Entropy dynamics in gravitational collapse: From local Minkowski breakdown to de Sitter thermodynamics

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The origin of black hole entropy remains one of the deepest mysteries in modern physics. Two recent developments offer complementary perspectives on this puzzle: the entropy release from regular Schwarzschild black holes with a de Sitter core (the Ovalle, Casadio, and Kamenshchik [OCK] construction), and the local thermodynamics of the de Sitter vacuum (Volovik). In the OCK framework, the inner horizon carries positive Bekenstein-Hawking entropy $S_{\rm inner} = \pi h_c^2$, whose formal contribution decreases as the core shrinks; the $n = 0$ Minkowski breaking obstructs a regular continuation within the ansatz. In Volovik's framework, the signed entropy assignment for a homogeneous de Sitter Hubble volume is $S_{\rm dS} = {\rm sgn}(H)\pi/H^2 = {\rm sgn}(H)A/4$: expanding de Sitter ($H > 0$) carries positive entropy, while contracting de Sitter ($H < 0$) carries negative entropy. We show that the sign of the Hubble parameter is the unifying element linking the two pictures. The OCK core is de Sitter only for $n > 2$; the Minkowski breaking is a classical kinematic obstruction, not a dynamical transition to a contracting de Sitter phase. Any connection between the expanding and contracting regimes might involve a quantum mechanism that remains speculative. Nevertheless, both frameworks embody the same thermodynamic principle: interior shrinkage changes the assigned entropy, with total entropy increase requiring an appropriate physical balance, and together they provide a consistent picture of entropy flow during gravitational collapse. The integer nature of the OCK parameter suggests a quantized entropy spectrum and a statistical interpretation of black hole entropy, while compatibility with the generalized second law is conditional on the entropy prescription and a justified environmental contribution.

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I. INTRODUCTION

The nature of black hole entropy stands as one of the most profound and enduring puzzles in modern theoretical physics. Since the pioneering work of Bekenstein and Hawking [1,2] established that black holes possess entropy proportional to their horizon area, $S = A/4$, the microscopic origin of this entropy has remained elusive despite decades of intensive investigation [3]. The quest to understand what degrees of freedom are counted by the Bekenstein-Hawking entropy has driven developments across general relativity, quantum field theory, string theory, and holography. Yet a complete, universally accepted microscopic description has not been achieved, suggesting that the resolution may require fundamentally new insights into the relationship between gravity, thermodynamics, and quantum mechanics.

Two recent developments, seemingly independent, offer complementary perspectives on this longstanding puzzle. The first comes from the construction of regular black hole solutions by Ovalle, Casadio, and Kamenshchik (OCK) [4,5]. These geometries provide an explicit family of Schwarzschild black holes with a de Sitter core, free from the central singularity that plagues the classical solution [6,7]. Remarkably, these regular black holes possess an inner Killing horizon that carries a formal Bekenstein-Hawking entropy $S_{\rm inner} = \pi h_c^2$ [8]. In the proposed collapse parametrization, the inner horizon shrinks and its formal area entropy decreases. The $n = 0$ "Minkowski breaking" concerns regular continuation within this family, rather than classical Schwarzschild formation in general. The OCK

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framework thus reveals a geometric setting for a proposed entropy-release interpretation and possible quantum effects, without establishing a radiative mechanism or quantum spectrum.

Subsequent work has extended these ideas in several directions. Casadio, Giusti, Kamenshchik, and Ovalle [9] analyzed the semiclassical energy-momentum tensor near the Minkowski breaking and showed, using the Madelung approximation for collapsing matter, that the quantum potential in the Raychaudhuri equation strongly opposes the collapse towards the Schwarzschild singularity after the Minkowski breaking. Aoki and Ovalle [10] provided an exact analytical model of the complete gravitational collapse, tracing the dynamical growth of the apparent horizon from microscopic scales until it stabilizes at $h = 2\mathcal{M}$; the model develops an integrable curvature singularity that remains hidden by the horizon, preserving weak cosmic censorship. Furthermore, the regular interior construction has been generalized to axisymmetric spacetimes, yielding a family of regular Kerr black holes that interpolate between regular geometries, integrable singularities, and the standard Kerr solution [11].

The second development comes from Volovik's local thermodynamics of the de Sitter vacuum [12]. Building on the observation that the de Sitter spacetime serves as a heat bath with local temperature $T = H/\pi$, twice the Gibbons-Hawking temperature [13], Volovik derived a remarkable result: the local entropy density $s_{\rm vac} = 3H/4$ changes sign with the Hubble parameter. The total entropy of a homogeneous de Sitter region is $S_{\rm dS} = {\rm sgn}(H)\pi/H^2 = {\rm sgn}(H)A/4$. Thus, expanding de Sitter ($H > 0$) carries positive entropy, while contracting de Sitter ($H < 0$) carries negative entropy. This led to the discovery that black hole gravastars, i.e., compact objects with a contracting de Sitter core inside their event horizon, can have zero total entropy because the negative core entropy cancels the positive horizon entropy exactly. Volovik's framework thus provides a thermodynamic interpretation of de Sitter entropy and its relation to horizon entropy, with the sign of the entropy determined by the direction of expansion.

More recently, Volovik [14] considered a simplified model of a black hole gravastar with a de Sitter core, showing explicitly that the static gravastar is thermodynamically unstable. By treating the interior as a homogeneous contracting de Sitter bubble of variable size, he demonstrated that the total entropy of the system increases monotonically as the bubble shrinks, from zero at the gravastar configuration to the Bekenstein-Hawking value $A/4$ when the bubble disappears and the central singularity forms. This result provides a thermodynamic ordering along the proposed path: the Schwarzschild black hole acts as the equilibrium state of maximal entropy, while the gravastar is only a metastable zero-entropy configuration. It also underscores the role of the contracting de Sitter phase as the intermediate state that drives the system toward the singular black hole.

At first glance, these two frameworks appear to describe different physical systems: the OCK framework addresses the thermodynamics of inner horizons in regular black holes, while Volovik's framework addresses the thermodynamics of de Sitter vacua and black hole gravastars. However, a closer examination reveals a deep connection. The distinction between the two frameworks lies in the sign of the Hubble parameter of the de Sitter core. The OCK solution contains a de Sitter-like core whose static geometry only fixes H^2; for convenience we associate it with an expanding phase ($H > 0$) and therefore with positive extra entropy. Volovik's black hole gravastar contains a contracting de Sitter core ($H < 0$) whose negative entropy cancels the horizon entropy. Both pictures illustrate a proposed thermodynamic correspondence, with total entropy increase conditional on the stated balance, either by releasing positive extra entropy or by canceling negative core entropy.

We stress that the two frameworks are *complementary paradigms*, not successive stages of a single classical collapse. The OCK core is de Sitter only for $n > 2$; for $0 < n \leq 2$ its center is curvature singular, rather than a regular anti–de Sitter core, and the local de Sitter thermodynamics of Volovik do not apply. The Minkowski breaking at $n = 0$ is a classical kinematic obstruction, not a mechanism that produces a contracting de Sitter gravastar. Any dynamical connection between the expanding and contracting phases might involve a quantum transition that has not been constructed. The identification of the Minkowski breaking as such a transition is therefore a speculation, not an established result. In this work, we demonstrate that the sign of the Hubble parameter is the unifying element that underlies both frameworks, and we carefully delineate the regime where each applies.

The paper is organized as follows: in Sec. II, we review the OCK regular Schwarzschild black holes, the entropy release mechanism, and the classical Minkowski breaking at $n = 0$. In Sec. III, we summarize Volovik's local de Sitter thermodynamics, focusing on the sign-dependent entropy and the distinction between black hole gravastars and horizonless configurations. In Sec. IV, we establish the complementarity of the two frameworks, and discuss the Minkowski breaking as a kinematic obstruction that could be resolved by a speculative quantum transition. Section V provides a quantitative entropy-release profile and a Landau-type entropic potential that separates the expanding and contracting de Sitter phases. In Sec. VI, we develop the unified thermodynamic description common to both scenarios, and in Sec. VII we summarize our findings and outline future directions. Throughout this work, we set $c = G = \hbar = k_{\rm B} = 1$.

