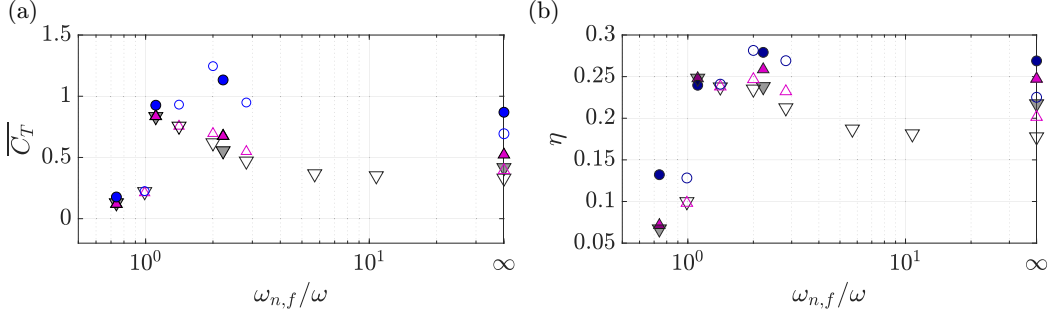


FIG. 9. (a) Mean C_T and (b) propulsive efficiency averaged over a cycle for wings with $AR = 2$ (open symbols) and $AR = 4$ (closed symbols). ▼: isolated wings. ▲: forewings. ●: hindwings. Taken from Martínez-Muriel *et al.* [70].

flexible case TandF2 [Fig. 8(c)], the LEV takes on a bowl-like shape, curving downstream at the center while remaining closer to the leading edge near the wingtips. In fact, for the forewings, the LEV shape is roughly the same as for the isolated wing; compare Figs. 7(a) and 8(a) for the rigid case, and Figs. 7(b) and 8(c) for the flexible cases IsolF2 and TandF2, respectively. This indicates that the presence of the hindwing has little influence on the flow development over the forewing.

The vortical interactions on the hindwing are significantly more complex, and we recommend viewing the corresponding animations for a clearer understanding of the temporal evolution and interplay of the vortical structures. The LEV shed by the forewing during the previous upstroke travels downstream to interact with the LEV that is being formed on the suction surface of the hindwing during downstroke. The interaction between these two vortices results in a dipole, which is shed into the wake during the upstroke. Note also that the wingtip vortices also play a role that leads to a loss of coherence of the LEVs and the dipoles along the whole span of the wings. As flexibility increases, vortical interactions appear to intensify. The displacement of the forewing's wingtips becomes more pronounced, leading to stronger wingtip vortices. Additionally, the delayed formation of the forewing's LEV alters the dipole structures over the hindwing, affecting both their intensity and chordwise location.

While the flow visualizations offer valuable insight into the underlying vortex dynamics, a quantitative analysis is necessary to assess their impact on the aerodynamic performance. Figure 9 presents the mean thrust coefficient and the propulsive efficiencies as a function of the wing flexibility. Forewings behave as isolated wings, achieving maximum performance at resonance. Hindwings exhibit distinctly different behavior, showing optimal performance in a broad region near $\omega_{n,f}/\omega = 2$ with moderate thrust increases of about 30% compared to rigid hindwings. The values of the mean thrust coefficient for forewings and hindwings for $AR = 4$ cases are summarized in Table II. The propulsive efficiency shows that flexibility yields moderate improvements for isolated wings, while tandem systems maintain relatively constant efficiency with minor improvements over rigid cases, suggesting that thrust increases are balanced by corresponding input power increases.

TABLE II. Mean thrust coefficient, $\overline{C_T}$ values for forewings and hindwings of wings with $AR = 4$ cases, and the difference in thrust between hindwings and forewings, ΔC_T . Adapted from Martínez-Muriel *et al.* [70].

