Letter

Imprecise Maxwell's demon with feedback delay: An exactly solvable information engine model

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A finite cycle time information engine based on a two-level system in contact with a thermal bath is studied analytically. The model for the engine incorporates an error in measuring the system's state and time delay between the measurement and the feedback process. The efficiency and power of the engine in steady state are derived as a function of level spacing, feedback delay time, engine cycle time, and measurement error. For a fixed value of level spacing and feedback delay, there is an upper bound on measurement error such that the engine can extract positive work. This threshold value of error is found to be independent of the cycle time. For a range of values of level spacing, feedback delay time and cycle time, efficiency has a non-monotonic dependence on the measurement error, implying that there is an optimal measurement error for the information engine to operate efficiently. The range of feedback delays within which the system functions as an engine, extracting positive work, depends on the measurement error and the level spacing.

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Maxwell's demon-like setups facilitate heat extraction from a thermal bath and its conversion into useful work through feedback control [1]. In the Szilard engine implementation of the Maxwell demon concept, the feedback control operates as follows: the demon measures whether a single molecule within a vessel, in contact with the thermal bath, is located on the left or right half of the vessel. This information is then utilized to extract work by inserting a partition into the box and a subsequent isothermal expansion of the volume containing the particle [2]. In the first analysis, this engine seems to violate the second law of thermodynamics. However, after nearly half a century of debates and discussions, it is now agreed that work can be extracted from such a system without contravening the second law of thermodynamics as long as the energy cost of the information processing performed by the demon is duly considered [1,3–6]. (For alternative perspectives on this debate, refer to the following sources [7–11].) Commonly referred to as information engines, they have been experimentally realized in various classical [12–15] and quantum [16–21] systems.

Two modifications can be considered for a practical engine assessment: (i) introducing a delay between measurement and feedback, and (ii) addressing errors in the measurement process. Regarding the Szilard engine version of Maxwell's demon, a feedback delay time implies a time lag between measuring the particle's position in the box and inserting the partition. In any experimental implementation of Maxwell's demon, it is a challenging task to achieve instantaneous feedback following the measurement [14]. Therefore, accounting for the delay between measurement and feedback in a practical information engine context is essential. Given this experimental constraint, it becomes imperative to investigate how the engine's capacity for work extraction and efficiency depends on the feedback delay. In the context of an information engine, efficiency quantifies the degree to which information is converted into work. Such an analysis helps determine the permissible range of feedback delay for the system to effectively utilize the information and operate as an engine [12,16]. As with feedback delay, measurement errors are unavoidable in practice [22–24]. Moreover, there is a cost associated with running more precise engines, which has to be kept in mind while optimizing the efficiency of the engine as a whole. Thus, it is essential to consider how the accuracy of the measurement influences the information engine's performance parameters.

Many recent works, both in experiments [14,25,26] and in theories [22,27,28], have looked at ways to improve the efficiency and power of information engines. Information engines based on colloidal particles moving through a harmonic potential [15,29,30] and periodic potentials [12,31] have been studied. These studies consider the possibility of extracting work or converting the information about the particle's position into work with the help of a feedback scheme. The simplest model that can be studied to investigate the nonequilibrium dynamics of an information engine is based on a two-level system. The efficiency and power of such a two-level information engine without feedback delay have been studied analytically in the limit of infinite cycles and relaxation time [23]. An analytical study of a similar engine based on a two-level system incorporating feedback delay but without error has recently been done [32]. The present work is an analytic study of the most general information engine of this type, which incorporates both feedback delay and measurement error. Further, the results are derived for the engine working with a finite cycle time, rather than assuming equilibrium at the beginning of each cycle. Under the assumption of a nonequilibrium steady state, the engine's power and efficiency are derived, and their dependence on cycle time, feedback delay time, measurement error, and the energy difference between the levels is studied.