II. ENTROPY RELEASE IN REGULAR SCHWARZSCHILD BLACK HOLES

The OCK family of regular Schwarzschild black holes provides a concrete geometric framework in which the thermodynamics of inner horizons can be studied explicitly. In this section, we review the key properties of these solutions and examine the proposed entropy-release interpretation that will be central to our unified thermodynamic picture.

A. Horizon structure and de Sitter core

The OCK regular Schwarzschild interior is described by the line element

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \tag{1}$$

with

$$f(r) = \begin{cases} 1 - \frac{2m(r)}{r}, & 0 < r \le h, \\ 1 - \frac{2\mathcal{M}}{r}, & r > h, \end{cases} \tag{2}$$

where $h = 2\mathcal{M}$ is the event horizon radius and $\mathcal{M}$ is the Arnowitt–Deser–Misner mass. The Misner-Sharp mass for the $N = 1$ case is [5]

$$m(r) = \frac{r}{2(n-2)}\left[\frac{r^2}{h^2}(n+1) - 3\left(\frac{r}{h}\right)^n\right], \quad n > 2, \tag{3}$$

which satisfies the matching conditions $m(h) = h/2 = \mathcal{M}$ and $m'(h) = 0$, ensuring continuity of the metric and its first derivative across the event horizon. The corresponding metric function is

$$f(r) = 1 - \frac{1}{n-2}\left[\frac{r^2}{h^2}(n+1) - 3\left(\frac{r}{h}\right)^n\right]. \tag{4}$$

B. The expanding de Sitter core

At the origin, the metric function behaves as

$$f(r) = 1 - \frac{n+1}{n-2}\frac{r^2}{h^2} + \cdots, \tag{5}$$

so that $f(0) = 1$ and the geometry approaches a de Sitter core with effective cosmological constant

$$\Lambda_{\text{eff}} = \frac{3}{h^2}\frac{n+1}{n-2}. \tag{6}$$

In the static chart, the metric of pure de Sitter space reads

$$ds^2 = -(1 - H^2 r^2)dt^2 + \frac{dr^2}{1 - H^2 r^2} + r^2 d\Omega^2, \tag{7}$$

and depends only on H^2; the Hubble parameter enters squared and its sign is a matter of convention. Taking the positive root by comparing with the small-r expansion of Eq. (5) gives

$$H = \frac{1}{h}\sqrt{\frac{n+1}{n-2}} > 0. \tag{8}$$

The static OCK geometry determines only H^2, not the sign of H; the choice $H > 0$ is a convenient convention that selects the expanding de Sitter branch. What matters for the thermodynamic discussion is that the inner horizon carries a positive geometric entropy $S_{\text{inner}} = \pi h_c^2$, irrespective of how the global time orientation is fixed. This is a crucial distinction from the black hole gravastar scenario, where the de Sitter core is contracting ($H < 0$) and carries negative entropy [12].

For $n > 2$, the metric function $f(r)$ possesses a second zero at $r = h_c < h$, corresponding to an inner (Cauchy) horizon. For the illustrative case $n = 3$, we have

$$h_c(n = 3) = \frac{1 + \sqrt{13}}{6} h \approx 0.7676\, h. \tag{9}$$

C. Entropy release during collapse

For a smooth stationary Killing horizon in Einstein gravity, including the idealized inner horizon, the Noether-Wald charge formalism assigns the area entropy [3,15,16]

$$S_{\text{inner}} = \frac{A_{\text{inner}}}{4} = \pi h_c^2. \tag{10}$$

This entropy is *hidden* from external observers in equilibrium: the inner horizon is causally disconnected from the outside world, so its area does not contribute to the entropy measured at infinity. Its disappearance does not by itself imply entropy radiation to infinity. We use its area as a formal geometric diagnostic; the physical interpretation and Cauchy-horizon limitations are discussed in Sec. VII.

Under the additive bookkeeping hypothesis, the assigned entropy of the regular black hole is

$$S_{\text{reg}} = S_{\text{outer}} + S_{\text{inner}} = \frac{A}{4} + \pi h_c^2, \tag{11}$$

where $A = 4\pi h^2$ is the area of the event horizon.

During gravitational collapse, the parameter n decreases monotonically, $\dot{n}(v) \le 0$, driven by the null convergence condition [5]. As n decreases from a regular value ($n > 2$) toward zero, the inner horizon shrinks continuously and its formal area-entropy contribution decreases correspondingly. The entropy difference between the regular black hole and the Schwarzschild state is

$$\Delta S(n) = S_{\text{reg}}(n) - S_{\text{Sch}} = \pi h_c^2. \tag{12}$$

For the illustrative case $n = 3$, we obtain

$$\Delta S(n = 3) = \pi h_c^2 \approx 0.589\pi h^2 = 0.589\frac{A}{4}. \quad (13)$$

Thus, a regular black hole with $n = 3$ has a formal additional area-entropy term equal to approximately 59% of the Bekenstein-Hawking entropy of the event horizon.

In the algebraic continuation, the inner horizon shrinks to zero as $n \to 0^+$, where Minkowski breaking occurs: the metric function at the origin jumps discontinuously to $f(0) = -1/2 \neq 1$, signaling a loss of local Minkowski structure. This obstructs a regular continuation within the ansatz; it does not prove that a quantum jump is necessary or rule out classical formation of a singular Schwarzschild black hole.

D. Concept of gravitational collapse

A potential source of confusion in the OCK framework is the apparent tension between the static nature of the geometry and the concept of gravitational collapse. The OCK metric is manifestly static, describing a stationary regular interior whose de Sitter core may be associated with an expanding congruence. Yet the entropy release mechanism relies on a dynamical process in which the parameter n decreases in time. Here, we resolve this apparent paradox by carefully distinguishing between the static geometry and the dynamical evolution that drives the system from one static configuration to another.

The static metric alone does not describe collapse; promoting n to a time-dependent function makes the geometry dynamical. In the dynamical setting, one promotes n to a function of the advanced time v, so that $n = n(v)$. The null convergence condition then imposes $\dot{n}(v) \leq 0$, meaning that n decreases monotonically during collapse [5]. This evolution drives the system from one static configuration (with a given n) to another (with a smaller n). A continuation preserving a regular centre fails before the algebraic Schwarzschild member at $n = -1$ is reached; this fact alone does not establish a necessary quantum jump.

The expanding de Sitter core ($H > 0$) is a feature of each static snapshot, not a contradiction with the collapse dynamics. The collapse is defined by the evolution of n, not by the sign of H. As n decreases, the inner horizon radius $h_c(n)$ shrinks, and its formal area-entropy contribution decreases.

One may speculate that a full quantum gravitational process at the Minkowski breaking point could change the interior state and the associated expansion, converting the expanding interior into a contracting de Sitter phase—the configuration that characterizes a black hole gravastar in Volovik's framework. However, within the purely classical OCK geometry, the core ceases to be de Sitter for $n < 2$ (the leading regular de Sitter expansion no longer applies) and no such transition is implied. The two frameworks are therefore complementary paradigms rather than successive stages of a single classical collapse; a quantum connection is a conjecture, not a consequence of the static geometry.

E. Quantum breakdown and critical value of n

The inner horizon surface gravity is defined as $\kappa_{\text{inner}} = -\frac{1}{2}f'(h_c)$, which is positive for the Cauchy horizon. Using the metric function (4), we obtain

$$\kappa_{\text{inner}} = \frac{1}{h(n-2)}\left[(n+1)\frac{h_c}{h} - \frac{3n}{2}\left(\frac{h_c}{h}\right)^{n-1}\right]. \quad (14)$$

As $n \to 0^+$, the inner horizon radius vanishes as $h_c/h \sim e^{-1/2}(2/3)^{1/n}$ and the surface gravity diverges as

$$\kappa_{\text{inner}} \sim \frac{ne^{1/2}}{2h}\left(\frac{3}{2}\right)^{1/n}. \quad (15)$$

A heuristic extrapolation scale is obtained when the formal inner-horizon temperature approaches the Planck scale, $\kappa_{\text{inner}} \sim 1/\ell_P$. Solving this condition yields $n \sim \ln(3/2)/\ln(h/\ell_P)$ at leading logarithmic order. For astrophysical black holes, $h/\ell_P \sim 10^{38}$, giving $n \simeq 4.35 \times 10^{-3}$ when the logarithmic prefactor is retained. This extrapolation already concerns centrally singular members with $n \leq 2$, and does not identify a universal quantum-transition threshold.