	TandR	TandF1	TandF2
$\overline{C_{T,F}}$	0.522	0.674	0.836
$\overline{C_{T,H}}$	0.869	1.133	0.926
ΔC_T	0.347	0.459	0.090

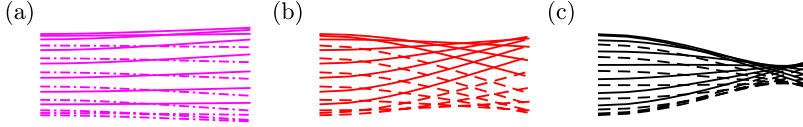


FIG. 10. Deflection of the midchord line of the hindwing during the cycle. Only half the wing is shown from root (left) to tip (right). Solid (dashed) lines correspond to the downstroke (upstroke). The following cases with increasing flexibility are shown: (a) TandF1 (—). (b) TandF2 (—). (c) TandF3 (—). Adapted from Martínez-Muriel *et al.* [70].

The differences in the aerodynamic performance of forewings and hindwings suggest that the underlying mechanisms at play in each wing are different. Flow visualizations and the values of thrust coefficient suggest that the structural response of the forewing should be like the isolated wing. This is confirmed in the analysis presented in Ref. [70], and it is not repeated here for the sake of brevity.

The structural response of the hindwing presents, however, significant differences with that of the forewing and isolated wings. Figure 10 displays the deflection patterns of the midline of the hindwings with different flexibility levels, showing the spanwise deformation during flapping cycles. The case TandF1, Fig. 10(a), exhibits deflection patterns very similar to the corresponding isolated wing, case IsolF1, Fig. 6(a). However, cases TandF2 and TandF3, Figs. 10(b) and 10(c), display smaller tip displacements compared to their isolated counterparts, Figs. 6(b) and 6(c). This explains the lower aerodynamic performance of these two cases.

These results suggest that the hindwing flexibility introduces a lag between the motion of the root (which in the study was chosen to provide an optimal wake capture) and the motion of the tips. The magnitude of this lag affects both the intensity of the wake capture and the timing between forces and pitching angle, which in turn affects the wing deformation (both amplitude and phase lag). Also, the performance of the hindwing is dependent on the flexibility of the forewing, since the latter impacts the strength of the vortices in the wake, and hence the maximum propulsive performance of the hindwing. Therefore, an excellent performance of the tandem system requires a precise coordination between vortical interactions and structural response.

For the cases studied, it was shown that the optimal performance of the forewings was found at resonance, while the optimal performance of the hindwings was found for somewhat stiffer wings. Therefore, we speculated that it could be possible to design a tandem system made of wings with different flexibilities that could outperform the systems made of equal wings. To test this hypothesis, nine additional simulations for mixed tandem systems were performed for wings of lower aspect ratio, $AR = 2$.

The mean thrust coefficients as a function of the corresponding wing flexibility are shown in Fig. 11. For the mixed systems, we need to distinguish between the natural frequency in the fluid of each wing. We do so by introducing the superscript F for forewing and H for hindwing. The results in Fig. 11(a) demonstrate that the forewing thrust coefficient $\overline{C}_{T,F}$ is roughly independent of the hindwing stiffness, with resonant forewings consistently providing maximum thrust regardless of the hindwing configuration. For rigid hindwings (i.e., $\omega_{n,f}/\omega = \infty$), the data in Fig. 11(b) show that $\overline{C}_{T,H}$ increases with fore-wing flexibility, an effect attributed to the larger wake velocities induced by more nonmonotonic forewings. The same trend is observed for hindwings with intermediate flexibility, but a nonmonotonic behavior is observed for the resonant hindwings. The maximum combined thrust $\overline{C}_{T,F} + \overline{C}_{T,H}$ is achieved by systems with resonant forewings and stiffer (intermediate) hindwings, confirming that mixed flexibility systems can outperform equally flexible configurations. The physical interpretation of this result is that the optimal arrangement consists of a resonant forewing that maximizes thrust generation and the wake intensity, paired with an intermediate-flexibility hindwing tuned for optimal wake capture.