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The model. The information engine consists of a system of two states, with energies $+U_0$ (the up-state) and $-U_0$ (the down-state). The system is in contact with a thermal bath at temperature T. In addition, a demon measures the system's state at regular intervals of time, $t = n\alpha$ (n an integer), where α is the cycle time. The demon initiates feedback depending on the outcome of the measurement. The feedback protocol follows: If the system is measured in the up-state, the demon switches the system's state to the down-state at a time $t = n\alpha + \epsilon$, extracting work equal to $2U_0$ in the process. ϵ is the feedback delay time and is less than the cycle time α. If the system is determined to be in the down-state during the measurement, the demon carries out no feedback process.

The measurement carried out by the demon is prone to error—i.e., there is a chance that during the measurement process, the demon measures the state to be down when the actual state of the system is up, and vice versa. We assume that the demon's error is such that the conditional probabilities are $P(M = d|X = u) = \delta_a$ and $P(M = u|X = d) = \delta_b$, where the variable X represents the actual state of the system, and the variable M represents the measurement outcome. u and d denote the up- and down-states, respectively. It is assumed that $\delta_a = \delta_b \equiv \delta$ for simplicity. It follows that $P(M = d|X = d) = P(M = u|X = u) = 1 - \delta$. It should be noted that the demon will initiate the feedback depending on the measurement outcome and not on the actual state of the system. Also, even with no measurement error and no feedback delay, this information engine does not yield 100% efficiency. This is because, for finite U_0, the post-feedback distribution of the system is not the equilibrium distribution [3,33].

The master equation for the process is given by

$$\frac{dp_u}{dt} = -(k_1 + k_2)p_u + k_2 , \tag{1}$$

where p_u is the probability of finding the system in the up-state. k_1 is the transition rate from up-state to down-state, and k_2 is the transition rate from down-state to up-state. The probability of the system being in the down-state is given by $p_d = 1 - p_u$. A detailed balance in equilibrium stipulates $\frac{k_1}{k_2} = e^{2\beta U_0}$. The relaxation time for the system to reach equilibrium starting from an arbitrary initial state is given by $\tau = \frac{1}{k_1+k_2}$. When the measurement outcome is up-state, the master equation must be integrated in two time segments: from $t = n\alpha$ to $t = n\alpha + \epsilon$, and then from $t = n\alpha + \epsilon$ to $t = (n + 1)\alpha$. This is because the state will be switched after a delay time of ϵ for this measurement outcome. The information engine will reach a steady state after many cycles of operation. The steady-state probability is derived below. From this, one can obtain the average work extracted and the average information processed during measurement, which are then used to determine the efficiency and power of the information engine.

The steady state solution. Let $p_u^{ss}(t)$ be the steady-state probability of the system to be in the up-state at time t. Then, for $\epsilon < t \leqslant \alpha$,

$$\begin{aligned} p_u^{ss}(t) = &\sum_{X'}[P_2(u;t|X';0)P_0(X',d)] \\ &+ \sum_{X',X''}[P_2(u;t|X'';\epsilon^+)P_1(X''|X')P_0(X',u)], \end{aligned} \tag{2}$$

where $P_0(X, M)$ is the joint probability for X (state) and M (measurement outcome) at the time of measurement ($t = 0$). $P_1(X''|X')$ is the probability for the state to be X'' at $t = \epsilon^+$ (i.e., right after feedback), given that the state was X' at $t = 0$ and the measurement outcome was $M = u$. $P_2(X\,; t_2\,|X'\,; t_1)$ is the probability of finding the system in state X at $t = t_2$, given that it was in X' at $t = t_1$ with no interference from the demon in the interval between t_1 and t_2 and is given by

$$P_2(X;t_2|X';t_1) = p^{\text{eq}}(X')(1 - e^{-(t_2-t_1)/\tau}) + \tilde{\delta}_{XX'}\, e^{-(t_2-t_1)/\tau} . \tag{3}$$