The OCK framework describes entropy release during gravitational collapse as a sequence of stages that naturally emerges from the geometry and its dynamics. The process begins with an initial regular black hole characterized by an expanding de Sitter core ($H > 0$) carrying positive inner-horizon entropy $S_{\text{inner}} = \pi h_c^2$. As the parameter n decreases monotonically, the inner horizon shrinks continuously, and the formal area-entropy contribution decreases. The extrapolated surface gravity becomes Planckian at very small n, but central singularity and Cauchy-horizon backreaction can invalidate the description earlier. The Minkowski breaking at $n = 0$ then marks a kinematic obstruction to regular continuation that motivates consideration of quantum effects, possibly linking to the contracting de Sitter phase of a gravastar, but no continuous regular transition is supplied by this ansatz. The Schwarzschild member has entropy $A/4$; a physical transition and the environmental entropy required by the additive interpretation remain to be calculated.

This picture will be complemented in the next section by a framework in which a contracting de Sitter core carries negative entropy that cancels the horizon entropy. The fundamental distinction between the two frameworks lies in the sign of the Hubble parameter: expanding cores yield positive entropy, while contracting cores yield negative entropy. Both frameworks, however, motivate a

thermodynamic comparison, whose physical second-law interpretation requires the assumptions specified below.

III. LOCAL THERMODYNAMICS OF DE SITTER VACUUM

Volovik's local thermodynamics of the de Sitter vacuum provide a complementary perspective on gravitational entropy, based on the observation that the de Sitter spacetime acts as a thermal bath for matter with a well-defined local temperature. In this section, we review the key elements of this framework, emphasizing the sign of the entropy and its relation to the expansion or contraction of the de Sitter state.

A. Local temperature and its physical origin

Volovik established that the de Sitter vacuum serves as a heat bath for matter with the local activation temperature $T = H/\pi$ [12], which is twice the Gibbons-Hawking temperature $T_{\rm GH} = H/2\pi$ [13]. This temperature governs local processes that take place well inside the cosmological horizon and have no direct relation to the horizon itself.

The physical origin of this temperature lies in the specific symmetry of the de Sitter spacetime. The de Sitter metric in Painlevé-Gullstrand form is

$$ds^2 = -dt^2 + (dr - Hr\,dt)^2 + r^2 d\Omega^2, \tag{16}$$

which is invariant under the modified translations $\mathbf{r} \to \mathbf{r} - e^{Ht}\mathbf{a}$. This symmetry, which reduces to ordinary translations in the limit $H \to 0$, ensures that all comoving observers at any point of the de Sitter space perceive the same local temperature. Thus, the de Sitter vacuum is unique among gravitational backgrounds in providing a thermal bath whose temperature does not violate the spacetime symmetry.

The temperature $T = H/\pi$ governs activation processes that are energetically forbidden in Minkowski spacetime but allowed in the de Sitter background. For example, the ionization rate of an atom in the de Sitter environment is $w \sim \exp(-E/T)$, where E is the ionization potential. Similarly, the decay of a composite particle into two particles with total mass exceeding the original mass is governed by the same temperature [12]. Note that for $H < 0$, the temperature becomes negative, which is a signed convention in this prescription, not evidence for population inversion or a universal activation-rate formula.

B. Local entropy density

From the Friedmann equations of general relativity, the vacuum energy density is

$$\epsilon_{\rm vac} = \frac{3}{8\pi}H^2 = \frac{3\pi}{8}T^2. \tag{17}$$

Assuming that the temperature $T = H/\pi$ is the local temperature of the de Sitter vacuum, we can determine the free energy density. In this prescription, choosing $s_{\rm vac}(0) = 0$ removes a possible linear-in-T integration term, and the free energy density is quadratic in temperature, as follows:

$$F = \epsilon_{\rm vac} - T s_{\rm vac}. \tag{18}$$

Using the thermodynamic relation $s_{\rm vac} = -\partial F/\partial T$, we obtain the local entropy density

$$s_{\rm vac} = -\frac{\partial F}{\partial T} = \frac{3\pi}{4}T = \frac{3}{4}H. \tag{19}$$

Note that Eq. (19) shows that the sign of the entropy density follows the sign of the Hubble parameter:

$$s_{\rm vac} = \begin{cases} \text{positive}, & H > 0 \ (\text{expanding de Sitter}), \\ \text{negative}, & H < 0 \ (\text{contracting de Sitter}). \end{cases} \tag{20}$$

This is a remarkable result: the entropy of the de Sitter vacuum is not an intrinsic positive quantity but changes sign with the direction of expansion. An expanding de Sitter universe ($H > 0$) has positive entropy, while a contracting de Sitter universe ($H < 0$) has negative entropy. This sign change is central to the thermodynamic interpretation of black hole gravastars and the complementarity with the OCK framework.

C. Modified Gibbs-Duhem relation

The conventional vacuum pressure $P_{\rm vac}$ obeys the equation of state $w = -1$ and enters the energy-momentum tensor as

$$T^{\mu\nu} = -\epsilon_{\rm vac} g^{\mu\nu}, \qquad P_{\rm vac} = -\epsilon_{\rm vac}, \tag{21}$$

where $\Lambda = 8\pi\epsilon_{\rm vac}$ and the orthonormal components are $\mathrm{diag}(\epsilon_{\rm vac}, P_{\rm vac}, P_{\rm vac}, P_{\rm vac})$ for signature $(-+++)$. In the de Sitter state, $P_{\rm vac} < 0$.

This pressure does not satisfy the standard thermodynamic Gibbs-Duhem relation $T s_{\rm vac} = \epsilon_{\rm vac} + P_{\rm vac}$, because the rhs is zero. The reason is that this relation does not account for the gravitational degrees of freedom. Earlier work has shown that gravity contributes to thermodynamics with a pair of thermodynamically conjugate variables: the gravitational coupling $K = 1/16\pi$ (in units $G = 1$) and the scalar curvature $\mathcal{R}$ [17–19]. The Einstein-Hilbert action contains the term $K\mathcal{R}$, and its contribution to thermodynamics is analogous to work density [20–23].

Including these gravitational variables, the modified Gibbs-Duhem relation takes the form

$$T s_{\rm vac} = \epsilon_{\rm vac} + P_{\rm vac} - K\mathcal{R}, \tag{22}$$

where $\mathcal{R} = -12H^2$ for the de Sitter spacetime. This equation is satisfied identically, since $\epsilon_{\rm vac} + P_{\rm vac} = 0$ and $-K\mathcal{R} = 12KH^2 = T s_{\rm vac}$.

Equation (22) suggests the introduction of an effective pressure that absorbs the gravitational degrees of freedom:

$$P = P_{\rm vac} - K\mathcal{R}. \tag{23}$$

Then the conventional Gibbs-Duhem relation is restored, as follows:

$$Ts_{\rm vac} = \epsilon_{\rm vac} + P. \tag{24}$$

The effective de Sitter pressure P is positive, $P = \epsilon_{\rm vac} > 0$, and satisfies the equation of state $w = 1$, which is characteristic of stiff matter introduced by Zeldovich [24]. Thus, due to the gravitational degrees of freedom, the de Sitter state behaves thermodynamically like a nonrelativistic Fermi liquid, where the thermal energy is proportional to T^2, and algebraically like stiff matter; the effective thermodynamic pressure does not determine a material sound speed. This effective pressure reveals the thermodynamic nature of the de Sitter vacuum as stiff matter ($w = 1$), a key property that will be used in the unified framework developed in Sec. IV. In that framework, the sign of the entropy, determined by the sign of the Hubble parameter, provides the direct link between the expanding core of the OCK black hole and the contracting core of the black hole gravastar.

