

Circular polarization effects induced by photon-axion mixing in astrophysical environments

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Axions and axionlike particles (ALPs) are compelling candidates for dark matter and new physics beyond the Standard Model. Photon-axion mixing in external magnetic fields not only modifies the photon energy spectrum and linear polarization state but also induces circular polarization signals. Compared to spectral and linear polarization methods, circular polarization benefits from lower astrophysical background contamination and weaker dependence on the intrinsic source spectrum, providing an independent probe for axion searches. In this work, we study the circular polarization induced by photon-axion mixing within the chiral basis framework. By analytically solving the evolution equations under the single-domain approximation, we derive an expression for the circular polarization degree P_C , applicable in the resonant, strong coupling, and weak coupling regimes. The opposite-phase coupling of the axion field to left- and right-handed circular polarization components generates phase differences and intensity asymmetries, thereby converting initially linearly polarized light into nonzero circular polarization signals. Within single-domain magnetic field models, we compare the energy-dependent circular polarization in four astrophysical environments (active galactic nucleus jets, the intracluster medium, the intergalactic medium, and the Galactic magnetic fields). We find that the x-ray to MeV band represents the most sensitive observational window for axion-induced circular polarization signals. In multidomain propagation models, using the blazar S4 0954 + 65 as a case study, phase accumulation in random magnetic domains causes the circular polarization degree to fluctuate with redshift and exhibit pronounced energy structures in the x-ray to MeV band. Using the optical circular polarization upper limit $P_C < 0.184\%$ (measured in the z-SDSS band) from this source, we statistically constrain $g_{a\gamma\gamma} \lesssim 3 \times 10^{-11} \text{ GeV}^{-1}$ (95% confidence level) for $m_a \sim 10^{-16} - 10^{-10} \text{ eV}$, with the strongest constraint reaching $g_{a\gamma\gamma} \sim 6 \times 10^{-12} \text{ GeV}^{-1}$ near $m_a \sim 10^{-14} \text{ eV}$. These results establish circular polarization as a complementary axion probe, and future high-energy circular polarization observations are expected to further strengthen constraints on the ultralight axion parameter space.

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I. INTRODUCTION

The particle nature of dark matter remains one of the central open questions in modern cosmology and particle physics. Among numerous candidate particles, axions and axionlike particles (ALPs) have attracted significant attention due to their compelling theoretical motivation. The QCD axion was originally introduced via the Peccei-Quinn mechanism to address the strong CP problem in quantum chromodynamics (QCD) [1]. More generally, theoretical frameworks beyond the Standard Model, such as string theory, predict a large number of light pseudoscalar particles, namely ALPs [2]. Unlike the QCD axion, the mass and coupling constant of ALPs can be independent of each other, thereby covering a much broader parameter

space and yielding diverse signatures across dark matter searches, cosmological evolution, and high-energy astrophysics [3]. Unless otherwise specified, we use axions throughout this paper to collectively refer to both the QCD axion and ALPs.

The feeble coupling between axions and electromagnetic fields enables their mutual conversion with photons in external magnetic fields, thereby modifying the photon flux, energy spectrum, and polarization state [4–8]. This mechanism underpins laboratory searches including the axion dark matter experiment (ADMX) [9], the Any Light Particle Search II (ALPS-II) [10], and the CERN Axion Solar Telescope (CAST) [11], and motivates searches in astrophysical environments with strong magnetic fields and long propagation baselines. Currently, astrophysical searches primarily rely on two classes of observables: spectral structure and polarization information. Spectral

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methods search for energy-dependent oscillations, spectral irregularities, or flux anomalies in the x-ray, gamma-ray, or radio bands, and have been applied to the Galactic Center, active galactic nuclei, super star clusters, and pulsars [12–17]. However, spectral constraints depend on assumptions about the intrinsic source spectrum, absorption backgrounds, and magnetic field configurations, which introduces unavoidable systematic uncertainties.

