

Polaritonic machine learning for graph-based data analysis

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(Received 26 September 2025; accepted 5 August 2026; published 26 August 2026)

Photonic and polaritonic systems offer a fast and efficient platform for accelerating machine learning (ML) through physics-based computing. To gain a computational advantage, however, polaritonic systems must meet the following: (1) exploit features that specifically favor nonlinear optical processing; (2) address problems that are computationally hard and depend on these features; and (3) integrate photonic processing within broader ML pipelines. In this work, we propose a polaritonic machine learning approach for solving graph-based data problems. We demonstrate how lattices of condensates can efficiently embed relational and topological information from point cloud datasets. This information is then incorporated into a pattern recognition workflow based on convolutional neural networks (CNNs), leading to significantly improved learning performance compared to physics-agnostic methods. Our extensive benchmarking shows that photonic machine learning achieves over 90% accuracy for Betti number classification and clique detection tasks, a substantial improvement over the 35% accuracy of bare CNNs. Our study introduces a distinct way of using photonic systems as fast tools for feature engineering, while building on top of high-performing digital machine learning.

DOI: [10.1103/fgy1-n4vp](https://doi.org/10.1103/fgy1-n4vp)

I. INTRODUCTION

Machine learning (ML) is a powerful tool for data processing tasks [1], ranging from natural language processing [2,3] to scientific discovery [4–6]. One part of ML's success comes from the development of models that can capture relevant features for different data types [6,7]. This corresponds to extracting patterns from images with convolutional neural networks (CNNs) [8–10], link predictions for molecular graphs with geometric machine learning [11,12], and tokenized amino acid sequences processed by transformers [4]. Another part of its success comes from the growth of computing power [e.g., graphical processing units (GPUs)], which allows running deep ML architectures with billions of weights [13]. At the same time, solutions based on conventional electronics scale extensively, leading to rapidly increasing energy consumption [14]. Proposing nature-inspired and physics-based solutions is considered a way forward for building ML pipelines in the future [15–19].

Physical systems based on optics and photonics emerged recently as promising platforms for ML, thanks to their favorable features that include large bandwidth and low-loss operation [20]. Based on propagating light, these systems can address various ML tasks bypassing conventional electronics [21,22]. This includes performing classification with diffraction neural networks [23], gaining a huge increase in convolutional capacities with frequency combs [24], and

insitu backpropagation [25,26]. Optical ML-based sensing capabilities were demonstrated in Ref. [27], achieving high accuracy and subphoton energy per operation [28]. Furthermore, optical neural networks were used for generative modeling [29–31]. As ML models crucially require nonlinearity, linear optical neural networks often incorporate it by analog-to-digital conversion or intensity saturation [27], showing significant improvement over bare linear performance. Hence, designing physically nonlinear devices for processing shall largely increase ML capabilities.

Nonlinear optical effects depend on coupling between light and matter, and are greatly enhanced when cavity-confined photons hybridize with excitons [32–34]. The resulting quasiparticles (exciton-polaritons) exhibit strong nonlinearity per unit volume [35,36], emerging from exciton-exciton scattering and nonlinear phase space filling [37–42]. The nonlinear response can be enhanced in the presence of dipolar interactions [43–47], Rydberg states [48–50], doping-related effects [51–54], moiré patterns for bilayers of two-dimensional (2D) materials [40,55–59], and biexcitonic resonances [60,61]. A distinct feature of polaritonic systems is their ability to form condensates as macroscopically coherent states [62–65], leading to many-body excitations in the form of quantized vortices [66–73], phase defects, and solitons [74–77]. These features of fluids of light [34] can also be paired with rich physics of polaritonic lattices created via patterning [78–85] and optical pumping [86–90]. These capabilities suggest promise for polaritonic ML applications.

To date, polaritonic approaches to ML concern models designed to mimic neural network-type structures, Ising-type optimizers, and reservoir computing (RC) devices [91]. Proposals include polaritonic RC for speech recognition [92], polariton neural networks for MNIST image classification

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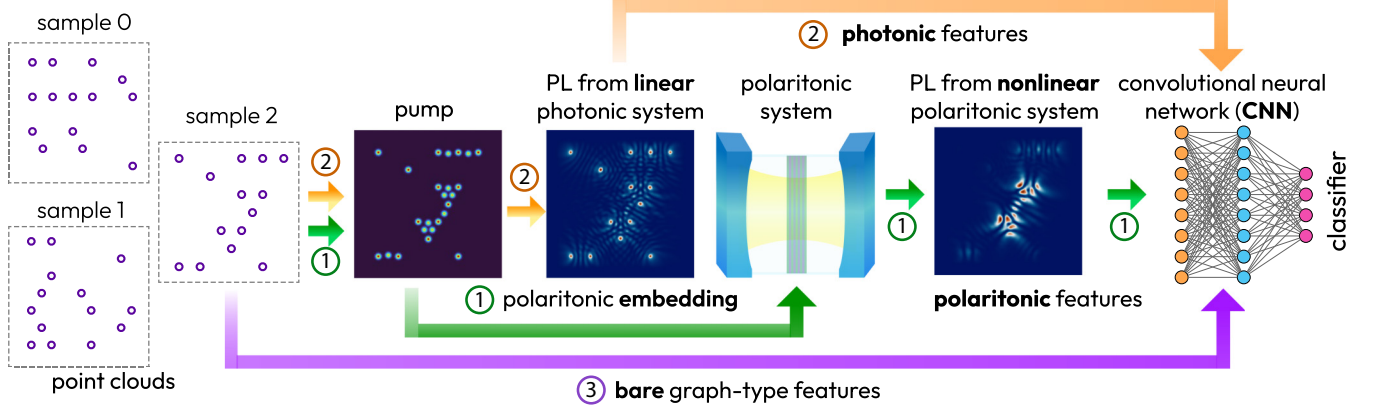