D. Total entropy in the Hubble volume

The cosmological horizon of a de Sitter spacetime is located at the radius $R_H = 1/|H|$, enclosing the volume $V_H = \frac{4\pi}{3}R_H^3 = \frac{4\pi}{3|H|^3}$. Integrating the local entropy density (19) over this volume gives the total gravitational entropy of the de Sitter vacuum, as follows:

$$S_{\rm dS} = s_{\rm vac}V_H = \frac{3}{4}H \cdot \frac{4\pi}{3|H|^3} = \frac{\pi H}{|H|^3} = {\rm sgn}(H)\frac{\pi}{H^2}. \tag{25}$$

Using the horizon area $A = 4\pi R_H^2 = 4\pi/H^2$, this can be written in the more familiar form

$$S_{\rm dS} = {\rm sgn}(H)\frac{A}{4}. \tag{26}$$

Thus, for an expanding de Sitter phase ($H > 0$) the total entropy is the Gibbons-Hawking entropy $A/4$, whereas for a contracting phase ($H < 0$) it is *negative*: $S_{\rm dS} = -A/4$.

This result reproduces the holographic bulk-surface correspondence: the entropy obtained from the local vacuum thermodynamics inside the horizon exactly matches the entropy attributed to the horizon itself, up to the sign given by the direction of expansion. Equation (25) [or its dimensional form $S_{\rm dS} = {\rm sgn}(H)\pi/(GH^2)$ when G is restored] is the correct unified entropy function for a homogeneous de Sitter region; earlier versions of this work contained an erroneous expression $S(H) \propto -1/(4H)$, which is dimensionally inconsistent and gives the wrong sign. This sign-dependent entropy provides the foundation for the unified entropy dynamics, where the sign of the entropy follows the sign of the Hubble parameter.

E. Gravastars and entropy cancellation

In Volovik's framework, the term "gravastar" refers to a black hole that possesses a de Sitter core inside its event horizon, which distinguishes it from the horizonless gravastars originally proposed in Refs. [25–27]. In such a black hole gravastar, the de Sitter core is contracting, as follows from the continuity of the shift vector in Painlevé-Gullstrand coordinates across the horizon. Consequently, with $H < 0$, the local entropy density is negative:

$$s_{\rm core} = \frac{3}{4}H < 0. \tag{27}$$

The total entropy of the de Sitter region inside the horizon is obtained from Eq. (26) with $H < 0$, as follows:

$$S_{\rm core} = -\frac{A}{4}. \tag{28}$$

This negative entropy cancels the positive Bekenstein-Hawking entropy of the black hole horizon, yielding

$$S_{\rm gravastar} = S_{\rm BH} + S_{\rm core} = \frac{A}{4} - \frac{A}{4} = 0. \tag{29}$$

The cancellation assumes that the core fills the horizon, $R_c = h = 1/|H|$. For a finite homogeneous contracting core, the same density instead gives $S_{\rm core} = -\pi|H|R_c^3$. In the fixed-mass bubble model of Ref. [14], $R_c^3 = 2\mathcal{M}/H^2$, so the combined assignment is $A/4 - 2\pi\mathcal{M}/|H|$.

Thus, the special black hole gravastar configuration has zero assigned entropy in this prescription; entropy cancellation alone does not establish absence of Hawking radiation. This entropy cancellation encapsulates the key distinction between the two frameworks: a contracting de Sitter core carries negative entropy that cancels the horizon entropy, while an expanding core carries positive entropy that is released during collapse. In both cases, the sign of the Hubble parameter determines the thermodynamic role of the core. This complementary relationship will be formalized in Sec. IV, where we show how the sign of H provides the direct link between the expanding core of the OCK black hole and the contracting core of the black-hole gravastar.

These results establish the thermodynamic foundation for the complementarity with the OCK framework. The key ingredients are the local temperature $T = H/\pi$ (which can be negative), the entropy density $s_{\rm vac} = 3H/4$ whose sign follows the sign of H, the sign-dependent total entropy $S_{\rm dS} = {\rm sgn}(H)A/4$, and the cancellation mechanism for black hole gravastars. Together, these elements provide

the essential thermodynamic structure that we will exploit in the next section to develop the unified description of gravitational collapse.

IV. COMPLEMENTARITY AND UNIFICATION

Having reviewed the OCK framework of entropy release from regular Schwarzschild black holes and Volovik's local thermodynamics of the de Sitter vacuum, we now establish the complementarity between these two approaches. The key observation is that both frameworks describe a closely related thermodynamic principle: the reduction of the interior contribution is associated with an increase of the total entropy, but from opposite perspectives—in the OCK framework through the disappearance of a positive inner-horizon entropy, and in Volovik's description through the cancellation of a negative bulk entropy. The thermodynamic correspondence between these descriptions is naturally organized by the sign of the Hubble parameter associated with the de Sitter sector.

The geometrical origin of this sign distinction becomes particularly transparent by comparing the static and Painlevé-Gullstrand representations of de Sitter spacetime. In static coordinates,

$$ds^2 = -(1 - H^2 r^2)dt^2 + \frac{dr^2}{1 - H^2 r^2} + r^2 d\Omega^2; \quad (30)$$

the geometry depends only on H^2 and therefore does not distinguish between expanding and contracting de Sitter patches. By contrast, in Painlevé-Gullstrand coordinates,

$$ds^2 = -dT^2 + (dr - v(r)dT)^2 + r^2 d\Omega^2, \quad v(r) = \pm Hr, \quad (31)$$

the two signs explicitly encode opposite time orientations: $v = +Hr$ describes the expanding patch, whereas $v = -Hr$ describes the contracting one.

This distinction has a direct thermodynamic counterpart. The horizon area depends only on H^2 and is therefore the same for the expanding and contracting descriptions,

$$A_H = \frac{4\pi}{H^2}, \quad (32)$$

whereas the sign-sensitive de Sitter entropy may be written as

$$S_{\rm dS} = \mathrm{sgn}(H)\frac{A_H}{4} = \mathrm{sgn}(H)\frac{\pi}{H^2}. \quad (33)$$

This sign belongs to the bulk thermodynamic prescription, not to conventional Wald entropy. Reversing the horizon-generating Killing field reverses both its Noether charge and its surface-gravity normalization, leaving the standard area entropy positive. Expansion and contraction therefore share the same intrinsic de Sitter geometry; $H \to -H$ does not change $\Lambda = 3H^2$.

A. Expanding versus contracting de Sitter

Table I summarizes the proposed entropy assignments and their geometric regimes.

In the OCK geometry, the $n > 2$ core is locally de Sitter and may be associated with the expanding branch, $H > 0$, whereas Volovik's black hole gravastar employs the contracting branch, $H < 0$. The thermodynamic significance of this distinction follows from Volovik's local relation

$$s_{\rm vac} = \frac{3H}{4}, \quad (34)$$

which assigns positive bulk entropy density to expanding de Sitter and negative bulk entropy density to contracting de Sitter.

The entropy quantities entering the two constructions should, nevertheless, be distinguished carefully. In the OCK geometry, the additional positive contribution is the geometric entropy $S_{\rm inner} = \pi h_c^2$ associated with the inner Killing horizon, whereas in Volovik's construction, the negative contribution is a bulk vacuum entropy obtained by integrating $s_{\rm vac}$ over the contracting de Sitter core. They are therefore not identical entropy functionals. Their complementarity lies instead in the thermodynamic role they play: reducing the interior contribution increases the total entropy, through the release of a positive contribution in the OCK description and through the removal of a negative contribution in Volovik's description.

TABLE I. Comparison of the OCK and Volovik frameworks.

Aspect	OCK framework	Volovik framework
Core type	Expanding de Sitter ($n > 2$)	Contracting de Sitter
Hubble parameter	$H > 0$ (for $n > 2$)	$H < 0$
Extra entropy	Positive: $+\pi h_c^2$ (inner horizon)	Negative: $-A/4$ (bulk)
Physical process	Entropy release as inner horizon shrinks	Entropy cancellation (horizon + core = 0)
Regime of validity	Regular core for $n > 2$; centrally singular continuation for $0 < n \leq 2$	Static configuration (by construction)
Quantum transition	Minkowski breaking at $n = 0$ (kinematic obstruction)	Not emphasized
Geometric setting	Regular Schwarzschild interior	Black-hole gravastar

TABLE II. Comparison of entropy dynamics in the two frameworks, under the proposed entropy assignments.