Polarization observations, which encode the coherent phase of the electromagnetic field and the anisotropic response of the propagation medium, offer complementary probes of axion-photon mixing through two distinct mechanisms. First, photon-axion conversion in an external magnetic field selectively affects the photon polarization component parallel to the transverse magnetic field, thereby inducing or modifying linear polarization. Magnetic white dwarfs, neutron-star magnetospheres, and gamma-ray bursts have been used in this context to constrain axion-photon couplings over different mass ranges [18–21]. Second, if ultralight axions constitute a coherent dark matter background, the oscillating axion field can induce time-dependent birefringence and periodic variations of the polarization angle [22]. Based on this mechanism, pulsar polarimetry arrays, long-term optical polarization monitoring of blazars, and millimeter-wave polarization calibration sources have all been employed to search for polarization angle oscillations induced by axion dark matter [23–28]. Nevertheless, linear polarization and polarization-angle observables are often degenerate with magnetic-field geometry in the emission region, intrinsic synchrotron polarization, Faraday rotation, and source variability. Therefore, it remains necessary to develop observables with lower backgrounds and complementary to linear polarization.

In many standard astrophysical emission processes, the degree of circular polarization is typically much lower than that of linear polarization. Circular polarization can therefore partly avoid the above degeneracies and provides a new channel for probing axions. Several recent works have examined axion-induced circular polarization from specific angles. Masaki *et al.* [29] showed how magnetic field geometry shapes polarization dynamics and the associated observational constraints. Yao *et al.* [30] derived analytical expressions for blazar circular polarization in the weak-mixing limit and constrained the photon-axion coupling from optical observations; a follow-up study demonstrated that coherent ALP dark matter oscillations can resonantly enhance this conversion via a Floquet mechanism [31]. Separately, propagation through stochastic magnetic fields was shown to generate nontrivial circular polarization even for initially unpolarized photon beams [32]. Collectively, these studies demonstrate that circular polarization serves not only as a direct propagation effect of axion-photon mixing, but also as a novel observable complementary to energy spectra, linear polarization, and time series of polarization angles.

Despite these advances, several aspects remain to be addressed. Existing analytical treatments are largely developed in the linear polarization basis or within the weak-mixing approximation, which obscures the chiral phase structure of the left- and right-handed circular polarization components. Furthermore, most studies are restricted to single-domain or idealized multidomain magnetic field configurations, leaving the cumulative evolution of circular polarization along realistic multienvironment lines of sight systematically unexplored. Observational constraints have also been obtained predominantly in the optical band, whereas the photon-axion mixing efficiency is strongly energy-dependent and is expected to peak in the x-ray to MeV regime.

Motivated by these considerations, in this work we investigate circular polarization induced by axion-photon mixing in astrophysical environments. We formulate the mixing problem in the chiral basis and derive analytical expressions for the circular polarization degree in the weak-mixing, strong-mixing, and resonant regimes, making explicit how the opposite phase responses of the left- and right-handed circular polarization modes generate circular polarization during propagation. We then examine the energy dependence of the signal in representative single-domain environments, including AGN jets, the intracluster medium, the intergalactic medium, and the Galactic magnetic field. To connect the analytical results with realistic propagation, we further construct a multi-domain model along the line of sight toward the blazar S4 0954 + 65 and use the optical upper limit on its circular polarization to constrain the axion parameter space.

The paper is organized as follows. In Sec. II, we develop the chiral-basis formalism and derive the analytical expressions for the circular polarization degree. In Sec. III, we apply the single-domain model to several representative astrophysical environments and examine the corresponding energy dependence. In Sec. IV, we study multidomain propagation toward S4 0954 + 65 and derive constraints from the optical circular-polarization limit. We summarize our results in Sec. V.

II. ANALYTIC MODEL FOR CIRCULAR POLARIZATION IN PHOTON-AXION MIXING

In this section, we present an analytic model for circular polarization in a photon-axion mixing system, formulated in a chiral basis. Solving the evolution equations for a single-domain magnetic field, we derive closed-form expression for the circular polarization degree $P_C(z)$ and discuss its simplified forms under several representative physical conditions.

The photon-axion coupling is described by the Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{ALP}} &= \frac{1}{2} \partial^\mu a \partial_\mu a - \frac{1}{2} m_a^2 a^2 - \frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} \\ &= \frac{1}{2} \partial^\mu a \partial_\mu a - \frac{1}{2} m_a^2 a^2 + g_{a\gamma\gamma} a \mathbf{E} \cdot \mathbf{B},\end{aligned}\quad (1)$$

where a is the axion field with mass m_a , $g_{a\gamma\gamma}$ is the photon-axion coupling constant, $F_{\mu\nu}$ is the electromagnetic tensor, and $\tilde{F}^{\mu\nu}$ is its dual. We consider a photon-axion state with energy E propagating along the z -axis through a magnetized medium. In the short-wavelength approximation, the evolution of the system is governed by a Schrödinger-like equation [5]