FIG. 1. Workflow for point cloud classification based on CNNs. Datasets are processed in three distinct ways, using bare graph-type images, PL of a linear photonic system, and PL of a nonlinear polaritonic system. Green arrow ①: point clouds are incorporated into a nonresonant pump profile featuring both source and barrier spots, triggering polariton condensation with nonlinear features. Corresponding PL is used in the CNN-based classification. Orange arrow ②: data embedded into a potential profile of linear photonic systems, with PL (light intensity) used as an input for CNNs. Purple arrow ③: bare point clouds are directly used as input for the CNN workflow.

[93] with RC-type offline learning, binarized and time-delay polariton networks for learning optical logical gates [94,95] and image recognition [96], and recently, a full polaritonic four-neuron network for pattern recognition [97]. The energy efficiency of polaritonic ML was quantified in Ref. [98]. In a parallel track, polaritonic lattices were suggested as effective quantum reservoirs for quantum state discrimination [99–102]; something that becomes within reach for quantum polaritonics [103–105]. The proposed approaches show promise, yet still operate at a small scale as compared to current deep learning.

In this work, we propose a polariton-based ML evolved in a distinct direction. First, instead of trying to reproduce deep learning with nonlinear photonics, we use polaritonic systems for feature engineering, while offloading postprocessing to known classical ML architectures (Fig. 1). Unlike reservoir computing approaches that exploit nonlinear dynamics with linear readouts [91–93], our method uses condensate physics where quantized vortices and coherent superfluid flow patterns emerge from nonlinear interactions to encode graph-structural information into photoluminescence (PL) images. This is similar to positional embeddings, proven to be state of the art [2,106,107]. Second, we choose a task that naturally fits polaritonic lattices yet remain unexplored, corresponding to the analysis of graph-type data and point clouds. Here, polaritonic systems can act as physical accelerators used for highlighting certain features that originate from topology in system’s response as well as the topology of data [108]. We show that by introducing nonlinear features from polariton condensation, CNN-based classification can significantly improve on purely graph-based analysis.

II. MODEL

A. Point cloud and graph analysis

We start by describing the data preparation and specifying the learning tasks. Consider a point cloud dataset $\mathcal{D}$ consisting

of n points with coordinates $\mathcal{D} = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ specified in Euclidean space $\mathbb{R}^d$. We aim to learn its topological properties [109–112], which can be difficult to access due to the combinatorial complexity for this task [113,114]. Inspired by the manifold reconstruction problem [115,116] and the simplicial finite element meshes in computational geometry [117,118], we start by generating a regular (equilateral) triangle mesh of the d -dimensional Euclidean space on which the dataset is defined, i.e., a discrete local approximation of the continuous space $\mathbb{R}^d$. We construct a triangular mesh with the edges of length equal to the average distance between data points. This choice, defining a trade-off between information loss due to space discretization and computational resources needed to run simulations, falls within the broader framework of mesh refinement [119]. We then map points of $\mathcal{D}$ to the closest vertices of the mesh, generating an approximated version of the dataset referred to as $\tilde{\mathcal{D}}$ with new coordinates $\tilde{\mathcal{D}} = \{\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, \dots, \tilde{\mathbf{x}}_n\}$ [120]. The approximated dataset can be associated with Čech simplicial complex [108] whose set of vertices (zero-simplices) corresponds to the approximated coordinates and whose edges exist between vertices that are nearest neighbors on the underlying mesh. The resulting graphs are simple by construction: each data point maps to a unique mesh vertex (precluding self-loops), and the nearest-neighbor adjacency relation is binary (precluding multiple edges between the same pair of vertices). The complex is built relative to a filtration distance ϵ where the equilateral triangles in the mesh have edges of length 2ϵ [114]. Thus, the study of the topological properties of the original dataset $\mathcal{D}$ translates into the evaluation of k th Betti numbers β_k (i.e., the “ k th order topological holes”) in the approximated dataset $\tilde{\mathcal{D}}$. In the following, we restrict the point cloud dimensionality to $d = 2$, matching the pump profile in our simulations. The resulting mesh construction is a triangular lattice whose constant is the average spacing between data points of $\mathcal{D}$.

