Nonminimally coupled scalar field, area quantization, and black hole entropy

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(Received 1 April 2026; accepted 1 June 2026; published 1 July 2026)

The enumeration of black hole entropy in candidate theories of quantum gravity utilizes the quantum properties of microstates residing on the black hole horizon. For example, in loop quantum gravity, the computation of entropy is based on the spectrum of the area operator, and one determines the possible number of area microstates corresponding to a given classical horizon area. In this paper, we derive the eigenspectrum of the horizon area operator for rotating/nonrotating black holes in a gravitational theory nonminimally coupled to scalar fields. Using the weak isolated horizon formalism, we show that the spectrum of the area operator follows unambiguously from the algebra of horizon symmetry. More precisely, from the quantum mechanical point of view, the horizon geometry must be naturally discrete, a conclusion that is arrived at directly, without the need for any particular theory of quantum gravity. The area spectrum depends on the Barbero-Immirzi parameter as well as the value of the scalar field on the horizon. The area spectrum is equidistant, which is consistent with the Bekenstein-Mukhanov proposal and gives rise to black hole entropy and their quantum corrections.

DOI: 10.1103/wb82-sq14

I. INTRODUCTION

Understanding the classical and quantum properties of the black hole horizon, and black hole entropy in particular, remains one of the most challenging problems of gravitational physics [1–4]. It is strongly believed that a better knowledge of the quantum properties of the horizon may allow us a peek into the microscopic world of quantum spacetime [5–8]. Furthermore, one also expects that the quantum formulation of geometry/gravity may eventually help to resolve the problems associated with classical gravitational physics [9–15]. Therefore, the first test of any candidate theory of quantum gravity is whether it successfully determines the value of black hole entropy, consistent with the first law of black hole mechanics. For general theory of relativity (GR), the black hole entropy is proportional to the area of the horizon cross section [16–18]. This value of entropy, together with Hawking's derivation of black hole radiance, provides consistency to Bekenstein's claim that the laws of black hole mechanics must be the same as the laws of thermodynamics [19,20]. More precisely, if quantum mechanics is taken into account, a black hole in GR with surface gravity κ must be considered as a thermal object with temperature $\mathcal{T} = (\kappa/2\pi)$, and entropy $\mathcal{S} = \mathcal{A}/4\ell_P^2$, where $\mathcal{A}$ is the area of the horizon cross section and ℓ_P is the Planck length. Note that with these identifications, the zeroth law, which states that the surface gravity must be constant on the horizon, becomes identical to the zeroth law of thermodynamics, whereas the first law of horizon mechanics, for a black hole of mass $\mathcal{M}$, is mapped to the first law of thermodynamics $d\mathcal{M} = \mathcal{T}d\mathcal{S}$. The second law of black hole mechanics, which states that the horizon area of a classical black hole must always increase, provides credence to the claim that $\mathcal{S}$ is proportional to $\mathcal{A}$ [21,22]. The two leading theories of quantum gravity, string theory and loop quantum gravity (LQG), have successfully obtained this leading value of entropy from the enumeration of horizon microstates [23–36]. For string theory, the microstate computations have been carried out for certain four-dimensional supersymmetric ($\mathcal{N} = 4$ and $\mathcal{N} = 8$) Bogomol'nyi-Prasad-Sommerfield black holes and five-dimensional Breckenridge-Myers-Peet-Vafa solutions, whereas for LQG, the computations are for extremal or nonextremal (weak isolated) horizons in four dimensions [27,37]. Furthermore, subleading corrections to entropy have also been obtained from such counting techniques and have provided fundamental clues toward quantum properties of the black hole horizon [8,29–31,36,38–40]. Both of these quantum theories agree that the general expression of black hole entropy, obtained from the microscopic computation, should have the following expansion (for large black holes, $\mathcal{A} \gg \ell_P^2$):

$$\mathcal{S} = \frac{\mathcal{A}}{4\ell_P^2} + a_1 \ln\frac{\mathcal{A}}{\ell_P^2} + a_2\frac{\ell_P^2}{\mathcal{A}} + \cdots, \tag{1}$$

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where a_i are coefficients in the expansion, but there is no general consensus on their values [29–31,38,41–43]. This general expectation for the value of entropy, Eq. (1), also follows from macroscopic computations carried out through the Wald formula or the entropy function formalism [44–48]. For small black holes, exponential corrections to the area value also arise for the black hole entropy [49,50]. Similar results for the leading and subleading corrections to black hole entropy have also been obtained from the AdS/CFT perspective with asymptotic and near horizon symmetries, Euclidean quantum gravity, and other semiclassical techniques like entanglement, the brick wall approach, or the tunneling methods [51–84].

The laws of black hole mechanics change if a scalar field (ϕ) is present in the spacetime, which couples not just to the spacetime metric, but also to the Ricci scalar through a function $f(\phi)$. Naturally, the classical first law for black holes in this theory is altered, and the horizon area is modified so that the entropy must be identified with $f(\phi_\Delta)\mathcal{A}$, where ϕ_Δ is the value of the field on the horizon [85–87], provided the temperature is $\kappa/2\pi$. The question now is to provide a microscopic derivation of this entropy value through counting of horizon microstates. The theory of LQG does provide some clues toward this value [88]. Since this method shall play an important role for the purposes of comparison with our approach, in the following, we provide a brief discussion of the LQG computation.

The calculation proceeds through several steps [85–89]. First, using the weakly isolated horizon (WIH) formalism, one constructs the classical phase space of black hole solutions of the gravitational theory with a nonminimally coupled scalar field (a discussion on the notion of WIH shall be given in the following sections). This space of solution contains all those black hole spacetimes whose inner boundary (which we shall denote by Δ) satisfies the conditions of a WIH and are asymptotically flat at spatial infinity (the asymptotic conditions do not play particular roles in the entropy computation and we shall not devote any further discussions on this point). More precisely, in four dimensions, Δ is assumed to be an expansion-free null hypersurface with topology $\mathbb{S}^2\times\mathbb{R}$ on which the equations of motion and the (dominant) energy conditions hold true. One may show that the null vector ℓ^a generating Δ is a Killing vector field *on* the horizon, although the immediate neighborhood may be nonstationary, which may not even admit any timelike Killing vector. If one restricts to spherically symmetric sector of the phase space, the gravitational symplectic structure cleanly separates into two parts, (a) a bulk part consisting of the conjugate pair of tetrads and spin connections, and (b) a boundary term that is completely determined through a one-form V. For nonrotating and nonextremal black holes, this boundary theory turns out to be topological, and more precisely, a $U(1)/SU(2)$ Chern-Simons theory corresponding to this one-form V_a [90–92]. The level of the Chern-Simons theory depends on the value of the scalar field on the horizon, and is given by $[f(\phi_\Delta)\mathcal{A}/4\pi\gamma\ell_P^2]$, where γ is the Barbero-Immirzi parameter. This completes the first step, where one associates, with the horizon Δ, a field theory whose quantization may provide the states responsible for obtaining the entropy. Note that matter fields, like the Maxwell or Yang-Mills, or minimally coupled scalar fields, do not contribute to the surface symplectic structure, the Chern-Simons field V_a is purely gravitational.

The second step is to quantize this spherically symmetric sector of the phase space [9,28,33,88]. One assumes that the (kinematical) Hilbert space is a tensor product of the bulk ($\mathcal{H}_V$) and the surface ($\mathcal{H}_S$) Hilbert spaces with the form $\mathcal{H}=\mathcal{H}_V\otimes\mathcal{H}_S$ (note that this is an assumption in line with the fact that the classical symplectic structure also has a similar decomposition). $\mathcal{H}_V$ is the Hilbert space of states of the bulk and describes polymer excitations of geometrical and matter fields. More precisely, $\mathcal{H}_V$ is spanned by *spin networks*, denoted by $|\mathscr{G};j,m\rangle$, which are characterized by a graph $\mathscr{G}$, with j labeling the edges and m labeling the vertices. The spin network is a particularly convenient basis since they are eigenstates of the area operator. In particular, given a closed surface $\mathbb{S}$ in the bulk, one may view the classical area of this surface as arising from the quantum excitations generated due to the intersection of the edges of $\mathscr{G}$ with $\mathbb{S}$. So, one may view edges as depositing quantized areas when they puncture $\mathbb{S}$. If there are n edges that puncture $\mathbb{S}$, the eigenvalue of the area operator is $[8\pi\gamma\ell_P^2/f(\phi_\Delta)]\sum_{i=1}^n\sqrt{j_i(j_i+1)}$ [28]. On the other hand, surface Hilbert space $\mathcal{H}_S$ contains space of states of the Chern-Simons theory on a punctured sphere, along with the prequantization condition that $[f(\phi_\Delta)\mathcal{A}/4\pi\gamma\ell_P^2]$ must be integer valued. To obtain the physical Hilbert space, one must reproduce the classical embedding of the horizon Δ in the bulk as a quantum condition. As a result, the states belonging to the physical Hilbert space must be such that they satisfy constraints in the bulk and the *quantum horizon condition* (sometimes called the $F-\Sigma$ equation) on the horizon. Let us summarize the picture of a quantum horizon Δ_Q: the states that characterize Δ_Q are the spin network states. Let a graph $\mathscr{G}$ of this spin network with edges labeled by j_i puncture the horizon cross section S_{Δ_Q}, with label m_i [with the requirement that m_i has $(2j_i+1)$ values, $-j_i\le m_i\le j_i$], then area of the quantum horizon $\mathcal{A}_Q$ is taken to be the value of the area that the bulk area operator assigns to punctures on S_{Δ_Q}, so that $\mathcal{A}_Q=[8\pi\gamma\ell_P^2/f(\phi_\Delta)]\sum_{i=1}^n\sqrt{j_i(j_i+1)}$, such that the additional *total projection* constraint, $\sum_i m_i=0$, is also satisfied. This completes the characterization of the quantum horizon, and we are ready for the third step, that of state counting. The third and final step is to obtain the entropy using the point of view of microcanonical ensemble. Given the classical area $\mathcal{A}$, the counting must obtain the number of states $\mathcal{N}$ such that for a small value of δ,

