

Thermal effects and finite-temperature cosmology in perturbatively stabilized large volume scenarios

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We analyze finite-temperature effects in the perturbative large volume scenario (LVS), examining the dynamics of thermalized Kähler moduli and the stability of thermally induced vacua. We determine the maximum decompactification temperature T_{max} , showing its dependence on winding-loop corrections in the effective theory. Moduli stabilization arises from perturbative logarithmic loop corrections to the Kähler potential, with higher-derivative F^4 terms used to test vacuum robustness. We derive cosmological implications, including bounds on reheating, and show that the model favors high scale inflationary scenarios with the heavy modulus decaying before dominating the energy density of the universe. We also analyze thermal metastability, demonstrating that the recovery of the $T = 0$ vacuum is sensitive to the postinflationary thermal history, potentially yielding either an Anti-de Sitter vacuum, a metastable phase, or a stable de-Sitter configuration. We conclude by briefly discussing the relevance of finite-temperature effects for entropy-assisted tunneling and the broader string landscape.

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I. INTRODUCTION

A long-standing problem of string phenomenology is the determination of dynamics of the uncharged massless fields, called moduli. Over the decades, significant advances have been made in tackling this problem. In type IIA string theory, turning on background fluxes can fix all geometric moduli [1–4], despite the fact that the mathematical study of fluxes is much more complicated compared to type IIB theory. Type IIB compactifications require additional contributions from both perturbative and nonperturbative effects [5,6]. In comparison, this scheme allows for a large set of solutions where flux backreaction is mostly captured by a warp factor, making moduli stabilization better understood in IIB Calabi-Yau flux compactifications. A prominent example of successful IIB moduli stabilization is the large volume scenario (LVS) introduced in [6], which has applications in both cosmology and particle physics. In this framework [7], α' and g_s corrections combine together with nonperturbative effects to stabilize the Kähler moduli, while background fluxes stabilize the dilaton and complex structure moduli. Unlike the KKLT approach [5], LVS does not require fine-tuning of flux values, and the Calabi-Yau volume is stabilized on

an exponentially large scale. This setup provides a trustworthy four-dimensional effective theory and allows for creation of hierarchies relevant for phenomenology.

Recently, several arguments have questioned the validity of constructing de Sitter vacua within a 4D effective supergravity framework. These concerns have contributed to the formulation of the swampland de Sitter conjecture [8–10]. Both of the aforementioned approaches in constructing stable vacua admit an Anti-de Sitter (AdS) vacuum, while a de-Sitter (dS) minimum emerges with a supersymmetric minimum at infinity after the uplifting. These two vacua are separated by a barrier V_b , whose magnitude is specified by the scale of the AdS point. Most swampland related criticisms focus on the reliability of nonperturbative effects, the role of uplift mechanisms involving anti-D3-branes, and the precision of quantum corrections. The modulus controlling the overall volume of the Calabi-Yau (CY) manifold interacts with all sources of energy due to the Weyl rescaling required to obtain the four-dimensional supergravity effective action in the Einstein frame. Consequently, any energy contribution that exceeds the height of the potential barrier can push the system toward decompactification. For example, during inflation, the energy of the inflaton may generate an additional uplift term, potentially causing an infinite volume runaway [11]. Similarly, after inflation, inflaton decay produces a high-temperature thermal plasma, leading to temperature-dependent corrections to the moduli potential, which could destabilize the moduli if they exhibit runaway behavior. The temperature at which the finite-temperature effects [12–17] dominate over the zero-temperature potential, i.e., the decompactification temperature, is approximately of

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order $T_{\max} \sim V_b^{1/4}$, with V_b denoting the height of the potential, and sets an upper bound on the reheating temperature. On the other hand, if the finite-temperature potential develops new minima rather than running away, the system may undergo phase transitions [18–21], potentially affecting early universe evolution and leaving observable imprints today. High-temperature minima could also influence the likelihood of the universe settling in a metastable state at $T = 0$. Studies of several toy models [22–27] suggest that despite the existence of a supersymmetric global minimum, systems starting in a high-temperature minimum often end up in a long-lived metastable state with broken supersymmetry at low temperatures. If analogous behavior occurs in realistic scenarios, it could provide conceptual insights into the present accelerated expansion of the universe.

In the current work, we examine the aforementioned questions under the perspective of perturbative moduli stabilization scheme [28–33] considering also higher derivative corrections and winding loop corrections. This combination results in a distinct hierarchy in the masses and couplings of the corresponding moduli, leading to thermalization of a direction transverse to the volume modulus as it is summarized in Sec. II. In Secs III and IV, the finite temperature effects augment the effective theory and their effects is to shift the dS vacuum to a new point in the moduli space, where we manage to provide analytic expression both for this dynamical phenomenon as well as the upper bound on temperature. To our knowledge, this is the first approach in the literature that unravels the association of $T_{\max}$ with winding loop effects and the uplifting mechanism, while we provide analytical evidence that the thermalized field decays before its energy density starts to overclose the universe. Finally in Sec. V, the potential thermal metastability of the underlying theory is discussed and a critical temperature is indicated, which signifies that Hawking-Moss (HM) [34] instanton solutions along the volume direction do not destabilize the thermal vacuum. Conversely, during the cooling of the universe, the reheating scale above this critical temperature could either induce a metastable dS augmented by an AdS vacuum—suggesting a possible first-order phase transition—or lead to a stable $T = 0$ vacuum via a continuous second-order-like transition. Finally, we offer some remarks on the plausible association between finite temperature effects and quantum-entropy-assisted phase transitions, highlighting their relevance with the fate metastable vacua of the string landscape.

II. THE STRUCTURE OF THE MODEL

In this paper, we consider an effective theory model within the type IIB string framework, where the dilaton and the complex structure moduli are stabilized by turning on 3-form fluxes. The Kähler potential receives corrections either from perturbative dynamics or from nonperturbative corrections in the superpotential. The well-known structures

of KKLT and LVS vacua incorporate nonperturbative corrections as the primary mechanism to stabilize the Kähler sector. The nonscale structure of effective theory is broken α' effects or considering g_s corrections in the Kähler potential. Apart from the BBHL α'^3 perturbative corrections [35], we currently consider 1-loop logarithmic effects scaling as $\sim \eta \log(\tau)$, stemming from intersecting D_7 -branes [36,37], where the final form of the Kähler potential in terms of the four-cycle moduli is written as,

$$\mathcal{K} = -2 \log \left(\sqrt{\tau_1 \tau_2 \tau_3} + \xi + \sum_i (\eta_i \log(\tau_i)) \right). \quad (2.1)$$

The form of the compactified space considered here is toroidal looking volume form and it follows from the proposal of [29,36], and the superpotential receives no nonperturbative corrections [38],

$$\mathcal{V} = \sqrt{\tau_1 \tau_2 \tau_3}, \quad (2.2)$$

$$\mathcal{W} = \mathcal{W}_0. \quad (2.3)$$

To validate the above proposal of the compactified space, one could revisit the Kreuzer-Skarke [39,40] database of CY's manifolds, where the three moduli case could descend from a manifold with $h^{1,1} = 3$. As pointed out in the analysis performed in [28], a manifold with the following toric data could potentially result into the proposed volume form of the current model. One such CY threefold corresponding to the polytope Id: 249 in the CY database of [40] can be defined by the following toric data:

Hyp	x_1	x_2	x_3	x_4	x_5	x_6	x_7
4	0	0	1	1	0	0	2
4	0	1	0	0	1	0	2
4	1	0	0	0	0	1	2
	$K3$	$K3$	$K3$	$K3$	$K3$	$K3$	SD

The Hodge numbers of the threefold are $(h^{2,1}, h^{1,1}) = (115, 3)$, the Euler number is defined by $\chi = -224$, where the Stanley-Reisner (SR) ideal is written as,

$$\text{SR} = \{x_1 x_6, x_2 x_5, x_3 x_4 x_7\}. \quad (2.4)$$

The structure of the divisor topologies has been analyzed with *cohomCalg* [41,42], where this framework can be represented by the following Hodge diamonds:

$$\begin{array}{ccccc}
 & & 1 & & 1 \\
 & 0 & & 0 & \\
 K3 \equiv 1 & 20 & 1, & SD \equiv 27 & 184 & 27. \\
 & 0 & & 0 & \\
 & & 1 & & 1
 \end{array} \quad (2.5)$$

The basis of smooth divisors $\{D_1, D_2, D_3\}$ define the following intersection polynomial, which admits only one nonzero triple intersection number:

$$I_3 = 2D_1D_2D_3, \quad (2.6)$$

while the second Chern class of the CY is given by,

$$c_2(CY) = 5D_3^2 + 12D_1D_2 + 12D_2D_3 + 12D_1D_3. \quad (2.7)$$

Afterward, the Kähler form can be derived through $J = \sum_{\alpha=1}^3 t^\alpha D_\alpha$, where the overall volume and the four-cycle volume moduli follow from,

$$\mathcal{V} = 2t^1 t^2 t^3, \quad \tau_1 = 2t^2 t^3, \quad \tau_2 = 2t^1 t^3, \quad \tau_3 = 2t^1 t^2. \quad (2.8)$$

Finally, the final form of the compactified volume could be expressed by the combination of the two cycles as,

$$\mathcal{V} = 2t^1 t^2 t^3 = t^1 \tau_1 = t^2 \tau_2 = t^3 \tau_3 = \frac{1}{\sqrt{2}} \sqrt{\tau_1 \tau_2 \tau_3}. \quad (2.9)$$

This validates the proposal for a toroidal like volume in the current model, where it possesses an exchange symmetry $1 \leftrightarrow 2 \leftrightarrow 3$ for the moduli descending from the exchange of the basis divisors $K3$. In the current work, we assume the simplified version of $\mathcal{V} = \sqrt{\tau_1 \tau_2 \tau_3}$ adapted in the aforementioned studies. Further, the Kähler cone for this setup is described by the conditions below,

$$\text{Kähler cone: } t^1 > 0, \quad t^2 > 0, \quad t^3 > 0. \quad (2.10)$$

The effective scalar potential can be expressed by the following form:

$$V_{\text{eff}} = e^{\mathcal{K}} \left(\sum_{I,J} D_I \mathcal{W} K^{IJ} D_J \bar{\mathcal{W}} - 3|W|^2 \right), \quad (2.11)$$

$$V_{\text{eff}} = \frac{3\mathcal{W}_0^2 g_s (-16\eta + \xi + 4\eta \log(\mathcal{V}))}{4\mathcal{V}^3} + \mathcal{O}\left(\frac{1}{\mathcal{V}^4}\right). \quad (2.12)$$

