Kinetic and fluid descriptions of Jeans instability in kappa-distributed suprathermal astrophysical plasmas

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The Jeans instability in an unmagnetized, collisionless self-gravitating plasma characterized by a kappa or Lorentzian velocity distribution is investigated in the framework of kinetic and fluid approaches. We have examined the role of suprathermality on the instability threshold wave number and growth rates. The kinetic analysis, based on the linearized Boltzmann-Vlasov and Poisson equations, yields a κ-dependent dispersion relation and a modified critical Jeans wave number, which reduces to the classical results in the Maxwellian limit ($\kappa \rightarrow \infty$) showing their applicability in high-energy tails. Kinetic theory predicts enhanced growth rates and a wider unstable wave number range due to resonant wave-particle interactions in the suprathermal limit (lower κ values). In contrast, the fluid description, which incorporates a κ-dependent equation of state, introduces a modified sound speed and predicts a stabilizing influence of suprathermal populations. Comparison of the two approaches shows that the enhanced growth rate of the kinetic Jeans instability due to suprathermality signifies the importance of kinetic effects in nonequilibrium astrophysical plasmas under specific conditions. The results are discussed in order to understand the role of suprathermality in the gravitational collapse of the warm-hot circumgalactic medium of the Milky Way.

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I. INTRODUCTION

The Jeans instability (which arises when self-gravity overcomes internal gas pressure) plays a central role in governing the structure formation across diverse astrophysical systems. A gravitating system becomes unstable for perturbations of wavelength $\lambda > \lambda_J$, where $\lambda_J = c_s(\pi/G\rho_0)^{1/2}$, is the Jeans length, c_s is the adiabatic sound speed, G is the universal gravitational constant, and ρ_0 is the density of the medium [1]. It drives the collapse and fragmentation of molecular clouds, setting the scales for star formation [2], formation of galactic disks [3], accretion disks [4], giant planet formation [5], circumgalactic medium (CGM) [6], halo satellite galaxies [7], etc. The Jeans instability has been investigated for static and expanding universes, accounting for viscous and thermal effects [8], charged dust grains [9], and dissipative self-gravitating Bose-Einstein condensates with applications in the formation of dark matter halos in cosmology [10]. Recent work generalizes the classical Jeans instability analysis to more realistic situations, including viscoelastic astrofluids [11], magnetized plasmas in dense stars [12,13], and magnetized dusty plasmas [14]. In space plasmas, a self-consistent kinetic model is developed to study solar coronal properties and the fast solar wind acceleration in magnetized plasma near the Sun [15]. Parker Solar Probe (PSP) measurements show that a strongly magnetized high-temperature solar corona expands the solar wind stream into space [16]. The most recent measurements show that the fundamental Alfvén waves in a magnetized plasma play a key role in heating the solar corona and solar wind [17].

Most of the work employs a fluid description based on hydrodynamic equations, which is adequate when the system is collisional, isotropic, and close to thermodynamic equilibrium. However, in many astrophysical environments, such as collisionless stellar systems, dark-matter halos, and non-Maxwellian plasmas, the fluid closure fails to capture a realistic picture of the system. In such cases, the kinetic description, formulated through the Vlasov-Poisson or Boltzmann-Poisson equations, provides a more accurate representation of the dynamics. It assumes a particle distribution function $f(\boldsymbol{r}, \boldsymbol{v}, t)$ (where $\boldsymbol{r}$ is the position vector and $\boldsymbol{v}$ is the particle velocity) which obeys the Vlasov-Poisson equations and characterizes the equilibrium and perturbed states of the system. The analytical solutions of the Vlasov-Poisson equation give the dispersion relation for an infinite homogeneous gravitating stellar system within the kinetic framework [18]. Thus, while the fluid approach, which incorporates macroscopic properties, is useful due to its simplicity, the kinetic theory provides a more rigorous and reliable framework for describing the Jeans instability in collisionless or weakly collisional astrophysical systems. In the present work, we present a unified kinetic-fluid model to study the Jeans instability in a κ-distributed suprathermal gas with astrophysical applications.

In past years, a significant contribution has been made to investigate the Jeans instability using kinetic theory in Maxwellian dusty plasmas [19], Lorentzian dusty plasma with Lennard-Jones (L-J) potential [20], and including Eddington-inspired Born-Infeld (EiBi) gravity [21]. The choice of different distribution functions in the kinetic description of

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the Jeans instability enhances its applicability across various plasma environments. Reporting some of them, Lima *et al.* [22] have investigated the Jeans instability in a collisionless plasma using nonextensive kinetic theory and demonstrated that the critical wavelength and mass are explicitly dependent on the nonextensive q parameter. Trigger *et al.* [23] considered the impact of collisions on the Jeans instability within the kinetic framework and showed that collisions do not affect the Jeans instability criterion. Analogously, the Jeans instability is studied in a self-gravitating system composed of dark and bright matter within the framework of nonextensive statistics and its associated kinetic theory [24]. Furthermore, it is observed that in κ-deformed "Kaniadakis-distributed" self-gravitating systems, the growth of the Jeans instability is significantly altered due to the influence of orbital angular momentum [25,26].

In numerous space plasma environments, a persistent challenge is understanding how suprathermal particle populations and non-Maxwellian velocity distributions influence wave dispersion, instability growth, and large-scale fluid behavior. The κ-distributed suprathermal populations significantly modify the growth rate of the whistler-mode instabilities in space plasmas [27], electron-acoustic instabilities, and Bohm-Gross dispersion [28,29]. Lazar *et al.* [30] have shown that suprathermal corrections enhance the electromagnetic electron cyclotron instability driven by the temperature anisotropy ($T_{e,\perp} > T_{e\parallel}$). Rufai *et al.* [31] have discussed the influence of suprathermal particles on low-frequency electrostatic solitary waves propagating obliquely to the magnetic field of the magnetosphere. Boro and Prajapati [32] have shown that incorporating suprathermal particle populations into the dispersion relations of cosmic-ray-driven magnetohydrodynamic (MHD) waves increases the phase speed of the fast mode and stabilizes the Jeans gravitational instability in astrophysical plasmas. Extending this work for radiation-pressure-driven instabilities in rotating magnetized plasmas, a new instability criterion has been derived [33].

Many space and astrophysical plasmas are weakly collisional and exhibit suprathermal particle populations that are well described by kappa (non-Maxwellian) distributions rather than Maxwellian distributions. Because Jeans instability depends sensitively on the velocity distribution, a kinetic study of Jeans instability in a κ-distributed gas provides a more realistic description of collapse in nonequilibrium astrophysical plasma environments of the interstellar medium (ISM) and intracluster plasmas. In previous studies [20–23,25], the kinetic Jeans instability has been investigated using various velocity distributions. However, to the best of our knowledge, none of the authors has so far presented both kinetic and fluid descriptions of the Jeans instability in astrophysical suprathermal gas characterized by kappa distribution. Therefore, for the sake of astrophysical applications, we analyze the Jeans instability in the κ-distributed, self-gravitating, collisionless suprathermal gas using both kinetic and fluid approaches. This study will provide a comprehensive understanding of the Jeans instability to identify the limits of fluid theory and reveal additional kinetic effects absent from macroscopic descriptions.