$$\mathcal{A}-\delta \leq [8\pi\gamma\ell_P^2/f(\phi_\Delta)]\sum_{i=1}^{\mathcal{N}}\sqrt{j_i(j_i+1)} \leq \mathcal{A}+\delta. \qquad (2)$$

Assuming the black hole area is large, the entropy is enumerated from $\mathcal{S}=\ln\mathcal{N}$ (using $k_B=1$ units), through which one obtains Eq. (1) with certain choices of parameters γ and $a_1=-1/2$.

How does this calculation fare in comparison to other methods? Do the leading and subleading values of entropy match with the LQG method?[1] The answer is both yes and no, which is natural since the methodologies for computing black hole entropy are vastly different. Over the years, several techniques have been developed to compute black hole entropy, and to understand its precise dependence on the horizon area. The first attempt has been through the Euclidean quantum gravity approach [69]. This method has also been used to obtain quantum corrections to black hole entropy, including the logarithmic terms, with coefficients matching the LQG results [70–76,93], although recent careful studies have disputed this claim [41,78,79,94–100]. Second, a popular approach to calculate black hole entropy from first principles relies on conformal field theory techniques, and the use of the Cardy formula. The CFTs on the boundaries (both at asymptopia and at the horizon) are obtained from the algebra of vector fields, which preserve the corresponding asymptotic symmetries [51–53,55,56,58,59]. Here also, the leading and the logarithmic corrections to the Cardy formula provide a successful matching with the LQG value of black hole entropy. On the holographic approach, the Ryu-Takayanagi method attempts to obtain the black hole entropy through geometry and computes entanglement entropy in the boundary CFT through bulk minimal surfaces, where the logarithmic terms appear due to high-order metric or minimal surface deformations in the bulk [101]. The semiclassical technique of the brick-wall approach is another popular method that has provided a successful matching with the leading and subleading terms in LQG [81,82], as do others where the logarithmic corrections are related to the trace anomaly [65,67]. However, the most popular approach to understand *macroscopic* black hole entropy has been the Wald formula [44–46,68], and the entropy function formalism [48]. For these approaches, too, computations have shown a general trend of Eq. (1), although coefficients are often widely different from those of LQG.

Let us now come back to our discussion of the LQG method and if the approach may be improved upon. Note that, given the developments in the LQG literature, there exist some major gaps, which we wish to address here so as to provide a better understanding of the origin of black hole entropy. Let us enumerate the list of issues. First, the proof that the surface part of the gravitational symplectic structure (defined on the classical phase space admitting a WIH) is that of a $U(1)/SU(2)$ Chern-Simons theory requires that the horizon cross section be spherically symmetric. Quite remarkably, distorted horizons and horizons with multipole structures have been studied as well [10,102], although the computations become quite involved, and a simpler method may provide better insights. Second, during the state counting, when the classical area is matched with the eigenvalue of the area operator [given by $8\pi\gamma\ell_P^2\sqrt{j(j+1)}$], one uses the expression obtained from the bulk. More precisely, this expression of the area eigenvalue is evaluated for a surface $\mathbb{S}$, which foliates the spacelike hypersurface Σ on the full spacetime. Therefore, during the LQG calculation, one has to assume that the eigenvalue of the area operator shall continue to hold true for a sphere foliating the null hypersurface as well. Naturally, a direct evaluation of the spectrum of the area operator on the horizon should give us a better perspective into the nature of horizon microstates. Third, the evaluation of horizon entropy requires that the black hole has a large area and, indeed, the leading and subleading values are evaluated using the Stirling approximation with $\mathcal{A}\gg\ell_P^2$. Therefore, one needs to understand the changes one may encounter if this *large area constraint* is removed. Studies on such Planck area–sized black holes have revealed important features, like oscillations in the entropy spectra with constant periodicity [103], which points to an effective model of the equidistant spectrum for entropy, as envisaged through the Bekenstein-Mukhanov model [6]. Fourth, and the one that we believe is an important question, is the following: is it possible to develop a microscopic model of the black hole horizon without using any formulation of quantum gravity (either LQG or string theory) through which the origin of black hole entropy be explained? How much is the area operator and its spectrum tied to the LQG formulation? Or in other words, is it possible to obtain a quantized area operator on the horizon independent of the theory of quantum gravity? We shall show that it is indeed possible to develop a model based on symmetries of the horizon where all these questions may be answered in the affirmative and the issues be resolved quite naturally.

We shall use the method advocated in [49] for GR, which we briefly review. First, one constructs the classical phase space of the first-order Holst action admitting WIH as an inner boundary (and is asymptotically flat at spatial infinity). The second step is to note that the WIH boundary, which admits a local Lorentz symmetry of $\mathrm{ISO}(2)\ltimes\mathbb{R}$, gives rise to observable charges proportional to the area of the horizon cross section. This is not unexpected since it is well known that local symmetries in gauge theory give rise to observable edge states on boundaries [104–106]. On the

[1]One must point out that even within the LQG framework, there are competing claims on the coefficient of the log term, depending on whether one uses the $U(1)$ or the $SU(2)$ Chern-Simons theory or how different spin configurations are being counted [29–31,36,38,42].

WIH, the observable (area of the horizon cross section, $\mathcal{A}$) is Hamiltonian charge due to the rotation and boost generators corresponding to local $\mathrm{ISO}(2) \ltimes \mathbb{R}$ transformations. Third, one may note that the algebra of $\mathrm{ISO}(2) \ltimes \mathbb{R}$ dictates that the angular momentum generators must be quantized with integers $n\hbar$, and quantum states residing on the horizon cross sections carry a representation of ISO(2). Since the horizon area $\mathcal{A}$ is also the generator of rotations, this provides a link to label the quantum states of the WIH by the integers $n\hbar$ or, in the language of quantum states, the eigenstates of the area operator. More precisely, one may visualize the classical horizon being formed out of quantized areas. The fourth step is to use the notion of microcanonical ensemble and determine the number of ways one may assimilate the quantized areas to obtain a value of classical area, the logarithm of which should provide the measure of entropy. Naturally, this counting also leads to the correct value of entropy and area but does not provide log corrections [107]. Our approach shall be to use these techniques to evaluate the black hole entropy, although major changes shall arise due to the presence of the nonminimal scalar field. In particular, we shall show that the area spectrum does depend on this scalar field, and hence the quantization and entropy counting does explicitly carry the imprint of this scalar field. As a result, the entropy shall depend both on the horizon area and the value of the scalar field on Δ.

We proceed as follows. In the next section, we discuss the boundary condition of a WIH, and develop the zeroth and the first laws of black hole mechanics for N dimensions using the first-order nonminimally coupled Palatini action based on tetrads and spin connections (although we will be working in N dimensions, we shall continue to refer to the orthogonal basis as tetrads). This action describes a gravitational theory coupled nonminimally to a scalar field. We shall construct the phase space of this theory and obtain the symplectic structure on the space of solutions, which admits a WIH as the internal boundary. We shall show that the zeroth law of horizon mechanics arises a result of the boundary conditions. We shall also derive the first law of horizon mechanics and show that the necessary and sufficient condition for the existence of the first law of horizon mechanics is that a well-defined Hamiltonian evolution is generated on the phase space. Moreover, the first law shall also show that, if thermodynamics is to be taken into account, one must include the contribution of the scalar field toward the black hole entropy. Then, we discuss the Holst action for the four-dimensional theory, where an additional term must be added to the Palatini action. This additional Holst term does not contribute to the first law of horizon mechanics but does alter the symplectic structure when boundaries are present. Using this new Holst symplectic structure, we shall derive the Hamiltonian charges corresponding to the local Lorentz rotation and boost transformations on the horizon Δ, and show that they are proportional to the cross sectional area of Δ. The next section contains a discussion on the quantum theory, the area spectrum, and the calculation of black hole entropy. A discussion is carried out in the last section.