Here, $\tilde{\delta}_{XX'}$ is the Kronecker delta function and $p^{\text{eq}}(X')$ is the equilibrium distribution without feedback. For $X = u$,

$$p^{\text{eq}}(X = u) = \frac{e^{-\beta U_0}}{2\cosh(\beta U_0)} \equiv p_u^{\text{eq}} , \tag{4}$$

and $p^{\text{eq}}(X = d) = 1 - p_u^{\text{eq}} \equiv p_d^{\text{eq}}$. Table I in the Supplemental Material [34] gives the set of various joint and conditional probabilities appearing in Eq. (2), where $p_{u0}^{ss} \equiv p_u^{ss}(t = 0)$ and $p_{d0}^{ss} \equiv 1 - p_{u0}^{ss}$.

Since $p_u^{ss}(t)$ is a periodic function with period α, p_{u0}^{ss} must be the same as $p_u^{ss}(\alpha)$. Using this fact in Eq. (2), one gets a self-consistent relation for p_{u0}^{ss} (see the Supplement). Solving for p_{u0}^{ss},

$$p_{u0}^{ss} = \frac{p_u^{\text{eq}}[1 + e^{-\alpha/\tau}(2\delta - 1) - 2\delta e^{-(\alpha-\epsilon)/\tau}] + \delta e^{-(\alpha-\epsilon)/\tau}}{1 + (2\delta - 1)\big(2p_u^{\text{eq}} - 1\big)[e^{-\alpha/\tau} - e^{-(\alpha-\epsilon)/\tau}]}. \tag{5}$$

The units are chosen such that $k_B T = 1$ and the relaxation time scale of the system is $\tau = 1$. Once p_{u0}^{ss} is known, one can use Eq. (2) to find $p_u^{ss}(t)$ for $\epsilon < t \leqslant \alpha$. For $0 < t \leqslant \epsilon$, $p_u^{ss}(t)$ can be found using the relation $p_u^{ss}(t) = \sum_{X',M} P_2(X = u, t|X'; t = 0)\, P_0(X', M)$.

The average work extracted in the steady state by the information engine is given by

$$\begin{aligned} -\langle W\rangle = &\, p_{u0}^{ss}(1 - \delta)[2U_0\tilde{p} - 2U_0(1 - \tilde{p})] \\ &+ p_{d0}^{ss}\delta[2U_0(1 - \tilde{p}') - 2U_0\tilde{p}'], \end{aligned} \tag{6}$$

where

$$\tilde{p} \equiv e^{-\epsilon/\tau}\big(1 - p_u^{\text{eq}}\big) + p_u^{\text{eq}} \tag{7}$$

is the conditional probability, $P_2(X = u; t = \epsilon^-|X' = u; t = 0)$. That is, given that the system's state is up at the beginning of the cycle, $\tilde{p}$ is the probability that it is in up-state just before $t = \epsilon$. Similarly,

$$\tilde{p}' \equiv e^{-\epsilon/\tau} p_u^{\text{eq}} + \big(1 - p_u^{\text{eq}}\big) \tag{8}$$

is the conditional probability $P_2(X = d, t = \epsilon^-|X = d, t = 0)$. The two terms on the right-hand side of Eq. (6) correspond to $X = u$ at the beginning of the cycle with measurement being correct and $X = d$ at the beginning of the cycle with the measurement being incorrect, respectively.

The power of the information engine is the average rate of work extracted and is given by

$$\Theta \equiv \frac{-\langle W\rangle}{\alpha} = \frac{2U_0 p_{u0}^{ss}(1 - \delta)(2\tilde{p} - 1) - 2U_0 p_{d0}^{ss}\delta(2\tilde{p}' - 1)}{\alpha} , \tag{9}$$

where the numerator is the simplified form of Eq. (6). The other important performance parameter of an engine is its efficiency. Although the demon succeeds in rectifying thermal fluctuations to do work, it comes at the cost of information processing. The thermodynamic cost of the erasure of memory bits associated with the measurement is given by k_BTI, where I is the quantity of information obtained. In the context of an information engine, efficiency is defined as the ratio of average work extracted to the average information processing cost. That is,