First, we consider a problem related to extraction of the first-order Betti number β_1 from simplicial complexes associated with the approximated dataset, that is, its 2D holes.

Note that in the regular triangle mesh described before, the evaluation of β_1 equates to solving the clique counting problem [121], where the cliques are defined as subsets of vertices that are fully connected. In this work, we exclusively consider three-cliques (triangles), i.e., sets of three mutually adjacent vertices. We note that decision problems based on the number of cliques are NP-complete [122], and their counting is in general #P-complete. In our case, this is framed as a classification problem: distinguishing point clouds containing zero, one, or two triangles (three-class task) or zero versus one triangle (two-class task). Second, going beyond Betti number classification, we introduce an asymmetry detection task to evaluate the ability to capture subtle structural differences (see Appendix B for an extended discussion).

B. CNNs for geometric data

Convolutional neural networks, originally designed for grid-structured data such as images, can be adapted to process point clouds by stacking convolutional layers with learnable filters [1,8,123]. These filters capture spatial hierarchies of features, enabling CNNs to recognize fundamental patterns such as edges, shapes, and textures. We use a CNN-based model with two convolutional layers (detailed in Appendix C), each followed by batch normalization [124] and max pooling operations. The depth is chosen such that for the limited sample size ($|\mathcal{D}| \leq 1280$ for two classes) CNNs do not overfit or suffer from training difficulties. The network processes 256×256 grayscale images using 3×3 kernels, rectified linear unit (ReLU) activations, and global average pooling [125], resulting in 83 331 trainable parameters. The full ML-based workflow for classification is shown in Fig. 1, visualizing the analysis with different inputs.

C. Polaritonic lattices

We proceed to model nonequilibrium polariton condensates, forming a lattice-type structure. Once propagated in time, the resulting PL patterns can be used similarly to positional embeddings in the ML workflow. The modeling can be performed by solving a generalized Gross-Pitaevskii equation (GPE), coupled to the exciton reservoir rate equation [34]. The nonlinear GPE reads [126]

$$i\hbar \frac{\partial}{\partial t} \Psi = \left\{ -\frac{\hbar^2}{2m} \nabla^2 + \alpha |\Psi|^2 + G \left[\mathcal{N} + \frac{\eta}{\Gamma} P(\mathbf{r}) \right] + i\frac{\hbar}{2} [R\mathcal{N} - \gamma] \right\} \Psi, \quad (1)$$

$$\frac{\partial}{\partial t} \mathcal{N} = -[\Gamma + R|\Psi|^2]\mathcal{N} + P(\mathbf{r}), \quad (2)$$

where m is an effective polariton mass (lower branch) and α and G denote the interaction strengths of the polariton-polariton and polariton-reservoir, respectively. For the reservoir described by the number of excitons $\mathcal{N}(\mathbf{r}, t)$, pumped continuously at rate $P(\mathbf{r})$, we introduce a scattering rate R coupling reservoir and condensates. Here, η is the ratio of dark excitons, and γ (Γ) is the decay rate of the polaritons (reservoir). While various polaritonic systems can be used,

we consider a planar stack of InGaAs quantum wells with negative detuning [88].

The dataset is embedded via pump configurations specified as Gaussian spots following point clouds, $P(\mathbf{r}) = p_s \sum_i G(\mathbf{r} - \mathbf{r}_i) + p_b \sum_j G(\mathbf{r} - \mathbf{r}_j)$, where the sums run over source spot positions $\{\mathbf{r}_i\}$ and barrier spot positions $\{\mathbf{r}_j\}$, with the respective pump rates p_s and p_b . The source spots are pumped above threshold to trigger condensation, while the barrier spots operate below threshold to create potential landscapes without forming independent condensates. Each spot follows a Gaussian distribution, $G(\mathbf{r}) = (2\pi\sigma^2)^{-1} \exp[-|\mathbf{r} - \mathbf{r}'|^2/(2\sigma^2)]$. For those data spots non-resonantly excited above threshold, the coupling between spots introduces nonlinear features that depend on the cloud geometry. We note that possible solutions include topologically nontrivial excitations like quantized vortices [66] and solitons. Here, the vortices correspond to integer phase winding numbers emerging from quantum flow circulation [34] and highlighting relevant topological information. The barrier spots appear in two configurations (see Fig. 3): the inner barriers within kagome triangles (4 μm center-to-edge spacing) and the outer barriers in a hexagonal arrangement at half the lattice constant from nearest source spots. These barriers facilitate vortex creation and contribute to interference patterns but do not modify the underlying graph structure because graph adjacency is determined solely by mesh topology before physical embedding. All details for the data embedding procedure and parameters are detailed in Appendix A.