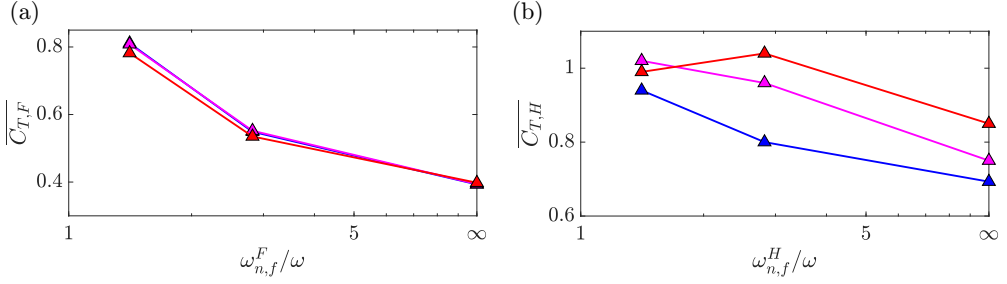


FIG. 11. Mean thrust coefficient $\overline{C_T}$ as a function of the flexibility of the wing, $\omega_{n,f}/\omega$, for (a) forewings and (b) hindwings. Colored lines connect cases where the opposite wing has a fixed flexibility: —: Rigid. —: Intermediate. —: Resonant. Taken from Martínez-Muriel *et al.* [70].

The findings of this section suggest that performance optimization in tandem systems is not achieved by maximizing the individual performance of each wing independently, but rather by tuning their mechanical properties to exploit flow-mediated coupling. Optimal coordination in flexible multiwing systems may require differing stiffness profiles for fore- and hindwings. While resonance emerges as the optimal design guide to choose the stiffness of the former, the effect of the stiffness in lagging the deformation of the hindwing (therefore enabling a productive wake capture) seems to be the most critical issue to select the stiffness of the latter.

V. EMERGENT HYDRODYNAMIC COORDINATION

This section focuses on self-propelled flappers modeled as chordwise-flexible plates. Unlike in previous sections, where the relative position of the wings was fixed, the flappers adjust their position dynamically through hydrodynamic interactions, enabling the study of the equilibrium positions, stability, and propulsion efficiency of a fully coupled system. The study [84] was inspired by collective locomotion observed in nature, such as fish schooling and bird flocking. The motivation stems from understanding how flow-mediated interactions between individuals can lead to more efficient propulsion [85–87], particularly in 3D settings where the wake dynamics differ significantly from 2D cases. Previous works had largely focused on 2D configurations [88–95], with few 3D studies [96], leaving open questions about the applicability of those findings to 3D scenarios where the wake pattern transitions from a reverse von Kármán vortex street in two dimensions to a diverging wake of vortex rings in three dimensions.

Two self-propelling plates in tandem configuration immersed in an otherwise undisturbed fluid were considered. Both plates were driven by a prescribed sinusoidal heaving motion at their leading edges (heaving amplitude $h_0 = 0.5c$), with a phase offset (ϕ) and a vertical offset (H) between them [Fig. 12(a)]. Each flapper was modeled as a flexible, rectangular body with chordwise flexibility, discretized into rigid segments connected by torsional springs [Fig. 12(b)]. The Reynolds number of the flow based on the maximum vertical velocity of the leading edge, $V = 2\pi h_0 f$, is $\text{Re} = 200$, the density ratio was set to $\tilde{\rho} = 10$, and the ratio of the first natural frequency of the flapper in vacuum and the angular frequency of the imposed motion was set to $\omega_n/\omega = 2.17$. These parameters were chosen to yield a good propulsive speed of the isolated flapper, following the work for 2D isolated flappers of Arora *et al.* [61]. The horizontal motion of the plates is not imposed but emerges from the fluid-structure interaction. The study systematically explored the parameter space of ϕ and H by performing 24 simulations to identify equilibrium positions and quantify hydrodynamic performance.