$$\eta = \frac{-\langle W\rangle}{k_BT\langle I\rangle}\,, \tag{10}$$

where $\langle I\rangle$ is the average information gathered during a cycle. If the states of the system at the beginning and end of the cycle were not correlated (which would be the case if $\alpha - \epsilon \gg 1$), the information cost would be determined by the average mutual information [35]

$$\langle I_a\rangle = \sum_{M,X} P_0(X,M)\ln\frac{P(M|X)}{P(M)}\,, \tag{11}$$

where $P(M|X)$ is the probability that the measured state of the system is M, given that the actual state of the system is X, and $P(M)$ is the probability for the measurement giving M. One can use the probabilities given in Table II in the Supplement to evaluate $\langle I_a\rangle$, found to be

$$\langle I_a\rangle = \Delta\ln\left(\frac{1-\Delta}{\Delta}\right) - \delta\ln\left(\frac{1-\delta}{\delta}\right) + \ln\left(\frac{1-\delta}{1-\Delta}\right), \tag{12}$$

where $\Delta = p_{u0}^{ss} + \delta - 2p_{u0}^{ss}\,\delta$ (see the Supplement).

Since the information engine's cycle time is finite, the states at the beginning and end of the cycle will be correlated. This implies that the mutual information between the state of the system at time $t = n\alpha$ and the measurement outcome of the system's state at $t = (n-1)\alpha$ will generally be nonzero [36]. This mutual information is given by $I_b(X, M_{pc}) = \ln\frac{P(X|M_{pc})}{P(X)}$, where X and M_{pc} denote the state of the system in the current cycle and the measurement outcome in the previous cycle, respectively. $P(X|M_{pc})$ is the probability that the state of the system in the current cycle is X, given that the measurement outcome of the previous cycle was M_{pc}. Averaging I_b over the joint distribution $P(X, M_{pc})$ gives us

$$\langle I_b\rangle = \sum_{M_{pc},X} P(X,M_{pc})\ln\frac{P(X|M_{pc})}{P(X)}. \tag{13}$$

The details of this calculation, as well as the final form of $\langle I_b\rangle$, are given in the Supplement. The minimum average cost of running the information engine would be proportional to the difference between $\langle I_a\rangle$ and $\langle I_b\rangle$. Assuming $\langle I_a\rangle$ to be larger than $\langle I_b\rangle$, the minimal cost is given by

$$k_BT\langle I\rangle = k_BT(\langle I_a\rangle - \langle I_b\rangle)\,. \tag{14}$$

Using the definition of efficiency given in Eq. (10) and substituting for the average work extracted, as well as the information cost found above,

$$\eta = \frac{2U_0\;p_{u0}^{ss}(1-\delta)(2\tilde{p}-1) - 2U_0\;p_{d0}^{ss}\;\delta(2\tilde{p}'-1)}{k_BT\langle I\rangle}\,. \tag{15}$$

Thus, the power and efficiency of the information engine in the steady state have been found in terms of the four key engine parameters: cycle time (α), measurement error (δ), the energy difference between levels ($2U_0$), and the feedback delay time (ϵ). There is ample parameter space to explore, and some of the most interesting results are presented in the rest of the paper. Since the dependence of efficiency and work done per cycle on U_0 and ϵ (for infinite cycle time and zero error in measurement) has been discussed in a recent work [32]. The focus here is on the other two design parameters: α and δ.