Aspect	OCK scenario	Volovik scenario
Initial state	Regular BH: $S_{\rm reg} = A/4 + \pi h_c^2$	Gravastar: $S_{\rm gravastar} = 0$
Final state	Schwarzschild: $S_{\rm Sch} = A/4$	Black hole: $S_{\rm BH} = A/4$
Entropy change (system)	$\Delta S_{\rm BH} = -\pi h_c^2 < 0$	$\Delta S_{\rm total} = A/4 > 0$
Entropy conservation	$\Delta S_{\rm BH} + \Delta S_{\rm rad} \geq 0$	Direct increase
Mechanism	Positive entropy release	Negative entropy cancellation

B. Entropy dynamics in the two paradigms

The conditional entropy balances are summarized in Table II.

Despite their apparent differences, both frameworks admit a comparison of their assigned entropy changes; physical entropy increase remains conditional on the required balance. This manifests differently in each scenario: in the OCK framework, as the inner horizon shrinks, the positive assigned contribution decreases; in Volovik's framework, a contracting de Sitter core carries negative entropy that cancels the positive horizon entropy.

Consider the entropy changes in each scenario. In the OCK case, the regular black hole has entropy $S_{\rm reg} = A/4 + \pi h_c^2$, while the Schwarzschild state has $S_{\rm Sch} = A/4$. The black hole entropy alone decreases by $\Delta S_{\rm BH} = -\pi h_c^2 < 0$, and an additive second-law interpretation would require an environmental contribution $\Delta S_{\rm rad} \geq \pi h_c^2$; no outgoing radiation state is calculated here. Under that additional hypothesis, the total entropy change would satisfy $\Delta S_{\rm total} = -\pi h_c^2 + \Delta S_{\rm rad} \geq 0$, preserving the generalized second law. In the Volovik case, the gravastar has zero total entropy, while the black hole state has entropy $S_{\rm BH} = A/4$. The entropy change is $\Delta S_{\rm total} = A/4 > 0$, with the entropy increasing directly as the negative-entropy core shrinks.

The two scenarios are thus complementary: in OCK, the black hole entropy decreases while the environment's entropy increases; in Volovik, the total entropy increases as the negative-entropy core disappears. Both can be compatible with a second-law interpretation under the stated entropy and environmental assumptions. This complementarity is traced to the sign of the Hubble parameter: the expanding core releases positive entropy, while the contracting core has negative entropy that cancels the horizon entropy.

C. Minkowski breaking and the limits of classical unification

It is tempting to regard the Minkowski breaking at $n = 0$ in the OCK analysis as a continuous transition from an expanding de Sitter core to a contracting one, thereby unifying the two frameworks into a single collapse sequence. However, this interpretation is not supported by the classical OCK geometry.

For $n > 2$, the interior is a genuine expanding de Sitter core, and the inner horizon carries positive entropy. As n decreases, the effective cosmological constant $\Lambda_{\rm eff}(n) = 3(n+1)/[h^2(n-2)]$ increases, diverging as $n \to 2^+$. For $0 < n < 2$, the r^n term dominates the r^2 term and curvature diverges as r^{n-2}; the center is not a regular anti–de Sitter core. At $n = 2$, the fixed-r limit is $f = 1 - x^2 + 3x^2 \ln x$, $x = r/h$, with logarithmically divergent central curvature. Therefore, the local de Sitter thermodynamics of Volovik (which relies on a positive cosmological constant and a well-defined Hubble parameter) does not apply in the regime $n < 2$. The inner horizon continues to shrink and its formal geometric entropy still decreases, but the process is no longer described by a de Sitter core with $H > 0$.

At $n = 0$, the metric function develops a discontinuity (Minkowski breaking), signaling failure of a locally Minkowskian central limit within this continuation. This is a kinematic obstruction, not a mechanism that converts the interior into a contracting de Sitter phase. Consequently, the OCK regular black hole and the Volovik black hole gravastar are complementary paradigms that share the same sign-dependent entropy principle, but they are *not* successive stages of a single classical collapse.

D. Speculative quantum connection

Although the classical OCK solution does not evolve into a contracting de Sitter gravastar, one may speculate that a full quantum gravitational process at the Minkowski breaking point might change the interior state and its congruence from expansion to contraction, without a sign change of $\Lambda = 3H^2$. The divergence of the inner-horizon temperature as $n \to 0^+$ and the breakdown of the semiclassical description at $n \sim 1/\ln(h/\ell_P)$ motivate studying quantum effects, without identifying a controlled transition near the breaking point. If an instanton exists that interpolates between an expanding de Sitter interior and a contracting one, the Minkowski breaking would indeed correspond to a quantum phase transition, and the gravastar would emerge as an intermediate state with zero total entropy before the core eventually collapses to the Schwarzschild singularity. In such a scenario, the total entropy would flow as

$$S = \frac{A}{4} + \pi h_c^2 \longrightarrow 0 \longrightarrow \frac{A}{4}.$$

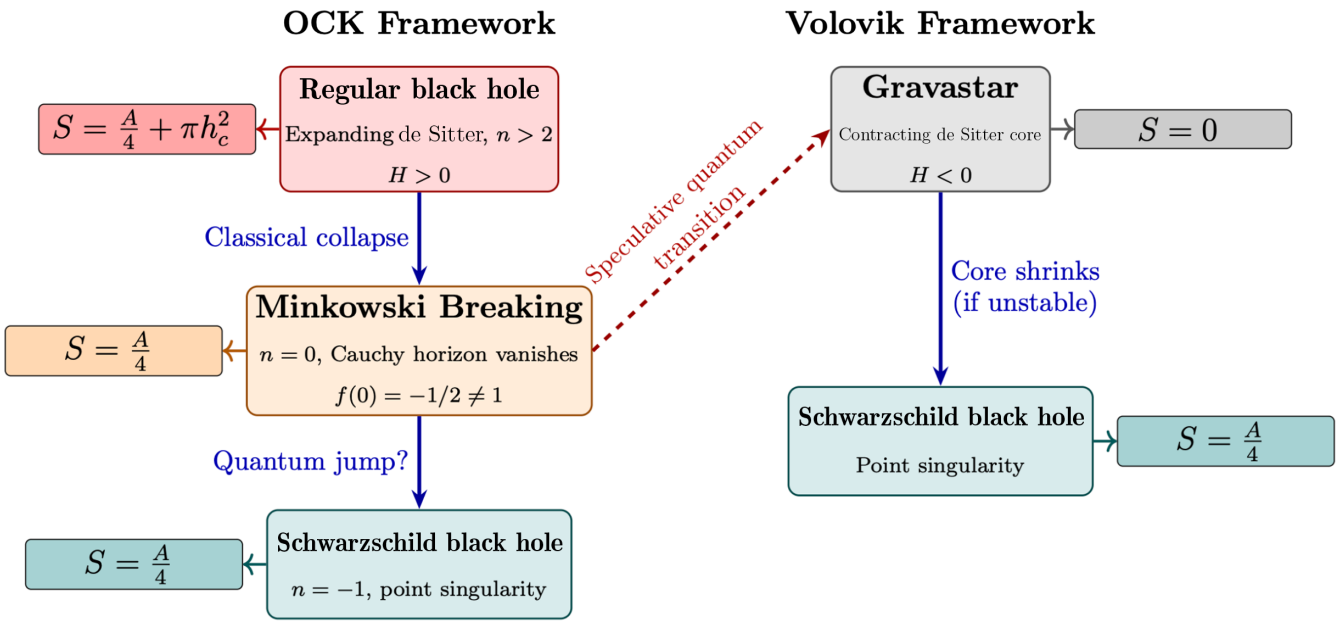


FIG. 1. Entropy flow in the two complementary frameworks. Left column: The OCK regular black hole contains an expanding de Sitter core ($H > 0$) and carries positive inner-horizon entropy $S = A/4 + \pi h_c^2$. Classical collapse drives the parameter n to zero; at $n = 0$ the inner horizon disappears and the metric function undergoes a Minkowski breaking, leaving $S = A/4$. Whether a further (quantum) jump to the Schwarzschild state is required remains an open question. Right column: In Volovik's picture, a black hole gravastar possesses a contracting de Sitter core ($H < 0$) whose negative bulk entropy exactly cancels the horizon entropy, giving $S = 0$. If the core shrinks (e.g., due to perturbations), the entropy rises to $S = A/4$, restoring the Schwarzschild black hole. The dashed red arrow indicates a *speculative* quantum transition that could connect the two paradigms by flipping the sign of the Hubble parameter at the Minkowski breaking point. No concrete dynamical mechanism for such a transition is known. The diagram summarizes proposed assignments; it establishes neither a radiation flux nor a generalized-second-law proof. The $n \leq 2$ continuation is centrally singular, so the arrows are not a sequence of regular collapse states.