Additionally, subleading string loop effects can be classified to two categories: the ones derived from KK corrections and the other from winding loop effects [43–46]. The Kähler corrections of this origin are written as,

$$\mathcal{K}_{g_s}^{KK} = g_s \sum_{\alpha} \frac{C_{\alpha}^{KK} t_{\perp}^{\alpha}}{\mathcal{V}}, \quad \mathcal{K}_{g_s}^W = \sum_{\alpha} \frac{C_{\alpha}^W}{\mathcal{V} t_{\perp}^{\alpha}}, \quad (2.13)$$

where the $\mathcal{C}_i$ parameters are functions that depend on the moduli of the complex structure and the dynamics of open strings, while t_i define the form of two cycles of the underlying geometry. In particular, two-cycle volume

moduli $t_{\perp}^{\alpha}$ denote the transverse space between the D_7 -branes and the O_7 planes. In contrast, $t_{\parallel}^{\alpha}$ stands for the volume of the curve at the intersection point of intersecting D_7 -branes in the background geometry. Moreover, higher derivatives corrections could also be included, whose origin is due to $\mathcal{O}(F^4)$ and their form on a given Calabi-Yau compactification is expressed as [47],

$$V_{F^4} = -\kappa^2 \frac{\lambda |\mathcal{W}_0|^4}{g_s^{3/2} \mathcal{V}^4} \Pi_i t^i, \quad \kappa = \frac{g_s}{8\pi}, \quad (2.14)$$

where λ is a constant with no moduli dependence and Π_i are topological numbers defined by the harmonic 2-form of the compactification. In the current setup, we assume the existence of winding loop corrections along with higher derivative corrections, where the Kaluza-Klein (KK) effects can be neglected since they parametrize the distance among the stacks of D_7 -branes and O_7 planes, and it can be assumed that $C^{KK} \ll 1$. Then, the contribution to the scalar potential arising from the aforementioned subleading effects is given as,

$$V_{F^4} = -\frac{3\lambda(t_1 + t_2 + t_3)\mathcal{W}_0^4 \sqrt{g_s}}{8\pi^2 \mathcal{V}^4},$$

$$V_w = -\frac{\mathcal{W}_0^2 g_s \left(\frac{c_1}{t_1} + \frac{c_2}{t_2} + \frac{c_3}{t_3} \right)}{8\pi \mathcal{V}^3}, \quad (2.15)$$

$$t_1 = \sqrt{\frac{\tau_2 \tau_3}{2\tau_1}}, \quad t_2 = \sqrt{\frac{\tau_1 \tau_3}{2\tau_2}}, \quad t_3 = \sqrt{\frac{\tau_1 \tau_2}{2\tau_3}}. \quad (2.16)$$

Beyond the structure of the scalar potential at zero temperature, one-loop finite temperature effects parametrize the interactions between moduli and gauge bosons, which lead to thermalization and eventually to loop corrections of following form:

$$V_{\text{eff}} = V_{\text{eff}}(T=0) + V^{\text{loop}}(T). \quad (2.17)$$

These finite temperature corrections $V^{\text{loop}}(T)$ have a generic loop expansion at one-loop and two-loop level, which is described by [48–50],

$$V^{1\text{-loop}} = \frac{T^2}{24} \left(\text{Tr}[M_{\phi}^2] + \text{Tr}[M_{\psi}^2] \right) - \frac{\pi^2 T^4}{90} (g_b + g_f), \quad (2.18)$$

$$V^{2\text{-loop}} = T^4 \left(\kappa_1 g_{MSSM}^2 + \kappa_2 g_{\phi XX}^2 m_{\phi}^2 + \kappa_3 g_{\psi XX}^2 m_{\psi}^2 \right) + \dots, \quad (2.19)$$

where g_b, g_f are the degrees of freedom of bosons and fermions, respectively, and M^2 moduli-dependent bosonic and fermionic mass matrices of all the particles forming the

thermal plasma. The leading order expansion of the 2-loop expansion is parametrized by contributions of loop diagrams. The terms in (2.19) descend from two loops involving MSSM particles or diagrams with one modulus and two gauge bosons X . The mass of the supersymmetric partners $\tilde{\phi}$ can be computed from the following formula:

$$m_{\tilde{\phi}}^2 = \text{Tr}[M_f^2] = e^G K^{i\bar{j}} K^{l\bar{m}} \left(\nabla_i G_l + \frac{G_l G_l}{2} \right) \times \left(\nabla_{\bar{j}} G_{\bar{m}} + \frac{G_{\bar{j}} G_{\bar{m}}}{2} \right), \quad (2.20)$$

where $G = \mathcal{K} + \log |\mathcal{W}|$ is the supergravity Kähler invariant potential and $\nabla_i G_j = G_{ij} - \Gamma_{ij}^l G_l$, with $\Gamma_{ij}^l = K^{l\bar{m}} \partial_i K_{j\bar{m}}$ being the geometric connection. In effective $\mathcal{N} = 1$ supergravity, fermion masses arise from the second derivatives of the superpotential and the Kähler potential, while due to the LVS behavior of the vacuum, we expect that these particles are much lighter than the scalar masses. Thus, we could safely assume that $m_{\tilde{\phi}}^2 \gg m_{\phi}^2$.

Before studying the behavior of the finite temperature effects, we first focus on the stability of the underlying theory. The minimization of the moduli can be more easily performed on the orthonormal following basis [51], which can be derived through the leading order diagonal (or large volume expansion) structure of the Kähler metric,

$$K_{ij} \cong \begin{pmatrix} \frac{1}{4\tau_1^2} & 0 & 0 \\ 0 & \frac{1}{4\tau_2^2} & 0 \\ 0 & 0 & \frac{1}{4\tau_3^2} \end{pmatrix} + \mathcal{O}(\eta, \xi),$$

$$K_{ij}^{-1} \cong \begin{pmatrix} 4\tau_1^2 & 0 & 0 \\ 0 & 4\tau_2^2 & 0 \\ 0 & 0 & 4\tau_3^2 \end{pmatrix} + \mathcal{O}(\eta, \xi), \quad (2.21)$$

$$t_i = \frac{1}{\sqrt{2}} \ln(\tau_i). \quad (2.22)$$

Since we want to study the volume modulus $\mathcal{V}$ direction, it is better to find the definition of the volume direction and the two perpendicular directions in the normalized fields basis. An option is to choose the following directions:

$$t = \frac{1}{\sqrt{3}} (t_1 + t_2 + t_3) = \frac{\sqrt{6}}{3} \ln(\mathcal{V}), \quad (2.23)$$

$$u = \frac{1}{\sqrt{2}} (t_1 - t_2), \quad (2.24)$$

$$v = \frac{1}{\sqrt{6}} (t_1 + t_2 - 2t_3). \quad (2.25)$$

In this basis, the effective potential reads as,

$$V_{\text{eff}}(T=0) = \frac{3}{4} e^{-3\sqrt{\frac{2}{3}}t} \mathcal{W}_0^2 g_s (\xi + 2\eta(\sqrt{6}t - 8)) - \frac{\mathcal{W}_0^2 g_s e^{-5\sqrt{\frac{2}{3}}t - u - \frac{2u}{\sqrt{3}}} (\mathcal{C}_3 e^u + e^{\sqrt{3}v} (\mathcal{C}_2 + \mathcal{C}_1 e^{2u}))}{8\pi} - \frac{3\lambda \mathcal{W}_0^4 \sqrt{g_s} (e^{u+\sqrt{3}v} + e^{2u} + 1) e^{-\frac{11u}{\sqrt{6}} - u - \frac{u}{\sqrt{3}}}}{8\pi^2}. \quad (2.26)$$

The minimal values of the three directions can be written at leading order as,

$$u_{\min} = \frac{1}{2} \log\left(\frac{\mathcal{C}_2}{\mathcal{C}_1}\right) + \mathcal{O}(\lambda), \quad v_{\min} = \frac{\log\left(\frac{\mathcal{C}_3}{\sqrt{\mathcal{C}_1 \mathcal{C}_2}}\right)}{\sqrt{3}} + \mathcal{O}(\lambda), \quad (2.27)$$

$$t_{\min} = \frac{52\eta - 3\xi + 36\eta W_0 \left(\frac{5(\frac{\mathcal{C}_1 \mathcal{C}_2 \mathcal{C}_3}{\eta^3})^{1/3} e^{\frac{\xi}{12\eta} - \frac{13}{9}}}{108\pi} \right)}{6\sqrt{6}\eta} + \mathcal{O}(\lambda). \quad (2.28)$$

The W_0 branch of the Lambert function denotes the minimum of the potential along the volume direction. We can observe that the volume modulus is mainly stabilized by the perturbative corrections, while the sub-leading winding loop corrections stabilize the transverse directions. The higher derivative corrections scaling as $\sim \mathcal{O}(F^4)$ are added to examine the robustness of the minima against subleading corrections. Now, we perform the diagonalization of the mass matrix in the original basis (τ_1, τ_2, τ_3) , in order to deduce the exact hierarchy in the particle spectrum and in the couplings with the gauge bosons, concluding that way whether the normalized fields couple sufficiently strongly with the thermal bath. To work out the canonical normalization [52], we can expand around the stabilized vacuum expectation value (vev) of each modulus as,

$$\tau_i = \langle \tau_i \rangle + \delta\tau_i, \quad (2.29)$$

where then the Lagrangian will be written as,

$$\mathcal{L} = \frac{1}{2} K_{ij} \partial_\mu \delta\tau_i \partial^\mu \delta\tau_j - V - \frac{1}{2} V_{ij} \tau_i \tau_j. \quad (2.30)$$

Writing the original moduli $\delta\tau_i$ in terms of the canonically normalized fields $\delta\phi_i$, then the Lagrangian is written as,

$$\mathcal{L} = \frac{1}{2} \sum_i \partial_\mu \delta\phi_i \partial^\mu \delta\phi_i - V - \frac{m_i^2}{2} \sum_i \delta\phi_i^2, \quad \delta\tau_i = \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix} \frac{\delta\phi_i}{\sqrt{2}}, \quad (2.31)$$

where χ_i are the components of the eigenvectors. The mass-squared matrix is computed by $M_{ij}^2 = \frac{1}{2} (K_{ik}^{-1}) V_{kj}$, while the eigenvectors are normalized by $\vec{\alpha}^T \cdot K \cdot \vec{\alpha} = 1$. The diagonal elements of the mass matrix are given below as,

$$M_{11}^2 = \frac{9\mathcal{W}_0^2 g_s (-272\eta + 15\xi + 60\eta \log(\mathcal{V}))}{8\mathcal{V}^3} - \frac{\mathcal{W}_0^2 g_s (\tau_1 \tau_2 (C_1 \tau_1 + 3C_2 \tau_2) + 3C_3 \mathcal{V}^2)}{2\pi \tau_1 \tau_2 \mathcal{V}^4} - \frac{15\lambda \mathcal{W}_0^4 \sqrt{g_s} (3\tau_1^2 \tau_2^2 + 3\tau_1 \mathcal{V}^2 + 7\tau_2 \mathcal{V}^2)}{16\pi^2 \tau_1 \tau_2 \mathcal{V}^5}, \quad (2.32)$$

$$M_{22}^2 = M_{11}^2 (C_1 \leftrightarrow C_2, \tau_1 \leftrightarrow \tau_2), \quad M_{33}^2 = M_{11}^2 \left(C_1 \leftrightarrow 3C_1, C_3 \leftrightarrow \frac{1}{3}C_3 \right) + \frac{15\lambda \mathcal{W}_0^4 \sqrt{g_s} (\mathcal{V}^2 - \tau_1^2 \tau_2)}{4\pi^2 \tau_1 \mathcal{V}^5}. \quad (2.33)$$