II. HORIZON GEOMETRY AND MECHANICS IN N DIMENSIONS

We begin by providing a motivation for developing the WIH formalism (a detailed discussion may be found in [37,108–112]). The power of the WIH formalism lies in the fact that it provides a realistic framework for evolution of black hole horizons, along with a broader foundation to understand the zeroth and the first laws of black hole mechanics. The WIH is a quasilocal notion of a stationary black hole horizon which, unlike the event horizon formalism, does not require knowledge of the future null asymptotic infinity to ascertain its location. A WIH is capable of describing a stationary horizon in a nonstationary spacetime, provided no energy-momentum crosses this surface. As a result, the space of solutions that admit a WIH is much larger compared to that of an event horizon. The WIH may be modeled as a codimension-1 null tube foliated by marginally trapped surfaces, and in the following, we shall develop this formalism in a similar fashion. More precisely, we shall consider a generic null hypersurface (Δ) in the spacetime, and impose certain conditions on Δ such that it acquires the properties of a stationary horizon.

Let us consider a N-dimensional spacetime $\mathscr{M}$ endowed with metric g_{ab} and signature $(-++\cdots)$, along with a metric compatible covariant derivative ∇_a. We shall use the orthonormal basis for our calculations and hence develop the Newman-Penrose formalism for higher dimensions, which may be easily adapted to the null hypersurfaces. We use the following notation for vectors and one-forms, respectively [113,114]:

$$m^{(0)} = n \quad m^{(1)} = \ell \quad m^{(i)} = \text{spacelike covectors}, \tag{3}$$

$$m_{(0)} = -\ell \quad m_{(1)} = -n \quad m_{(i)} = \text{spacelike vectors}, \tag{4}$$

such that the following orthonormality conditions hold: $\ell\cdot\ell = 0, n\cdot n = 0, \ell\cdot n = -1, \ell\cdot m^{(i)} = 0, n\cdot m^{(i)} = 0$, and $m^{(i)}\cdot m^{(j)} = \delta^{(ij)}$. Note that a common label $m^{(a)}$, such that the indices a, b, c run from 0 to $(D-1)$, is used for all the frame vectors. The indices i, j, k run from 2 to $(N-1)$ and, therefore, represent only spacelike vectors $m^{(i)}$. The spacetime metric is given by

$$g_{ab} = -2\ell_{(a}n_{b)} + m_a^{(i)} m_b^{(j)} \delta_{(ij)}. \tag{5}$$

Let Δ be a $(N-1)$-dimensional null hypersurface in $\mathscr{M}$, generated by a future directed null normal ℓ^a, and endowed with a degenerate metric $q_{ab} \triangleq g_{\underleftarrow{ab}}$ (the symbol $\triangleq$ shall denote equalities that hold only on Δ, and the underarrows show the indices are pulled back to Δ). We assume that Δ is foliated by spacelike $v = \text{constant}$ $(N-2)$ surfaces S_v. For a simplified Newman-Penrose-like description in higher dimensions, we choose the two null normals to S_v as ℓ^a and n^a, with the one-form n_a being such that $n_a = -(dv)_a$. The vector field $\ell^a = (\partial_v)^a$ is then tangent to Δ. The spacelike vectors $m^a_{(i)}$ form the basis spacelike cross sections of S_v. Let us define the following Ricci rotation coefficients:

$$\nabla_b \ell_a = L_{cd}\, m_a^{(c)} m_b^{(d)}. \tag{6}$$

These Ricci coefficients are not all independent or nonzero. To obtain the constraints on these functions we first use the orthonormality condition $\ell \cdot \ell = 0$ to obtain

$$L_{0a} = 0. \tag{7}$$

Second, since Δ is null, the generator ℓ^a must be geodetic. From Eq. (6),

$$\ell^b \nabla_b \ell_a = L_{c0} m_a^{(c)} = L_{10} \ell^a + L_{i0} m_a^{(i)}. \tag{8}$$

Since the geodetic condition requires that the right side be proportional to ℓ^a, it then follows that $L_{i0} \triangleq 0$. Using $L_{0a} = 0$ and $L_{i0} = 0$ we can now write Eq. (6) as

$$\nabla_{\underleftarrow{b}} \ell^a \triangleq \left[L_{10} n_b + L_{1i} m_b^{(i)}\right] \ell^a + L_{ij} m_b^{(j)} m^{(i)a}. \tag{9}$$

The matrix L_{ij} can be decomposed in the symmetric (and trace-free) and the antisymmetric parts in the following manner: $L_{ij} = S_{ij} + A_{ij}$, where $S_{ij} = L_{(ij)} = \sigma_{ij} + \theta/(N-2)\delta_{ij}$, and $A_{ij} = L_{[ij]} = \omega_{ij}$ is the twist corresponding to the generator ℓ^a. If the surface Δ is a twist-free, shear-free, and expansion-free null surface, then $L_{ij} \triangleq 0$, and it further follows from Eqs. (7) and (8) that the following quantities vanish identically on Δ:

$$L_{0a} \triangleq L_{i0} \triangleq L_{ij} \triangleq 0. \tag{10}$$

So, once the dust settles, Eq. (9) may be written as

$$\nabla_{\underleftarrow{b}} \ell^a \triangleq \omega_b \ell^a, \tag{11}$$

with $\omega = [L_{10} n + L_{1i} m^{(i)}]$. This implies that $\nabla_{\underleftarrow{(a}} \ell_{b)} \triangleq 0$, so that ℓ^a is a Killing vector field on the Δ, although the immediate neighborhood of the hypersurface may be completely nonstationary.

Given this general description of a twist-free, shear-free, and expansion-free null surface Δ, let us now define the notion of a WIH. The null hypersurface Δ in $\mathcal{M}$ is called a WIH if the following conditions hold:

(1) Δ is topologically $\mathbb{S} \times \mathbb{R}$, where $\mathbb{S}$ is a compact and connected $(N-2)$ surface.
(2) The expansion scalar of the null normal ℓ^a, given by $\theta_{(\ell)} = q^{ab} \nabla_a \ell_b$, is vanishing on Δ, that is, $\theta_{(\ell)} \triangleq 0$.
(3) $-T_a{}^b \ell^a$ is future directed and causal, and all field equations of matter and gravitational fields hold on Δ.
(4) The quantity $[\pounds_\ell, \nabla_{\underleftarrow{a}}]\ell^a \triangleq 0$ acting on all vector fields tangential to Δ.
(5) The scalar field ϕ is Lie dragged along Δ, $\pounds_\ell \phi \triangleq 0$.

Let us provide a brief discussion on these conditions and how they might be helpful in modeling a black hole horizon. The first condition is on the topology of the horizon cross section. As is well known, Hawking's theorem restricts the topology of black hole horizons (for asymptotically flat spacetimes) in four dimensions to S^2, although it may admit complicated topologies in higher dimensions [21,115]. The topological requirement on Δ is stated to represent general situations. The condition that Δ is expansion-free indicates that no matter or gravitational field crosses the horizon, and this is the defining requirement representing a nonexpanding isolated null surface through which no energy momentum passes through. Note that not all null hypersurfaces represent a WIH, and expansion-freeness is crucial—the Minkowski null cone, for example, is not a WIH. The third condition demands that the field equations hold on the WIH, and that the energy-momentum tensor satisfy the dominant energy condition, with $-T_a{}^b \ell^a$ being a future directed and null vector field. This may then be connected to the Ricci tensor through the Einstein equations. However, in higher curvature theories, the field equations do not connect the R_{ab} to T_{ab} and, therefore, one may directly impose such a restriction on R_{ab}. Given these three conditions, the Δ only represents a nonexpanding horizon (NEH). The NEH cannot satisfy the laws of black hole mechanics, and one needs to restrict the geometry further to extract meaningful physics. The fourth condition restricts the connection components on the normal bundle. More precisely, if ω_a is the connection corresponding to the vector field ℓ^a in Eq. (11), this condition requires that it be Lie dragged along Δ, $\pounds_\ell \omega_a \triangleq 0$. Since we shall be working with a nonminimally coupled scalar field (ϕ), we shall impose the restriction that the field is essentially constant along the horizon generator. The last condition becomes essential for a proof of the first law.