We first briefly describe the behavior of the isolated flapper for the kinematic, inertial, and flexibility parameters described in the previous paragraph. After an initial transient, it reaches a steady state with a mean speed, $\overline{U}_{p,s} = 0.88V$, shedding a vortex ring during each stroke. The vortex rings move away from the flapper with an oblique trajectory due to their own induced velocity

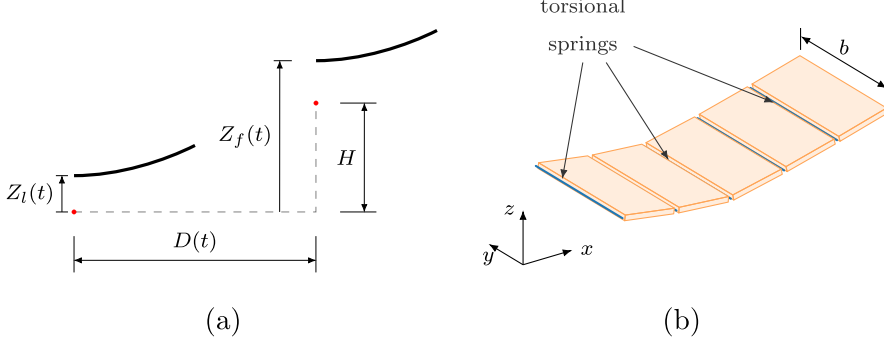


FIG. 12. (a) Side view of the configuration. The leading edge of each flapper has a prescribed heaving motion about a fixed position (represented as a red dot in the figure). The vertical offset between the follower's and leader's pivoting position is denoted as H . $D(t)$ is the instantaneous horizontal distance between the flappers' leading edges. (b) 3D representation of the model of a flapper. Adapted from Arranz *et al.* [84].

(not shown here), leading to a bifurcating wake [97]. The flapper exhibits symmetric deflection during both upstroke and downstroke. At the beginning of a stroke, the flapper is almost horizontal, whereas the largest deflection occurs at midstroke. Consequently, there is a phase offset of $\sim \pi/2$ between the imposed heaving motion and the passive deflection of the flapper. Such a phase offset is characteristic of oscillating foils with low mass ratios and is linked to an increase of the fluid forces during the midstroke, which dominate over inertia for low mass ratios [61,98]. Indeed, Yeh and Alexeev [99] found a similar phase offset for a flexible self-propelling plate of finite span when its plunging frequency was adjusted to yield maximum propulsive speed.

For the tandem system, we found that the system achieved stable equilibrium configurations, which we characterized in terms of their mean propulsive speed ($\bar{U}_p$) and the averaged horizontal separation between leader and follower, $\bar{D}$. Flow visualizations for two representative tandem configurations reveal distinct interaction regimes. In Fig. 13(a) ($H = 0.6c$, $\phi = 180^\circ$), the follower swims at a moderate horizontal gap behind the leader, interacting with the leader's wake vortices. The leader's flow field remains largely unaffected by the follower, while the follower traverses the leader's vortex rings, resulting in a modified wake structure downstream. In contrast, Fig. 13(b) ($H = 0$, $\phi = 0^\circ$) shows a compact configuration where the plates are nearly touching, and their wakes merge, resembling a single larger flapper. These two cases exemplify the regular and compact configurations, respectively, analogous to the 3D counterparts of similar patterns reported for 2D tandem plates [88]. Interestingly, regular configurations result in propulsive speeds that are roughly equal to those of isolated flappers, while some of the compact configurations result in marginally higher (up to 2%) propulsive speeds (see Ref. [84] for more details).

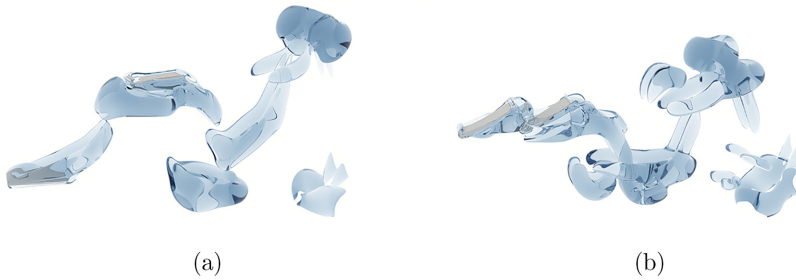


FIG. 13. Flow visualization around the flappers in tandem formation. Flow is visualized in terms of isosurfaces of $Q/f^2 = 0.5$. (a) $H = 0.6C$, $\phi = 180^\circ$ and (b) $H = 0$, $\phi = 0^\circ$. Adapted from Arranz *et al.* [84].