Working regimes of the engine. The work extracted, $(-\langle W\rangle)$, should be positive for the demon to function as an engine. $-\langle W\rangle = 0$ gives us the boundary surface in the four-dimensional parameter space that defines the working regime of the engine. Since δ is a parameter that affects work extraction adversely, $-\langle W\rangle$ will decrease as δ increases. The maximum possible δ that the demon can tolerate and still extract positive work (see the Supplement) is given by

$$\delta_{\max} = \frac{p_u^{\rm eq}(2\tilde{p}-1)}{1-4p_u^{\rm eq}(1-\tilde{p})}, \tag{16}$$

which is independent of the cycle time α. For $U_0 \to \infty$, $p_u^{\rm eq} \to 0$ and $\delta_{\max} \to 0$. When $U_0 \to 0$, $p_u^{\rm eq} \to \frac{1}{2}$ and $\delta_{\max} \to \frac{1}{2}$. This suggests that, with a fixed value of U_0, precise measurement is crucial for extracting positive work at low temperatures, while it becomes less critical at high temperatures. The values of U_0 and ϵ for which $\delta_{\max} < 0$ correspond to the situation where the demon cannot extract positive work even without measurement error.

Power and efficiency behavior. The dependence of power and efficiency on various engine design parameters is shown in Figs. 1 and 2. We list below some of the key observations (a more detailed study of the parameter space with discussions will be presented in a long paper):

(i) For the cases where the device works as an engine ($\Theta > 0$), the power decreases with increasing cycle time [see Fig. 1(a)]. Even though the work extracted per cycle increases with cycle time, the engine's power is compromised for longer cycle times because the increase in work grows sub-linearly with cycle time.

(ii) Unlike power, efficiency increases with cycle time [see Fig. 2(a)].

(iii) For large values of U_0 (or an equivalent at low temperatures), even a small error in the measurement leads to negative values for power. As seen in Fig. 1(b), zero crossing of power happens at smaller δ values for larger U_0. This behavior is true of efficiency as well, as is seen by the abrupt shift of η to negative values when δ is increased from 0 to 0.1 with $\epsilon = 0.1$ and $U_0 = 1.1$ [see Fig. 2(a)].

(iv) The power exhibits a non-monotonic dependence on U_0. This behavior is evident from the variation of Θ along the line AB in Fig. 1(c), which represents the values of power in the $U_0 - \epsilon$ plane for $\alpha = 3$ and $\delta = 0.05$.

(v) The range of ϵ over which the engine extracts positive work decreases with increasing U_0. This is seen in Fig. 1(c), where the $\Theta = 0$ curve in $U_0 - \epsilon$ has negative slopes everywhere. This feature has also been observed in an error-free information engine with a cycle time much larger than the relaxation time [32]. This property of the engine is due to the reduced resident time in the up-state with larger U_0 values.

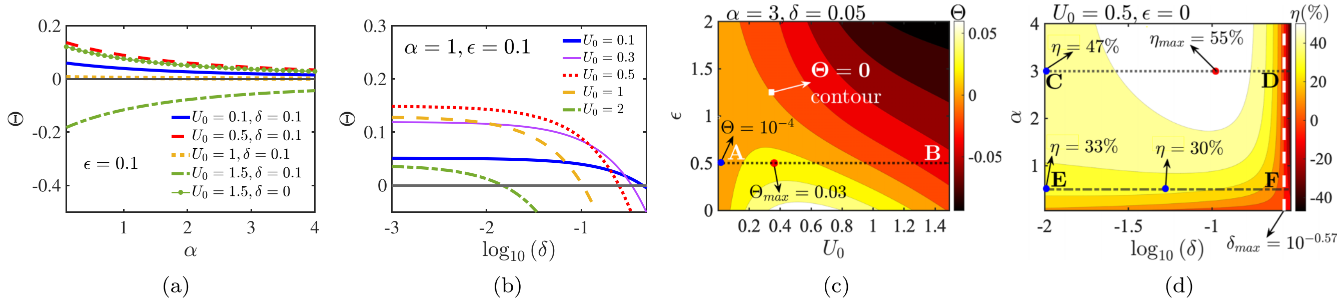

FIG. 1. Variations of power (Θ) and efficiency (η) with various engine design parameters are shown. (a) Θ decreases monotonically with cycle time (α) in the regimes where the system works as an engine. (b) As expected, Θ reduces with increasing measurement error (δ). At low values of level spacing ($2U_0$), the engine can function with more significant errors. (c) Variations of Θ in the $\epsilon - U_0$ plane for the case when $\delta = 0.05$ and $\alpha = 3$. For fixed ϵ, there is a particular value of U_0 at which power is a maximum. (d) Dependence of η on α and δ for $U_0 = 0.5$ and $\epsilon = 0$. For large enough values of α, η is maximum at an intermediate value of δ.