Now that we have a horizon Δ, let us begin our proof of the laws of thermodynamics. The first is the *zeroth law*: the curvature of the connection component (also called the rotation one-form) ω will play an important role. Note that the rotation one-form ω_a in Eq. (11) may be written as

$$\omega_a \triangleq -\kappa_{(\ell)} n_a + \tilde{\omega}_a, \tag{12}$$

where $\tilde{\omega}_a$ is pullback of the one-form on the cross sections, and if the horizon is nonrotating, then $\tilde{\omega}_a = 0$. Also, from $\nabla_{[a}\nabla_{b]}\ell^c = R_{ab}{}^{c}{}_{d}\ell^d$, one gets that

$$R_{\underleftarrow{ab}}{}^{c}{}_{d}\ell^d \triangleq (d\omega)_{\underleftarrow{ab}}\ell^c. \tag{13}$$

From Eq. (13) it is easy to show that $d\omega$ is a purely spatial form, and hence it can be written as

$$d\omega \triangleq C_{ij}\, m^{(i)} \wedge m^{(j)}, \tag{14}$$

where the scalars C_{ij} are related to the Weyl tensor [114]. From the fourth boundary condition $\pounds_\ell \omega \triangleq 0$, it follows that

$$d\,\kappa_{(\ell)} \triangleq 0, \tag{15}$$

where we have used Eq. (12), and that $\kappa_{(\ell)}$ is the acceleration of the vector field ℓ^a on the horizon. This proves the zeroth law: *surface gravity is a constant on the horizon* Δ. We see that the zeroth law emerges directly from the geometrical structure of the horizon and the requirement that the connection component on the normal bundle is *time* independent, or in other words, is Lie dragged along the horizon generator.

Let us now obtain the first law, which requires the knowledge of the dynamics of gravitational fields. The Lagrangian N-form for the nonminimally coupled scalar field on a N-dimensional manifold M is

$$\begin{aligned}16\pi G L = f(\phi)\Sigma_{IJ} \wedge F^{IJ} - \frac{1}{2}K(\phi)(d\phi \wedge^* d\phi)\\ - V(\phi)^{N-2}\epsilon - d[f(\phi)\Sigma_{IJ} \wedge A^{AJ}],\end{aligned} \tag{16}$$

where the quantity Σ_{IJ} is constructed out of tetrads e^I, whereas $F_{IJ} = dA_{IJ} + A_{IK} \wedge A^K{}_J$ is the field strength corresponding to the connection A_{IJ}. The quantity $K(\phi)$ is a function of the scalar field, which may be chosen to match the action to the second-order metric-based action [116]. The action also contains a boundary term, which is similar to the Gibbons-Hawking boundary action,

$$\Sigma_{IJ} = \frac{1}{(N-2)!}\epsilon_{IJI_1I_2\ldots I_{N-2}} e^{I_1} \wedge e^{I_2} \wedge \ldots \wedge e^{I_{N-2}}. \tag{17}$$

The variation with respect to the connection A_{IJ} gives us the following equation of motion:

$$D\tilde{\Sigma}_{IJ} = (-1)^{N+1} d\tilde{\Sigma}_{IJ} + \tilde{\Sigma}_{IK} \wedge A^K{}_J + \tilde{\Sigma}_{KJ} \wedge A^K{}_I = 0, \tag{18}$$

where D is the gauge covariant derivative operator, $D\lambda^I = \partial\lambda^I + A^I{}_J\lambda^J$, for any internal vector λ^I. Also,

$$\tilde{\Sigma}_{IJ} \coloneqq f(\phi)\Sigma_{IJ}. \tag{19}$$

The variation with respect to the tetrads gives us the Einstein equations up to a boundary term. Let us make the following rescaling of the cotetrad fields: $e^I \longrightarrow \hat{e}^I = p^{-1}e^I$, such that $p = 1/f(\phi)^{\frac{1}{N-2}}$. We find that A is the unique Lorentz connection compatible with $\hat{e}^I$. Further, when the equation of motion (18) is satisfied, the first-order action is equivalent to the standard nonminimally coupled Einstein-Hilbert action up to a surface term. The on-shell variation of the action leads to the following integral:

$$\delta S = \int_{\partial M = \Delta \cup i^o} (-1)^N (\delta\tilde{\Sigma}_{IJ} \wedge A^{IJ}) - K(\phi)^* d\phi\delta\phi, \tag{20}$$

which has to vanish for the action principle to be well defined. The quantities at the asymptotic boundary vanish by appropriate fall-off conditions on the fields at i^0, whereas those on the horizon need to be properly evaluated. We show that this integral vanishes on Δ. To show this, let us first calculate the expression of A^{IJ},

$$\begin{aligned}\nabla_{\underleftarrow{b}}\ell^a &= \nabla_{\underleftarrow{b}}(e^a_I\ell^I) = (\nabla_{\underleftarrow{b}}e^a_I)\ell^I + e^a_I\nabla_{\underleftarrow{b}}\ell^I\\ &= (\nabla_{\underleftarrow{b}}e^a_I)\ell^I + e^a_I A_{\underleftarrow{b}}{}^I{}_J.\end{aligned} \tag{21}$$

But the ∇_a is compatible with $\hat{e}^a_I$, i.e., $\nabla_b\hat{e}^a_I = 0$, which is true on the horizon. Also, since $\hat{e}^a_I = pe^a_I$, we get that $\nabla_{\underleftarrow{b}}e^a_I = -(\nabla_{\underleftarrow{b}}\ln p)e^a_I$. One may then rewrite

$$\omega_b\ell^a \triangleq \nabla_{\underleftarrow{b}}\ell^a = -(\nabla_{\underleftarrow{b}}\ln p)e^a_I\ell^I + e^a_I A_{\underleftarrow{b}}{}^I{}_J \tag{22}$$

and, therefore,

$$(\omega_b + \nabla_{\underleftarrow{b}}\ln p) \triangleq A_{\underleftarrow{b}}{}^I{}_J\ell^I n^J. \tag{23}$$

So, we can write

$$\underleftarrow{A}_{IJ} \triangleq -2(\omega + d\ln p)\ell_{[I}n_{J]} + C_{IJ}, \tag{24}$$

such that $C_{IJ}\ell^I \triangleq 0$. On the other hand, the expression for Σ_{IJ} on the horizon in terms of internal vectors can be written as

$$\underleftarrow{\Sigma}_{IJ} \triangleq 2^{N-2}\epsilon\ell_{[I}n_{J]} + 2\alpha_k\ell_{[I}m^{(k)}_{J]}, \tag{25}$$

where $\alpha_{(k)}$ is a $(N-2)$-form. Using the expressions of A_{IJ} from Eq. (24) and Σ_{IJ} from (25), we get

$$\delta\tilde{\Sigma}_{\underleftarrow{IJ}} \wedge \underleftarrow{A}^{IJ} \triangleq 2\delta(f(\phi)^{N-2}\epsilon) \wedge (\omega + d\ln p). \tag{26}$$

So, we can write (20) as follows:

$$\delta S = \int_\Delta [2\delta\{f(\phi)^{N-2}\epsilon\} \wedge (\omega + d\,\ln p)] - K(\phi)^* d\phi\,\delta\phi. \tag{27}$$

To show that the above integral vanishes on Δ, note that on some initial cross section S_- of Δ, $\delta\phi = 0$ and $\delta({}^{N-2}\epsilon) = 0$. Since both ϕ and ${}^{N-2}\epsilon$ are Lie dragged along the horizon generator ℓ and also $\delta\ell \triangleq c\ell$, $\delta\phi \triangleq 0$ and $\delta({}^{N-2}\epsilon) \triangleq 0$. This proves that the surface term is zero due to the boundary conditions, on the horizon, and the action principle is well defined, hence equations of motion follow from $\delta S = 0$.

To obtain the symplectic structure, we proceed as follows. A variation of the Lagrangian n-form (in n dimensions) gives the equation of motion (EOM) and a boundary term: $\delta L = (\mathrm{EOM})\delta(\mathrm{fields}) + d\theta(\delta)$, with $\theta(\delta)$ being the symplectic potential, a spacetime $(N-1)$-form, but a one-form on phase space. The symplectic current is a closed two-form on phase space, obtained from the antisymmetric combination of symplectic potential [117,118]: $J(\delta_1, \delta_2) = \delta_1\theta(\delta_2) - \delta_2\theta(\delta_1)$. From the Lagrangian in (16), the symplectic potential is $\theta(\delta) = \delta\tilde{\Sigma}_{IJ} \wedge A^{IJ}$. The symplectic current can be written as

$$\begin{aligned} 16\pi G J(\delta_1, \delta_2) &= (-1)^N (\delta_2\tilde{\Sigma}_{IJ} \wedge \delta_1 A^{IJ} - \delta_1\tilde{\Sigma}_{IJ} \wedge \delta_2 A^{IJ}) \\ &\quad + K(\phi)(\delta_2{}^* d\phi\delta_1\phi - \delta_1{}^* d\phi\delta_2\phi). \end{aligned} \tag{28}$$

Let the manifold $\mathcal{M}$ be such that it is bounded by the horizon Δ, two partial Cauchy slices M_+ and M_-, and the spatial infinity i^0 (see Fig. 1). Since $J(\delta_1, \delta_2)$ is closed, on $\mathcal{M} = \Delta \cup M_+ \cup M_- \cup i^0$ we get

$$\left[\int_{M+} - \int_{M_-} - \int_\Delta\right] J(\delta_1, \delta_2) = 0, \tag{29}$$

where the quantity on i^0 is taken to vanish by choice of appropriate fall-off conditions on the fields. On M_+ or M_-,