In the above expressions, we have implicitly denoted the exchange of coefficients/moduli with $\leftrightarrow$ that is needed for the rest of the diagonal elements. The off-diagonal elements are expressed as,

$$M_{12}^2 = \frac{9\tau_1 \mathcal{W}_0^2 g_s (-56\eta + 3\xi + 12\eta \log(\mathcal{V}))}{8\tau_2 \mathcal{V}^3} - \frac{\mathcal{W}_0^2 g_s (\tau_1 \tau_2 (C_1 \tau_1 + C_2 \tau_2) + 2C_3 \mathcal{V}^2)}{\sqrt{2}\pi \tau_2^2 \mathcal{V}^4} - \frac{9\lambda \mathcal{W}_0^4 \sqrt{g_s} (3\tau_1^2 \tau_2^2 + 5\tau_1 \mathcal{V}^2 + 5\tau_2 \mathcal{V}^2)}{16\pi^2 \tau_2^2 \mathcal{V}^5}, \quad M_{21}^2 = M_{12}^2 (\tau_2 \leftrightarrow \tau_1), \quad (2.34)$$

$$M_{13}^2 = \frac{9\tau_1^2 \tau_2 \mathcal{W}_0^2 g_s (-56\eta + 3\xi + 12\eta \log(\mathcal{V}))}{8\mathcal{V}^5} - \frac{\tau_1 \mathcal{W}_0^2 g_s (\tau_1 \tau_2 (C_1 \tau_1 + 2C_2 \tau_2) + C_3 \mathcal{V}^2)}{\sqrt{2}\pi \mathcal{V}^6} - \frac{9\lambda \tau_1 \mathcal{W}_0^4 \sqrt{g_s} (5\tau_1^2 \tau_2^2 + 3\tau_1 \mathcal{V}^2 + 5\tau_2 \mathcal{V}^2)}{16\pi^2 \mathcal{V}^7}, \quad M_{31}^2 = M_{13}^2 \left(g_s \mathcal{W}_0^2 \tau_1 \rightarrow g_s \mathcal{W}_0^2 \frac{\tau_3^2}{\tau_1}, \lambda \rightarrow \lambda \frac{\tau_3^2}{\tau_1} \right), \quad (2.35)$$

$$M_{23}^2 = \frac{9\tau_1 \tau_2^2 \mathcal{W}_0^2 g_s (-56\eta + 3\xi + 12\eta \log(\mathcal{V}))}{8\mathcal{V}^5} - \frac{\tau_2 \mathcal{W}_0^2 g_s (\tau_1 \tau_2 (2C_1 \tau_1 + C_2 \tau_2) + C_3 \mathcal{V}^2)}{\sqrt{2}\pi \mathcal{V}^6} - \frac{9\lambda \tau_2 \mathcal{W}_0^4 \sqrt{g_s} (5\tau_1^2 \tau_2^2 + 5\tau_1 \mathcal{V}^2 + 3\tau_2 \mathcal{V}^2)}{16\pi^2 \mathcal{V}^7}, \quad M_{32}^2 = M_{23}^2 \left(g_s \mathcal{W}_0^2 \tau_2 \rightarrow g_s \mathcal{W}_0^2 \frac{\tau_3^2}{\tau_2}, \lambda \rightarrow \lambda \frac{\tau_3^2}{\tau_2} \right). \quad (2.36)$$

The eigenvalues of the mass matrix can be computed using the invariants of the mass matrices, the trace and the determinant. More specifically, the characteristic polynomials and the corresponding eigenvalues are expressed by,

$$\text{Tr}[M^2] = \sum_i m_i^2, \quad \frac{\text{Det}[M^2]}{\text{Tr}[M^2]^2} \sim \frac{m_1^2 m_2^2 m_3^2}{m_1^4} \sim \frac{m_2^2 m_3^2}{m_1^2}. \quad (2.37)$$

The heaviest eigenvalue ($m_1^2 \gg m_{2,3}^2$) is effectively parametrized by the scale of the perturbative corrections and it is given as,

$$m_1^2 \cong \text{Tr}[M^2] \cong \frac{9}{8} e^{-3\sqrt{\frac{2}{3}}t} \mathcal{W}_0^2 g_s (-272\eta + 15\xi + 30\sqrt{6}\eta t) - \frac{7\mathcal{W}_0^2 g_s e^{-5\sqrt{\frac{2}{3}}t - u - \frac{2u}{\sqrt{3}}} (C_3 e^u + e^{\sqrt{3}u} (C_2 + C_1 e^{2u}))}{2\pi}. \quad (2.38)$$

The two smallest eigenvalues, given the large hierarchy, can be given by the algebraic expressions of the 2×2 submatrix, where the trace of the M^2 matrix specifies the quantities as,

$$m_{2,3}^2 \cong \frac{1}{2} \left(\text{Tr}[M^2] - \frac{1}{2} (\text{Tr}[M^2]^2 - \text{Tr}[M^4])^{1/2} \right), \quad A = (\text{Tr}[M^2]^2 - \text{Tr}[M^4])^{1/2}, \quad (2.39)$$

$$\begin{aligned}
A = & \frac{1}{64} e^{-3\sqrt{6}t} \mathcal{W}_0^4 g_s^2 (17649\xi^2 + 8\eta^2(52947t^2 - 160104\sqrt{6}t + 726208) + 12\eta\xi(5883\sqrt{6}t - 53368)) \\
& - \frac{117\lambda(e^{2u} + 1)\mathcal{W}_0^6 g_s^{3/2}(-2368\eta + 129\xi + 258\sqrt{6}\eta t) e^{-10\sqrt{\frac{4}{3}}t - u - \frac{v}{\sqrt{3}}}}{64\pi^2} \\
& - \frac{63\mathcal{W}_0^4 g_s^2(-272\eta + 15\xi + 30\sqrt{6}\eta t) e^{-\frac{19}{\sqrt{6}}u - \frac{2v}{\sqrt{3}}}(\mathcal{C}_3 e^u + e^{\sqrt{3}v}(\mathcal{C}_2 + \mathcal{C}_1 e^{2u}))}{8\pi}.
\end{aligned} \tag{2.40}$$

Let us now derive the corresponding eigenvectors. We start with the first eigenvalue m_1^2 as,

$$M^2 \vec{\alpha}_1 = m_1^2 \vec{\alpha}_1 \Rightarrow M_{11}^2 \chi_1 + M_{12}^2 \chi_2 + M_{13}^2 \chi_3 = m_1^2 \chi_1. \tag{2.41}$$

Due to the requirement that we would like to have a hierarchy in the masses and the internal geometry, we demand that $\langle \tau_1 \rangle \ll \langle \tau_2, \tau_3, \mathcal{V} \rangle$. This hierarchy can be safely achieved by tuning the fluxes $\mathcal{W}_0$ and the $\mathcal{C}_i$ parameters, as these are given in the minima of Eq. (2.27). Thus, we need to observe the scaling of the ratio between the entries of the mass matrix and the eigenvalue.

$$\begin{aligned}
\frac{M_{11}^2}{m_1^2} &\sim \mathcal{O}(1), & \frac{M_{12}^2}{m_1^2} &\sim \mathcal{O}\left(\frac{\tau_1}{\tau_2}\right), & \frac{M_{13}^2}{m_1^2} &\sim \mathcal{O}\left(\frac{\tau_1^2 \tau_2}{\mathcal{V}^2}\right), \\
\frac{M_{12}^2}{m_1^2} &\sim \mathcal{O}\left(\frac{\tau_2}{\tau_1}\right), & \frac{M_{22}^2}{m_1^2} &\sim \mathcal{O}(1), & \frac{M_{23}^2}{m_1^2} &\sim \mathcal{O}\left(\frac{\tau_1 \tau_2^2}{\mathcal{V}^2}\right), \\
\frac{M_{31}^2}{m_1^2} &\sim \mathcal{O}\left(\frac{\mathcal{V}^2}{\tau_1^2 \tau_2}\right), & \frac{M_{32}^2}{m_1^2} &\sim \mathcal{O}\left(\frac{\mathcal{V}^2}{\tau_1 \tau_2^2}\right), & \frac{M_{33}^2}{m_1^2} &\sim \mathcal{O}(1).
\end{aligned} \tag{2.42}$$

The large hierarchy in the mixing is required for the thermalization procedure, since a parametrically small coupling cannot contribute sufficiently to maintain in thermal equilibrium the moduli with the MSSM thermal bath. Next, we are going to analyze the derivation of the first eigenvector, and the rest will follow applying the same recipe. For the heavy eigenvalue, the following constraints descend from Eq. (2.41), taking into the ratios defined in (2.42):

$$\frac{M_{11}^2}{m_1^2} \chi_1 + \frac{M_{12}^2}{m_1^2} (\chi_2 + \chi_3) = \chi_1 \Rightarrow \chi_1 = \frac{M_{12}^2}{m_1^2 - M_{11}^2} (\chi_2 + \chi_3), \tag{2.43}$$

$$\frac{M_{21}^2}{m_1^2} \chi_1 + \frac{M_{22}^2}{m_1^2} (\chi_2 + \chi_3) = \chi_2 \Rightarrow \chi_3 = \chi_2 \left(-1 + \frac{m_1^2}{1 + \frac{M_{12}^2 M_{21}^2}{M_{22}^2 (m_1^2 - M_{11}^2)}} \right). \tag{2.44}$$

From these two equations and using the ratios in Eq. (2.42), we can determine two components of the eigenvector, while the third will be specified by normalization of the eigenvectors. So,

$$\vec{\alpha}^T \cdot K \cdot \vec{\alpha} = \frac{1}{4\tau_1^2} \chi_1^2 + \frac{1}{4\tau_2^2} \chi_2^2 + \frac{1}{4\tau_3^2} \chi_3^2 \cong \frac{1}{4\tau_2^2} (\chi_2^2 + \chi_3^2) = 1 \Rightarrow (\chi_2^2 + \chi_3^2) \cong 4\tau_3^2 \Rightarrow \tag{2.45}$$

$$\chi_2 \cong \chi_3 \cong \sqrt{2}\tau_3, \tag{2.46}$$

where in the last step, we have applied our requirement that the vev of τ_1 modulus is smaller compared to the other two moduli. This will be justified in the next section, where we explicitly derive the vev of the moduli at the stable vacuum. Consequently, the first eigenvector can be described by the following form:

$$\vec{\alpha}_1 = \{\chi_1, \chi_2, \chi_3\}. \tag{2.47}$$

The other two eigenvectors characterize the degenerate light eigenvalues. So, we follow the same procedure to derive their scaling,

$$\chi_1 + \frac{M_{12}^2}{M_{11}^2}(\chi_2 + \chi_3) = \frac{m_2^2}{M_{11}^2}\chi_1 \Rightarrow \chi_1 = \frac{M_{12}^2}{m_2^2 - M_{11}^2}(\chi_1 + \chi_3), \quad (2.48)$$

$$\chi_1 + \frac{M_{22}^2}{M_{11}^2}(\chi_2 + \chi_3) = \frac{m_2^2}{M_{11}^2}\chi_2 \Rightarrow \chi_3 = \chi_2 \left(-1 + \frac{m_2^2 M_{21}^2 (m_2^2 - M_{11}^2)}{M_{22}^2 (m_2^2 - M_{11}^2) + M_{12}^2 M_{21}^2} \right). \quad (2.49)$$

Similarly exploiting the ratios between the eigenvalues and the matrix elements at the vacuum, the components of the eigenvectors can be approximated by the following formula:

$$\chi_3 \cong \chi_2 \cong \sqrt{2}\tau_3, \quad (2.50)$$

$$\vec{\alpha}_2 = \{\chi_1, \chi_3, \chi_2\}, \quad \vec{\alpha}_3 = \{\chi_2, c\chi_3, \chi_1\}, \quad c \sim \mathcal{O}(1). \quad (2.51)$$

The constant c is introduced in the computation of the “light” eigenvectors due to the degeneracy of the masses, and it can be perceived as the splitting factor of the light eigenvalues. The diagonalization of the mass matrix and the ordering of the eigenvalues lead to the following pattern of mixing between the moduli τ_i and the normalized fields ϕ_i ,

$$\delta\tau_1 = \chi_1(\vec{\alpha}_1)\delta\phi_1 + \chi_1(\vec{\alpha}_2)\delta\phi_2 + \chi_2(\vec{\alpha}_3)\delta\phi_3, \quad (2.52)$$

$$\delta\tau_2 = \chi_2(\vec{\alpha}_1)\delta\phi_1 + c\chi_3(\vec{\alpha}_2)\delta\phi_2 + \chi_3(\vec{\alpha}_3)\delta\phi_3, \quad (2.53)$$

$$\delta\tau_3 = \chi_3(\vec{\alpha}_1)\delta\phi_1 + \chi_2(\vec{\alpha}_2)\delta\phi_2 + \chi_1(\vec{\alpha}_3)\delta\phi_3. \quad (2.54)$$