Such a sequence can satisfy the generalized second law provided the first step is accompanied by radiation (or other environmental degrees of freedom) that carries an entropy at least $\frac{A}{4} + \pi h_c^2$, just as in the pure OCK scenario. Without specifying this environmental contribution the sequence alone does not manifestly obey the second law. At present this remains a conjecture, and constructing the relevant instanton is an open problem. The connection with quantum cosmic censorship and the distinct question of Cauchy-horizon predictability are discussed in Sec. VII.

E. Summary

The two frameworks are complementary paradigms governed by the same sign-dependent entropy principle (see Fig. 1): an expanding de Sitter core ($H > 0$) carries positive extra entropy that is released during collapse (the OCK scenario), whereas a contracting de Sitter core ($H < 0$) carries negative entropy that exactly cancels the horizon entropy, yielding a zero-entropy gravastar (Volovik's scenario). Compatibility with a physical generalized second law remains conditional on those assumptions. The Minkowski breaking at $n = 0$ is a classical kinematic obstruction, not a continuous transition to a contracting phase; a quantum connection between the two pictures is a hypothesis whose mechanism remains unspecified. The sign of the Hubble parameter thus acts as the unifying order parameter that dictates the thermodynamic role of the interior.

V. THERMODYNAMIC ANALYSIS OF THE ENTROPY FLOW

The complementarity between the OCK and Volovik frameworks can be sharpened by two concrete analyses: (i) tracking the entropy released as the inner horizon shrinks, both for classical continuous evolution and for a discrete quantum cascade, and (ii) formulating a Landau-type thermodynamic potential that exposes the role of the Hubble parameter as an order parameter distinguishing the expanding and contracting de Sitter phases.

A. Entropy-release profile in the OCK collapse

For the $N = 1$ OCK solution, the inner-horizon entropy is $S_{\text{inner}}(n) = \pi h_c^2(n)$, where $h_c(n)$ is the smaller positive root of $f(r) = 0$ in Eq. (4). Although a closed analytic form for h_c is available only for special values of n, the function is easily computed numerically. Table III lists h_c/h and the entropy fraction $\Delta S/(A/4) = (h_c/h)^2$ for several values of the (static) parameter n.

TABLE III. Inner-horizon radius and entropy fraction for the $N = 1$ OCK solution.

n	h_c/h	$\Delta S/(A/4)$
3	0.7676	0.589
4	0.8165	0.667
5	0.8484	0.720
6	0.8708	0.758
7	0.8875	0.788
10	0.9188	0.844
100	0.9913	0.983
∞	1	1

During a classical continuous collapse, $n(v)$ evolves monotonically from a large initial value toward zero. The inner horizon shrinks and $S_{\rm inner}$ decreases smoothly; a physical interpretation as radiated entropy would require a transport calculation and an independently justified generalized-entropy balance. The entropy release per unit decrease of n is $dS_{\rm inner}/dn$, which remains finite for all $n > 0$.

If, instead, the transition between the static integer-n equilibrium states occurs via quantum jumps, the entropy is emitted in finite steps as the parameter decreases from one integer to the next. Let us define the step from $n+1$ to n (with $n = 3, 4, 5, \ldots$) as

$$\Delta S_n = S_{\rm inner}(n+1) - S_{\rm inner}(n) (n \geq 3). \tag{35}$$

Using the values in Table III, one finds

$$\Delta S_3 \approx 0.077 \frac{A}{4}, \qquad \Delta S_4 \approx 0.053 \frac{A}{4},$$
$$\Delta S_5 \approx 0.039 \frac{A}{4}, \qquad \Delta S_6 \approx 0.029 \frac{A}{4},$$

and the steps continue to diminish as n increases. Each step becomes smaller as n increases, reflecting the approach of the inner horizon to the event horizon. A pulse train is a possible hypothesis, but its existence and amplitudes would require transition dynamics and a radiation model; the integer area differences alone do not predict it. The sum of all integer steps from $n = \infty$ down to $n = 3$ does not account for the full inner-horizon entropy; a final, larger jump from $n = 3$ to $n = 0$ (where the inner horizon disappears) would remove the remaining formal area-entropy fraction $S_{\rm inner}(3) \approx 0.589A/4$. In the continuous evolution picture, the integral of $dS_{\rm inner}/dn$ from $n = 0$ to $n = \infty$ recovers the full inner-horizon entropy $\pi h^2 = A/4$.

B. Landau-type potential and order parameter

The sign of the Hubble parameter controls the sign of the total de Sitter entropy, $S_{\rm dS} = {\rm sgn}(H)A/4$. This suggests that H can be viewed as an order parameter that distinguishes two different thermodynamic phases. To make this analogy concrete, we construct a phenomenological Landau-type analogy using $F(H)$, without interpreting it as a dynamical stability potential or tunneling action.

For a homogeneous de Sitter region of Hubble radius $R_H = 1/|H|$, the internal energy and entropy are

$$E = \epsilon_{\rm vac} V_H = \frac{3H^2}{8\pi} \frac{4\pi}{3|H|^3} = \frac{1}{2|H|},$$
$$S = {\rm sgn}(H) \frac{\pi}{H^2}.$$

Using the local temperature $T = H/\pi$ (which can be positive or negative), the free energy $F = E - TS$ becomes

$$F(H) = \frac{1}{2|H|} - \frac{H}{\pi} {\rm sgn}(H) \frac{\pi}{H^2}$$
$$= \frac{1}{2|H|} - \frac{1}{|H|} = -\frac{1}{2|H|}. \tag{36}$$

Equation (36) depends only on $|H|$ and is always negative, decreasing without bound as $|H| \to 0$. It does not exhibit a minimum at any finite H, which reflects the fact that a free energy constructed from the static de Sitter patch does not capture the dynamical stability of the de Sitter state. To obtain a potential that distinguishes the sign of H, one must instead consider the entropy difference between configurations as the driver of quantum transitions. A more useful quantity is therefore the *entropic potential,*

$$V(H) \equiv -S(H) = -{\rm sgn}(H) \frac{\pi}{H^2}. \tag{37}$$

A formal entropic weight for two assigned states with Hubble parameters H_i and H_f may be written as $\Gamma \propto \exp[-(V(H_f) - V(H_i))] = \exp[S(H_f) - S(H_i)]$, but an absolute transition rate additionally requires dynamics, an action, and prefactors.

The potential $V(H)$ is plotted schematically in Fig. 2. For $H > 0$, $V(H) = -\pi/H^2$ is negative and dives to $-\infty$ as $H \to 0^+$; for $H < 0$, $V(H) = +\pi/H^2$ is positive and rises to $+\infty$ as $H \to 0^-$. The branches diverge at $H = 0$ because the Hubble integration volume becomes infinite; this is not a physical barrier. In this picture, the classical OCK collapse drives the system along the positive-H branch toward larger H (the inner horizon shrinks and the core entropy decreases), eventually leaving the regime where the de Sitter description applies. A quantum transition may be conjectured, but the plot identifies neither a classically forbidden region nor a required tunneling mechanism. While the shape of $V(H)$ is purely phenomenological, it illustrates the distinction between the signed assignments, without proving that a quantum process connects the paradigms. The divergence at $H = 0$ reflects the infinite Hubble volume and should not be interpreted as a literal infinite tunneling barrier. The potential simply illustrates that the sign-dependent entropy separates the expanding and contracting de Sitter phases; it does not by itself

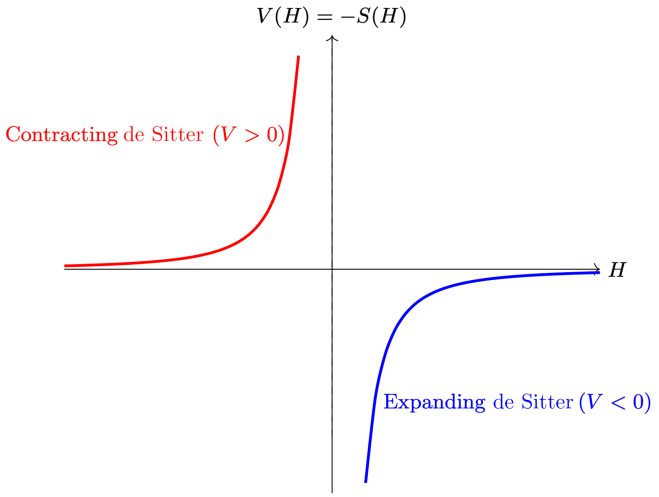


FIG. 2. Entropic potential $V(H) = -S(H)$ as a function of the Hubble parameter. The expanding phase ($H > 0$) has $V < 0$, the contracting phase ($H < 0$) has $V > 0$. The divergence at $H = 0$ reflects the infinite Hubble volume, not a physical tunneling barrier; no transition dynamics are inferred.

constitute a Landau free energy nor does it identify $n = 0$ with $H = 0$.