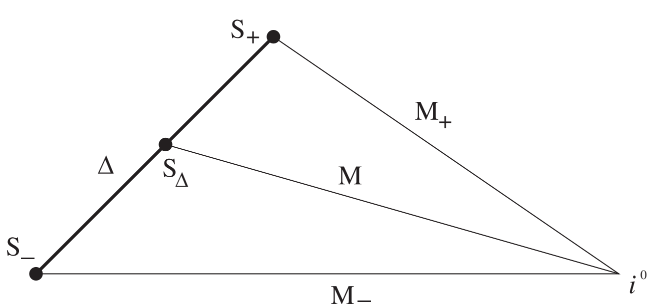

FIG. 1. $M_\pm$ are two partial Cauchy surfaces enclosing a region of spacetime and intersecting Δ in the 2-spheres $S_\pm$, respectively, and extend to spatial infinity i^0. Another Cauchy slice M is drawn, which intersects Δ in S_Δ.

the value of $J(\delta_1, \delta_2)$ is as given in (28), but on Δ, we can show that J is an exact form. To do so, let us introduce a scalar potential ψ for surface gravity κ,

$$\pounds_\ell \psi = \kappa_\ell, \quad \text{such that } \psi|_{S_-} = 0. \tag{30}$$

Note that ψ is not unique, as we can define $\psi' = \psi + \xi$, such that $\pounds_\ell \xi = 0$, hence $\pounds_\ell \psi' = \kappa_\ell$. We choose $\xi = -\ln p$ and, consequently, may show that

$$\begin{aligned} &(-1)^N 16\pi G J(\delta_1, \delta_2)|_\Delta \\ &\quad \triangleq 2d[\delta_2\{f(\phi)^{N-2}\epsilon\}\delta_1\psi - \delta_1\{f(\phi)^{N-2}\epsilon\}\delta_2\psi]. \end{aligned} \tag{31}$$

The above result shows that the symplectic current reduces to an exact form on the horizon such that $J(\delta_1, \delta_2)|_\Delta = dj(\delta_1, \delta_2)$. It follows from (29) that the conserved symplectic structure can be written as

$$16\pi G\Omega(\delta_1, \delta_2) = \int_M J(\delta_1, \delta_2) - \int_{S_\Delta} j(\delta_1, \delta_2). \tag{32}$$

Therefore, the symplectic structure on the phase space of N-dimensional Palatini action with a nonminimally coupled scalar field admitting a WIH as an inner boundary is

$$\begin{aligned} 16\pi G\Omega(\delta_1,\delta_2) = &\int_M (-1)^N (\delta_2\tilde{\Sigma}_{IJ} \wedge \delta_1 A^{IJ} - \delta_1\tilde{\Sigma}_{IJ} \wedge \delta_2 A^{IJ}) \\ &+ \int_M K(\phi)(\delta_2{}^* d\phi\delta_1\phi - \delta_1{}^* d\phi\delta_2\phi) \\ &- 2\oint_{S_\Delta} \delta_2[f(\phi)^{N-2}\epsilon]\delta_1\psi - \delta_1[f(\phi)^{N-2}\epsilon]\delta_2\psi. \end{aligned} \tag{33}$$

Note that this symplectic structure is independent of the hypersurface foliating the spacetime.

To determine the first law for an axisymmetric horizon, we introduce the vector field on the spacetime $t^a = B_{(\ell,t)}\ell^a - \Omega_t \varrho^a$, where ϱ^a is the angular Killing vector field corresponding to rotational symmetry of the cross section. More precisely we assume that the following conditions hold true: $\pounds_\varrho \omega \triangleq 0$, $\pounds_\varrho q_{ab} \triangleq 0$, $\pounds_\varrho \phi \triangleq 0$, and $\pounds_\ell \varrho \triangleq 0$. Let $\Omega(\delta, \delta_t) = X_t(\delta)$, and then using the equations of motion, after a few lines of calculations, one gets

$$\begin{aligned} X_t(\delta) = \frac{1}{8\pi G}\left[\oint_{S_\Delta} - \oint_{S_\infty}\right] \delta[{}^{N-2}\epsilon f(\phi)]\kappa_t \\ - \Omega_t\delta[\varrho \cdot \omega f(\phi)^{N-2}\epsilon] + \delta E^t_\infty. \end{aligned} \tag{34}$$

Here, E^t_∞ is the contribution to the Arnowitt-Deser-Misner (ADM) energy. In order to show that δ_t is Hamiltonian, we must show that $X_t(\delta) = \delta H_t$ is a closed form. This requires that the surface gravity κ_t and angular velocity Ω_t must be functions of area and angular momentum, respectively. Naturally, this may be written in the following form, and here, E^t_∞ is the contribution to the ADM energy:

$$\delta E^t_\Delta = \frac{1}{8\pi G}\kappa_t \delta\tilde{a}_\Delta + \Omega_t\, \delta J_\Delta, \tag{35}$$

where $\kappa_{(t)}$ is the surface gravity associated with t^a. The other quantities are defined as follows: $H^t = E^t_\infty - E^t_\Delta$, while the quantity behaving like the area $\tilde{a}_\Delta$ and the angular momentum J_Δ is given by

$$\tilde{a}_\Delta = \oint_{S_\Delta} f(\phi)^{N-2}\epsilon, \tag{36}$$

$$J_\Delta = -\frac{1}{8\pi G}\oint_{S_\Delta} (\varrho\cdot\omega)[f(\phi)^{N-2}\epsilon]. \tag{37}$$

The requirement that $X_t(\delta)$ be Hamiltonian further implies that the following condition holds true:

$$\frac{\partial\kappa_t(\tilde{a}_\Delta, J_\Delta)}{\partial J_\Delta} = 8\pi G\frac{\partial\Omega_t(\tilde{a}_\Delta, J_\Delta)}{\partial\tilde{a}_\Delta}. \tag{38}$$

Such a constraint ensures that the one-form $X_t(\delta)$ is closed, and that Ω_Δ and κ_t are functions of the angular momentum J_Δ and the horizon area $\tilde{a}_\Delta$.

A. Alternate action for four dimensions and covariant phase space

Let us now restrict our attention to four-dimensional spacetimes. The Palatini action in Eq. (16) with $N = 4$ is not the only Lagrangian in four-dimensional spacetimes that describes a theory with a scalar field coupled nonminimally to gravity. A Lagrangian similar to Holst's modification of the GR action [119] may also be considered [86]. Note that the Holst action is the starting point for LQG, and although we shall not be utilizing the quantum framework of LQG, the action has important ramifications for classical spacetimes with boundaries, as we shall discuss below. The nonminimal scalar coupled Holst action is given by

$$\begin{aligned}-16\pi G\gamma\, L = {}&\gamma f(\phi)\Sigma_{IJ}\wedge F^{IJ} - f(\phi)e_I\wedge e_J\wedge F^{IJ} - \gamma d\{f(\phi)\Sigma_{IJ}\wedge A^{IJ}\}\\ &+ d\{f(\phi)e_I\wedge e_J\wedge A^{IJ}\} - 8\pi G K(\phi)^* d\phi\wedge d\phi + 16\pi G V(\phi)\epsilon,\end{aligned} \tag{39}$$

where $V(\phi)$ is a potential for the scalar field, ϵ is the four-dimensional volume element, and the quantity $K(\phi)$ is a scalar function given by

$$K(\phi) = [1 + (3/16\pi G)(f'^2(\phi)/f(\phi))]. \tag{40}$$

But why is it worthwhile to consider this γ-dependent modification of the gravitational action? To begin, let us discuss some notable features of this action. First, this Lagrangian is well defined for four-dimensional spacetimes only, which follows due to the peculiar contraction labels of the Lorentz indices appearing in the tetrads (e^I) and the field strength (F_{IJ}). Second, note that, as compared to the Palatini action in Eq. (16), the theory contains a term dependent on the parameter γ, called the Barbero-Immirzi parameter. The addition of this γ-dependent action does not alter the classical equations of motion since they vanish due to the Bianchi identities [9,86]. Therefore, the classical phase space or the symplectic structure obtained from the Palatini action of Eq. (16) is equivalent to the one obtained due to the Holst action of Eq. (39). (However, in the quantum theory, the canonical transformations generated due to these γ-dependent quantities do not act unitarily and, therefore, the quantum theory admits inequivalent γ sectors [9].) So, although the action of Eq. (39) is equivalent to the Palatini action of Eq. (16), and the first law of black hole mechanics does not change, the presence of an inner boundary admitting residual local Lorentz symmetries alters the situation (to be discussed in the next section): the γ-dependent action does give rise to additional degrees of freedom and surface charges, which has important ramifications for the quantum structure of horizons.