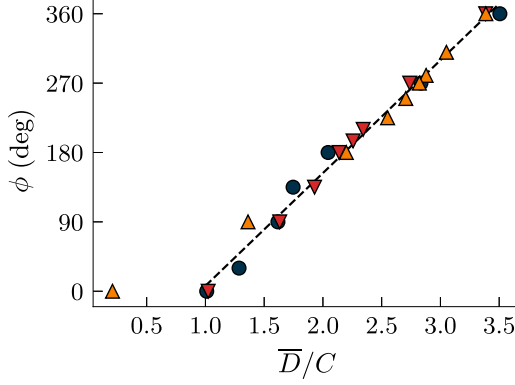


FIG. 14. Phase angle ϕ vs equilibrium distance $\bar{D}$. Symbols correspond to the vertical offset: (●) $H/C = 0$; (▼) $H/C = 0.3$; and (▲) $H/C = 0.6$. Dashed black line is the linear regression. Adapted from Arranz *et al.* [84].

Figure 14 shows the mean equilibrium distance as a function of ϕ for all the cases considered. Since the data for all the vertical offsets are included in the figure, the data show that the main dependence of equilibrium distance is on the phase shift, with a linear relation between the two. There is a single data point that does not follow the overall trend. It corresponds to a case where the two flappers, instead of exhibiting the behavior shown in Fig. 13, swim in a vertically aligned configuration. The vertical offset does not seem to influence the equilibrium distance; however, as we will see below, it influences the flapper power performance.

The performance of each flapper is computed in terms of its average power consumption over a flapping cycle, $\bar{P}_i$; see Ref. [84] for specific details on how this quantity is computed. The power ratio of the i flapper is defined as $\Pi_i = \bar{P}_i/\bar{P}_s$, where $\bar{P}_s$ is the averaged power of the isolated flapper. When the power ratio is smaller than 1, the flapper saves energy compared to the isolated flapper. The performance of all the simulated configurations is summarized in Fig. 15, distinguishing the input power required by the leader and the follower as compared to that required by the isolated flapper. Dashed lines are used to link configurations with the same phase shift, ϕ . Figure 15(a) shows that the leader's required input power remains virtually unchanged from isolation for configurations where $\bar{D}/C \geq 1.25$, with variations within $\pm 1\%$ of the isolated flapper value. Only in compact configurations does the leader experience slightly higher power requirements than in isolation. Figure 15(b) highlights that the follower's power requirements are significantly affected by the

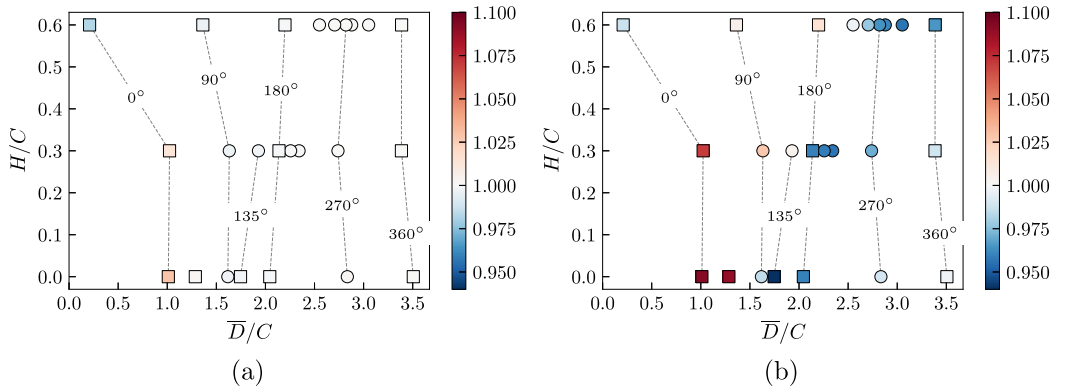


FIG. 15. (a) Input power ratio of the leader (Π_l), (b) input power ratio of the follower (Π_f). Symbols stand for the resolution of the simulation: □, $\Delta x = c/80$; ○, $\Delta x = c/50$. Adapted from Arranz *et al.* [84].