From the above mixing, we can clearly see a geometric separation of the internal geometry, which means that $\delta\tau_1 \sim \chi_2(\vec{\alpha}_3)\delta\phi_3$ while $\delta\tau_{2,3} \sim \chi_3(\vec{\alpha}_3)\delta\phi_{1,2}$. Regarding the ordering of the eigenvalues, they are given by this form $m_i^2 = \{m_3^2, m_2^2, m_1^2\}$. Turning our attention now on the coupling of the moduli to the gauge bosons X , these can be read from the gauge kinetic function [53],

$$f_i = \frac{T_i^{MSSM}}{4\pi} + h_i(F)S, \quad (2.55)$$

where T_i denotes the Kähler modulus on which the MSSM resides, while h_i is a topological function of the internal world-volume fluxes of the compactification. Taking this into account, we can assume that the small modulus τ_1 characterizes the visible sector, so the following term of the Lagrangian can be written as,

$$\mathcal{L} = -\frac{\tau_1}{M_{\text{pl}}} F_{\mu\nu} F^{\mu\nu} = -G_{\mu\nu} G^{\mu\nu} - \frac{\delta\tau_1}{\langle\tau_1\rangle M_{\text{pl}}} G_{\mu\nu} G^{\mu\nu} = -G_{\mu\nu} G^{\mu\nu} - \frac{\chi_2(\vec{\alpha}_3)}{\langle\tau_1\rangle M_{\text{pl}}} \phi_3 G_{\mu\nu} G^{\mu\nu}, \quad \chi_2(\vec{\alpha}_3) = \sqrt{2}\tau_3, \quad (2.56)$$

where in the last step, we expanded around the minimum of τ_1 and normalized the gauge fields $G_{\mu\nu} = \sqrt{\langle\tau_1\rangle} F_{\mu\nu}$. Consequently, we can observe that the heaviest field will be thermalized under specific conditions with respect to the magnitude of the coupling, which will be addressed in subsequent sections.

During the expansion of the universe, a specific particle species will remain in thermal equilibrium if the interaction rate Γ with the particle content of the thermal bath is greater than the expansion rate $H \sim g_*^{1/2} T^2 / M_{\text{pl}}$ of the universe, with g_* being the degrees of freedom in the thermal bath. The processes that contribute to the above thermalization are $2 \leftrightarrow 2$ (scattering and annihilations) processes and $1 \leftrightarrow 2$ decays and inverse production. Starting with $2 \leftrightarrow 2$ processes, we

can introduce the thermally averaged interaction rate

$$\Gamma \sim \frac{1}{\langle t_c \rangle}, \quad t_c \sim \frac{1}{n\sigma v}, \quad (2.57)$$

where t_c is the mean time between collisions, n is the number density, σ is the cross section, and v is the relative velocity between the particles of the thermal bath. The relativistic nature of the particles dictates the velocity to be of order $v \sim c \equiv 1$ and the number density $n \sim T^3$. We are interested in processes that include gravitational interactions, in order to see if under specific geometric enhancement through the moduli couplings, these interactions could become relevant for the thermalization of $2 \leftrightarrow 2$ processes. The cross sections for the aforementioned interactions are given by [54],

(i) Diagram with two gravitational vertices,

$$\langle \sigma \rangle \sim \mu \frac{T^2}{M_{\text{pl}}^4} \Rightarrow \langle \Gamma \rangle \sim \mu \frac{T^5}{M_{\text{pl}}^4}. \quad (2.58)$$

(ii) Diagram with one renormalizable and one gravitational vertex,

$$\langle \sigma \rangle \sim \sqrt{\mu} \frac{g^2}{M_{\text{pl}}^2} \Rightarrow \langle \Gamma \rangle \sim \sqrt{\mu} \frac{g^2 T^3}{M_{\text{pl}}^4}, \quad (2.59)$$

where μ is a moduli-dependent factor, more specifically the mixing of moduli with the gauge bosons, and g is the gauge coupling of the theory. Comparing these interaction rates with the expansion rate of the universe, we can derive the freeze-out temperature of the particular particle species. The freeze-out temperatures for the cases described above are expressed as,

$$i) \langle \Gamma \rangle > H \Rightarrow T > g_*^{1/6} \frac{M_{\text{pl}}}{\mu^{1/3}}, \quad (2.60)$$

$$ii) \langle \Gamma \rangle > H \Rightarrow T > g_*^{1/2} \frac{M_{\text{pl}}}{g^2 \mu^{1/2}}. \quad (2.61)$$

Turning our attention to decay processes, gravitational interactions mediated by massive particles at $T < M$ (with M being the Planck mass) result in a decay rate of the following form:

$$\Gamma \sim \mu \frac{m^3}{M_{\text{pl}}^2}. \quad (2.62)$$

In order to compare the above quantity with the expansion rate, we need to consider the thermally averaged decay rate,

$$\langle \Gamma' \rangle = \Gamma \frac{m}{\langle E \rangle}, \quad (2.63)$$

where we introduced a relativistic factor, in order to switch from the rest frame to the laboratory frame. This factor in the relativistic regime is given by $T \gtrsim m$, $\gamma = \langle E \rangle / m = \langle T \rangle / m$, while in the opposite limit for $T \lesssim m$, $\gamma \sim 1$. Regarding the inverse decay, they require temperatures higher than the mass $T \gtrsim m$, otherwise the final states do not have enough energy to reverse the decay. To summarize the requirements for the current case study, we write down the decay rates at $T \lesssim M$,

$$i) \langle \Gamma_{1 \rightarrow 2} \rangle \cong \begin{cases} \mu \frac{m^4}{M_{\text{pl}}^2 T}, & T \gtrsim m, \\ \mu \frac{m^3}{M_{\text{pl}}^2}, & T \lesssim m, \end{cases} \quad (2.64)$$

$$ii) \langle \Gamma_{2 \rightarrow 1} \rangle \cong \begin{cases} \mu \frac{m^4}{M_{\text{pl}}^2 T}, & T \gtrsim m, \\ \mu \frac{m^3}{M_{\text{pl}}^2} \left(\frac{m}{T} \right)^{3/2} e^{-m/T}, & T \lesssim m, \end{cases} \quad (2.65)$$

where, again, the μ factor is modulus dependent and the Boltzmann suppression $e^{-m/T}$ is introduced for processes that have insufficient energy to recreate the particle ($T \lesssim m$). Therefore, the thermal equilibrium for $T \gtrsim m$ is maintained only if,

$$\langle \Gamma \rangle > H \Rightarrow 1 < \frac{T}{m} < \left(\frac{\mu m}{g_*^{1/2} M_{\text{pl}}} \right)^{1/3}. \quad (2.66)$$

III. ACHIEVING A DS SPACE

On the basis of the preceding analysis, we observe that the previous vacuum is AdS and it is necessary to introduce an uplifting D term [55–57]. The relevant term we could implement as an uplifting term scales as,

$$V_{\text{up}} = \frac{d}{\mathcal{V}^2}. \quad (3.1)$$

The effective potential, under consideration, can now be formulated as,

$$V_{\text{eff}} = \frac{3\mathcal{W}_0^2 g_s (-16\eta + \xi + 4\eta \log(\mathcal{V}))}{4\mathcal{V}^3} - \frac{\mathcal{W}_0^2 g_s \left(\frac{\mathcal{C}_3 \mathcal{V}^3}{\tau_1 \tau_2} + \mathcal{C}_1 \tau_1 \mathcal{V} + \mathcal{C}_2 \tau_2 \mathcal{V} \right)}{8\pi \mathcal{V}^5} - \frac{3\lambda \mathcal{W}_0^4 \sqrt{g_s} \left(\tau_1 \tau_2 + \frac{\mathcal{V}^2}{\tau_1} + \frac{\mathcal{V}^2}{\tau_2} \right)}{8\pi^2 \mathcal{V}^5} + \frac{d}{\mathcal{V}^2}. \quad (3.2)$$

This potential can effectively stabilize all the moduli at Minkowski/dS spaces without introducing any tachyonic directions. Again we can change our basis to find the vacua in the orthonormal basis, where the potential is written as,

$$V_{\text{eff}} = \frac{3}{4} e^{-3\sqrt{\frac{2}{3}}t} \mathcal{W}_0^2 g_s (\xi + 2\eta(\sqrt{6}t - 8)) - \frac{\mathcal{W}_0^2 g_s e^{-5\sqrt{\frac{2}{3}}t - u - \frac{2v}{\sqrt{3}}}}{8\pi} \left(\mathcal{C}_3 e^u + e^{\sqrt{3}v} (\mathcal{C}_2 + \mathcal{C}_1 e^{2u}) \right) - \frac{3\lambda \mathcal{W}_0^4 \sqrt{g_s} \left(e^{u+\sqrt{3}v} + e^{2u} + 1 \right) e^{-\frac{11t}{\sqrt{6}} - u - \frac{v}{\sqrt{3}}}}{8\pi^2} + d e^{-\sqrt{6}t}. \quad (3.3)$$

The first modulus u can be stabilized at,

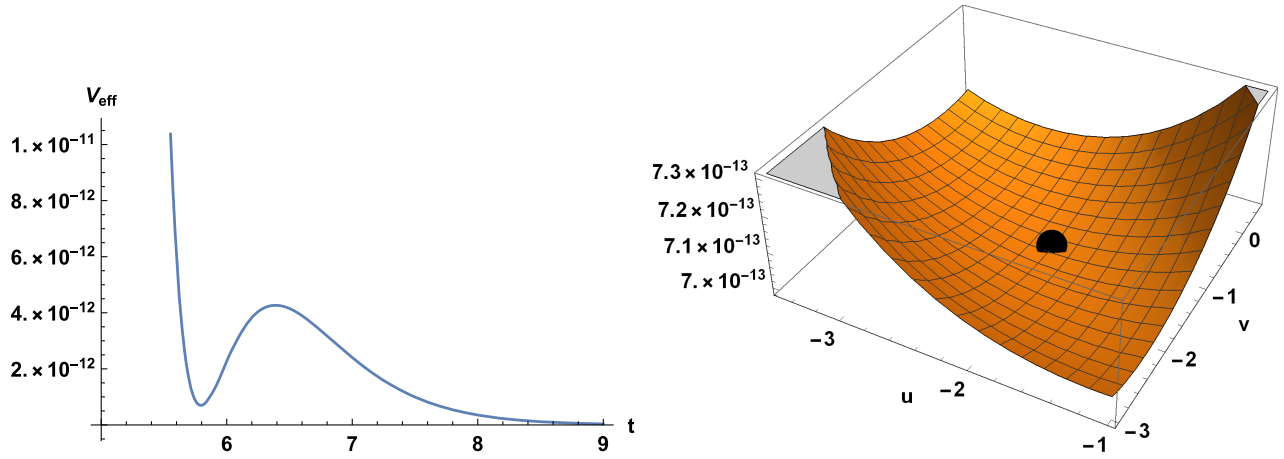


FIG. 1. Vacuum of the potential (3.3) along each direction. The black dot represents the minimum.

$$\frac{\partial V_{\text{eff}}}{\partial u} = 0 \Rightarrow u_{\min} = \frac{1}{2} \log \left(\frac{\pi C_2 \sqrt{g_s} e^{\frac{t}{\sqrt{6}} + \frac{2v}{\sqrt{3}}} + 3\lambda \mathcal{W}_0^2}{\pi C_1 \sqrt{g_s} e^{\frac{t}{\sqrt{6}} + \frac{2v}{\sqrt{3}}} + 3\lambda \mathcal{W}_0^2} \right). \quad (3.4)$$