$$i \frac{d}{dz} \psi(z) = \mathcal{H}(E, z) \psi(z). \quad (2)$$

Photon-axion mixing occurs in the transverse magnetic field $\mathbf{B}_T$ perpendicular to the propagation direction. Taking $\mathbf{B}_T$ to be oriented along the y -axis, the mixing matrix in the linear polarization basis $\psi = (|x\rangle, |y\rangle, |a\rangle)^T$ reads [7]

$$\mathcal{H}_{\text{lin}} = \begin{pmatrix} \Delta_{\perp} & 0 & 0 \\ 0 & \Delta_{\parallel} & \Delta_{a\gamma} \\ 0 & \Delta_{a\gamma} & \Delta_a \end{pmatrix}, \quad (3)$$

where the matrix elements include plasma effects, the QED vacuum polarization, the photon-axion interaction, and the axion mass effect. To investigate the circular polarization evolution of photons, we transform the system into the chiral basis defined by

$$|R\rangle = \frac{1}{\sqrt{2}}(|x\rangle + i|y\rangle), \quad |L\rangle = \frac{1}{\sqrt{2}}(|x\rangle - i|y\rangle). \quad (4)$$

The corresponding unitary transformation matrix is

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i & 0 \\ 1 & -i & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}. \quad (5)$$

In the chiral basis $\psi = (|R\rangle, |L\rangle, |a\rangle)^T$, the mixing matrix is obtained via the unitary transformation

$$\psi_{\text{chiral}} = U \psi_{\text{lin}}, \quad H_{\text{chiral}} = U H_{\text{lin}} U^\dagger. \quad (6)$$

With this convention, one obtains

$$\mathcal{H}_{\text{chiral}} = \begin{pmatrix} \frac{\Delta_{\perp} + \Delta_{\parallel}}{2} & \frac{\Delta_{\perp} - \Delta_{\parallel}}{2} & \frac{i\Delta_{a\gamma}}{\sqrt{2}} \\ \frac{\Delta_{\perp} - \Delta_{\parallel}}{2} & \frac{\Delta_{\perp} + \Delta_{\parallel}}{2} & -\frac{i\Delta_{a\gamma}}{\sqrt{2}} \\ -\frac{i\Delta_{a\gamma}}{\sqrt{2}} & \frac{i\Delta_{a\gamma}}{\sqrt{2}} & \Delta_a \end{pmatrix}, \quad (7)$$

where the photon-axion coupling term is $\Delta_{a\gamma} \equiv g_{a\gamma\gamma} B_T / 2$ and the axion mass term is $\Delta_a \equiv -m_a^2 / (2E)$. The photon-medium interaction terms $\Delta_{\perp}$ and $\Delta_{\parallel}$ are defined as [33]

$$\Delta_{\perp} \equiv -\frac{\omega_{\text{pl}}^2}{2E} + \frac{2\alpha}{45\pi} \left(\frac{B_T}{B_{\text{cr}}} \right)^2 E, \quad (8)$$

$$\Delta_{\parallel} \equiv -\frac{\omega_{\text{pl}}^2}{2E} + \frac{7\alpha}{90\pi} \left(\frac{B_T}{B_{\text{cr}}} \right)^2 E, \quad (9)$$

where α is the fine-structure constant, $\omega_{\text{pl}} = \sqrt{4\pi\alpha n_e / m_e}$ is the plasma frequency, and $B_{\text{cr}} = 4.41 \times 10^{13}$ G is the critical magnetic field.

For the astrophysical scenarios considered here (magnetic fields from μG to G, photon energies in the keV-MeV range), the QED vacuum polarization term is negligible compared with the plasma term $\Delta_{\text{pl}} = -\omega_{\text{pl}}^2 / (2E)$. Consequently, one may approximate $\Delta_{\perp} = \Delta_{\parallel} = \Delta_{\text{pl}}$, and the mixing matrix can be rewritten as

$$\mathcal{H}_{\text{chiral}} = \begin{pmatrix} \Delta_{\text{pl}} & 0 & \frac{i\Delta_{a\gamma}}{\sqrt{2}} \\ 0 & \Delta_{\text{pl}} & -\frac{i\Delta_{a\gamma}}{\sqrt{2}} \\ -\frac{i\Delta_{a\gamma}}{\sqrt{2}} & \frac{i\Delta_{a\gamma}}{\sqrt{2}} & \Delta_a \end{pmatrix}. \quad (10)$$

This form shows explicitly that left- and right-circularly polarization states couple to the axion with opposite phases. Such a chirality-dependent coupling can convert an initially linearly polarized photon beam into a state with nonzero circular polarization, providing an observable handle for axion detection.