II. KINETIC DESCRIPTION OF JEANS INSTABILITY IN A κ-DISTRIBUTED GAS

Consider an infinitely gravitating, unmagnetized, and collisionless plasma consisting of suprathermal populations described by the kappa (non-Maxwellian) distribution function. The kinetic description of a many-particle system within the nonrelativistic gravitational framework is presented for the analytical treatment of the Jeans instability. The dynamics of suprathermal particles are described by the coupled Vlasov and Poisson equations. For a distribution function $f(\boldsymbol{v}, \boldsymbol{r}, t)$, the Vlasov-Poisson equations are given by [18,22],

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{r}} = \nabla\phi \cdot \frac{\partial f}{\partial \boldsymbol{v}}, \tag{1}$$

where ϕ is the gravitational potential which obeys the Poisson equation

$$\nabla^2\phi = 4\pi G \int f(\boldsymbol{v}, \boldsymbol{r}, t) d^3\boldsymbol{v}. \tag{2}$$

The distribution of the mass density is given by $\rho(\boldsymbol{r}, t) = \int f(\boldsymbol{v}, \boldsymbol{r}, t) d^3\boldsymbol{v}$. The Vlasov-Poisson system described by Eqs. (1) and (2) is perturbed about the homogeneous, time-independent equilibrium, such that

$$f(\boldsymbol{v}, \boldsymbol{r}, t) = f_0(\boldsymbol{v}) + f_1(\boldsymbol{v}, \boldsymbol{r}, t), \quad \text{where } f_1 \ll f_0,$$
$$\phi(\boldsymbol{r}, t) = \phi_0(\boldsymbol{r}) + \phi_1(\boldsymbol{r}, t), \quad \text{where } \phi_1 \ll \phi_0, \tag{3}$$

where $f_0(\boldsymbol{v})$, $\phi_0(\boldsymbol{r})$ are the equilibrium parts and f_1, ϕ_1 are the perturbed parts of the distribution function and the gravitational potential.

Let us now linearize Eqs. (1) and (2) by substituting the perturbations defined in Eq. (3) keeping the first-order perturbed terms only and neglecting the higher-order terms. In this process, we neglect the gravitational effect of the uniform background density in an infinitely homogeneous medium. Considering only perturbations to study instability, which is referred to as "Jeans swindle" phenomenon. Mathematically, a direct substitution of Eq. (3) in Eqs. (1) and (2) provides both equilibrium and perturbed parts. In a homogeneous medium, if density ρ_0 is constant and equilibrium velocity $\boldsymbol{v}_0 = 0$, then Eq. (1) implies $\nabla\phi_0 = 0$. On the other hand, Poisson's equation requires that $\nabla^2\phi_0 = 4\pi G\rho_0$. These two are inconsistent unless $\rho_0 = 0$. Thus, to construct an artificial equilibrium for a homogeneous self-gravitating medium, one can invoke the "Jeans swindle." The standard approach is imposed that the unperturbed gravitational field vanishes, i.e., $\nabla\phi_0 = 0$ or $\nabla^2\phi_0 = 0$ [18].

Thus, under this assumption, the linearized perturbed form of Eqs. (1) and (2) can be written as

$$\frac{\partial f_1}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f_1}{\partial \boldsymbol{r}} = \nabla\phi_1 \cdot \frac{\partial f_0}{\partial \boldsymbol{v}}, \tag{4}$$

$$\nabla^2\phi_1 = 4\pi G \int f_1(\boldsymbol{v}, \boldsymbol{r}, t) d^3\boldsymbol{v}. \tag{5}$$

To solve the above equations, we use the plane wave solutions, according to which the perturbed quantities vary as

$$f_1(\boldsymbol{r}, \boldsymbol{v}, t) = \delta f(\boldsymbol{v}) \exp i(\boldsymbol{k} \cdot \boldsymbol{r} - \omega t),$$
$$\text{and } \phi_1(\boldsymbol{r}, t) = \delta\phi \exp i(\boldsymbol{k} \cdot \boldsymbol{r} - \omega t), \tag{6}$$

where $\delta f(\boldsymbol{v})$ and $\delta\phi$ represent the amplitude in the corresponding perturbed quantities. We substitute the above perturbations into Eqs. (4) and (5), and get the following relations:

$$(\boldsymbol{k}\cdot\boldsymbol{v}-\omega)\delta f(\boldsymbol{v})-\delta\phi\,\boldsymbol{k}\cdot\frac{\partial f_0}{\partial \boldsymbol{v}}=0, \tag{7}$$

$$k^2\delta\phi=-4\pi G\int\delta f(\boldsymbol{v})d^3\boldsymbol{v}. \tag{8}$$

Finally, we substitute the value of $\delta f(\boldsymbol{v})$ from Eq. (7) in Eq. (8), and get a standard dispersion relation between ω and k in the following form:

$$1+\frac{4\pi G}{k^2}\int\frac{\boldsymbol{k}\cdot\frac{\partial f_0}{\partial \boldsymbol{v}}}{(\boldsymbol{k}\cdot\boldsymbol{v}-\omega)}d^3\boldsymbol{v}=0. \tag{9}$$

Equation (9) represents the general dispersion relation of the Jeans instability derived using kinetic theory. In previous work, the authors have modified this dispersion relation due to the presence of particle collisions [23], L-J potential [20], EiBI gravity [21], and dark matter [24]. In the work of Lima *et al.* [22], the authors consider the distribution function $f_0(\boldsymbol{v})$ within the q-nonextensive framework proposed by Tsallis [34] and obtain a modified expression for the critical Jeans wave number associated with the Jeans instability. However, our concern is to consider the suprathermal particles characterized by the kappa distribution function and derive the full analytical dispersion relation of the kinetic Jeans instability.

The generalized Lorentzian or kappa distribution function $f_\kappa(v)$ provides a physical description of nonequilibrium plasmas where particle populations deviate from the classical Maxwellian behavior. This accounts for the presence of suprathermal particles (high-energy tails) that are often observed in space and astrophysical plasmas, such as the solar wind, planetary magnetospheres, and ISM. Thus, the generalized Lorentzian distribution is given by [35,36]

$$f_\kappa(v)=\frac{\rho_0}{(\pi\kappa\Theta^2)^{\frac{3}{2}}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}\left(1+\frac{v^2}{\kappa\Theta^2}\right)^{-(\kappa+1)}, \tag{10}$$

where $\Theta=[(2\kappa-3)/\kappa]^{1/2}(k_BT_{\rm eq}/m)^{1/2}$ is the modified thermal speed, k_B is the Boltzmann constant, $T_{\rm eq}$ is the equivalent temperature of the suprathermal particles, and m is the particle mass. Here $\Gamma(\kappa)$ denotes the κ-dependent Gamma function that arises when evaluating moments of the κ-velocity distribution, reflecting how statistical properties depend explicitly on κ. Physically, it quantifies the influence of suprathermal particles on macroscopic plasma behavior, modifying quantities such as pressure, temperature, and instability growth rates relative to the Maxwellian limit.