Regarding the volume direction t , it can be effectively described by leading order corrections (log-loop corrections and D terms) as opposed to our previous analysis. In this procedure, we assumed that the minima of u, v are mainly given by (2.27), since the subleading F^4 corrections are a minimal effect. This leads us to the following formula:

$$\frac{\partial V_{\text{eff}}}{\partial t} = 0 \Rightarrow t_{\min} \cong \frac{-12\sqrt{6}\eta W_{0/-1} \left(\frac{2de^{\frac{13}{3} - \frac{\xi}{4\eta}}}{9\eta \mathcal{W}_0^2 g_s} \right) + 52\sqrt{6}\eta - 3\sqrt{6}\xi}{36\eta}, \quad (3.5)$$

where $W_{0/-1}$ stands for the Lambert function, where the upper branch denotes the maximum of the potential and the lower branch the minimum. As for the last direction, it can be approximated by,

$$\frac{\partial V_{\text{eff}}}{\partial v} = 0 \Rightarrow v_{\min} \cong -\frac{\sqrt{2}t}{2} - \frac{\log \left(\frac{1024\pi^6 C_1^5 C_2^5 C_3^2 g_s^3}{((C_1 + C_2)\lambda \mathcal{W}_0^2 + 3\pi(-C_1 C_2 C_3)^{2/3} e^{\frac{t}{\sqrt{6}} \sqrt{g_s}})^6} \right)}{2\sqrt{3}}. \quad (3.6)$$

It is evident that the volume is mainly expressed by the log-loop effects, while the subleading corrections (winding loop effects) stabilize the transverse directions, generating in that way a geometrical hierarchy both in the masses and mixing. While our analysis for the mixing in the previous

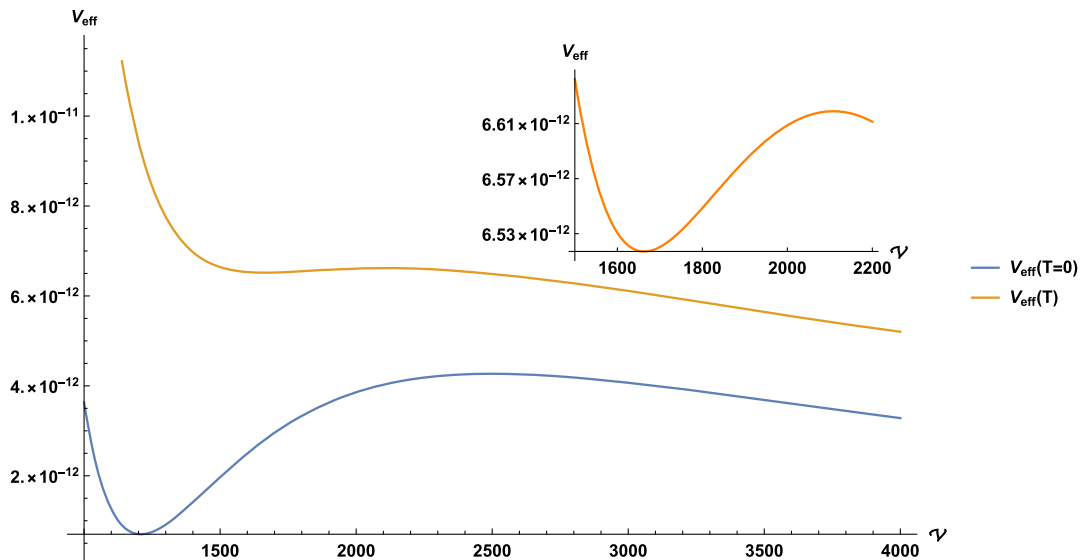


FIG. 2. Plot of the potential (4.5) along the volume direction under thermal effects.

TABLE I. Parameters used in the plots above.

g_s	$ \mathcal{W}_0 $	$\hat{\xi}$	$ \hat{\eta} $	$ \lambda $	d	$ \mathcal{C}_1 $	$ \mathcal{C}_2 = \mathcal{C}_3 $
0.1	1	5	0.6	1×10^{-3}	1.52×10^{-3}	0.5	1×10^{-3}

TABLE II. Vevs of the moduli at the vacuum and their corresponding masses.

τ_1	τ_2	$\mathcal{V}$	$m_{\tau_1}^2$	$m_{\tau_2}^2$	$m_{\mathcal{V}}^2$
3	650	1208	1.5×10^{-10}	7.7×10^{-14}	2.4×10^{-15}

section performed taking into account only the F term, the introduction of the D term affects only the eigenvalue of the lightest mode. Figures 1 and 2 depict graphically the global vacuum along the orthonormal basis (t, u, v) , where Table I and Table II provide the parameters used for the current vacuum along with the vevs and masses of the moduli.

It is important for someone to observe that the D terms only shift the eigenvalues at smaller values and never create a tachyonic direction. Regarding the observed asymmetry of the $\mathcal{C}_i$ parameters, this aspect favors the small vev of the corresponding modulus while it enhances the thermalization effects of this direction as it will be proved in the following section. Previous studies have assumed identical coefficients, although an explicit computation is needed to unravel their origin and effects after the stabilization of complex-structure moduli. Finally, the validity of the existence of a dS vacuum is translated as an upper bound of the uplift parameter d . This upper bound can be found from the Lambert function and the requirement that both branches exist ($W_{0/-1}(x)$, $-1/e \leq x < 0$).

$$-\frac{1}{e} \leq \frac{2de^{\frac{13}{3} + \frac{\xi}{4\eta}}}{9g_s\mathcal{W}_0^2\eta} < 0 \Rightarrow -\frac{9}{2}g_s\mathcal{W}_0^2\eta e^{-\frac{16}{3} + \frac{\xi}{4\eta}} \geq d > 0. \quad (3.7)$$

As for the lower bound, this can be derived by demanding the positivity of the potential at the global minimum. The potential there can approximated by,

$$V_{\text{eff}}^{\text{min}} \cong \frac{2d^3 \left(2 + 3W_0 \left(\frac{2de^{\frac{14}{3} + \frac{\xi}{4\eta}}}{27\eta\mathcal{W}_0^2g_s} \right) \right)}{729g_s^2\mathcal{W}_0^4\eta^2W_0 \left(\frac{2de^{\frac{14}{3} + \frac{\xi}{4\eta}}}{27\eta\mathcal{W}_0^2g_s} \right)^3} + \mathcal{O}(\mathcal{C}_i, \lambda). \quad (3.8)$$

Now, the lower bound is given by demanding that the parentheses in the numerator has negative sign, in order to compensate the negative sign from the Lambert function in the denominator,

$$\frac{2de^{\frac{14}{3} + \frac{\xi}{4\eta}}}{27\eta\mathcal{W}_0^2g_s} < -\frac{2}{3}e^{-\frac{2}{3}} \Rightarrow d \gtrsim -3g_s\mathcal{W}_0^2\eta e^{-\frac{15}{3} + \frac{\xi}{4\eta}}. \quad (3.9)$$

Consequently, the final bounds on the d parameter can be expressed by the following very stringent inequality:

$$-\frac{9}{2}g_s\mathcal{W}_0^2\eta e^{-\frac{16}{3} + \frac{\xi}{4\eta}} \geq d \gtrsim -3g_s\mathcal{W}_0^2\eta e^{-\frac{15}{3} + \frac{\xi}{4\eta}}. \quad (3.10)$$

IV. FINITE TEMPERATURE INDUCED VACUA AND RELATED COSMOLOGICAL IMPLICATIONS

Moving on, the next thing is to consider if the heaviest modulus can thermalize at sub-Planckian temperatures. The τ_1 modulus describes the heaviest normalized field ϕ_3 as indicated by the analysis of Sec. II, which means that finite temperature corrections could potentially be introduced along this direction at the level of the effective potential. Before studying their dynamics, we need to establish the point in cosmological times, in which the thermal equilibrium is achieved. Scattering or pair production processes, either with two gravitational vertices [Eq. (2.60)] or with one renormalizable and one gravitational vertex [Eq. (2.61)], can achieve thermal equilibrium of particles species with a thermal bath above the following freeze-out temperatures T_f ,

$$T^{\text{grav}} > T_f \equiv g_*^{1/6} \frac{M_{\text{pl}}}{2^{2/3}\tau_3^{4/3}}, \quad T^{\text{renor}} > T_f \equiv g_*^{1/2} \frac{M_{\text{pl}}}{2g^2\tau_3}, \quad (4.1)$$

where for the modulus dependent factor μ , we used the coupling of ϕ_3 field to the gauge bosons, as given by Eq. (2.56) $g_{\phi_3 XX} \sim \chi_2(\vec{\alpha}_3) = \sqrt{2}\tau_3$, since the vev of $\langle \tau_1 \rangle$ modulus is of order one. Both of these temperatures are below the Planck scale and in principle thermalize the vacuum after reheating T_{rh} . This implies an indirect lower bound on reheating, since a correct implementation of the inflationary scenario would require $T_f < T_{\text{rh}}$, otherwise finite temperature corrections have no effect. A similar argument can be applied for the decay and inverse decay processes, where we demand that the temperature T exceed the mass of ϕ_3 , in order to probe $X + X \rightarrow \phi_3$ processes,

$$T > m_{\phi_3} \equiv \left(12de^{2W_0 \left(\frac{2de^{\frac{13}{3} + \frac{\xi}{4\eta}}}{9\eta\mathcal{W}_0^2g_s} \right) + \frac{\xi}{2\eta} - \frac{26}{3}} - \frac{4d^3 \left(5W_0 \left(\frac{2de^{\frac{13}{3} + \frac{\xi}{4\eta}}}{9\eta\mathcal{W}_0^2g_s} \right) + 1 \right)}{27\eta^2\mathcal{W}_0^4g_s^2W_0 \left(\frac{2de^{\frac{13}{3} + \frac{\xi}{4\eta}}}{9\eta\mathcal{W}_0^2g_s} \right)^3} \right)^{1/2}. \quad (4.2)$$

Now, this temperature has to be confronted by the equilibrium temperature for inverse decays, whose definition

descends from Eq. (2.66). Since the $\mu = \chi_2(\vec{\alpha}_3)^2$ factor denotes the coupling for the field with the gauge bosons in this process, we can write,

$$T < T_{\text{eq}} \equiv \left(2^{1/3} e^{\frac{2^{1/3}(\sqrt{3}t-2v)}{3\sqrt{3}}} \frac{2^{1/3} m_{\phi_3}^{1/3}}{M_{\text{pl}}^{1/3} g_*^{1/6}} \right) m_{\phi_3} \cong \left(\underbrace{\frac{2^{1/3} e^{\frac{5}{27} \left(-12W_0 \left(\frac{2de^{\frac{13}{9\eta} \frac{\xi}{4\eta}}}{\mathcal{W}_0^2 g_s} \right) - \frac{3\xi}{\eta} + 52 \right)}}{g_*^{1/6}}}_{y} \frac{m_{\phi_3}^{1/3}}{M_{\text{pl}}^{1/3}} \right) m_{\phi_3}, \quad (4.3)$$

which is clearly greater than the activation temperature (4.2) for the decays even at leading order, due to the fact that $y > 1$ at the dS vacuum presented in Table II. Inevitably, decay and inverse decays do contribute to the thermalization of the modulus.