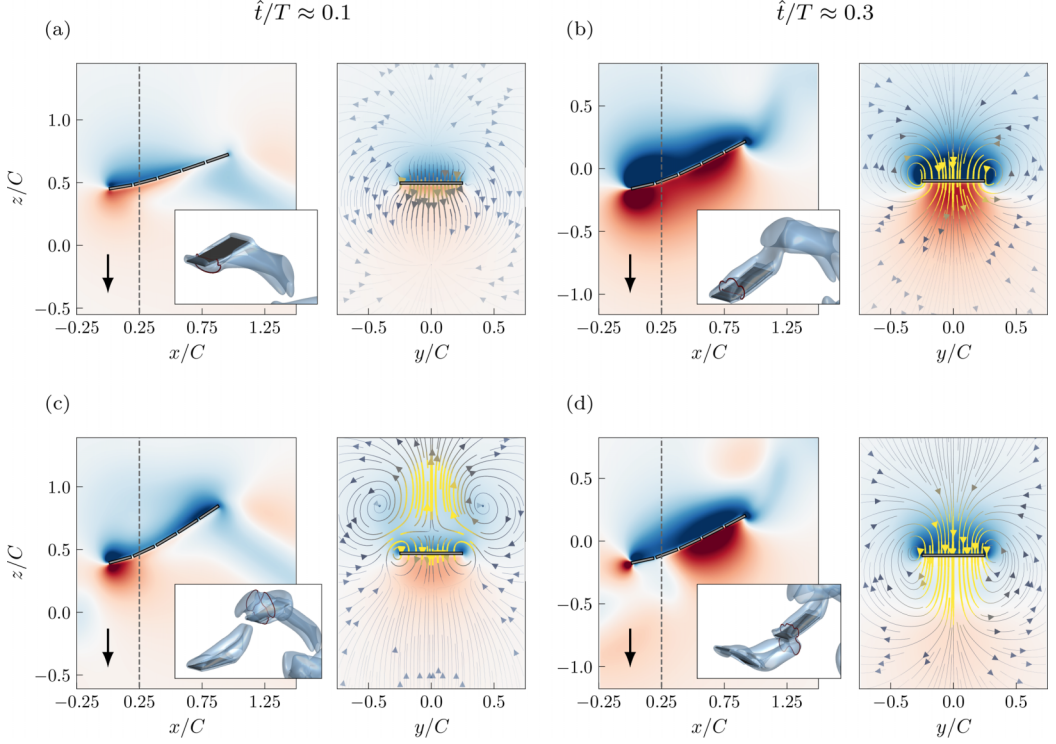


FIG. 16. Flow visualization of (a), (b) isolated flapper; (c), (d) tandem case with $H = 0$ and $\phi = 135^\circ$; the time instants in each panel are (a), (c) $\hat{t}/T \approx 0.1$ and (b), (d) $\hat{t}/T \approx 0.3$. For panels (a)–(d), the contour on the left corresponds to the pressure field at the midplane of the flappers. The black arrow indicates the direction of the flapper motion. Contour on the right displays the pressure in the transverse plane indicated with a dashed line in the left panel, and the instantaneous streamlines of the *in-plane* velocity. The streamlines are colored with the local velocity magnitude. An inset is added to each panel displaying the isosurfaces of $Q/f^2 = 0.5$ of the corresponding case. Adapted from Arranz *et al.* [84].

tandem arrangement, varying up to $\pm 10\%$ depending on ϕ and H . For certain phase offsets, the follower can extract energy from the leader's wake, reducing its required power below that of an isolated flapper. The transition to beneficial interaction occurs roughly at $\phi > 30^\circ$, 135° , and 180° for $H = 0$, $0.3c$, and $0.6c$, respectively.

Based on the comparative analysis of the required input power as a function of time (not shown here), it was observed that, irrespective of the final power ratio, the follower always requires more power than the isolated flapper during the first half of each stroke (i.e., downstroke and upstroke), and less during the second half of each stroke. Detailed flow field analyses at specific instants during the flapping cycle clarify the mechanisms underlying performance differences. This analysis is illustrated in Fig. 16, comparing the case of the isolated flapper to the optimal case ($\phi = 135^\circ$, $H = 0$) at two instants during the downstroke; see also animations provided in the Supplemental Material of Ref. [84].