Once the generalized GPE is solved (propagated until time $t_f \gg 1/\gamma$), the resulting PL profile $|\Psi(\mathbf{r}, t_f)|$ is used as an input for CNN. Other features include the phase profile, $\arg[\Psi(\mathbf{r}, t_f)]$. In Fig. 2, we show simulations for different pump profiles (and point clouds) with the corresponding PL and phase profiles.

D. Linear photonics

Together with nonlinear (polaritonic) feature embedding, we provide a recipe for purely photonic feature engineering, also serving as a reference point. Here, relevant features also appear within interference patterns and can yield the learning procedure (see Fig. 1). In this case, the underlying GPE reads

$$i\hbar \frac{\partial}{\partial t} \Psi = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) - i\frac{\hbar}{2} \gamma \right] \Psi + \frac{1}{L} \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} E(\mathbf{r}, \mathbf{r}_{0,n}) e^{-i[\omega t - \mathbf{k}_l(\mathbf{r} - \mathbf{r}_{0,n})]}, \quad (3)$$

where V is a real potential of barrier spots, N is the number of activated source spots, and the other parameters remain the same as in Eq. (1). Since we do not have condensates in the linear case (exciton-exciton interaction is absent), the data embedding is modified and uses a set of resonant pumps. Specifically, E is the profile map shaped into different normalized Gaussian spots at position $\mathbf{r}_{0,n}$ with the same full width at half maximum (FWHM) as the nonresonant pump profile in the polaritonic case; ω represents the energy of the driving source; $\mathbf{k}_l = (2m\omega/\hbar)^{1/2} [\cos(2\pi l/L)\hat{x} + \sin(2\pi l/L)\hat{y}]$ is the resonant wave vector pointing at different directions

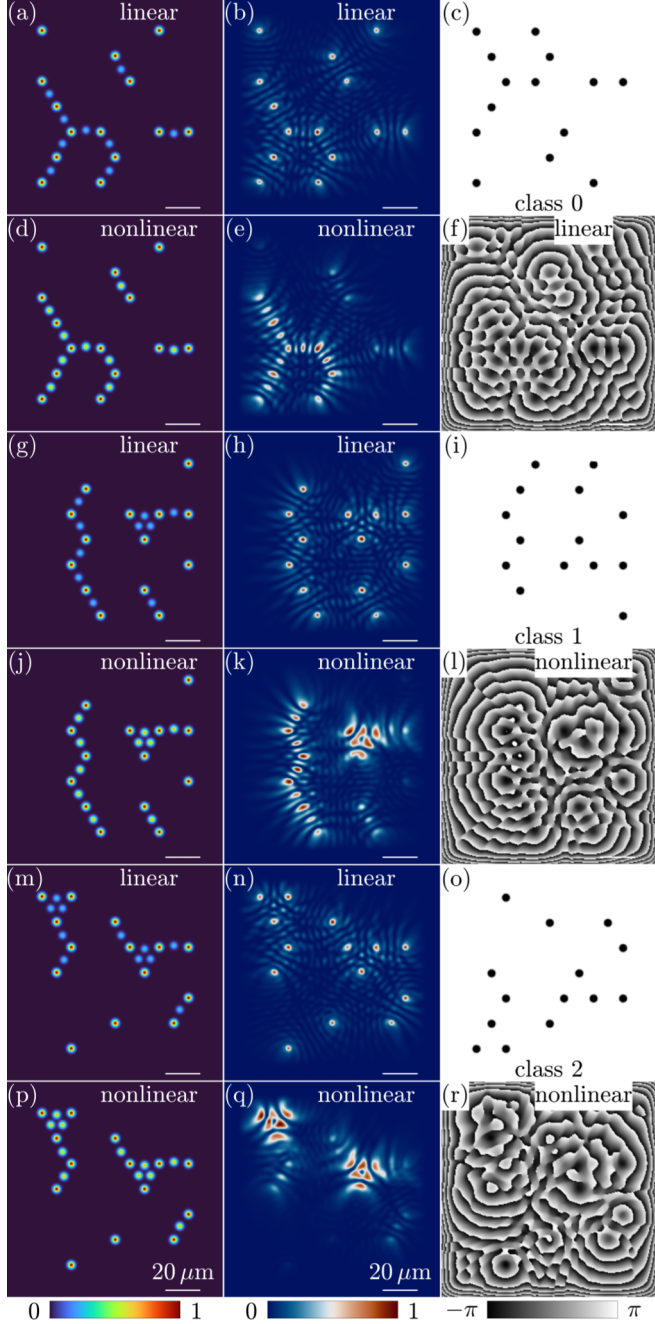


FIG. 2. Examples of pump profiles for (a), (g), (m) linear and (d), (j), (p) nonlinear photonic embedding of point clouds shown in (c), (i), (o). PL profiles are shown in (b), (e), (h), (k), (n), (q), and the corresponding phase profiles are presented in (f), (l), (r) for nonlinear operation. Topological features of polaritonic flow can be clearly observed in (k) and (q).