The range of the κ index varies from $3/2<\kappa<\infty$. The parameter κ quantifies the deviation from thermal equilibrium: for large κ ($\kappa\to\infty$), the distribution approaches the Maxwellian form, recovering thermal equilibrium. The smaller κ values correspond to stronger nonequilibrium states with enhanced high-energy tails representing the suprathermal limit. The modified thermal speed (Θ) ensures that the kappa distribution yields the same mean energy (or effective temperature) as the Maxwellian case in the high-κ limit. Thus, the kappa distribution provides a consistent and generalized framework to model both the core thermal population and the suprathermal tail, bridging the gap between equilibrium and nonequilibrium plasma states.

As $\kappa\to\infty$, the Lorentzian kappa distribution function (10) reduces to the Maxwellian distribution

$$f_M(v)=\frac{\rho_0}{\pi^{\frac{3}{2}}v_{\rm th}^3}\exp\left(-\frac{v^2}{v_{\rm th}^2}\right), \tag{11}$$

where $v_{\rm th}=(k_BT/m)^{1/2}$ is the thermal speed.

Due to the vast applications of the Lorentzian kappa distribution function (10) in space plasmas, its utility can also be generalized in self-gravitating astrophysical systems. Let us now substitute the generalized kappa distribution function (10) into the dispersion relation (9) of the kinetic Jeans instability, and after solving, we get the following relation:

$$1-\frac{\omega_J^2}{k\Theta^5}\frac{2}{(\pi\kappa)^{3/2}}\frac{\Gamma(\kappa+1)(\kappa+1)}{\kappa\Gamma\left(\kappa-\frac{1}{2}\right)}\times\int\frac{v\left(1+\frac{v^2}{\kappa\Theta^2}\right)^{-(\kappa+2)}}{(\boldsymbol{k}\cdot\boldsymbol{v}-\omega)}d^3\boldsymbol{v}=0, \tag{12}$$

where $\omega_J=(4\pi G\rho_0)^{1/2}$ is the Jeans frequency, i.e., the characteristic dynamical (free-fall) rate set solely by the background self-gravity. It is the inverse of the gravitational timescale $t_J\sim\omega_J^{-1}$ for a uniform medium, and it provides the natural scale against which pressure competes with the gravitational effect.

The dispersion relation (12) illustrates the linear self-gravitating response of a collisionless gas with a suprathermal (κ) velocity distribution in equilibrium. Physically, the left-hand side is $1+\chi(k,\omega)$ in which the second term is the gravitational susceptibility produced by the kinetic response of the κ-distributed particles. The integral in velocity space contains two crucial terms: (1) the full non-Maxwellian shape of the equilibrium through the algebraic tail $[1+v^2/(\kappa\Theta^2)]^{-(\kappa+2)}$ and (2) the resonant denominator $(\boldsymbol{k}\cdot\boldsymbol{v}-\omega)$ that generates kinetic Landau effects. It shows a singularity with infinite blowup solutions for the condition $\omega=\boldsymbol{k}\cdot\boldsymbol{v}$.

In order to visualize the Jeans instability criterion and the growth rate of the Jeans instability, let us solve the kinetic dispersion relation (12). The velocity-space integral is evaluated by transforming it to cylindrical coordinates and reducing to the one-dimensional κ plasma dispersion function (with the Landau prescription for the resonant pole). A detailed analysis is provided in Appendix A. The formal solution of Eq. (12) gives the following dispersion relation:

$$1-\frac{\omega_J^2}{(\pi\kappa)^{3/2}}\frac{2a}{\Theta^3}\frac{\pi\Gamma(\kappa+1)}{k^2\Gamma\left(\kappa-\frac{1}{2}\right)}Z_\kappa(z)=0, \tag{13}$$

where $a^2=\kappa\Theta^2$.

The Jeans instability criterion can be obtained using the marginal stability condition setting $\omega=0$. It provides the critical Jeans wave number at which the dynamical behavior of the system changes. Solving the integral (A3) given in Appendix A, putting $\omega=0$ and substituting in the dispersion relation (13). In this way, the Jeans instability condition is obtained, which shows that the system will be gravitationally unstable for all perturbation wave numbers,

$$k < k_J^{(\mathrm{kin})}, \quad \text{where } k_J^{(\mathrm{kin})} = \frac{\omega_J}{\Theta}\left(\frac{2\kappa-1}{\kappa}\right)^{1/2}. \tag{14}$$

This represents the κ modified Jeans instability criterion in a suprathermal gravitating gas derived using the kinetic description. Here the classical thermal speed (v_{th}) is replaced by the κ modified thermal speed (Θ). The above expression demonstrates how suprathermal particle populations, represented by the κ distribution, alter the classical gravitational instability threshold. In the Maxwellian limit ($\kappa \to \infty$, $T_{\mathrm{eq}} = T$), the factor $[(2\kappa-1)/\kappa]^{1/2} \to \sqrt{2}$, and $\Theta \to \sqrt{2k_BT/m}$. Thus, the expression reduces to the standard Jeans wave number, $k_J = \omega_J/v_{\mathrm{th}}$ for a Maxwellian gas, which is identical to the classical Jeans instability criterion for a neutral gas cloud, as obtained by Chandrasekhar [1]. Obviously, expression (14) of the Jeans wave number shows a significant deviation from the nonextensive q-parametrized critical wave number $k_q^2 = k_J^2(3q-1)/2$ as derived from Lima *et al.* [22]. The above instability criterion (14) is also identical to the condition (13) of Qian *et al.* [20], neglecting the contributions of the dust grains and L-J potential in their work.

It is evident that the factor $[(2\kappa-1)/\kappa]^{1/2}$ is a decreasing function of κ, lower κ values (i.e., stronger suprathermal effects) reduce the effective Jeans wave number. This means that the system becomes unstable only for comparatively smaller wave numbers. Physically, suprathermal particles raise the effective pressure through their extended velocity tails, thereby suppressing gravitational collapse at intermediate scales. Furthermore, the dependence on the modified thermal speed Θ further adjusts the instability threshold. Since Θ incorporates κ-dependent corrections to the thermal motion, it redefines the balance between gravitational attraction and kinetic pressure. A larger Θ representing enhanced effective thermal agitation by lowering $k_J^{(\mathrm{kin})}$, again indicating increased stabilization against collapse. Therefore, both Θ and κ jointly control the onset of instability through modifications of the effective pressure support.