Regarding the finite temperature corrections at the effective potential, it was pointed out in [54] that 1-loop effects are subleading compared to 2-loop corrections taking into account that the MSSM is supported by one modulus. This also holds in our model, since the mass scale of both fermions and bosons does not bring any new moduli dependence in the effective theory. This could be understood from the fact that these eigenvalues are derived from the mass matrix of the current model. We schematically prove our argument by writing the potential up to 1-loop effects as,

$$V_{\text{eff}} = V(T=0) + \frac{T^2}{M_{\text{pl}}^2} (m_{\phi_3}^2) \sim e^{-3\sqrt{\frac{2}{3}}t} \mathcal{W}_0^2 \left(1 + \underbrace{\frac{T^2}{M_{\text{pl}}^2}}_{\ll 1} \right) + \dots, \quad (4.4)$$

where in the above equation we used the t dependent part of the $T=0$ potential at (3.3), while for the mass of the modulus we applied Eq. (2.38). Thus, in order to bring into the table corrections that could compete with the potential at zero temperature, we examine the 2-loop thermal

corrections. Since τ_1 modulus is thermalized, from now on we consider the following potential:

$$V_{\text{tot}} = V_{\text{eff}} + V_{\text{up}} + T^4 (\kappa_1 g_{\text{MSSM}}^2 + \kappa_2 g_{\phi_3 XX}^2 m_{\phi_3}^2) \cong V_{\text{eff}} + V_{\text{up}} + T^4 \left(\frac{1}{\tau_1} + (\chi_2(\vec{\alpha}_3))^2 \text{Tr}[M^2] \right), \quad (4.5)$$

where we assume that $\kappa_{1,2}$ are constants of order 1 and are absorbed. This can be justified by considering the fact that the MSSM are created by D_7 -branes associated with the τ_1 modulus. Then, the coefficient κ_1 is defined by the rank of the gauge group $SU(N)$,

$$\kappa_1 = \frac{(N^2 - 1)(N - N_f)}{64}, \quad (4.6)$$

where N_f is the number of fermions [58]. Despite the fact that previous studies have not managed to find new vacua due to thermal effects, in the current model we are going to show that new thermally induced vacua can be obtained. In our approach, it will be shown that the T-dependent contributions are comparable to the dominant contribution of the effective potential at $T=0$. This contradicts previous studies [54], where the nonperturbative corrections along the thermalized direction are parametrically larger than the finite temperature effects. On the contrary, in our perturbative stabilization scheme, the transverse directions to the volume are stabilized by subleading effects (winding loop corrections), which can compete with the finite-T induced corrections, leading to a reshape of the potential. Thus, we are obliged to take into account the full restabilization of the total potential and search for vacua that are connected to the $T=0$ dS vacuum. This argument will become clear, when we will prove that the compactification volume only slightly shifts while the displacement of the other moduli is more robust. The new ingredients could in principle provide us with further bounds on the free parameters of the model, while it could be possible to associate the decompactification temperature T_{max} with the uplifted T-dS vacua. The shift in the vevs of the moduli can be achieved by restabilizing the potential. Starting from the u direction,

$$\frac{\partial V_{\text{tot}}}{\partial u} \cong 0 \Rightarrow u_{\text{min}} \cong \frac{1}{2} \log \left(\frac{2\pi C_2 \mathcal{W}_0^2 g_s e^{\frac{t}{\sqrt{6}} + \frac{2v}{\sqrt{3}}} + 3\lambda \mathcal{W}_0^4 \sqrt{g_s} - 16\pi^2 e^3 \sqrt{\frac{2}{3}} t^4}{2\pi C_1 \mathcal{W}_0^2 g_s e^{\frac{t}{\sqrt{6}} + \frac{2v}{\sqrt{3}}} + 3\lambda \mathcal{W}_0^4 \sqrt{g_s}} \right). \quad (4.7)$$

We can see that a new factor proportional to the temperature is appended in the previously derived vacuum. This factor comes with an exponent $\sim T^4 e^{-3\sqrt{3}t/2}$ greater than the leading order exponent $\sim C_2 e^{t/\sqrt{6}}$ of the previous vacuum solution. This indicates a large shift due to finite temperature corrections. Moving to the volume direction t , an approximate formula at leading order is given by,

$$\frac{\partial V_{\text{tot}}}{\partial t} = 0 \Rightarrow \frac{e^{-3\sqrt{\frac{2}{3}}t} \left(-24de^{\sqrt{\frac{2}{3}}t} \left(8T^4 e^{\frac{2(\sqrt{2}t-2v)}{\sqrt{3}}} + 1 \right) - 9\mathcal{W}_0^2 g_s (-52\eta + 3\xi + 6\sqrt{6}\eta t) - 8T^4 e^{\frac{7t}{\sqrt{6}} - u - \frac{v}{\sqrt{3}}} \right)}{4\sqrt{6}} + \frac{5\mathcal{W}_0^2 g_s e^{-5\sqrt{\frac{2}{3}}t - u - \frac{2v}{\sqrt{3}}} (\mathcal{C}_3 e^u + e^{\sqrt{3}v} (\mathcal{C}_2 + \mathcal{C}_1 e^{2u}))}{4\sqrt{6}\pi} \cong 0. \quad (4.8)$$

$$t_{\min} \cong \frac{-12\sqrt{6}\eta W_0 \left(-\frac{e^{\frac{13}{3} - \frac{\xi}{4\eta}} (\sqrt{\frac{2}{3}} \mathcal{W}_0 \sqrt{-\mathcal{C}_1 g_s T^4 - 2d})}{9\eta \mathcal{W}_0^2 g_s} \right) + 52\sqrt{6}\eta - 3\sqrt{6}\xi}{36\eta}. \quad (4.9)$$

It is evident that the thermally induced vacuum slightly shifts the volume due to the suppressed term of order $\sim T^2$. In an interesting way at high temperatures, the winding loop effects are the mediator of the novel contributions along the visible sector, a fact that is analogous to the role of loop-induced threshold corrections in heterotic string models [59–61]. Moreover, the $\mathcal{C}_1$ parameter, which accompanies T , is a loop effect along the τ_1 direction and it brings out the necessary asymmetry needed along the winding loop effects that was mentioned before in the stabilization procedure. Despite being a fair assumption in our approach, the emergence of this asymmetry can be traced back to the dynamics of the complex structure moduli and the geometric properties of the background geometry. This behavior shares some interesting features with the work of [62], where the scalars are stabilized dynamically during the thermal cosmological evolution of the universe. Regarding the v direction, we apply the approximate expressions for the minima of transverse directions and due to the complexity of the equations, numerically we find that,

$$\frac{\partial V_{\text{tot}}}{\partial v} \cong 0 \Rightarrow v_{\min} \cong -4v_{\min}(T=0). \quad (4.10)$$

This thermal displacement along the u, v directions, which in practice parametrizes the dynamics of τ_1, τ_2 moduli, can be traced back to the coupling of the MSSM-branes $g_{\text{MSSM}}^2 \sim \tau_1^{-1} \sim e^{-u-v/\sqrt{3}}$. This coupling exchanges the behavior of the vev between the τ_1 and τ_2 directions, since the thermal effects are turned on along the “small” modulus τ_1 and the compactification volume is fixed by the leading order effects. Thus, the above behavior for the stability of the vacuum is expected. To support our argument, the following plots (Fig. 2 and Fig. 3) illustrate the vacua along each direction, to validate our approximations.

As is already known in the literature, finite temperature corrections can destabilize the compactification above a critical point. Therefore, it is important to characterize the decompactification temperature $T_{\max}$, which differs from the rough estimate that states that the critical temperature is given by $T_{\max} \sim V_b^{1/4}$ with V_b denoting the height of the potential barrier at $T=0$. Now, the maximum temperature can be extracted from the maximum point in the barrier, which is given by the W_{-1} branch of the Lambert function,

$$t_{\max} \cong \frac{-12\sqrt{6}\eta W_{-1} \left(-\frac{e^{\frac{13}{3} - \frac{\xi}{4\eta}} (\sqrt{\frac{2}{3}} \mathcal{W}_0 \sqrt{-\mathcal{C}_1 g_s T^4 - 2d})}{9\eta \mathcal{W}_0^2 g_s} \right) + 52\sqrt{6}\eta - 3\sqrt{6}\xi}{36\eta}. \quad (4.11)$$

A few comments need to be made here regarding the nature of this solution. First, the maximum is expressed by the lower branch of the Lambert function, while the prefactor of temperature effects slightly decreases compared to the minimum vev (4.9). This fact can be understood as a hint that the thermal effects push the solutions close to the boundary of the range of validity of the Lambert function, which physically means that the temperature has the tendency to flatten the potential. By the above definition, we can extract the decompactification temperature $T_{\max}$ in terms of the free parameters, through requiring a real

solution with the argument of the Lambert function satisfying $x \geq -1/e$,

$$T_{\max}^2 \leq \frac{\sqrt{\pi}}{\sqrt{-\mathcal{C}_1 g_s}} \left(\frac{2d}{\mathcal{W}_0} + 9\eta g_s \mathcal{W}_0 e^{\frac{\xi}{4\eta} - \frac{16}{3}} \right). \quad (4.12)$$

This bound clearly implies that the decompactification temperature is well below the Planck scale. It is worth noticing also that the number of fluxes $\mathcal{W}_0$ do affect severely the cosmological evolution, since exponentially small fluxes would lead to a smaller decompactification

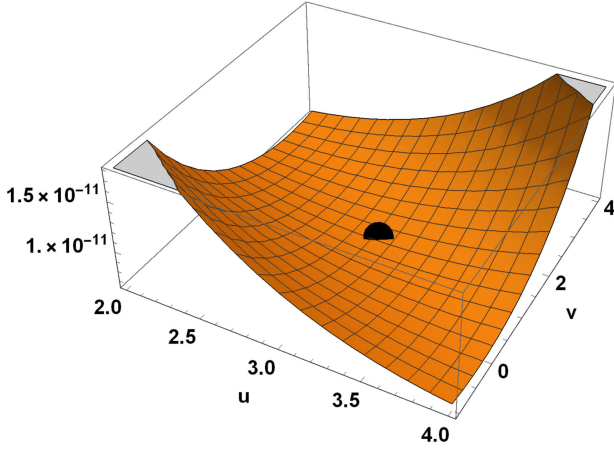


FIG. 3. Plot of the potential (4.5) along the transverse space under thermal effects.

temperature. This can be inferred from Eq. (3.10), where a smaller number of fluxes require smaller uplift d and this consequently would lead to decompactification for high scale inflation. Thus, the above bound could be recognized, also, as an indirect cue to unravel the correlation between fluxes and inflation scale in type IIB LVS compactifications. One last check, to validate and quantify our claim, is to check the “thermal” force R that drives the displacement of the moduli, and it is given by the steepness of the potential at the thermal vacuum,

$$R = \left| \frac{\partial_{\tau_1} V_T}{\partial_{\tau_1} V_0} \right| \sim 1. \quad (4.13)$$

In the above equation we have used Eq. (4.5), where V_0 the $T = 0$ part for the potential while V_T describes the finite temperature corrections. Since the thermal energy can compete with the zero temperature potential, the restabilization of the effective theory was required and it offers an alternative to the cosmological evolution of the universe. Previous analyses of finite temperature effects in the LVS assumed that the modulus that couples to the thermal bath is stabilized by nonperturbative contributions to the superpotential. In this case, the stabilizing force dominates over thermal corrections, enforcing the hierarchy

$$\partial V_T \ll \partial V_0. \quad (4.14)$$

In perturbative LVS models, where Kähler moduli are stabilized by loop-induced and higher-derivative corrections rather than non-perturbative effects, this hierarchy could in principle fail. As a consequence, thermal corrections can compete with the stabilizing potential, leading to the appearance of thermally induced vacua. This regime lies outside the assumptions of previous analyses and has not been explored before. As a result, the standard conclusion that finite-temperature effects cannot significantly

reshape the vacuum structure does not apply in perturbative LVS compactifications.