Figure 16(c) depict the pressure field and the velocity field near the follower at the beginning of the downstroke ($\hat{t}/T \approx 0.1$). The corresponding visualization for the isolated flapper is shown in Fig. 16(a). At this time instant, the instantaneous required power of the follower exceeds that of the isolated flapper. The follower interacts with the vortex ring (VR) shed during the leader's upstroke, which induces an upwards velocity jet. Due to the phase offset, the VR is located above the follower and it pushes fluid against the flapper motion. This results in a flow pattern with a saddle point on

the suction side of the follower. Note that this saddle point does not occur in the case of the isolated flapper [Fig. 16(a)].

Figures 16(b) and 16(d) display the flow after the mid-downstroke ($\hat{t}/T \approx 0.3$), when the follower requires less power than the isolated flapper. Figure 16(d) shows that the VR shed during the leader's upstroke has traveled downstream, while the VR shed during its downstroke (with opposite circulation) starts interacting with the follower. Since both the follower's wingtip vortices and the VR have the same circulation, they seem to merge near the leading edge, and no saddle point is observed. This yields a downwards, high-velocity jet, which decreases the pressure on the follower's lower surface, thus explaining the lower power required by the follower compared to the isolated flapper. In summary, in the regular configurations, beneficial interactions occur when the follower moves in the same direction as the jet induced by the vortex rings, while detrimental interactions arise when the jet opposes the follower's motion. In compact configurations, the interaction is less favorable, with the follower often encountering opposing flow (not shown here), increasing its energy expenditure.

Based on the analysis of Fig. 16, it was possible to develop a predictive framework based on the flow field of an isolated flapper, offering a route to estimate follower performance in tandem without full tandem simulations. The details of that analysis are not included here for the sake of brevity, and they can be found in Ref. [84].

The findings reviewed in this section illustrate a general principle: flow-mediated interactions can spontaneously organize flapping bodies into coordinated, energy-efficient configurations, even without direct mechanical coupling. The emergence of equilibrium spacing highlights the potential for passive hydrodynamic synchronization in collective propulsion, a mechanism that could be exploited in the design of autonomous bioinspired systems.

VI. CONCLUSIONS

This review examined our recent work on hydrodynamic interactions in tandem flapping wing systems. Beginning with the transition from 2D to 3D aerodynamics in rigid wings with prescribed motion, we showed how three-dimensional effects alter vortical structures and force generation mechanisms compared to their two-dimensional counterparts. Rigid wing configurations reveal significant performance penalties in three-dimensional flow, with thrust reductions of 20–30 % for hindwings due to wingtip vortices and vortex breakdown phenomena. However, aspect ratio selection can mitigate these losses, with mixed aspect ratio systems showing superior performance by preserving quasi-2D vortical structures over the hindwing's span.

We next examined spanwise-flexible wings in tandem configurations, highlighting the complex interplay between structural dynamics and hydrodynamic interactions. For isolated flexible wings, optimal performance typically occurs near fluid-structure resonance, when the flapping frequency approaches the wing's natural frequency in the fluid. In tandem arrangements, however, forewings and hindwings display distinct optimal flexibility characteristics: forewings perform best at resonance, while hindwings achieve peak performance at slightly higher stiffness values. Notably, mixed-flexibility configurations, featuring resonant forewings paired with moderately stiff hindwings, yield enhanced overall thrust compared to systems with uniform flexibility.

Finally, we examined emergent hydrodynamic coordination in self-propelled chordwise-flexible flappers, where dynamic positioning through flow-mediated interactions enables the study of equilibrium behavior and energy savings. The phase relationship between tandem flappers determines whether interactions are beneficial or detrimental. Followers can achieve significant energy savings of up to 10% by exploiting the leader's wake structures, with the transition to beneficial interaction occurring at specific phase offsets depending on vertical positioning.

These findings reveal that optimal tandem performance requires a subtle balance between vortex interactions and structural dynamics, providing fundamental insights for the design of bioinspired multiwing propulsion systems.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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