To solve the dispersion relation (13) for calculating the growth rate, in Appendix B, $Z_\kappa(z)$ is solved using some integral identities. The solution gives the simplified form of the dispersion relation (13) as follows:

$$1 - \frac{\omega_J^2}{(\kappa)^{3/2}}\frac{2a}{k^2\Theta^3}\frac{\Gamma\left(\kappa+\frac{1}{2}\right)}{\Gamma\left(\kappa-\frac{1}{2}\right)}[1 + z\mathcal{Z}_\kappa(z)] = 0. \tag{15}$$

Using the property of Γ function, $\Gamma(\kappa+1/2)/\Gamma(\kappa-1/2) = (\kappa-1/2)$ after simplifications, we get

$$1 - \frac{\omega_J^2}{k^2a^2}(2\kappa-1)[1 + z\mathcal{Z}_\kappa(z)] = 0. \tag{16}$$

The purely growing mode or the Jeans instability is analyzed by assuming $\omega = i\gamma$ (where $\gamma > 0$ is the growth rate of the instability. Thus, $z = i\eta$, with $\eta = \gamma/ka$. For a small phase velocity ($|z| \ll 1$), the κ dispersion function satisfies

$$\mathcal{Z}_\kappa(i\eta) \approx i\sqrt{\pi}\frac{\Gamma(\kappa)}{\Gamma(\kappa-\frac{1}{2})}\eta. \tag{17}$$

Thus,

$$z\mathcal{Z}_\kappa(z) \approx -\sqrt{\pi}\frac{\Gamma(\kappa)}{\Gamma\left(\kappa-\frac{1}{2}\right)}\eta^2. \tag{18}$$

Substituting the above value into Eq. (16), the growth rate equation is obtained as

$$1 - \frac{\omega_J^2}{k^2v_{\mathrm{th}}^2}\frac{(2\kappa-1)}{(2\kappa-3)}\left[1 - \sqrt{\pi}\frac{\Gamma(\kappa)}{\Gamma\left(\kappa-\frac{1}{2}\right)}\frac{\gamma^2}{k^2v_{\mathrm{th}}^2}\frac{\kappa}{(2\kappa-3)}\right] = 0. \tag{19}$$

Finally, this shows the general dispersion relation of the Jeans instability in collisionless suprathermal plasmas following a rigorous analytical calculation. It gives an exact similar Jeans instability criterion (14), which is derived by setting the marginal stability condition. The second term in the square brackets refers to the growth rate (γ), indicating that the instability growth is governed by kinetic rather than purely fluid effects. The appearance of the Gamma function ratio is a direct consequence of velocity-space integration over the κ distribution. This term quantitatively captures the contribution of suprathermal particles to the resonant interaction between gravitational perturbations and particle motion. Overall, Eq. (19) makes explicit how suprathermal (κ-distributed) particles in the astrophysical system, such as CGM, modify both the Jeans threshold [via the $(2\kappa-1)/(2\kappa-3)$ factor] and the growth rate (via the Γ-function term), compared with a Maxwellian plasma. This allows us to quantify when long-wavelength perturbations become unstable and how quickly they grow once kinetic effects are included.

To visualize the effect of the κ parameter on the growth rate of the kinetic Jeans instability, solving for γ^2, and taking normalized parameter $\gamma^* = \gamma/\omega_J$ and $k^* = kv_{\mathrm{th}}/\omega_J$, we get the normalized dispersion relation of the growth rate

$$\gamma^{*2} = \left(\frac{2\kappa-3}{\kappa}\right)\frac{1}{\sqrt{\pi}}k^{*2}\frac{\Gamma\left(\kappa-\frac{1}{2}\right)}{\Gamma(\kappa)}\left[1 - k^{*2}\frac{(2\kappa-3)}{(2\kappa-1)}\right]. \tag{20}$$

Figure 1 illustrates the normalized growth rate (Re γ^*) of the Jeans instability as a function of the normalized wave number k^* for different values of the κ parameter, namely, $\kappa = 1.75, 2.0, 3.0$, and the Maxwellian limit $\kappa \to \infty$. The κ index characterizes the deviation of the particle velocity distribution from a Maxwellian; smaller κ corresponds to stronger suprathermal (high-energy) tails, while larger κ approaches thermal equilibrium. It is shown that for all values κ, the growth rate exhibits characteristic Jeans-type behavior starting from zero at $k^* = 0$, increasing with wave number, reaching a maximum at an intermediate k^*, and then decreasing to zero at a critical cutoff wave number. This cutoff marks the boundary between unstable long-wavelength modes and stable short-wavelength modes.

An important observation evident from the figure is that both the unstable wave number range and the maximum growth rate increase as κ decreases. In the suprathermal regime (lower κ), the instability extends to significantly larger values of k^* and exhibits a higher peak growth rate compared to the Maxwellian case. Conversely, as κ increases toward

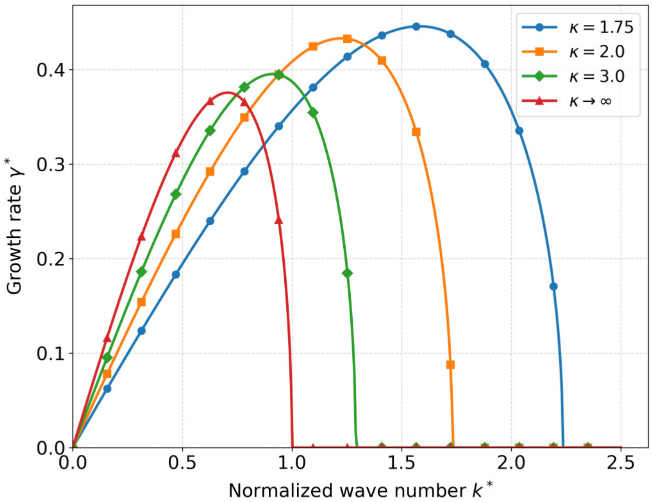

FIG. 1. Growth rate of kinetic Jeans instability (Re γ^*) versus normalized wave number (k^*) for different values of κ index.

the Maxwellian limit, the unstable region contracts, and the growth rate is reduced. This behavior reflects the inherently kinetic nature of the Jeans instability in κ-distributed plasmas. The presence of suprathermal particles modifies the velocity-space response and reduces the effective Landau-type damping associated with resonant particles. As a result, self-gravity becomes more efficient in amplifying density perturbations, leading to higher growth rates and a wider unstable spectrum in the low-κ regime. In addition, the position of the maximum growth rate shifts toward higher wave numbers as κ decreases, indicating that shorter-wavelength modes become increasingly unstable in strongly suprathermal plasmas. This trend is absent in fluid descriptions, highlighting the importance of kinetic effects in determining both the scale and the strength of gravitational instabilities.