Finally, let us comment on the decays of the modulus ϕ_3 during radiation domination. Since the volume direction provides a more fertile ground to study inflation due to its shape, we assume that ϕ_3 is not responsible for initial reheating after the inflationary era. These decays could, in principle, wash out any previous cosmological relics if the modulus dominated era T_{dom} precedes the decay temperature T_D of the modulus. It is imperative to explore this scenario, because a modulus dominated era has important implications on the cosmological history concerning the baryon asymmetry, dark radiation [63–65], and dark matter abundances [66,67]. The correct implementation of an inflationary scenario in the current framework requires that $T_f < T_{\text{rh}} < T_{\text{max}}$, while a correct estimate/computation of baryon asymmetry and Big Bang nucleosynthesis is beyond the scope of this work. To this point, we need to compute the freeze-out temperature for the $2 \rightarrow 2$ processes, which can be taken out of Eqs. (2.60), (2.61) where the μ factor is $\mu = (\chi_2(\vec{a}_3))^4 = 8\tau_3^4$,¹

$$T_f > \frac{M_{\text{pl}}(\tau_1\tau_2)^{4/3}g_*^{1/6}}{2^{2/3}\mathcal{V}^{8/3}} \gtrsim 10^{15} \text{ GeV}, \quad (4.15)$$

where the parameters used are listed in Table II, and $g_* \sim 100$. Decays and inverse decay processes provide a similar bound on the order of magnitude for their freeze-out temperatures. Clearly, this bound indicates that finite temperature corrections for moderate values of volume tend to favor a high scale of decompactification temperature, as it is argued also in [54]. In addition, we prove in the final part of this section that decays of thermalized moduli cannot dilute the preexisting relic abundances in the current setup. First, we compute the temperature at which the modulus starts to oscillate,

$$T_{\text{osc}} \sim g_*^{-1/4} \sqrt{m_{\phi_3} M_{\text{pl}}}. \quad (4.16)$$

The energy density of the oscillating modulus can be characterized by its mass and initial amplitude of oscillation, and it can be written as,

$$\rho_{\phi_3} \sim m_{\phi_3}^2 \langle \tau_1 \rangle^2. \quad (4.17)$$

Now, we can compare it with the energy density of radiation at the oscillating temperature, which scales as $\sim T^4$,

¹Each vertex $\phi F_{\mu\nu} F^{\mu\nu}$ carries two derivatives (two powers of momentum) and for a process $2 \leftrightarrow 2$ its amplitude $|A|^2$ scales as $|A|^2 \sim g^4$. Here we use the reduced Planck mass $M_{\text{pl}} = 2.4 \times 10^{18} \text{ GeV}$.

$$\left(\frac{\rho_{\phi_3}}{\rho_r}\right)\bigg|_{T_{\text{osc}}} \sim \frac{m_{\phi_3}^2 \langle \tau_1 \rangle^2}{g_* T_{\text{osc}}^4} \sim \frac{\langle \tau_1 \rangle^2}{M_p^2}. \quad (4.18)$$

At the moment where the two energy densities are comparable, this signals the modulus domination temperature T_{dom} ,

$$\left(\frac{\rho_{\phi_3}}{\rho_r}\right)\bigg|_{T_{\text{dom}}} \sim 1. \quad (4.19)$$

Using all the above, we can derive the domination temperature and define it as,

$$T_{\text{dom}} \left(\frac{\rho_{\phi_3}}{\rho_r}\right)\bigg|_{T_{\text{dom}}} \sim T_{\text{osc}} \left(\frac{\rho_{\phi_3}}{\rho_r}\right)\bigg|_{T_{\text{osc}}} \Rightarrow T_{\text{dom}} \sim T_{\text{osc}} \frac{\langle \tau_1 \rangle^2}{M_{\text{pl}}^2}. \quad (4.20)$$

Consequently, the following ratio can determine if the cosmological evolution includes a modulus dominated cosmological era,

$$\frac{T_D}{T_{\text{dom}}} \sim \frac{g_*^{-1/4} \sqrt{\frac{g^2 m_{\phi_3}^3}{64\pi M_{\text{pl}}}}}{g_*^{-1/4} \frac{\langle \tau_1 \rangle^2}{M_{\text{pl}}^2} \sqrt{m_{\phi_3} M_{\text{pl}}}} \sim \frac{M_{\text{pl}} m_{\phi_3}}{\langle \tau_1 \rangle^2} \gg 1. \quad (4.21)$$

Obviously, the thermalized modulus decays well before its energy density starts to dominate the universe. The cosmological implications of such a result are directly related to early universe phenomena, such as baryon asymmetry and Big Bang nucleosynthesis. Our result is in sharp contrast with moduli-dominated cosmologies, where late decays typically erase any previously generated asymmetry and require new mechanisms for baryogenesis [68–72]. Thus, given our high scale freeze-out temperature and the need for a high scale inflation, various baryogenesis or leptogenesis can be implemented incorporating heavy Majorana neutrinos [73] or through the Affleck-Dine mechanism [74]. We intend to return to these open questions in a future work.

V. THERMAL METASTABILITY AND PHASE TRANSITIONS IN THE EARLY UNIVERSE

In the previous section, we proved that the thermal corrections do not completely destabilize the vacuum, although the theory predicts a decompactification temperature above which the fate of the effective theory is at stake. However, it would be interesting to examine the possibility of the thermal metastability of the vacuum. This aspect of the string landscape has raised a fruitful discussion regarding the generation of gravitational waves [75,76] and a potential interpretation of dark energy as quintessence [77–79]. Previous studies have shown that the lifetime of these vacua is long enough to surpass the age of the universe, either considering Coleman–De Lucia (CdL) [80,81] or HM instanton solutions [34]. This fact can be attributed to the large enough barrier that separates the false vacuum and the true vacuum, which in the most cases is the true vacuum at infinity. The indirect effect of the D-term uplift is that the uplifted vacuum flattens faster than the barrier shrinks, thereby preventing the formation of CdL instantons that would potentially trigger strong first-order transitions in the early universe.

Having this in mind, in this section, we try to probe the temperature that signifies the emergence of HM instanton solutions in the presence of finite temperature corrections. These events are of particular importance due to their classical picture of the modulus jumping to the top of the potential and rolling classically toward the real vacuum. In order to do so, we first examine whether there exists a critical temperature $T_{\text{crit}} < T_{\text{max}}$ at which the reduction of the barrier height outpaces the flattening of the vacuum. As a result, the tunneling channels open before the decompactification temperature is reached, making moduli tunneling the preferred decay mode. The analysis in the previous section comes in handy, especially the definition of the T_{max} , where as the temperature increases, we expect in some region the two extrema (minimum and maximum) to coincide. The approximate formula for the minimum we found before effectively describes a small vicinity of the temperatures, away from the boundaries of the Lambert function. As the temperature increases, the prefactor of the $\mathcal{O}(T^2)$ term should decrease to reach the maximum point. Thus, a critical temperature T_{crit} can be found by demanding the following inequality:

$$\frac{dB}{dT} = \frac{d\Delta V}{dT}, \quad \Delta V = V(t_{\text{max}}, T) - V(t_{\text{min}}, T), \quad B: V(t_{\text{max}}, T) \text{ barrier height}, \quad (5.1)$$

$$\left| \frac{dB}{dT} \right| > \left| \frac{dV(t_{\text{min}}, T)}{dT} \right|. \quad (5.2)$$

Now, as discussed before, the parameter that characterizes the critical temperature is the n parameter in the definition of a minimal value,

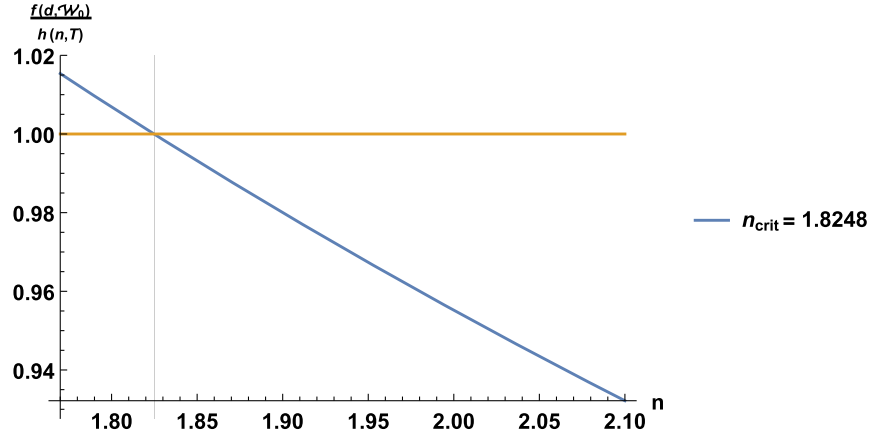


FIG. 4. Ratio of functions in inequality (5.5) characterizing the critical thermal effects on the vacuum. The critical temperature is given $T(n_{\text{crit}}) = 0.011$ and a slightly increased $\lambda = 6 \times 10^{-3}$ is needed, in order to satisfy the criterion (5.2). The rest of the parameters are given by Table I.

$$t_{\min}(n) \cong \frac{-12\sqrt{6}\eta W_0 \left(-\frac{e^{\frac{13-\xi}{3-4\eta}} (\sqrt{\frac{n-1}{\pi}} \mathcal{W}_0 \sqrt{-\mathcal{C}_1 g_s T^4 - 2d})}{9\eta \mathcal{W}_0^2 g_s} \right) + 52\sqrt{6}\eta - 3\sqrt{6}\xi}{36\eta}, \quad (5.3)$$

$$t_{\max}(n) \cong \frac{-12\sqrt{6}\eta W_{-1} \left(-\frac{e^{\frac{13-\xi}{3-4\eta}} (\sqrt{\frac{n-1}{\pi}} \mathcal{W}_0 \sqrt{-\mathcal{C}_1 g_s T^4 - 2d})}{9\eta \mathcal{W}_0^2 g_s} \right) + 52\sqrt{6}\eta - 3\sqrt{6}\xi}{36\eta}. \quad (5.4)$$

So, the critical temperature T_{crit} is bounded by the critical value of n , which can be observed by the following figures,

$$2d + 9g_s \eta \mathcal{W}_0^2 e^{+\frac{\xi}{4\eta} - \frac{16}{3}} = f(d, \mathcal{W}_0) \leq \sqrt{\frac{n}{\pi}} \sqrt{-\mathcal{C}_1 g_s} \mathcal{W}_0 T_{\text{crit}}^2 = h(n, T). \quad (5.5)$$

This bound can be perceived as a deviation in the prefactor of the term $\mathcal{O}(T^2)$, which accounts for the shift of the vacuum. Now, the reason why this critical temperature T_{crit} satisfies inequality (5.2) is that the solution must be pushed sufficiently close to the boundary of the Lambert W_{-1} branch, where thermal effects can compete with (and possibly overcome) the uplift. The D-term uplift scales differently from the thermal uplift and, in particular, it cannot reduce the barrier width efficiently enough to trigger tunneling. In Fig. 4, we display the critical value of the parameter n within the minimal range considered, which corresponds to the critical temperature T_{crit} . For temperatures above T_{crit} , the inequality (5.2) is satisfied, indicating that in this regime Hawking-Moss instantons dominate over CdL instantons.