(vi) For low values of U_0, efficiency has a non-monotonic dependence on δ, as can be seen in the cases $U_0 = 0.1$, $U_0 = 0.2$, and $U_0 = 0.5$ [solid blue curve, thin purple curve, and dashed red curve, respectively, in Fig. 2(b)]. This means the measurement should have a non-zero error for optimal performance.

(vii) The non-monotonic variation of η in δ depends also on the cycle time, as seen in Fig. 1(d), where η is given in the $\alpha - \delta$ plane for $U_0 = 0.5$ and $\epsilon = 0$. For example, when $\alpha = 3$ (line CD), η has a maximum at an intermediate δ value. But for $\alpha = 0.5$ (line EF), η decreases with δ.

In an experimental setup, there is bound to be a limit on the minimum value possible for delay time dictated by the implementation of the feedback process. Similarly, some errors would be unavoidable in the measurement process. In Fig. 3, a comparison is made of the power and efficiency in the $\alpha - U_0$ plane between an ideal scenario, where $\epsilon = 0$ and $\delta = 0$, and a more practical situation, where $\epsilon = 0.5$ and $\delta = 0.1$ are assumed. As expected, both performance parameters decrease with non-zero ϵ and δ. The parameters for maximal power with efficiency being at least 20% of the maximum efficiency for both cases are highlighted (blue diamonds in Fig. 3). Similarly, the parameters for maximal efficiency with power being at least 20% of the maximum power available for both cases are marked (violet squares in Fig. 3). Additionally, parameter values for which the power and efficiency are not significantly compromised are indicated (green circles in Fig. 3), with both power and efficiency being at least 50% of their maximum values for the corresponding ϵ and δ values. With the introduction of error and delay times, one observes appreciable changes in the operating points of the information engine.

Summary. In this work, Maxwell's demon based on a two-level system has been studied. To the best of our knowledge, this is the first time that the performance parameters of an

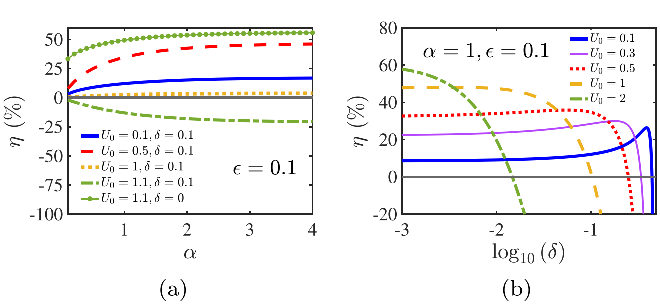

FIG. 2. Dependence of efficiency (η) on various engine design parameters is shown. (a) η shows a monotonic increase with cycle time (α) in the regimes where the system works as an engine. (b) η has a non-monotonic dependence on the measurement error (δ) at low values of level spacing ($2U_0$), implying that at high temperatures, maximal efficiency is obtained by working with an imprecise measurement.