Furthermore, it is noted that the above trends differ from those reported by Qian *et al.* [20], who have observed that suprathermality stabilizes the kinetic Jeans instability. It is obvious because in the present work, a different dispersion relation is obtained than in Ref. [20], in which they have considered the Lorentzian dusty plasma with L-J potential. In the present model with the considered assumptions, the dispersion relation (20) shows that as κ decreases, the Gamma-function ratio increases, amplifying the destabilizing term and leading to a higher growth rate of the Jeans instability. This result implies that nonthermal velocity distributions can significantly accelerate gravitational collapse in collisionless astrophysical environments under specific conditions.

III. FLUID DESCRIPTION OF JEANS INSTABILITY IN A SUPRATHERMAL GAS

In this section we present a macroscopic study of the κ modified equation of state on the linear gravitational instability. Let us now consider the hydrodynamic fluid behavior of the self-gravitating suprathermal gas characterized by the kappa distribution. The expression of the number density for a plasma obeying a κ distribution has been formulated in earlier works [37,38]. Following this, the density distribution of a suprathermal self-gravitating gas of density (ρ) is taken into account, and the equation of state is derived by Boro and Prajapati [32], which is given by

$$P_s^g = \left(\frac{3-\chi_\kappa}{2\chi_\kappa}\right)\left[\frac{2}{f}\left(\frac{\rho}{\rho_0}\right) + \left(\frac{\rho}{\rho_0}\right)^{\chi_\kappa} - (1-\chi_\kappa)\right]\rho_0 \times \left(\frac{k_B T}{m}\right), \tag{21}$$

where $\chi_\kappa = (\kappa - 3/2)/(\kappa - 1/2)$, is the deformation parameter and f is the degrees of freedom.

Because the kappa spectral index satisfies $3/2 < \kappa < \infty$, the parameter χ_κ lies within $0 < \chi < 1$, and in the Maxwellian limit ($\kappa \to \infty$), $\chi_\kappa \to 1$, giving the pressure relation $P = \gamma n k_B T$, with $\gamma = (2/f) + 1$ as adiabatic index. In the limit of large degrees of freedom ($f \to \infty$), $\gamma \to 1$, it reduces to the ideal gas equation $P = n k_B T$. The κ-dependent terms in Eq. (21) thus represent suprathermal corrections to the polytropic relation, accounting for non-Maxwellian effects in the dispersion characteristics.

Let us differentiate Eq. (21) with respect to the mass density ρ, which gives the expression of sound speed in a suprathermal gas

$$\frac{dP}{d\rho} = C_{\rm st}^2 = \left[\frac{6-(2-3f)\chi_\kappa - f\chi_\kappa^2}{2f\chi_\kappa}\right] v_{\rm th}^2. \tag{22}$$

This analysis demonstrates that the sound speed in a suprathermal gas is explicitly dependent on the parameters f and χ_κ. Different choices of these parameters reproduce the corresponding polytropic exponents for various types of gases. For example, in the limit $\chi_\kappa \to 1$ with $f = 3$ translational degrees of freedom, the expression reduces to $c_s = \sqrt{5/3}\, v_{\rm th}$, which is the standard sound speed for a monoatomic Maxwellian gas. Similarly, choosing $f = 5$ (three translational and two rotational degrees of freedom) gives the sound speed appropriate for a diatomic gas, while $f = 6$ (three translational and three rotational modes) corresponds to a triatomic gas.

The hydrodynamic fluid description of a suprathermal gas is characterized by the basic fluid equations. The momentum transfer or equation of motion for a suprathermal gravitating gas can be written as

$$\rho\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla\right)\boldsymbol{v} = -\nabla P_s^g - \rho\nabla\phi. \tag{23}$$

The continuity equation is given by

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\boldsymbol{v}) = 0. \tag{24}$$

The Poisson equation for gravitational potential is given by

$$\nabla^2\phi = 4\pi G\rho. \tag{25}$$

Finally, the equation of state is given by

$$\nabla P_s^g = C_{\rm st}^2 \nabla\rho. \tag{26}$$

Equations (23)–(26) represent the governing equations to describe the dynamic behavior of a self-gravitating suprathermal gas. These equations are linearized assuming first-order perturbations in each physical quantity, such as $\boldsymbol{v} = \boldsymbol{v_0} + \boldsymbol{v_1}$, $\rho = \rho_0 + \rho_1$, and $\phi = \phi_0 + \phi_1$. Here $\boldsymbol{v}_0$, ϕ_0, and ρ_0 represent the unperturbed parts of the fluid velocity, fluid density, and gravitational potential. The fluid is initially assumed to be at rest, i.e., $\boldsymbol{v}_0 = 0$. $\boldsymbol{v_1}$, ρ_1, and ϕ_1 denote the perturbed parts of the fluid velocity, fluid density, and gravitational potential, respectively. The linearized form of the above basic equations is obtained as

$$\frac{\partial \boldsymbol{v_1}}{\partial t} = -\frac{C_{\rm st}^2}{\rho_0}\nabla \rho_1 - \nabla \phi_1, \tag{27}$$

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot \boldsymbol{v_1} = 0, \tag{28}$$

and

$$\nabla^2 \phi_1 = 4\pi G \rho_1. \tag{29}$$

Let us assume that each perturbed quantity has space-time dependent variations of the form $A_1(\boldsymbol{r}, t) \propto \exp(i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega t)$, where ω is the perturbation frequency and $\boldsymbol{k}=k_x\hat{\boldsymbol{x}} + k_z\hat{\boldsymbol{z}}$ is the wave vector. Such perturbations provide the derivatives $\nabla = i\boldsymbol{k}$ and $\partial/\partial t = -i\omega$. On solving the above equations, we get the following dispersion:

$$\omega^2 = \left[\frac{6 - (2-3f)\frac{(2\kappa-3)}{(2\kappa-1)} - f\left[\frac{(2\kappa-3)}{(2\kappa-1)}\right]^2}{2f\frac{(2\kappa-3)}{(2\kappa-1)}}\right] v_{\rm th}^2 k^2 - \omega_J^2. \tag{30}$$

The above equation represents the dispersion relation for the Jeans instability in a suprathermal kappa-distributed gas, derived from the fluid description. This shows the generalization of the fundamental Jeans instability criterion of a self-gravitating gas as derived by Chandrasekhar [1] in the suprathermal limit. The discussion appropriately discusses the dispersion properties of a suprathermal self-gravitating gas in the high-energy tails of the kappa distribution. Thus, we have two distinct dispersion relations derived by kinetic theory [see Eq. (19)] and fluid theory [see Eq. (30)].