The above observation leads us to consider the implications of the HM tunneling of the current vacuum. The criterion that has to be satisfied for the emergence of the Hawking-Moss instanton, is described by the following equation [82,83]:

$$H_c^2 = \frac{V''(t_{\max})}{4} + \frac{\Delta V}{3}, \quad H_{\pm} = \sqrt{\frac{V(t_{\min}, t_{\max})}{3}}, \quad (5.6)$$

where H is the Hubble parameter and H_c denotes the critical Hubble parameter that discriminates between the different instanton solutions. Our finite temperature induced vacuum is almost flat, thus the ΔV term can be safely neglected and, then, the criterion reads as [82,83],

$$\frac{H_c^2}{H_-^2} = \frac{3}{4} \frac{V''(t_{\max})}{V(t_{\min})}. \quad (5.7)$$

For $H_- > H_c$, the CdL instantons cannot be justified, which leads to a classical description of the field jumping above the barrier. The Euclidean action of the instanton solution in a symmetric $O(4)$ symmetric background can be written as,

TABLE III. Vevs of the moduli at the vacuum and their corresponding masses. $T_{\max}$ is the decompactification temperature.

τ_1	τ_2	$\mathcal{V}$	$m_{\tau_1}^2$	$m_{\tau_2}^2$	$m_{\mathcal{V}}^2$	T	$T_{\max}$ (GeV)
7100	17	1656	7×10^{-12}	4.5×10^{-15}	2×10^{-21}	0.01075	3×10^{16}

$$S_E(t) = -2\pi^2 \int d\rho \frac{1}{H_{\pm}^3} \sin^3(H_{\pm}\rho) V(t) = -\frac{24\pi^2}{V(t_{\pm})},$$

$$ds^2 = d\rho^2 + \chi(\rho)^2 d\Omega^2, \quad (5.8)$$

where ρ is the time variable and χ is the scale factor of the sphere. Returning to the criterion of Eq. (5.7), we compute it for the parameters of Table III, except ($T = 0.011, \lambda = 6 \times 10^{-3}$), where the minima of the potential are described by (5.3), (5.4).

$$\left. \frac{H_c^2}{H_-^2} \right|_{dS} \sim 0.88. \quad (5.9)$$

The tunneling coefficient for the process can be numerically expressed by,

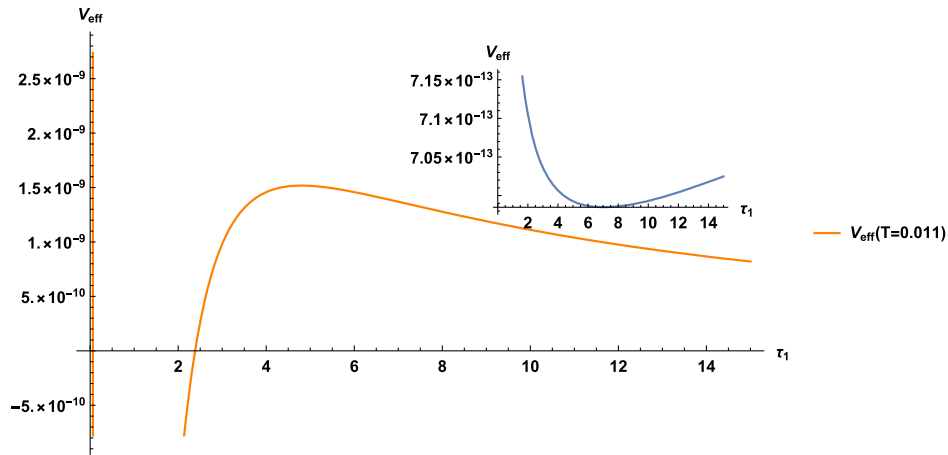
$$\Gamma = Ae^{-B}, B|_{dS} = -24\pi^2 \left(\frac{1}{V(t_{\min})} - \frac{1}{V(t_{\max})} \right) \Big|_{dS} \sim 10^{11}. \quad (5.10)$$

It is evident that the finite temperature corrections cannot destabilize the vacuum. So, the volume direction t which is the lightest direction does not provide any new physics. Thus, we expect that as the temperature decreases, a classical continuous second-order-like phase transition will occur toward the $T = 0$ vacuum. More interestingly, let us plot the potential along the thermalization direction, which is the τ_1 direction. The true vacuum at $T = 0$ lies at

$\tau_1 = 7$, where upon the inclusion of finite temperature corrections, the slope of the potential pushes the vacuum to large τ_1 . However, finite temperature corrections create a barrier (Fig. 5) between a new thermal AdS and the thermal metastable dS minimum described above. This fact opens up a new possibility to explain the metastability of the dS vacua in string compactifications, since plausible first-order phase transitions could emerge as the universe cools down.

Consequently, we could denote two separate cases that associate the metastability of the vacuum with the reheating scale.

- (i) $H_{\inf} < T_{\text{crit}}$: If the inflation scale is very high and the reheating process is instantaneous, then the finite temperature corrections displace the vacuum very far away from the $T = 0$ minimum, where the volume direction is safe from tunneling. In this case, a continuous second-order-like phase transition will occur toward the return of the moduli at the true vacuum at $T = 0$. The generation of gravitational waves in this picture cannot be attributed to a phase transition, but the possibility of an oscillon condensation is very likely due to the oscillation of the field in the thermalized vacuum [84,85]. Earlier studies discussed the emergence of these instabilities of the thermal bath, which lead to low frequency gravitational waves closer to the currently observable range.
- (ii) $H_{\inf} > T_{\text{crit}}$: Finite temperature corrections again displace the vacuum away from the $T = 0$ minimum, but the potential generates a second AdS

FIG. 5. Plot of the potential (4.5) (orange curve) with $T \neq 0$ along the τ_1 direction, while the blue curve shows the minimum at $T = 0$.

vacuum at small τ_1 values. Thus, in this region of parameter space, first-order phase transitions or thermal bounces could lead to an interesting cosmological history, where the fate of the universe will result in a big crunch. However, the supergravity approximation and the loop corrections need to be further understood, in order to make definite statements about the metastability nature of the proposed vacuum structure.

Since the τ_1 modulus is the heaviest field, the transition should be treated as a multifield process, while the bounces can be treated as thermal bubbles [86,87]. A further indication that first-order thermal transitions could arise in our setup comes from recent developments in the study of vacuum transitions in the string landscape.² In particular, the Hamiltonian analysis of [91–93] suggests that the standard Euclidean CdL picture may underestimate the effective connectivity between vacua. The Euclidean approach typically treats de Sitter, Minkowski, and AdS vacua as pure states with vanishing entropy, which tends to suppress or entirely forbid certain transitions. By contrast, in the Hamiltonian formalism these vacua behave as mixed states with nonzero entropy, and the corresponding transition rates need not vanish. As argued in [91], both up- and down-tunneling processes may become allowed once entropic contributions are included, and many transitions can be viewed as limits of transitions involving small black holes in the $M \rightarrow 0$ limit. From this perspective, the thermal bath that displaces the τ_1 modulus in our model may play a role analogous to the entropy contribution of a small black hole: it effectively promotes the finite-temperature de Sitter configuration to a mixed state with an increased number of accessible microstates. This raises the possibility that the thermal metastable vacuum generated at large τ_1 could undergo transitions into the competing AdS region even in situations where a naïve Euclidean CDL analysis would indicate extreme suppression. Although such transitions remain model dependent, the entropic reasoning of [91] suggests that they need not be entirely negligible in a finite-temperature cosmological environment.

In our setup, the thermal corrections generate an additional AdS extremum at small τ_1 , separated from the displaced de Sitter minimum by a temperature-dependent barrier. The arguments of [91] may offer qualitative support for this interpretation: since the thermal de Sitter phase carries an entropy proportional to $H^{-2}(T)$, detailed balance may allow transitions from the high-entropy thermal configuration toward the lower-entropy AdS minimum, including processes that would be highly suppressed at zero temperature. Furthermore, the possible presence of primordial black holes

in the early universe could enhance such effects, as their microstates modify tunneling amplitudes in a similar manner as identified in [91]. The generation of primordial black holes at the early universe has been studied extensively in the past [94–97], where their presence could have implications on baryogenesis as well as on their role as catalyst for phase transitions. Therefore, the thermal AdS vacuum emerging in the τ_1 direction may be viewed as potentially accessible through entropy-assisted vacuum transitions, consistent with the broader quantum-gravitational picture emerging from the Hamiltonian formulation. We do not state that our thermal entropy of the dS vacuum is a direct analog of the horizon entropy, but we tried to schematically provide a conceptual motivation.

Taken together, these considerations point toward a scenario in which the two cases discussed above, $H_{\text{inf}} < T_{\text{crit}}$ and $H_{\text{inf}} > T_{\text{crit}}$, might correspond to different possible cosmological histories within the landscape. In the first case, the thermal displacement of τ_1 could remain in a regime where entropic effects are insufficient to overcome the stabilizing potential, leading the modulus to roll back smoothly to the $T = 0$ vacuum. In the second case, however, the thermal state may enter the region where entropy-assisted transitions of the type described in [91] could become relevant, opening a possible pathway for bubble nucleation into the emergent thermal AdS vacuum and initiating a big-crunch cosmology. Such a mechanism would provide a landscape-motivated explanation for how different reheating scales might lead to different cosmological outcomes, with the metastability of de Sitter vacua influenced not only by the zero-temperature potential but also by their thermal and entropic properties.

VI. CONCLUSIONS

In this work, we have analyzed the finite-temperature cosmological dynamics of a perturbatively stabilized LVS compactification including logarithmic loop corrections, winding effects, and F^4 higher-derivative contributions. We demonstrated that one of the transverse Kähler moduli can thermalize after reheating, inducing a temperature-dependent shift of the vacuum and generating a metastable de Sitter configuration at high energies. Within this framework, we derived analytic expressions for the thermally shifted minima and the corresponding decompactification temperature T_{max} , explicitly relating these quantities to the microscopic parameters of the model. Moreover, we demonstrated that finite temperature effects on the stabilization process emerge through the winding-loop sector, where the corresponding factor $\mathcal{C}_1$ parametrizes the aforementioned contributions to the effective theory. This opens up questions on the dynamics of the complex structure moduli sector at high energies, where the position of the branes and the flux background will specify the order of magnitude for the coefficients $\mathcal{C}_i$. Our analysis shows that, although the volume modulus remains safely stabilized for

²Similar analysis showed some indications for high energy phase transitions in heterotic string theory [88,89]. A more recent study correlates the Festina Lente Swampland bound with the thermal effects [90].

all temperatures below T_{max} , thermal corrections in the transverse direction may create an additional AdS extremum separated by a shrinking barrier. This opens the possibility that, for temperatures above a critical value T_{crit} , plausible first-order transitions or thermal bounces could temporarily alter the cosmological evolution before the system relaxes back to the $T = 0$ vacuum. Furthermore, we examined the temperatures where the thermalized moduli will decay, which lead to the observation that their energy density cannot dominate the universe and so moduli-dominated eras are disfavored for high scale reheating energies in the current framework.

We also discussed how these features may connect with recent developments in the Hamiltonian description of vacuum transitions. In particular, the work of [91] suggests that the entropic nature of finite-temperature (and black-hole-seeded) configurations can modify the effective connectivity of the string landscape, allowing transitions that are strongly suppressed in the Euclidean CDL framework. While our model remains within the controlled supergravity regime, the appearance of a thermally generated AdS vacuum provides a concrete setting in which such entropy-assisted transitions could, in principle, play a role. Future directions include a full multifield bounce

computation in the field space, an exploration of whether black-hole-induced channels enhance or suppress the plausible transitions identified here, and a study of how different reheating scales may select different cosmological histories within the landscape taking into account the flux background with its implications on dilaton and complex structure moduli. These questions could help clarify whether thermal metastability in perturbative LVS might contribute to a broader understanding of vacuum selection and the fate of de Sitter solutions in string theory.

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DATA AVAILABILITY

This is a purely mathematical work and no data was created or analyzed in this study. All figures can be reproduced directly from the presented equations.

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