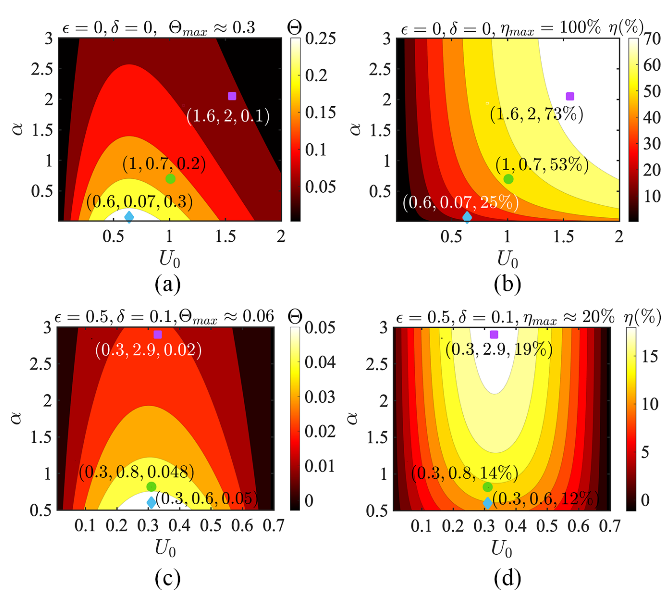

FIG. 3. Contour plots for power (Θ) and efficiency (η) in the $U_0 - \alpha$ plane for the case of no feedback delay time (ϵ) and no measurement error (δ) [(a) & (b), respectively] and with $\epsilon = 0.5$ and $\delta = 0.1$ [(c) & (d) respectively]. The locations marked by violet squares (blue diamonds) give us the choice of α and U_0 for maximum efficiency (power) with power (efficiency) being at least 20% of its maximum value. The green circle indicates the choice of α and U_0 for performance that does not compromise either efficiency or power. The values given in brackets are the locations of the points and the corresponding power (efficiency) value ($U_0, \alpha, \Theta[\eta]$).

information engine that incorporates feedback delay time and measurement error and operates in finite cycle time have been solved analytically. The closed-form expression for power and efficiency allows for complete engine characterization. Note that the model does not employ a reversible protocol, which implies heat dissipation between the system and the bath [3]. Some of the interesting consequences have been highlighted, including the non-monotonic dependence of efficiency on the error in measurement at high temperatures and the independence of cycle time on the engine's error tolerance. The efficiency calculation assumes the optimal use of information, accounting for the correlation between the measurement and the state across consecutive cycles. However, the method to achieve this is not explicitly discussed. One possible approach is to use Bayesian inference methods, as described in [37]. The predictions of this study are verifiable in various experimental systems already implemented. Specifically, this work provides a framework for a class of experimental realizations of information engines, such as those based on colloidal particles [12], single-electron boxes (SEBs) [16,38], and quantum Maxwell's demons using superconducting qubits [39]. These experimental systems inherently operate on principles similar to the two-level model studied here, making them candidates for both validating the predictions and exploring broader applications of the framework.