In the Maxwellian limit ($\kappa \to \infty$), with degrees of freedom $f = 3$, Eq. (30) reduces to

$$\omega^2 = c_s^2 k^2 - 4\pi G \rho_0. \tag{31}$$

From the dispersion relation (30), one can get the Jeans instability criterion for a suprathermal gravitating gas. One of the two roots of Eq. (30) will be imaginary, providing an instability if the following condition is satisfied:

$$\left[\frac{6 - (2-3f)\frac{(2\kappa-3)}{(2\kappa-1)} - f\left[\frac{(2\kappa-3)}{(2\kappa-1)}\right]^2}{2f\frac{(2\kappa-3)}{(2\kappa-1)}}\right] v_{\rm th}^2 k^2 - 4\pi G \rho_0 < 0. \tag{32}$$

This gives the perturbation wave numbers $k_J^{\rm (fluid)}$ for which instability will be observed, where $k_J^{\rm (fluid)}$ is the critical wave number given by

$$k_J^{\rm (fluid)} = \frac{\omega_J}{v_{\rm th}}\left[\frac{6 - (2-3f)\frac{(2\kappa-3)}{(2\kappa-1)} - f\left[\frac{(2\kappa-3)}{(2\kappa-1)}\right]^2}{2f\frac{(2\kappa-3)}{(2\kappa-1)}}\right]^{-1/2}. \tag{33}$$

The system represented by Eq. (30) will be gravitationally unstable for all perturbation wave numbers $k_J^{\rm (fluid)}$ or Jeans length $\lambda > \lambda_J^{\rm fluid} (= 2\pi/k_J^{\rm (fluid)})$. In the Maxwellian limit, i.e., $\kappa \to \infty$, with $f = 3$, the above expression gives the classical Jeans wave number relation $k_J = \omega_J/c_s$. Equation (33) shows that the critical Jeans wave number $k_J^{\rm (fluid)}$ is controlled by the ratio of the Jeans frequency ω_J to the characteristic thermal speed $v_{\rm th}$, modulated by a dimensionless correction factor that depends on the distribution parameters κ and f. Physically, the bracketed factor inside the square root encodes how departures from a Maxwellian velocity distribution (parameterized by κ) of the suprathermal component modify the effective pressure support. A larger contribution from suprathermal particles (or a smaller κ, indicating heavier high-energy tails) alters the critical Jeans wave number and the corresponding critical Jeans length scale.

The dependence of $k_J^{\rm (fluid)}$ on the spectral index κ reveals that increasing κ (approaching the Maxwellian limit) leads to a moderate increase in $k_J^{\rm (fluid)}$ and a corresponding decrease in the critical Jeans wavelength. Physically, this indicates that suprathermal populations (low κ) enhance effective pressure and thermal support, thereby suppressing gravitational instability. In contrast, as the distribution tends toward the Maxwellian (high κ), this stabilizing influence weakens, allowing smaller-scale gravitational condensation to occur more readily.

The critical Jeans wave number obtained from kinetic theory $k_J^{\rm (kin)}$ in expression (14) differs significantly from that of the expression obtained in fluid theory $k_J^{\rm (fluid)}$ in expression (33). Fluid theory assumes local thermodynamic equilibrium and a single isotropic sound speed, which tends to overestimate the effective pressure support against gravitational collapse. In contrast, kinetic theory incorporates the full velocity distribution, which reduces stabilization and typically leads to a larger critical wave number ($k_J^{\rm (kin)}$). As a result, kinetic systems become gravitationally unstable at smaller scales than predicted by fluid theory. The difference arises because collisionless particles cannot provide the instantaneous isotropic pressure response assumed in hydrodynamic fluid theory.

Now, the dispersion relation (30) is analyzed to understand the Jeans instability mechanism in warm-hot CGM of the Milky Way observed by eROSITA (extended ROentgen Survey with an Imaging Telescope Array) as a representative example of a self-gravitating astrophysical medium containing suprathermal particles. The relevant plasma parameters for CGM masses $M_{\rm CGM} = 10^{11} M_\odot$ (where $M_\odot$ is the solar mass) is the number density $n = 7.2 \times 10^{-5}\,{\rm cm}^{-3}$ and temperature $T = 2 \times 10^6$ K [39,40]. For the chosen halo parameters and assuming a pure hydrogen plasma, the thermal velocity is measured as $v_{\rm th} \approx 1.28 \times 10^7\,{\rm cm\,s}^{-1}$. The Jeans and Jeans timescales are calculated, respectively, as $\omega_J \approx 1.0 \times 10^{-17}\,{\rm s}^{-1}$ and $t_J = \omega_J^{-1} \approx 3.15 \times 10^9$ yr. Taking $\kappa = 2$ and $f = 3$, we get the critical Jeans wave number $k_J^{\rm (fluid)} = 3.9 \times 10^{-25}\,{\rm cm}^{-1}$ and corresponding Jeans length $\lambda_J \approx 1.6 \times 10^{25}$ cm.

These estimates imply that the warm-hot CGM is globally stable against rapid gravitational collapse due to its high thermal speed and extremely long Jeans timescale, which is comparable to cosmological timescales. However, the presence of density inhomogeneities arising from

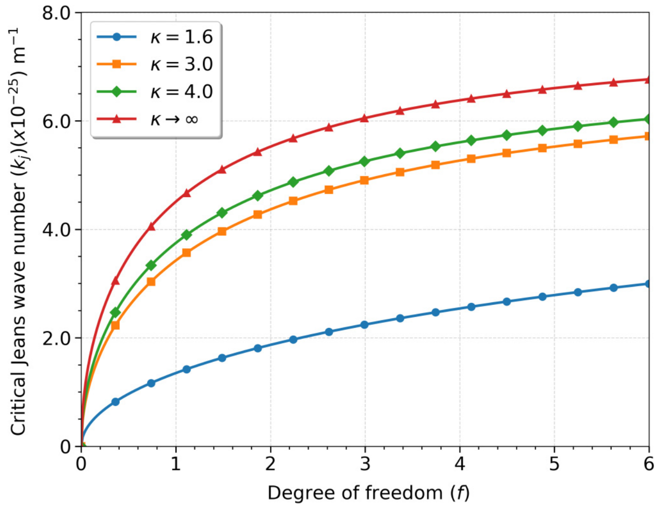

FIG. 2. Critical Jeans wave number k_j versus degree of freedom with different values of κ parameter.

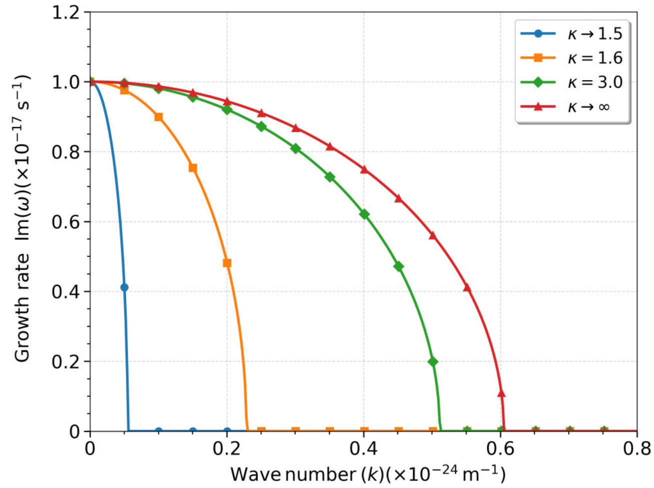

FIG. 3. Growth rate of Jeans instability versus wave number with different values of κ parameter (using fluid theory). The constant value of the degree of freedom is taken to be $f = 3$.

galactic feedback processes, accretion flows, or turbulence can locally enhance ρ, thereby increasing ω_J and reducing t_J, enabling gravitational instability on smaller scales. Such localized collapse may lead to the formation of cooler, denser structures embedded within the hot medium, contributing to the multiphase nature of the CGM and potentially supplying cold gas to the galactic disk. Thus, while the eROSITA observations support a quasi-hydrostatic large-scale CGM, gravitational instability remains an important mechanism for structure formation and gas cycling in the galactic environment.