defined by angles $2\pi/L$ for an integer L . Here, $(\hat{x}, \hat{y})$ refers to the 2D unit of $\mathbf{r}$. We consider two possible driving configurations. The first corresponds to a multidirectional resonant driving ($L = 365$), closer to condensate-based processing from the feature engineering perspective, while more challenging experimentally (many driving lasers are needed). The second configuration corresponds to unidirectional driving ($L = 1$) that is easier to implement in practice, but requires

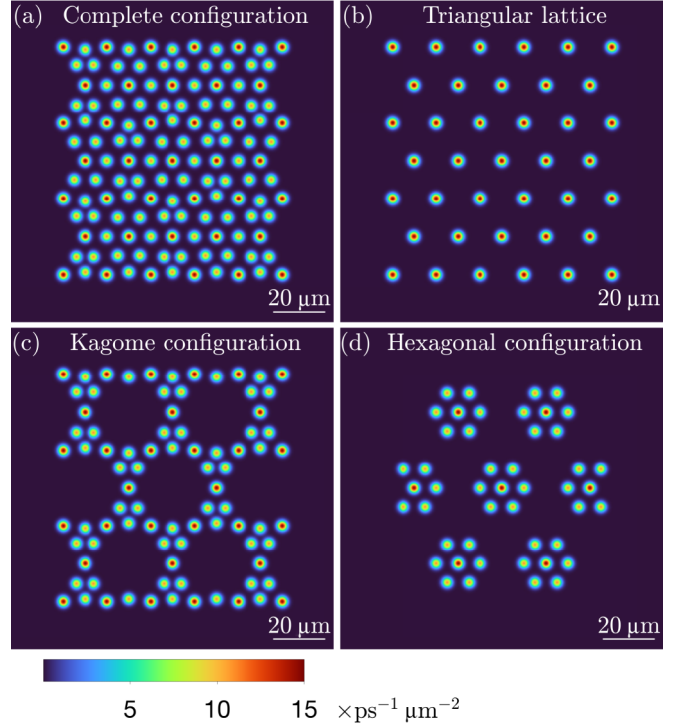


FIG. 3. (a) Complete pump configuration is produced with (b) triangular lattice made of only source spots and the rest barrier spots. This configuration can also be formed by combining a (c) kagome pattern with additional (d) hexagonal configuration.

breaking the concentric symmetry. Simulation parameters for the linear photonic model are in Appendix A.

III. RESULTS

We apply the proposed polariton-enriched CNNs for classification tasks based on clique detection and asymmetry detection. For clique detection, we test both binary (two-class) and three-class classification settings, with configurations containing zero to one triangle and zero to two triangles, respectively. For the clique detection and asymmetry detection tasks, we use 640 and 500 samples per class, respectively. In each case, the results are averaged over ten independent runs using different random seeds. The dataset is split into training, validation, and testing subsets with ratios of 70%, 15%, and 15%, respectively. Stratified sampling is used to maintain class balance. Our results are summarized in Table I.

For the two-class clique detection task, polaritonic PL provided the highest test accuracy of $91.51\% \pm 2.97\%$, substantially outperforming linear photonic systems ($L = 1$: $80.73\% \pm 13.95\%$; $L = 365$: $75.00\% \pm 7.18\%$) and pure graph images ($52.08\% \pm 3.14\%$). This boost in performance can be traced to the formation of vortices in polaritonic systems, where topological defects appear when pump spots form a clique-shaped configuration (e.g., triangle). These vortices, generated by the nonlinear interaction term in the generalized GPE, serve as a signature of clique presence. Importantly, vortex formation requires activated

TABLE I. Mean and standard deviation of test accuracy for several classification tasks and different data processing.

No. of classes (task)	No. of samples	No. of parameters	PL (polaritonics)	PL (photonics, $L = 1$)	PL (photonics, $L = 365$)	Graph images
2 (clique det.)	640	83 331	0.9151 ± 0.0297	0.8073 ± 0.1395	0.7500 ± 0.0718	0.5208 ± 0.0314
3 (clique det.)	640	83 331	0.8931 ± 0.0167	0.9330 ± 0.0255	0.9097 ± 0.0359	0.3507 ± 0.0211
2 (asym. det.)	500	83 331	0.8313 ± 0.0288	0.7053 ± 0.0971	0.9247 ± 0.0888	0.6893 ± 0.0411

source spots to lie within the kagome lattice region where the source-barrier configuration supports flow circulation. The triangles formed outside this region, or spanning disconnected lattice parts, do not produce vortices despite constituting three-cliques in the graph. The polaritonic system thus provides a physically filtered representation: PL patterns encode both clique presence and spatial embedding within the lattice geometry. For the asymmetry detection task, where all configurations lie within the proper kagome region, vortex count directly corresponds to triangle count. The coherent superfluid flow between condensate spots further creates distinct wake patterns that encode graph connectivity information into time-integrated PL images. In the three-class clique detection task, the performance hierarchy shifts: the linear photonic system with $L = 1$ achieves the best accuracy at $93.30\% \pm 2.55\%$, followed by the case with $L = 365$ ($90.97\% \pm 3.59\%$) and the polaritonic system ($89.31\% \pm 1.67\%$). This reversal indicates that when distinguishing between multiple clique configurations, the directional interference patterns generated by $L = 1$ pump configuration provide features that are easier to digest. The asymmetric intensity distribution effectively encodes the number and arrangement of triangles, while the isotropic $L = 365$ case generates concentric interference. All photonic configurations largely outperform the baseline of point cloud images ($35.07\% \pm 2.11$).