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[1] K. Maruyama, F. Nori, and V. Vedral, Rev. Mod. Phys. **81**, 1 (2009).
[2] L. Szilard, Behavioral Science **9**, 301 (1964).
[3] J. M. R. Parrondo, J. M. Horowitz, and T. Sagawa, Nat. Phys. **11**, 131 (2015).
[4] C. H. Bennett, Int. J. Theor. Phys. **21**, 905 (1982).
[5] T. Sagawa and M. Ueda, Phys. Rev. Lett. **102**, 250602 (2009).
[6] H. Leff and A. F. Rex, *Maxwell's Demon 2 Entropy, Classical and Quantum Information, Computing* (CRC Press, Boca Raton, Florida, 2002).
[7] J. Earman and J. D. Norton, Stud. Hist. Philos. Sci. Part B: Stud. **29**, 435 (1998).
[8] J. Earman and J. D. Norton, Stud. Hist. Philos. Sci. Part B: Stud. **30**, 1 (1999).
[9] M. Hemmo and O. Shenker, J. Philos. **107**, 389 (2010).
[10] J. D. Norton, Stud. Hist. Philos. Sci. Part B: Stud. **42**, 184 (2011).
[11] L. B. Kish and C.-G. Granqvist, Europhys. Lett. **98**, 68001 (2012).
[12] S. Toyabe, T. Sagawa, M. Ueda, E. Muneyuki, and M. Sano, Nat. Phys. **6**, 988 (2010).
[13] A. Bérut, A. Arakelyan, A. Petrosyan, S. Ciliberto, R. Dillenschneider, and E. Lutz, Nature (London) **483**, 187 (2012).
[14] T. K. Saha, J. N. E. Lucero, J. Ehrich, D. A. Sivak, and J. Bechhoefer, Proc. Natl. Acad. Sci. **118**, e2023356118 (2021).
[15] G. Paneru, D. Y. Lee, T. Tlusty, and H. K. Pak, Phys. Rev. Lett. **120**, 020601 (2018).
[16] J. V. Koski, V. F. Maisi, T. Sagawa, and J. P. Pekola, Phys. Rev. Lett. **113**, 030601 (2014).
[17] B.-L. Najera-Santos, P. A. Camati, V. Métillon, M. Brune, J.-M. Raimond, A. Auffèves, and I. Dotsenko, Phys. Rev. Res. **2**, 032025(R) (2020).
[18] D. V. Averin, M. Möttönen, and J. P. Pekola, Phys. Rev. B **84**, 245448 (2011).
[19] P. A. Camati, J. P. S. Peterson, T. B. Batalhão, K. Micadei, A. M. Souza, R. S. Sarthour, I. S. Oliveira, and R. M. Serra, Phys. Rev. Lett. **117**, 240502 (2016).
[20] M. Naghiloo, J. J. Alonso, A. Romito, E. Lutz, and K. W. Murch, Phys. Rev. Lett. **121**, 030604 (2018).
[21] K. Chida, S. Desai, K. Nishiguchi, and A. Fujiwara, Nat. Commun. **8**, 15301 (2017).
[22] L. Dinis and J. M. R. Parrondo, Entropy **23**, 8 (2021).
[23] J. Um, H. Hinrichsen, C. Kwon, and H. Park, New J. Phys. **17**, 085001 (2015).
[24] J. Hoppenau and A. Engel, Europhys. Lett. **105**, 50002 (2014).
[25] G. Paneru, D. Y. Lee, J.-M. Park, J. T. Park, J. D. Noh, and H. K. Pak, Phys. Rev. E **98**, 052119 (2018).
[26] M. Rico-Pasto, R. K. Schmitt, M. Ribezzi-Crivellari, J. M. R. Parrondo, H. Linke, J. Johansson, and F. Ritort, Phys. Rev. X **11**, 031052 (2021).
[27] J. N. E. Lucero, J. Ehrich, J. Bechhoefer, and D. A. Sivak, Phys. Rev. E **104**, 044122 (2021).
[28] P. S. Pal, S. Rana, A. Saha, and A. M. Jayannavar, Phys. Rev. E **90**, 022143 (2014).
[29] D. Abreu and U. Seifert, Europhys. Lett. **94**, 10001 (2011).
[30] M. Bauer, D. Abreu, and U. Seifert, J. Phys. A: Math. Theor. **45**, 162001 (2012).
[31] K. V. and T. Joseph, Phys. Rev. E **106**, 054146 (2022).
[32] K. V. and T. Joseph, Phys. Rev. E **109**, 034121 (2024).
[33] J. M. Horowitz and J. M. R. Parrondo, New J. Phys. **13**, 123019 (2011).
[34] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevE.111.L042104 for detailed analytical derivations, additional results, and supporting figures.
[35] T. M. Cover and J. A. Thomas, *Elements of Information Theory* (Wiley, New York, 1991).
[36] T. Sagawa and M. Ueda, Phys. Rev. Lett. **109**, 180602 (2012).
[37] D. A. S. Johan du Buisson and J. Bechhoefer, Adv. Phys.: X **9**, 2352112 (2024).
[38] J. V. Koski, V. F. Maisi, J. P. Pekola, and D. V. Averin, Proc. Natl. Acad. Sci. **111**, 13786 (2014).
[39] N. Cottet, S. Jezouin, L. Bretheau, P. Campagne-Ibarcq, Q. Ficheux, J. Anders, A. Auffèves, R. Azouit, P. Rouchon, and B. Huard, Proc. Natl. Acad. Sci. **114**, 7561 (2017).