Figure 2 illustrates the dependence of the critical Jeans wave number $k_J^{(\mathrm{fluid})}$ on the degree of freedom f for various values of the κ index. The curves show that for all κ values, the critical Jeans wave number increases monotonically with the degree of freedom. For large values of f, a higher wave number is required for gravitational collapse. This behavior indicates that systems with higher internal degrees of freedom are more resistant to Jeans gravitational instability. Physically, lower κ values correspond to distributions with stronger suprathermal tails. These suprathermal populations provide additional nonthermal pressure that supplements the thermal support against gravity. As a result, for low kappa, e.g., $\kappa = 1.6$, the critical wave number is significantly reduced, implying that gravitational collapse can occur on larger spatial scales. In contrast, for large kappa, the distribution approaches the Maxwellian limit, the nonthermal tail contribution diminishes, and the Jeans wave number becomes larger. Thus, the figure demonstrates that both the kappa index and the degree of freedom have a profound influence on the onset of gravitational instability. Suprathermal particles systematically lower the threshold for collapse, while increasing degrees of freedom tend to stabilize the medium by raising the instability threshold. These combined effects highlight the importance of using non-Maxwellian models in environments such as galaxy halos, molecular cloud interfaces, or collisionless astrophysical plasmas where suprathermal tails are prevalent.

Taking the plasma parameters of the CGM, Eq. (30) is solved to calculate the growth rate of the Jeans instability (Imω) for various values of κ parameter. Figure 3 shows how the growth rate of the instability changes with the wave number for different values of the suprathermal parameter κ. The constant values are taken to be $v_{\mathrm{th}} \approx 1.28 \times 10^7$ cm s^{-1} and $\omega_J \approx 1.0 \times 10^{-17}$ s^{-1}. Each curve corresponds to a κ value that represents how strongly the particle distribution deviates from a normal Maxwellian distribution. The growth rate decreases with increasing wave number, indicating that long-wavelength perturbations grow more efficiently than short-wavelength perturbations. As κ increases, the instability region increases significantly, showing that in the Maxwellian limit system, it tends to be more gravitationally unstable. On the other hand, in the suprathermal limit, the instability region shifts towards the lower values of wave number, thus showing its stabilizing influence on the growth rate. It is noted that for $\kappa = 2$, $k_J^{(\mathrm{fluid})} = 3.9 \times 10^{-25}$ cm^{-1}, which increases to $k_J^{(\mathrm{fluid})} \sim 6 \times 10^{-25}$ cm^{-1} for $\kappa \to \infty$ which confirms the predictions of Fig. 2 showing that increase in κ increases the critical Jeans wave number.

To visualize the growth rate of the Jeans instability in the entire $k - \kappa$ plane, let us illustrate the contour plot. Figure 4 shows the contour map of the Jeans growth rate, ω, plotted as a function of the wave number k and the suprathermal index κ. The color scale represents the magnitude of the growth rate: yellow regions correspond to the highest instability, orange and magenta shades denote intermediate growth, and dark blue regions indicate stability where Im$(\omega) = 0$. For low values of κ approaching the suprathermal limit, the stable region increases, indicating that in the suprathermal limit, the growth rate of the Jeans instability is suppressed. As the wave number and the parameter κ increase, the growth rate of the instability also increases. In the Maxwellian limit (large κ), the unstable region is observed to be larger. This trend suggests that the system becomes increasingly susceptible to gravitational instability as the distribution function becomes more thermalized. Hence, the contour map quantitatively confirms that both the wave number and the spectral index critically regulate the growth dynamics.

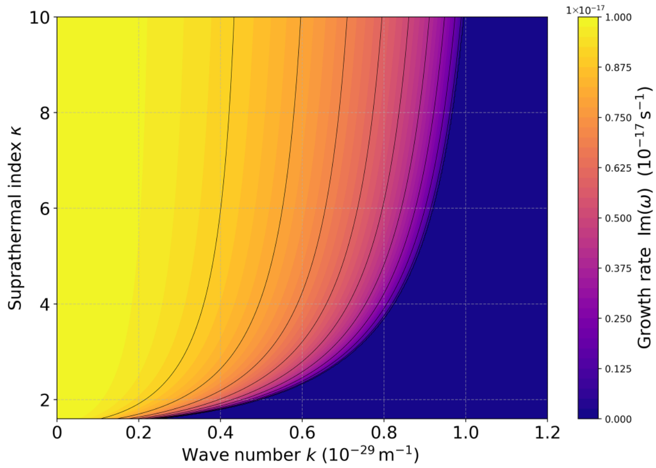

FIG. 4. Contour map of the instability growth rate $\mathrm{Im}(\omega)$ as a function of k and κ. The color scale represents the magnitude of the growth rate; yellow regions correspond to the highest instability and dark blue regions indicate stability.

IV. CONCLUSIONS

In this work the Jeans instability has been investigated in a self-gravitating suprathermal gas characterized by a Lorentzian (κ) velocity distribution using both kinetic and fluid approaches. The kinetic analysis, based on the linearized Boltzmann-Vlasov and Poisson equations, yields a generalized dispersion relation that incorporates suprathermal corrections through the κ-dependent velocity distribution. This formulation reduces to the classical Maxwellian case in the limit $\kappa \to \infty$, thereby validating the consistency of the model. The κ-dependent Jeans instability criteria and growth rates are determined under specific plasma conditions.

In kinetic theory, the growth rate of the Jeans instability is strongly influenced by the κ index through the detailed velocity-space structure of the distribution function. A decrease in κ, corresponding to an enhanced population of suprathermal particles, leads to a significant increase in the growth rate of the instability. This enhancement arises from resonant wave-particle interactions and nonlocal kinetic effects, which effectively reduce the stabilizing role of thermal pressure and allow gravitational perturbations to extract free energy from the high-energy tails of the distribution. As a result, collisionless plasmas described by κ distributions become unstable over a broader range of wavelengths and exhibit faster gravitational collapse compared to the Maxwellian limit.