For the binary asymmetry detection task, which requires distinguishing between regular and twisted triangular configurations (with one spot extended by $2.0 \mu\text{m}$), the photonic system with $L = 365$ achieves the highest accuracy at $92.47\% \pm 8.88\%$. The polaritonic system follows with $83.13\% \pm 2.88\%$, while the photonic system with $L = 1$ ($70.53\% \pm 9.71\%$) performs only marginally better than graph images ($68.93\% \pm 4.11\%$). The sensitivity of vortex formation to geometric symmetry explains the slightly lower performance of polaritonic embedding, where the twisted configuration disrupts the vortex formation in the kagome geometry. Isotropic interference patterns of the $L = 365$ system are most effective at capturing subtle geometric asymmetries. This task was intentionally included to probe the limitations of different embedding methods, demonstrating that no single approach universally dominates across all classification tasks.

IV. DISCUSSION

The results of the photonics-based ML demonstrate that incorporating physical features significantly enhances CNN performance for graph-based tasks compared to using raw graph representations. The evolution of nonlinear photonic systems and their PL profiles can be seen as an effective feature engineering for graphs embedded as pump profiles. In

polaritonic systems with nontrivial many-body dynamics, the generation of quantized vortices emerges as the key nonlinear feature, highlighting cliques and topological structures. At the same time, we note that features arising from interference (also present in linear photonic systems) can be suitable for some classification tasks.

We note that both two-class clique detection and asymmetry tasks are characterized by a large standard deviation (e.g., $\pm 9.93\%$ for photonics). This is due to the limited number of training samples (640 per class) combined with high sample diversity. However, polaritonic systems show remarkably low deviation for the two-class task ($\pm 2.97\%$), three-class task ($\pm 1.67\%$), and asymmetry task ($\pm 2.88\%$) indicating that nonlinear features are reliable when the task complexity increases. This consistency across ten random seeds which account for combined effects of dataset sampling, training and testing splits, and network initialization provides evidence of robustness. For the asymmetry detection, the $L = 365$ photonic system outperforms polaritonics (92.47% versus 83.13%), though with a larger standard deviation ($\pm 8.88\%$ versus $\pm 2.88\%$). Here, the twisted configuration disrupts vortex formation, while isotropic interference patterns of $L = 365$ capture this geometric asymmetry more effectively. The poor performance of $L = 1$ photonics (70.53%) for this task, comparable to graph images (68.93%), indicates that unidirectional flow provides insufficient information for detecting subtle geometric distortions.

Experimental implementation will require attention to positioning tolerances and intensity stability. However, stable polariton condensate lattices have been demonstrated using spatial light modulators with sufficient precision for lattice formation [87–89,127], and the time-integrated detection scheme inherently averages out dynamic fluctuations faster than the integration timescale.

V. CONCLUSION

We demonstrated that polaritonic systems can be used for ML by performing photonic feature engineering based on their nonlinear and topological properties. We showed that lattices of condensates can encode relational and topological information from point cloud data in a physically meaningful way. When these photonic features were used as input to CNNs, the resulting models achieved around 90% accuracy in Betti number classification and clique detection tasks, compared to 35% accuracy from models without photonic preprocessing. These results show that nonlinear photonic systems can act as efficient and task-relevant feature processors for ML pipelines, enabling faster, energy efficient [98], and more accurate processing of graph-structured data.

ACKNOWLEDGMENT

The authors acknowledge the support from UK EPSRC Grant No. EP/X017222/1.

DATA AVAILABILITY

The data and code that support the findings of this study are openly available in Zenodo [128].

APPENDIX A: SIMULATION PARAMETERS AND PUMP CONFIGURATION

We consider a planar stack of InGaAs quantum wells with slightly negatively detuned cavities [88]. The detuning between the exciton and photon modes controls the interaction terms as $\alpha = g|\chi|^4$ and $G = 2g|\chi|^2$, with $g = g_0/N$, where g_0 is the exciton-exciton interaction strength and N_{QW} is the number of quantum wells (QWs). The term $|\chi|^2$, representing the fraction of exciton in the polariton, is defined by the Hopfield coefficient [130] for the excitonic branch.