In summary, the explicit entropy-release profile quantifies the graduality (or discreteness) of the collapse, while the Landau-type entropic potential offers an intuitive thermodynamic picture of the order-parameter character of H and the distinction between the expanding and contracting signed-entropy branches. Together they reinforce the main message of this work: the sign of the Hubble parameter governs the thermodynamics of gravitational collapse, and a complete dynamical unification of the expanding and contracting phases remains an open quantum problem.

VI. UNIFIED ENTROPY DYNAMICS AND IMPLICATIONS

Having established the complementarity of the OCK and Volovik frameworks above, we now consolidate their insights into a unified thermodynamic picture. We stress that the two frameworks are complementary paradigms, not successive stages of a single classical collapse. A quantum connection is a possible hypothesis, but no dynamical bridge has been constructed. The central quantity in this complementarity is the Hubble parameter H, which determines both the sign of the entropy and the direction of the thermodynamic flow.

A. Sign-dependent entropy and its consequences

For a homogeneous de Sitter region, the total gravitational entropy is

$$S_{\mathrm{dS}}(H) = \mathrm{sgn}(H)\frac{\pi}{H^2} = \mathrm{sgn}(H)\frac{A}{4}, \tag{38}$$

where $A = 4\pi/H^2$ is the horizon area. Expanding de Sitter ($H > 0$) therefore has positive entropy $A/4$, while contracting de Sitter ($H < 0$) has negative entropy $-A/4$. This sign dependence is the fundamental insight that links the two frameworks.

In the OCK solution (for $n > 2$), the interior is an expanding de Sitter core with $H > 0$. The associated positive extra entropy is carried by the inner Killing horizon, as follows:

$$S_{\mathrm{inner}}(n) = \pi h_c^2(n) \qquad (n > 0). \tag{39}$$

As n decreases during collapse, the inner horizon shrinks and the formal area entropy πh_c^2 decreases. Because the OCK geometry is only approximately de Sitter, the exact relation between S_{inner} and H is not a simple algebraic function of H; only in the limit $n \gg 1$ (where $h_c \to h$) does S_{inner} approach the full de Sitter value $\pi/H^2 = A/4$. For finite n, the Wald inner-horizon entropy and the homogeneous de Sitter bulk entropy are different (e.g., at $n = 3$, they differ by a factor $\sim$2.36). The asymptotic agreement at $n \to \infty$ provides a conceptual bridge, but the two constructions probe different aspects of the geometry. Establishing a common microscopic origin remains an open problem.

In Volovik's black hole gravastar, the interior is a contracting de Sitter core with $H < 0$. Its total entropy is exactly the negative value

$$S_{\mathrm{core}} = -\frac{A}{4}, \tag{40}$$

which cancels the Bekenstein-Hawking entropy of the event horizon, yielding a zero-entropy object.

Both proposed interpretations can support a second-law picture only with a justified entropy balance: the shrinking of the interior region drives the system toward increased total entropy. The sign of H dictates the bulk entropy sign in Volovik's prescription; the OCK inner-area assignment remains positive irrespective of that sign.

B. Limits of classical unification and quantum connection

The OCK solution does not provide a classical path to a contracting de Sitter gravastar. For $n > 2$, the core is de Sitter, but for $0 < n \leq 2$, the center is curvature singular rather than a regular anti–de Sitter core; Volovik's local de Sitter thermodynamics do not apply. The Minkowski breaking at $n = 0$ is a kinematic obstruction—a noncommuting central limit that obstructs regular continuation within the ansatz—not a mechanism that flips the sign of H. Therefore, the two frameworks are complementary paradigms, not successive stages of a single classical collapse.

Although no classical evolution connects the OCK regular black hole to the Volovik gravastar, a quantum transition at the Minkowski breaking point could, in principle, change the interior state and expansion, without changing the sign of Λ merely through $H \to -H$. The divergence of the inner-horizon temperature and the breakdown of the semiclassical description at $n \sim 1/\ln(h/\ell_P)$ indicate that quantum effects dominate near $n = 0$. If an instanton exists that interpolates between an expanding de Sitter interior and a contracting one, the Minkowski breaking would correspond to a quantum phase transition, and the black-hole gravastar would emerge as an intermediate state with zero total entropy. The complete entropy flow would then be

$$S = \frac{A}{4} + \pi h_c^2 \longrightarrow 0 \longrightarrow \frac{A}{4},$$

consistent with the generalized second law at every step, provided the environmental entropy is included as discussed in Sec. IV. We stress that this is a conjecture; constructing the relevant instanton remains an open problem. The entropic plot in Sec. V is schematic and does not establish a physical tunneling barrier.

Even without a quantum connection, a conditional additive entropy balance can be formulated. In the OCK interpretation, removing the inner-area term would require a compensating environmental contribution,

$$\Delta S_{\rm BH} + \Delta S_{\rm env} = -\pi h_c^2 + \Delta S_{\rm rad} \geq 0, \tag{41}$$

where $\Delta S_{\rm rad} \geq \pi h_c^2$ is a consistency requirement of the additive interpretation, not a calculated flux or an exterior generalized-second-law theorem. In Volovik's scenario, the total entropy increases directly as the negative-entropy core shrinks, as follows:

$$\Delta S_{\rm total} = \frac{A}{4} - 0 = \frac{A}{4} > 0. \tag{42}$$

These assignments alone do not prove monotonicity of the physical generalized entropy; the matter state and environmental contribution remain necessary inputs.

C. Quantum breakdown and entropy quantization

The Minkowski breaking at $n = 0$ is a kinematic limiting obstruction that motivates a quantum investigation. The inner-horizon surface gravity diverges as $n \to 0^+$ [see Eq. (15)], implying

$$T_{\rm inner} = \frac{\kappa_{\rm inner}}{2\pi} \to \infty. \tag{43}$$

The extrapolated surface-gravity scale becomes Planckian at $n \sim \ln(3/2)/\ln(h/\ell_P)$, with the qualifications given in Sec. II. The discontinuity in the metric function at $n = 0$ signals a loss of local Minkowski structure. These limits do not by themselves select a quantum mechanism or exclude all classical evolutions outside the regular ansatz.

At the same time, the integer nature of the OCK parameter n motivates a proposed discrete entropy spectrum for the regular black holes,

$$S_{\rm reg}(n) = \frac{A}{4} + \pi h_c^2(n), \tag{44}$$

with $h_c(n)$ taking discrete values. This invites comparison with Bekenstein's conjecture that the horizon area is quantized [28]. In the present framework, the proposed quantization would be associated with the discrete internal structure of the regular black hole. Each value of n corresponds to a different interior configuration, all of which share the same exterior Schwarzschild metric. The number of distinct regular states for a fixed macroscopic mass is given by the number of allowed integer tuples,

$$\Omega(\mathcal{M}) = \#\{(n_1, \ldots, n_N) : 2 < n_1 < \ldots < n_N \leq n_{\max}\}, \tag{45}$$

where $n_{\max}$ is a hypothetical cutoff to be supplied by quantum gravity. A microscopic interpretation would require establishing

$$S_{\rm BH} = \ln \Omega(\mathcal{M}). \tag{46}$$

This offers a concrete statistical-mechanics proposal for the origin of black hole entropy, but neither a physical state count nor a mass-dependent cutoff reproducing the area law has been derived.