One may proceed as in the previous section to obtain the phase space from the action Eq. (39). For the four-dimensional spacetimes, one may use the standard Newman-Penrose null tetrads $(\ell^a, n^a, m^a, \bar{m}^a)$, such that $\ell\cdot n = -1$ and $m\cdot\bar{m} = 1$, with all other products being zero. In this basis, the spacetime metric is given by $g_{ab} = -2\ell_{(a}n_{b)} + 2m_{(a}\bar{m}_{b)}$. The horizon Δ is a three-dimensional hypersurface generated by the null vector field ℓ^a. Δ is foliated by 2-spheres with $(m, \bar{m})$ being tangential to these cross sections, and the area two-form on S^2 is given by ${}^2\epsilon = im\wedge\bar{m}$. One may determine the tetrads and spin connections for this spacetime in a manner similar to our discussions on n dimensions,

$$\underleftarrow{e^I_a} \triangleq -n_a\ell^I + m_a\bar{m}^I + \bar{m}_a m^I, \tag{41}$$

$$\begin{aligned}\underleftarrow{e^I_a}\wedge\underleftarrow{e^J_b} \triangleq {}&-2(n\wedge m)_{ab}\ell^{[I}\bar{m}^{J]}\\ &-2(n\wedge\bar{m})_{ab}\ell^{[I}m^{J]} + 2im^{[I}\bar{m}^{J]2}\epsilon_{ab},\end{aligned} \tag{42}$$

$$\underleftarrow{\Sigma^{IJ}} \triangleq 2\ell^{[I}n^{J]2}\epsilon + 2n\wedge(im\ell^{[I}\bar{m}^{J]} - i\bar{m}\ell^{[I}m^{J]}), \tag{43}$$

$$\underset{\leftarrow}{A}_{IJ} \triangleq -2(\omega + d\ln p)\ell_{[I}n_{J]} + C_{IJ}. \tag{44}$$

The symplectic structure corresponding to the Holst action of Eq. (39) may be obtained following the steps leading to Eq. (45) (see [86] for a detailed derivation),

$$\begin{aligned}\Omega(\delta_1,\delta_2) = &\frac{1}{8\pi G\gamma}\int_M [\delta_1\{f(\phi)e^I\wedge e^J\}\wedge\delta_2 A^{(H)}_{IJ} - \delta_2\{f(\phi)e^I\wedge e^J\}\wedge\delta_1 A^{(H)}_{IJ}]\\ &+\frac{1}{8\pi G\gamma}\int_{S_\Delta}[\delta_1\{f(\phi)^2\epsilon\}\delta_2\{\mu_{(m)}+\gamma\psi_{(\ell)}\} - \delta_2\{f(\phi)^2\epsilon\}\delta_1\{\mu_{(m)}+\gamma\psi_{(\ell)}\}]\\ &+\int_M K(\phi)[\delta_1({}^*d\phi)\delta_2\phi - \delta_2({}^*d\phi)\delta_1\phi],\end{aligned} \tag{45}$$

where $A^{(H)}_{IJ} = (1/2)[A_{IJ} - (1/2\gamma)\epsilon_{IJ}{}^{KL}A_{KL}]$. This phase space does give rise to the standard laws of black hole mechanics, as has already been shown in [86], and shall not be repeated here. We shall use this symplectic structure for our calculations for the quantum theory in the next section.

III. THE LOCAL LORENTZ SYMMETRIES ON WIH AND HAMILTONIAN CHARGES

The Lagrangian of Eq. (39) is a first-order theory based on the tetrad connection formalism. This theory is equivalent to the second-order metric theory if one imposes the zero torsion condition. However, the first-order theory is more natural since gravity is known to be invariant under diffeomorphisms as well as local Lorentz transformations. The first-order theory of Eq. (39) cleanly captures the action of both the diffeomorphisms as well as the local Lorentz group of transformations $\mathrm{SL}(2,\mathbb{C})$. The cotetrads and the connection transform under a Lorentz transformation in the following way:

$$e^I \to \Lambda^I{}_J e^J, \tag{46}$$

$$A^{IJ} \to (\Lambda^{-1})^I{}_K A^{KL}\Lambda_L{}^J + (\Lambda^{-1})^I{}_K d\Lambda^{KJ}, \tag{47}$$

where $\Lambda^I{}_J$ is the Lorentz transformation matrix. When boundaries are present, the full symmetry group is broken and only the residual transformations must be considered, which preserve the boundary conditions. This leads to the emergence of new physical degrees of freedom (called edge states) associated specifically to these boundaries. For example, mass and angular momentum may be considered as global charges arising out of broken diffeomorphisms at the boundaries, like the asymptotic infinities. Similarly, the presence of a WIH (considered as an inner boundary of the spacetime) also breaks $\mathrm{SL}(2,\mathbb{C})$ to $\mathrm{ISO}(2)\ltimes\mathbb{R}$ [49,120]. The question is the following: what are residual symmetries that map $\Delta\mapsto\Delta$? What are the charges that emerge from these broken gauge symmetries?

Let us answer the first question. The set of all Lorentz transformations belonging to the group $\mathrm{SL}(2\mathbb{C})$ consists of the following set of six transformations: (a) a boost in the null $(\ell - n)$ plane, (b) rotation in the null plane of $(m-\bar{m})$, (c) a boost and a rotation in the $(\ell - m)$ plane with n fixed, and (d) a boost and a rotation in the $(n-m)$ plane (keeping ℓ fixed). One may show [49,120,121] that only the first three transformations are useful and map a WIH to a WIH. The Lorentz matrices associated with these transformations are, respectively,

$$\Lambda_{IJ} = -\xi\ell_I n_J - \xi^{-1}n_I\ell_J + 2m_{(I}\bar{m}_{J)}, \tag{48}$$

$$\Lambda_{IJ} = -2\ell_{(I}n_{J)} + (e^{i\theta}m_I\bar{m}_J + \text{c.c.}), \tag{49}$$

$$\begin{aligned}\Lambda_{IJ} = &-\ell_I n_J - (n_I - cm_I - \bar{c}m_I + |c|^2\ell_I)\ell_J\\ &+(m_I - \bar{c}\ell_I)\bar{m}_J + (\bar{m}_I - c\ell_I)m_J.\end{aligned} \tag{50}$$

The generators corresponding to these transformations are given by the following quantities:

$$B_{IJ} = (\partial\Lambda_{IJ}/\partial\xi)_{\xi=1} = -2\ell_{[I}n_{J]}, \tag{51}$$

$$R_{IJ} = (\partial\Lambda_{IJ}/\partial\theta)_{\theta=0} = 2im_{[I}\bar{m}_{J]}, \tag{52}$$

$$P_{IJ} = (\partial\Lambda_{IJ}/\partial\mathrm{Re}c)_{c=0} = 2m_{[I}\ell_{J]} + 2\bar{m}_{[I}\ell_{J]}, \tag{53}$$

$$Q_{IJ} = (\partial\Lambda_{IJ}/\partial\,\mathrm{Im}\,c)_{c=0} = 2im_{[I}\ell_{J]} - 2i\bar{m}_{[I}\ell_{J]}, \tag{54}$$

where B and R generate the transformations (a) and (b), respectively, while P and Q generate (c). A straightforward calculation gives their Lie brackets

$$\begin{aligned}&[R,B]=0, \qquad [R,P]=Q, \qquad [R,Q]=-P,\\ &[B,P]=P, \qquad [B,Q]=Q, \qquad [P,Q]=0,\end{aligned} \tag{55}$$

where the symbol $[M,N]_{IJ} = M_{IK}N^K{}_J - N_{IK}M^K{}_J$. This is the Lie algebra of $\mathrm{ISO}(2)\ltimes\mathbb{R}$, where the symbol $\ltimes$ stands for the semidirect product. Note that ISO(2) is the little group of the Lorentz group that maps a null vector to itself and, therefore, it is not surprising that the boundary conditions on Δ are invariant only under this subgroup of the local Lorentz group. So, to summarize, R generates Euclidean rotations in the $(m-\bar{m})$ plane, P generates rotation in the $(\ell-m)$, Q generates rotation in the $(\ell-\bar{m})$, while B generates $\mathbb{R}$, scaling transformations of $(\ell-n)$.