In the fluid description, a κ-dependent equation of state has been used to derive modified expressions for the pressure, sound speed, and dispersion relation. The resulting κ-dependent Jeans wave number k_J defines the instability threshold, revealing that small κ values (i.e., enhanced suprathermal tails) decrease the critical Jeans wave number and hence reduce the growth rate of the instability by favoring the gravitational collapse in astrophysical plasmas. Consequently, fluid models predict the stabilizing influence of suprathermal particles, in direct contrast to kinetic predictions. This disparity highlights the fundamental limitations of fluid descriptions in collisionless regimes and underscores the necessity of kinetic theory to accurately capture the growth dynamics of the Jeans instability in nonthermal astrophysical plasmas. The kinetic results presented here are obviously model dependent, which differ slightly from previous results, e.g., Qian *et al.* [20], due to the inclusion of the κ-dependent Gamma function in the dispersion relation (20), whose value increases due to the reduction in κ index, showing an enhanced growth rate in the suprathermal limit.

The present kinetic-fluid framework provides a generalized theoretical basis for understanding structure formation and instability evolution in nonequilibrium astrophysical systems, such as interstellar clouds, stellar atmospheres, and galactic environments, where high-energy suprathermal particles play a crucial role. The present analysis may be extended to include the effects of temperature anisotropy and EiBI gravity in the magnetized plasmas. This may certainly modify the firehose and mirror-like instabilities, which are applicable to the solar atmosphere. The incorporation of EiBI gravity may further alter the gravitational coupling, introducing new stability regimes and cutoff scales. Together, these effects provide a more realistic framework for studying structure formation in anisotropic and nonthermal magnetized astrophysical plasmas.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: SOLUTION OF VELOCITY-SPACE INTEGRAL IN EQ. (12)

Equation (12) consists of a velocity-space integral, which is simplified here. Since the equilibrium distribution depends only on the magnitude of the velocity, the system lacks a preferred direction, allowing us to consider $\boldsymbol{k}$ along the flow direction for simplicity. Physically, this reflects the fact that only the component of particle motion along the propagation direction contributes to the wave-particle interaction, as in Landau damping. Thus, to solve this complex integral taking $\boldsymbol{k} \cdot \boldsymbol{v} = k v_\parallel$ and $d^3\boldsymbol{v} = 2\pi\, v_\perp\, dv_\perp\, dv_\parallel$. The integrand factor, depending only on $v^2 = v_\parallel^2 + v_\perp^2$, allows the integral of $v_\perp$ to be done first.

Let us consider

$$J_\perp(v_\parallel) = \int_0^\infty v_\perp \left(1 + \frac{v_\parallel^2 + v_\perp^2}{a^2}\right)^{-(\kappa+2)} dv_\perp. \tag{A1}$$

With $u = v_\perp^2$, $du = 2v_\perp dv_\perp$, and we get

$$\begin{aligned} J_\perp &= \frac{1}{2}\int_0^\infty \left(1 + \frac{v_\parallel^2 + u}{a^2}\right)^{-(\kappa+2)} du \\ &= \frac{a^2}{2(\kappa+1)}\left(1 + \frac{v_\parallel^2}{a^2}\right)^{-(\kappa+1)}. \end{aligned} \tag{A2}$$

Now we collapse the three-dimensional integral in Eq. (12) into a one-dimensional integral. Let us start from

$$I = \int \frac{\frac{\Gamma(\kappa+1)(\kappa+1)}{\Gamma(\kappa-\frac{1}{2})}\left(1 + \frac{v^2}{a^2}\right)^{-(\kappa+2)}}{kv_\parallel - \omega}\, d^3\boldsymbol{v}. \tag{A3}$$

Using the $v_\perp$ result of Eq. (A1) and the 2π angular factor in Eq. (A3) gives

$$I = 2\pi \frac{\Gamma(\kappa+1)(\kappa+1)}{\Gamma\left(\kappa - \frac{1}{2}\right)} \int_{-\infty}^\infty \frac{J_\perp(v_\parallel)}{kv_\parallel - \omega}\, dv_\parallel. \tag{A4}$$

Substituting the value of $J_\perp$ from Eq. (A2), we get

$$I = \frac{\pi a^2 \Gamma(\kappa+1)}{\Gamma\left(\kappa - \frac{1}{2}\right)} \int_{-\infty}^\infty \frac{(1 + v_\parallel^2/a^2)^{-(\kappa+1)}}{kv_\parallel - \omega}\, dv_\parallel. \tag{A5}$$

Let us now write the above integral in dimensionless form, putting $x = v_\parallel/a$ and $z = \omega/ka$. Then $dv_\parallel = a\,dx$ and $kv_\parallel - \omega = ka(x - z)$. Hence,

$$I = \frac{\pi a^2 \Gamma(\kappa+1)}{k\,\Gamma\left(\kappa - \frac{1}{2}\right)} Z_\kappa(z), \tag{A6}$$

where the one-dimensional integral, as the kappa plasma dispersion function, is given by

$$Z_\kappa(z) = \int_{-\infty}^\infty x \frac{(1 + x^2)^{-(\kappa+1)}}{x - z}\, dx. \tag{A7}$$

APPENDIX B: SOLUTION OF THE FUNCTION $Z_\kappa(z)$

The function $Z_\kappa(z)$, in Eq. (13) is solved using the identity

$$\frac{x}{x - z} = 1 + \frac{z}{x - z}.$$

Then the integral becomes

$$Z_\kappa(z) = \underbrace{\int_{-\infty}^\infty (1 + x^2)^{-(\kappa+1)} dx}_{I_0} + z \underbrace{\int_{-\infty}^\infty \frac{(1 + x^2)^{-(\kappa+1)}}{x - z} dx}_{I_1}. \tag{B1}$$

The solution of the first integration yields

$$I_0 = \sqrt{\pi}\, \frac{\Gamma\left(\kappa + \frac{1}{2}\right)}{\Gamma(\kappa+1)}. \tag{B2}$$

The second integral, which defines κ plasma dispersion function, is simplified by defining

$$\mathcal{Z}_\kappa(z) = \frac{1}{\sqrt{\pi}} \frac{\Gamma(\kappa+1)}{\Gamma\left(\kappa + \frac{1}{2}\right)} \int_{-\infty}^\infty \frac{(1 + x^2)^{-(\kappa+1)}}{x - z} dx. \tag{B3}$$

Therefore,

$$I_1 = \sqrt{\pi}\, \frac{\Gamma\left(\kappa + \frac{1}{2}\right)}{\Gamma(\kappa+1)} \mathcal{Z}_\kappa(z). \tag{B4}$$

In this way, the final form of $Z_\kappa(z)$ is obtained as

$$Z_\kappa(z) = \sqrt{\pi}\, \frac{\Gamma\left(\kappa + \frac{1}{2}\right)}{\Gamma(\kappa+1)} [1 + z\,\mathcal{Z}_\kappa(z)]. \tag{B5}$$

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