D. Summary and thermodynamic arrow of time

The sign of the Hubble parameter determines the sign of the gravitational entropy: expanding de Sitter carries positive entropy, contracting de Sitter carries negative entropy. The OCK framework describes the release of positive extra entropy from an expanding core, while Volovik's framework describes the cancellation of negative entropy in a contracting core. The two paradigms are complementary, with no causal connection established; a speculative quantum transition at the Minkowski breaking point could link them dynamically, but such a mechanism has not been constructed.

Despite this open question, the sign-dependent entropy principle unifies the thermodynamics of both scenarios and motivates a conditional thermodynamic arrow of time. An entropy difference can enter a statistical weight, but it does not determine an absolute transition rate. For the additive OCK system assignment, the change towards Schwarzschild is $-\pi h_c^2$, so an enhanced rate $\Gamma \propto e^{+\pi h_c^2}$ cannot be inferred without additional dynamics and environmental entropy. In Volovik's framework, the transition

from the gravastar to the black hole state is similarly favored by the entropy increase $\Delta S = A/4 > 0$. In both cases, Schwarzschild is a candidate endpoint in the proposed thermodynamic interpretation, rather than a dynamically established attractor. The sign-dependent entropy thus provides not only a unified thermodynamic description but also a possible microscopic interpretation and a proposed direction of entropy flow during collapse.

VII. CONCLUSIONS AND OUTLOOK

We have established a thermodynamic complementarity between two independent perspectives on gravitational entropy: the entropy release from regular Schwarzschild black holes described by the OCK construction, and the local thermodynamics of the de Sitter vacuum developed by Volovik. The central insight is that the sign of the gravitational entropy of a de Sitter region is determined by the sign of its Hubble parameter: expanding de Sitter ($H > 0$) carries positive entropy, while contracting de Sitter ($H < 0$) carries negative entropy. This simple rule reveals that the OCK and Volovik frameworks are not contradictory but represent two complementary manifestations of the same thermodynamic principle.

In the OCK framework, the regular black hole possesses an expanding de Sitter core (for $n > 2$) whose inner horizon carries a positive Bekenstein-Hawking entropy $S_{\text{inner}} = \pi h_c^2$. As the collapse parameter n decreases, the inner horizon shrinks and this its positive assigned contribution gradually decreases, with a possible quantum resolution of the $n = 0$ limiting obstruction remaining open. In Volovik's framework, a black hole gravastar contains a contracting de Sitter core whose total entropy is $-A/4$; this negative entropy exactly cancels the horizon entropy, yielding a zero-entropy static object. Both pictures embody a proposed entropy-flow interpretation, subject to the stated physical qualifications, either by releasing positive extra entropy (OCK) or by canceling negative core entropy (Volovik).

The total gravitational entropy of a homogeneous de Sitter region is $S_{\text{dS}} = \text{sgn}(H)\pi/H^2 = \text{sgn}(H)A/4$. This expression corrects an earlier erroneous formula, $S(H) = 1/(4H)$, that appeared in a previous version of this work.

The Minkowski breaking at $n = 0$ in the OCK solution is a classical kinematic obstruction; it does not, by itself, provide a dynamical transition to a contracting de Sitter gravastar. For $0 < n \leq 2$, the center is curvature singular rather than a regular anti–de Sitter core, and the local de Sitter thermodynamics no longer apply. Any connection between the two frameworks would require specified dynamics; a quantum change of the interior state is one conjectured possibility, not a sign flip of $\Lambda = 3H^2$. The divergence of the inner-horizon temperature and the breakdown of the semiclassical description at $n \sim 1/\ln(h/\ell_P)$ motivate studying quantum effects, without identifying a controlled transition near the breaking point, making such a transition plausible, but at present it remains a conjecture.

With this important caveat, the complementarity of the two frameworks offers a coherent thermodynamic picture. The integer nature of the OCK parameter n motivates a proposed discrete entropy spectrum $S_{\text{reg}}(n) = A/4 + \pi h_c^2(n)$, inviting comparison with Bekenstein's conjecture and a possible statistical interpretation of black hole entropy as the logarithm of the number of internal geometric configurations that are indistinguishable from the outside: $S_{\text{BH}} = \ln \Omega(\mathcal{M})$. Such entropy weights require a physical state count and dynamics before they can be interpreted as transition probabilities.

Several open questions and limitations of the present work should be acknowledged. The complementarity framework is kinematic rather than fully dynamical; a complete time-dependent description that would dynamically connect the two paradigms remains an open problem. The additive entropy rule $S_{\text{reg}} = S_{\text{outer}} + S_{\text{inner}}$ is a working hypothesis, and alternative prescriptions are not explored. The quantum interpretation of the Minkowski breaking is motivated by limiting divergences rather than derived from a controlled transition calculation, but a full quantum gravitational treatment is not attempted. These limitations reflect the scope of the present work and highlight the richness of the questions it identifies.

An important qualification concerns the physical meaning of the inner-horizon entropy. A Cauchy horizon bounds the region uniquely determined by the chosen initial data; the initial-value problem remains meaningful within the maximal globally hyperbolic development, while continuation beyond it need not be unique. Moreover, perturbative blueshift and mass inflation warn against assuming a stable smooth inner horizon [29,30]. Equation (10) is therefore used as a formal stationary geometric diagnostic, not as the entropy of a demonstrated equilibrium subsystem. Its variation is useful for comparing idealized interiors and identifying assumptions to be tested by backreaction. The additive rule (11) and any interpretation as released entropy remain conditional: disappearance of the inner area alone establishes neither radiation to infinity nor a generalized-second-law result.

The speculative quantum correction is also related to quantum cosmic censorship. An early proposal suggested that particle creation and its backreaction might prevent the emergence of a naked singularity [31]; semiclassical studies of Cauchy horizons likewise motivate examining quantum stress energy and predictability [32]. These are distinct questions: preventing an exposed singularity, destabilizing a Cauchy horizon, and obtaining a regular quantum interior are not equivalent outcomes. The cited results do not provide a quantum completion of the OCK construction or an instanton connecting the two paradigms. Such a claim would require a specified quantum state, a backreaction calculation, and an analysis of the resulting

causal structure. Quantum cosmic censorship is thus a relevant motivation for future work, rather than a mechanism established by the present entropy comparison.

Looking ahead, several promising directions emerge naturally. An explicit dynamical solution that interpolates between an expanding regular core and a contracting gravastar core—if it exists—would provide the missing link between the two pictures. The Painlevé-Gullstrand coordinate system, which naturally accommodates both signs of the Hubble parameter, is a natural framework for such a construction. A complete quantum treatment of the Minkowski breaking point, possibly via the Euclidean path integral, loop quantum gravity, or the stochastic semiclassical Einstein-Langevin equation, could determine whether a sign flip of H is dynamically allowed and compute the associated transition rate. The framework should be extended to rotating black holes, higher dimensions, and modified gravity theories, where the interplay between internal structure and thermodynamics may reveal qualitatively new features. The holographic bulk-surface correspondence in Volovik's framework suggests deep connections to de Sitter holography, while observational signatures of the entropy release during collapse—such as gravitational wave echoes or bursts of radiation—could provide experimental tests. Finally, a deeper exploration of the statistical interpretation $S_{\rm BH} = \ln \Omega(\mathcal{M})$ could provide a microscopic foundation for black hole thermodynamics, with key questions including the nature of the degrees of freedom counted by the integers n_i, the precise relation between the microstate count and the Bekenstein-Hawking entropy, and the role of the UV cutoff $n_{\rm max}$ imposed by quantum gravity.

In summary, the sign of the Hubble parameter is the unifying element that underlies the thermodynamics of both regular black holes with expanding de Sitter cores and black hole gravastars with contracting cores. The two frameworks are complementary paradigms that illuminate different facets of the same sign-dependent entropy principle. Whether a quantum transition can dynamically bridge them remains an open and intriguing question.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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