To answer the second question, we would like to construct the Hamiltonian charges for these transformations. We shall show next that the charges corresponding to B and R, which should generate boost and angular momentum on the phase space, respectively, are related to the horizon area. To obtain this, let us consider the variations of the cotetrads and the connection due to infinitesimal Lorentz transformations, $\Lambda^I{}_J = (\delta^I{}_J + \lambda^I{}_J)$. These are given by

$$\delta_\lambda e^I = \lambda^I{}_J\, e^J, \tag{56}$$

$$\delta_\lambda A^{IJ} = -d\lambda^{IJ} - A^{IK}\lambda_K{}^J - A^{JK}\lambda^I{}_K, \tag{57}$$

where ϵ_{IJ} are generators of (51)–(54). We also require the expression for the variation of Σ_{IJ} and that of $(e_I \wedge e_J)$. After a bit of algebra, one can show that [49]

$$\delta_\lambda \Sigma_{IJ} = \lambda_{KI}\Sigma_J{}^K - \lambda_{KJ}\,\Sigma_I{}^K, \tag{58}$$

$$\delta_\lambda(e_I \wedge e_J) = \lambda_{IK}\, e^K \wedge e_J + \lambda_{JK}\, e_I \wedge e^K. \tag{59}$$

Using these transformations for the tetrads and the connection variables under Lorentz transformations, we determine the Hamiltonian charges as follows. We use the bulk part of the symplectic structure of Eq. (45), keeping in mind that the scalar field is a constant on the horizon Δ. The contribution from the surface symplectic structure is sometimes important for the proof of the first law of black hole mechanics and is crucial [122], although for Lorentz transformations, all contributions from the boundary symplectic structure vanish. So the bulk part of the symplectic structure of Eq. (45), along with Eqs. (58) and (59), gives us

$$\Omega_B(\delta_\lambda, \delta) = -\frac{1}{16\pi G\gamma}\int_{S_\Delta} \delta[f(\phi_0)e_I \wedge e_J - \gamma f(\phi_0)\Sigma_{IJ}]\lambda^{IJ}. \tag{60}$$

Note that here, we have fixed the value of the scalar field $\phi = \phi_\Delta$ on the horizon. For $\lambda_{IJ} = R_{IJ} = 2im_{[I}\bar{m}_{J]}$, the only contribution comes through the γ-dependent symplectic structure, and one obtains the Hamiltonian charge for local Lorentz rotations,

$$\Omega_B(\delta_R, \delta) = -\frac{1}{8\pi G\gamma}\int_{S_\Delta} \delta^2\epsilon = -\delta\left[\frac{f(\phi_\Delta)\mathcal{A}}{8\pi G\gamma}\right] \equiv \delta(-\mathcal{J}), \tag{61}$$

where, for notational simplification, we denote the charge by $(-\mathcal{J})$, negative of the angular momentum corresponding to local Lorentz rotations. So, if we define $f(\phi_\Delta)\mathcal{A} = \tilde{\mathcal{A}}$, and call it the modified area, then

$$\mathcal{J} = \frac{\tilde{\mathcal{A}}}{8\pi G\gamma} \tag{62}$$

is the generator of local Lorentz rotations on the phase space of isolated horizons. For $\lambda_{IJ} = B_{IJ} = -2l_{[I}n_{J]}$, we shall denote the charge by $\mathcal{K}$ (boost) on the horizon, and the only contribution comes through the γ-independent symplectic structure in Eq. (60),

$$\Omega_B(\delta_B, \delta) = \frac{1}{8\pi G}\int_{S_\Delta} \delta^2\epsilon = \delta\left[\frac{\tilde{\mathcal{A}}}{8\pi G}\right] \equiv \delta(\mathcal{K}). \tag{63}$$

Again, $\mathcal{K} = (\tilde{\mathcal{A}}/8\pi G)$ is the generator of local Lorentz boosts on the phase space of isolated horizons. The Hamiltonian charges for $\lambda_{IJ} = P_{IJ}$ or $\lambda_{IJ} = Q_{IJ}$, which we shall denote by $\mathcal{P}$ and $\mathcal{Q}$, respectively, vanish on the WIH phase space and, therefore, do not generate evolutions.

From the equations, Eq. (62) and Eq. (63), one obtains the *linear simplicity constraint* $\mathcal{K} = \gamma\mathcal{J}$, which has important implications for quantum gravity, but has been argued from a completely different perspective [123]. Second, the area of the horizon, modified by the scalar field, is linked to the angular momentum $\tilde{\mathcal{A}} = 8\pi G\gamma\mathcal{J}$, as well as the local Lorentz boost, $\tilde{\mathcal{A}} = 8\pi G\mathcal{K}$. These equations shall play a fundamental role in our understanding of the quantum structure of horizons. We shall show that the quantum states residing on the horizon carry the finite dimensional representation of ISO(2), and are eigenstates of $\mathcal{J}$. These states are labeled by integers and, consequently, the equation $\tilde{\mathcal{A}} = 8\pi G\gamma\mathcal{J}$ gives rise to a discrete and equidistant spectrum of $\tilde{\mathcal{A}}$.

IV. AREA SPECTRUM AND ENTROPY OF WIH

To obtain the area spectrum, we proceed as follows. First, we note that the algebra of vector fields is mapped to the algebra of Hamiltonian charges on the phase space, and hence there is no central charge. In other words, the Poisson brackets of the Hamiltonian charges, obtained from the symplectic structure (45), follow the following algebra:

$$\{\mathcal{J},\mathcal{K}\}=0, \quad \{\mathcal{J},\mathcal{P}\}=\mathcal{Q}, \quad \{\mathcal{J},\mathcal{Q}\}=-\mathcal{P}. \tag{64}$$

The corresponding quantum algebra is given by

$$[\mathcal{J},\mathcal{K}]=0, \quad [\mathcal{J},\mathcal{P}]=i\hbar\mathcal{Q}, \quad [\mathcal{J},\mathcal{Q}]=-i\hbar\mathcal{P}. \tag{65}$$

Naturally, on the horizon cross section, the quantum states are in the representation of ISO(2). The operator $\mathbb{P} \equiv \mathcal{P}^2 + \mathcal{Q}^2$ is a Casimir of the algebra and, therefore, $\mathbb{P}$ and $\mathcal{J}$

commute. Hence, the states must be labeled by the eigenvalues of $\mathcal{P}$ and $\mathcal{J}$. Let the eigenvalues of $\mathbb{P}$ and $\mathcal{J}$ be p and j, respectively, therefore, these states shall be denoted by $|p,j\rangle$. The operators $\mathcal{P}$ and $\mathcal{Q}$ may be combined to create ladder operators for the algebra $\mathcal{P}_{\mp} = \mathcal{P} \mp i\mathcal{Q}$, which steps up and down the eigenvalues of $\mathcal{J}$, such that $\mathcal{P}_{\mp}|p,j\rangle = \mp\hbar|p, j\mp 1\rangle$ [49]. We have already argued in the previous section that for the WIH, the generators corresponding to P_{IJ} and Q_{IJ}, denoted by $\mathcal{P}$ and $\mathcal{Q}$, respectively, vanish. This is a reflection of the fact that by definition, the horizon area of WIH does not increase. Therefore, all the physical states of the horizon must satisfy this constraint (that area should not increase) for which the states must have $p=0$, and must be labeled by j only. So, the irreducible representations of the ISO(2) algebra useful for the WIH are one-dimensional. The states belonging to the horizon are the eigenstates of $\mathcal{J}$ labeled by integer $j=n$, with $n\in\mathbb{N}$ [124]. To summarize, we have obtained the states that belong exclusively on the horizon (are independent of the bulk or how the horizon is embedded) and carry integer labels.

Second, as we have shown above, on the phase space containing WIH as an inner boundary, the states on the horizon are eigenstates of the operator $\mathcal{J}$ with integer values. This label may be used to determine the spectrum of the area operator. Using Eq. (61), we obtain (using $c=1$ units)

$$\tilde{\mathcal{A}}|j\rangle = 8\pi G\gamma\mathcal{J}|j\rangle = 8\pi n\gamma\ell_P^2|j\rangle. \tag{66}$$

The area eigenvalues (which we shall denote by $\mathcal{A}$), are then given by

$$\mathcal{A} = \frac{8\pi n\gamma\ell_P^2}{f(\phi_\Delta)}. \tag{67}$$

This is similar to the result of [33], with the eigenvalue $[(8\pi\gamma\ell_P^2)/f(\phi_\Delta)]\sqrt{j(j+1)}$, where j is half-integer valued. Such a condition arises from quantization requirement on the level of the Chern-Simons theory at the horizon boundary, and is essential for quantizing this topological theory. In the present scenario, the area spectrum arises naturally due to geometry of the WIH formalism. The spectrum of the area operator is equidistant, which is consistent with the proposal in [6,125] or that obtained from quasinormal modes [126].

Third, we now use the area spectrum to obtain the entropy of a WIH with a fixed classical cross sectional area $\tilde{\mathcal{A}}_\Delta$. Given the description of a quantum state on WIH elaborated above, we shall replace the notion of a classical horizon and assume that the horizon WIH is a quantum state $|J\rangle$. Since all states of WIH belong to the representation of ISO(2), the label of the state $J\in\mathbb{N}$. Naturally, the classical area is an eigenvalue of the area operator Eq. (66), such that

$$\tilde{\mathcal{A}}|J\rangle = [8\pi\gamma\ell_P^2/f(\phi_\Delta)]\mathcal{N}|J\rangle \equiv \tilde{\mathcal{A}}_\Delta|J\rangle, \tag{68}$$

where the number $\mathcal{N}$ is a large integer for which the classical area is obtained. We assume that the surface S_Δ of the WIH is tessellated by patches of area, much like the surface of a soccer ball or a map of the Earth. Since the tessellated patches are at particular positions on the 2-sphere, they must be subjected to diffeomorphism constraints [33]. We shall fix this constraint by coloring these tessellations, which makes these patches distinguishable. Since each patch of area corresponds to a particular state belonging to ISO(2), we view the black hole horizon as a composite system created out of these states. The Hilbert space of the horizon surface is a tensor product of all these states. The area of each such tessellated patch is labeled by integers, so that the total state $|J\rangle$ must be described by a tensor product structure $|J\rangle = \otimes_i |j_i\rangle$, where i labels the patches. The area operator is taken to be acting on tessellations as follows: $\tilde{\mathcal{A}} = \oplus_i \tilde{\mathcal{A}}_i$, so that each patch contributes an area $8\pi\gamma\ell_p^2 n_i$. Note that $\mathcal{N} = \sum_i n_i$. To obtain the black hole entropy, we shall use the microcanonical ensemble and determine the number of independent ways the configuration $\{n_i\}$ is obtained such that for fixed $\mathcal{N}$ (fixed area phase space), the equation $\sum_i n_i = \mathcal{N}$ holds true.