The parameters chosen to emulate the experimental observations of polariton condensates are $m = 0.28 \text{ meV ps}^2 \mu\text{m}^{-2}$, $|\chi|^2 = 0.4$, $N_{\text{QW}} = 6$, $g_0 = 0.01 \text{ meV } \mu\text{m}^2$, $\hbar R = 10g$, $\eta = 2$, and $\gamma^{-1} = \Gamma^{-1} = 5.5 \text{ ps}$. The power density of the source and barrier for all simulations of polariton condensation is $p_s = 15 \text{ ps}^{-1} \mu\text{m}^{-2}$ and $p_b = 11 \text{ ps}^{-1} \mu\text{m}^{-2}$, respectively. For simulation of the linear photonic system, $\hbar\omega = 0.6 \text{ meV}$, and the highest intensity of the real-valued Gaussian-distributed potential $V(\mathbf{r})$ is fixed at 0.3 meV .

For pump configuration, $\sigma \approx 1.274 \mu\text{m}$, which corresponds to the FWHM of each Gaussian spot being $3.0 \mu\text{m}$ is used for all the Gaussian spots in this paper. As shown in Fig. 3, the geometric parameters are as follows. For the kagome configuration, the center-to-edge spacing within the equilateral triangles is $10 \mu\text{m}$ for the source spots and $4 \mu\text{m}$ for the inner barrier spots. For the hexagonal units, the distance between a source spot and its nearest barrier spots is half of the triangular lattice constant, with the lattice constant itself being approximately $17.3 \mu\text{m}$. The complete pump configuration thus combines two barrier types: (i) inner barriers within kagome triangles that facilitate vortex formation at clique locations, and (ii) outer barriers in a hexagonal arrangement that contribute to interference patterns between triangular units.

The PL images for nonlinear polaritonic system and the phase images are obtained, respectively, from the time-integrated $|\Psi|^2$ and the phase of the final-time Ψ through numerical solutions of the generalized GPE coupled with reservoir rate equation [see Eqs. (1) and (2) of the main text]. PL maps for linear photonic system are obtained from the final-time solution $|\Psi|^2$ of Eq. (3) from the main text. The simulations are performed using internally prepared GPU-executed code for propagating generalized GPEs.

APPENDIX B: DATASET GENERATION AND TASK DEFINITIONS

1. Graph construction

As described in the main text, the edges in the graph connect vertices that are nearest neighbors on the triangular mesh.

This nearest-neighbor rule on the mesh topology is distinct from distance-threshold methods (which connect any sites within a fixed radius) or Vietoris-Rips constructions (which build simplices from pairwise distances).

The pump configuration (Fig. 3) encodes graph information through source and barrier spots, but the graph adjacency is determined solely by the mesh topology. Multiple optical pathways exist in the physical system through direct coupling between source spots and interference mediated by both inner and outer barrier spots, yet these pathways contribute to feature-rich PL patterns without modifying the underlying graph structure. Consequently, no edge-collapsing procedure is required when processing the physical system's output.

2. Dataset statistics and image generation

The ML results presented in Table I from the main text are based on datasets randomly sampled from 14 of the 39 available source points within the lattice. Each dataset per category comprises $640 \times 3 = 1920$ images with dimensions of 256×256 pixels per image, where 640 is the total number of images per category per class and 3 represents the total number of classes. All images used for training are in single-channel grayscale format, including point cloud images, phase maps, linear photonic PL, and nonlinear polaritonic time-integrated PL.

The specific configuration (14 out of 39 sites, fixed lattice geometry) is deliberately chosen to match experimental realizability. Polariton condensate lattices created via continuous-wave all-optical pumping typically operate with tens of sites due to practical constraints including defects in semiconductor samples and laser power limitations. This configuration is thus both computationally interesting and experimentally accessible.

3. Clique detection tasks

Class labels are determined by counting three-cliques (triangles) formed when three activated source spots occupy mutually adjacent mesh vertices. For the two-class clique detection task, class 0 contains configurations with zero cliques and class 1 contains configurations with one or more cliques. For the three-class task, classes correspond to zero, one, or two cliques, respectively.

The raw dataset generation follows a two-stage procedure that avoids selection bias. First, a large pool of samples is generated by uniformly random activation of 14 out of 39 available lattice sites, performed independently of any graph property, with the duplicated activation sets removed. Second, the number of triangles in each sample is counted by enumerating all combinations of three activated sites and checking mutual adjacency on the mesh; the total count determines the class label. Samples are then grouped by triangle count and subsampled to equal class sizes (640 per class), while samples containing three or more triangles are excluded due to insufficient sample size for balanced training. Since no rejection or regeneration is conditioned on the latent graph structure, no selection bias is introduced, and stratified training, validation, and testing splits are performed after labeling.