Now, to obtain the counting, we first note that the horizons have a fixed area (or fixed $\mathcal{N}$). We assume that in the partition of $\mathcal{N}$, the number n_i is shared among s_i patches. This implies that $\sum_i s_i n_i = \mathcal{N}$, where $\sum_i s_i$ is the total number of tessellations. So, the total number of independent configurations is given by

$$\Omega(\mathcal{A}) = \frac{(\sum_i s_i)!}{\prod_i s_i!}. \tag{69}$$

To obtain the most probable configurations, we vary $\ln\Omega$ subject to the constraint $\delta\sum_i s_i n_i = 0$. One obtains

$$s_i = \left(\sum_i s_i\right)\exp(-\lambda n_i), \tag{70}$$

where λ may be determined from the constraint $\sum_i \exp(-\lambda n_i) = 1$, where $n_i = 1,\ldots,\mathcal{N}$. This sum may be easily evaluated and gives

$$\lambda = \ln 2 - 2^{-\mathcal{N}} + o(2^{-2\mathcal{N}}) \tag{71}$$

for large $\mathcal{N}$. One may obtain the entropy from $\mathcal{S} = \lambda\mathcal{N}$. Assuming that the black hole has a large area, the entropy is given by

$$\mathcal{S}(\mathcal{A}) = \frac{\tilde{A}_\Delta \ln 2}{8\pi\gamma\ell_p^2}. \tag{72}$$

For the choice $\gamma = \ln(2)/2\pi$, the leading-order Bekenstein-Hawking result is obtained. For a small area, one also obtains an exponentially suppressed correction to the classical result of $\exp[-\tilde{\mathcal{A}}_\Delta \ln 2/8\pi\gamma\ell_P^2]$. This exponential suppression has been obtained in [49] and in string computations through nonperturbative corrections [50].

How does this calculation and the value of black hole entropy compare with previous calculations, particularly with those of LQG? Here, we must point out that a direct comparison is not easy since there is a difference between our approach presented here and those followed in the LQG literature. First, as pointed out in the introductory section [the paragraph containing Eq. (2)], the LQG method requires a Hilbert space comprising both the bulk and the boundary Hilbert spaces, $\mathcal{H} = \mathcal{H}_V \otimes \mathcal{H}_S$. The surface Hilbert space contains states of the $U(1)/SU(2)$ Chern-Simons theory on a punctured sphere, whereas the bulk Hilbert space is formed out of the spin network states. One gets the physical Hilbert space by imposing the $F - \Sigma$ equation on these states. On the other hand, our approach followed here completely decouples the bulk, both classically as well as quantum mechanically. More precisely, the WIH formalism allows us to construct an isolated null hypersurface mimicking a classical black hole horizon in equilibrium, whereas our construction of horizon quantum states also does not require any dependence on the bulk geometry and follows quite naturally from the algebra of charges on the horizon. Therefore, in contrast to the LQG Hilbert space, the Hilbert space we use contains states localized on the horizon only. Second, the microcanonical calculation of LQG uses the square root area spectrum of LQG, Eq. (2), along with the total projection constraint $\sum_i m_i = 0$. Using these equations, the leading value of black hole entropy has been calculated following a variety of techniques [31,36,38,42,83,127,128] and leads to different values for the γ parameter. Our calculation, in contrast, is a straightforward application of the equidistant area spectrum and does not need additional constraint on the states. One may argue that the formulation does not arise from an underlying theory of quantum gravity and may only be considered as an effective quantum description of the localized horizon microstates, which explains the origin of black hole entropy. However, our results show that whatever the theory of quantum gravity may be, the quantum horizon must be formed out of discrete areas, a fundamental feature that follows directly from local Lorentz symmetry of the horizon.

V. DISCUSSIONS

This paper obtains a number of results in the context of a theory of gravity where the scalar field is coupled nonminimally to gravitational fields. For this theory, we discuss the formulation of classical laws of horizon mechanics, and in particular the zeroth and the first laws due to the WIH formulation in N dimensions. The zeroth law arises due of some constraints imposed on the connection on the normal bundle, while the first law is a necessary and sufficient condition for a well-defined Hamiltonian evolution on the phase space admitting WIH as an inner boundary. Our proof of the first law for higher dimensions follows a similar methodology as for the four dimensions. For four dimensions, we show that the phase space of WIH admits Hamiltonian charges for local Lorentz transformations as well. More precisely, we show that local Lorentz rotations and boosts belonging to $\mathrm{ISO}(2) \ltimes \mathbb{R}$ (the symmetry algebra on Δ) give rise to charges proportional to the horizon cross sectional area modified by the scalar field (other minimally coupled fields do not contribute since they do not appear in the boundary symplectic structure). Using this definition of area in terms of the Lorentz generators, we have established that the spectrum of the area operator is naturally discrete even for a nonminimally coupled theory. This area spectrum is given by $[(8\pi\gamma\ell_P^2)/f(\phi_\Delta)]n$, where n carry integer values. The conclusion is reached without the need for any particular quantum theory of gravity, like LQG or string theory. Furthermore, the eigenstates of the area operator are attached to the WIH boundary and do not depend on the bulk geometry. This is an advantage of this formulation since one may argue that the fields in the bulk do not affect the black hole entropy. Such a feature is useful since our formulation, unlike string theory or LQG, requires that the states on the horizon remain isolated from the bulk, both classically as well as quantum mechanically. The computation of entropy using the microcanonical fixed area ensemble reveals the standard Bekenstein-Hawking area law for large black holes as well as an exponential suppression for small Planck size black holes. Note the absence of log corrections. This is not obvious, but one may argue that such corrections only arise (at least in LQG) if one considers a large area with a high Chern-Simons level [107]. How does our computation of the area spectrum compare with other proposals? Our calculation shows that the simplified *it from bit* picture is a consistent formulation. Also, our area spectrum is similar to the Bekenstein-Mukhanov picture of horizon microstates [6] or the proposals based on quasinormal modes [126] or from [125]. Alternative proposals have been studied in [129,130].

There is a major drawback of our approach, which is it is tied to four dimensions, and an extension to higher dimensions shall be useful. At this point, let us emphasize the role of spacetime dimensions in our calculations. First, our proof of the first law of black hole mechanics has been carried out for arbitrary N-dimensional spacetime, Eq. (35). This proof uses the Palatini action for N-dimensional spacetime, which shows that the first law holds naturally for all spacetime dimensions. Hence, one would expect that the interpretation of the black hole as a thermal object should also continue to hold for arbitrary spacetime dimensions as well. This would require that black holes have an entropy proportional to their area and admit temperatures proportional to their surface gravity, and it is here that our formalism breaks down. We could only argue the existence of black hole entropy arising out of horizon microstates in four-dimensional spacetime. More precisely, our formalism fails to give a microscopic explanation of the origin of black hole entropy beyond four dimensions. The reason for this should be obvious from our calculations in the previous sections: first, the Holst-like action written in Eq. (39) only exists when spacetime dimensions $N = 4$. A consequence of this is that, in the presence of inner boundaries that admit residual ocal Lorentz invariance, one obtains the horizon area as surface charges. Furthermore, the algebra of these charges also follow the algebra of vector fields generating these charges and follow the ISO(2) algebra peculiar to the representations of lightlike vector fields in a four-dimensional spacetime [124]. Naturally, the quantum states residing on the horizon are labeled by the representation of this ISO(2) algebra. The counting of microstates and the enumeration of the black hole entropy, which utilizes these states, is also based on four-dimensional spacetime. So, to extend this formalism to higher-dimensional spacetimes, new variables may be needed that are conducive to the quantization techniques used here.

ACKNOWLEDGMENTS

A. G. acknowledges the financial support in the form of the fellowship from Central University of Himachal Pradesh. S. D. acknowledges the financial support provided by DST vide Grant No. DST/INSPIRE Fellowship/2020/IF200537. A. C. acknowledges the support from DAE-BRNS vide their project number 58/14/25/2019-BRNS.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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