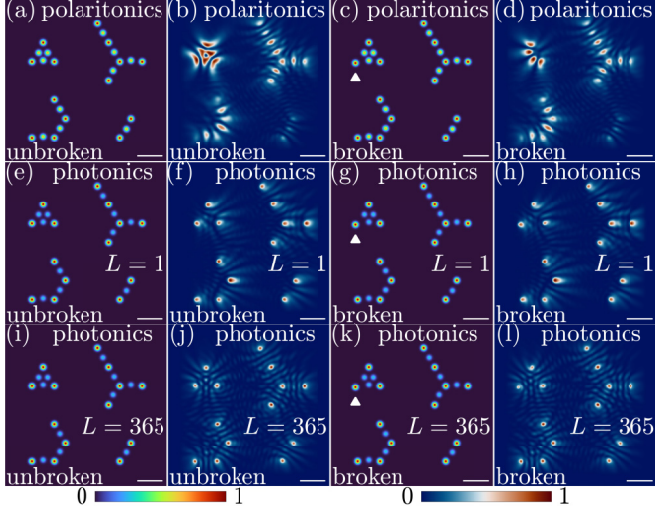


FIG. 4. Comparison of PL images for (a), (e), (i) unbroken and (c), (g), (k) broken pumping configurations in the (b), (d) nonlinear polaritonic system, (f), (h) linear photonic system with $L = 1$, and (j), (l) linear photonic system with $L = 365$. White triangle marks the twisted spot in the broken configuration. The spot is extended by $2.0 \mu\text{m}$ along its centroid line, resulting in a twisted configuration. White scale bar marks $20.0 \mu\text{m}$.

4. Asymmetry detection task

We introduce an asymmetry detection task to evaluate the ability to capture subtle structural differences. We create binary datasets where both classes contain triangular patterns in a kagome lattice configuration (Fig. 3).

The key distinction between the two classes is a geometric transformation: one class maintains regular triangular spot configurations, while the second class features *twisted* triangular spots. This twist is implemented by extending one spot by $2.0 \mu\text{m}$ along its centroid line, creating a subtle structural modification that breaks the symmetry required for vortex formation (as illustrated in Fig. 4). This task is particularly challenging since the geometric difference between classes is minimal and requires learning fine-grained geometric features with nearly degenerate configurations.

APPENDIX C: NEURAL NETWORK ARCHITECTURE AND TRAINING PROCEDURE

We use a CNN with two layers (detailed in the code listing), each followed by batch normalization [124] and max pooling operations. The architecture progressively increases feature maps from 64 to 128 channels while maintaining spatial resolution through padding. Each convolutional layer employs 3×3 kernels with stride 1 and padding 1, followed by ReLU activation functions. The network processes 256×256 grayscale images, with feature map dimensions reducing from 128×128 after the first pooling layer to 64×64 after the second. To address the parameter efficiency required for small datasets, we incorporate global average pooling [125] before the fully connected layers, reducing the total trainable parameters to 83 331.

```
class CNN(nn.Module):
    def __init__(self, num_classes=2, dropout_rate=0):
        super(CNN, self).__init__()

        self.conv1 = nn.Conv2d(1, 64,
                                kernel_size=3, stride=1,
                                padding=1)
        self.bn1 = nn.BatchNorm2d(64)

        self.conv2 = nn.Conv2d(64, 128,
                                kernel_size=3, stride=1,
                                padding=1)
        self.bn2 = nn.BatchNorm2d(128)
        self.max_pool = nn.MaxPool2d(
            kernel_size=2, stride=2)
        self.relu = nn.ReLU()
        self.dropout_conv = nn.
            Dropout2d(p=0)
        self.dropout_fc = nn.Dropout(p=
            dropout_rate)
        self.global_avg_pool = nn.
            AdaptiveAvgPool2d(1)

        self.fc1 = nn.Linear(128, 64)
        self.fc2 = nn.Linear(64,
                                num_classes)

    def forward(self, x):
        # Convolutional block 1
        x = self.conv1(x)
        x = self.bn1(x)
        x = self.relu(x)
        x = self.max_pool(x)
        x = self.dropout_conv(x)

        # Convolutional block 2
        x = self.conv2(x)
        x = self.bn2(x)
        x = self.relu(x)
        x = self.max_pool(x)
        x = self.dropout_conv(x)

        x = self.global_avg_pool(x)
        x = x.view(x.size(0), -1)

        x = self.relu(self.fc1(x))
        x = self.dropout_fc(x)
        x = self.fc2(x)
        return x
```

The listing above shows the exact code of the CNN architecture in PYTORCH.

All models are trained using the Adam optimizer [131] with a batch size of 32 and maximum of 1000 epochs. The learning rate serves as the primary hyperparameter tuned for each task based on validation performance. We employ a ReduceLROnPlateau learning rate scheduler with factor 0.8

and patience 20 epochs, and early stopping with patience of 200 epochs and minimum improvement threshold of 10^{-6} to terminate training when validation performance plateaus. The best model is selected based on validation accuracy. The dropout rate is set to 0 for all reported

results to ensure a fair comparison across embedding methods. Each configuration is evaluated across ten independent runs with different random seeds controlling network initialization and stratified training, validation, and testing splits (70%/15%/15%).

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