The evolution equation Eq. (2) is solved by diagonalizing the mixing matrix (see Appendix A for more details). Considering a uniform magnetic domain with no initial axion component, the initial photon state is taken to be

$$|\psi(0)\rangle = C_R |R\rangle + C_L |L\rangle, \quad C_a = 0. \quad (11)$$

The degree of circular polarization after the photon has propagated a distance z is defined by

$$P_C(z) = \frac{|R(z)|^2 - |L(z)|^2}{|R(z)|^2 + |L(z)|^2}, \quad P_C \in [-1, 1]. \quad (12)$$

For an initially linearly polarized photon state with polarization angle β relative to the x -axis, the circular polarization degree $P_C(z)$ can be expressed as

$$P_C(z) = \frac{2 \sin(2\beta) (A \sin \phi_{12} + A' \sin \phi_{13})}{\cos^2 \beta + 4 \sin^2 \beta (A^2 + A'^2) + 8 A A' \sin^2 \beta \cos \phi_{23}}, \quad (13)$$

with

$$A = \frac{2\Delta_{a\gamma}^2}{(\Delta_a - \Delta_{\text{pl}} - \Delta_{\text{osc}})^2 + 4\Delta_{a\gamma}^2},$$

$$A' = \frac{2\Delta_{a\gamma}^2}{(\Delta_a - \Delta_{\text{pl}} + \Delta_{\text{osc}})^2 + 4\Delta_{a\gamma}^2}.$$

The phase differences ϕ_{ij} arising from interference between distinct eigenmodes read

$$\begin{aligned}\phi_{12} &= \frac{1}{2}(\Delta_a - \Delta_{\text{pl}} - \Delta_{\text{osc}})z, \\ \phi_{13} &= \frac{1}{2}(\Delta_a - \Delta_{\text{pl}} + \Delta_{\text{osc}})z, \\ \phi_{23} &= \Delta_{\text{osc}}z,\end{aligned}$$

with oscillation frequency

$$\Delta_{\text{osc}} = \sqrt{(\Delta_a - \Delta_{\text{pl}})^2 + 4\Delta_{a\gamma}^2}.$$

It follows from Eq. (13) that $P_C(z)$ is maximized for $\beta = \pi/4$, and vanishes identically for $\beta = 0$ or $\beta = \pi/2$.

For an initially circularly polarized photon state, the analytic expression for $P_C(z)$ is

$$P_C(z) = \frac{\cos^2 \theta A'_{12} + \sin^2 \theta A'_{13}}{A_0 + A'_{23} \cos \phi_{23}}, \quad (14)$$

with

$$\begin{aligned}A_{12}' &= \frac{1}{2}(C_R^2 - C_L^2) \cos \phi_{12}, \quad A_{13}' = \frac{1}{2}(C_R^2 - C_L^2) \cos \phi_{13}, \\ A_{23}' &= \frac{1}{2} \sin^2 \theta \cos^2 \theta (C_R - C_L)^2, \\ A_0 &= \frac{1}{4} \left[(C_R + C_L)^2 + \left(1 - \frac{1}{2} \sin^2 2\theta \right) (C_R - C_L)^2 \right].\end{aligned}$$

Here θ is the photon-axion mixing angle, which characterizes the coupling strength and satisfies

$$\tan 2\theta = \frac{2\Delta_{a\gamma}}{\Delta_a - \Delta_{\text{pl}}}. \quad (15)$$

Considering a purely right-circularly polarized initial state ($C_R = 1$, $C_L = 0$), Eq. (14) can be rewritten as

$$P_C(z) = \frac{\cos^2 \theta \cos \phi_{12} + \sin^2 \theta \cos \phi_{13}}{1 - \sin^2 \theta \cos^2 \theta + \sin^2 \theta \cos^2 \theta \cos \phi_{23}}. \quad (16)$$

Defining the detuning parameter as $\Delta = \Delta_a - \Delta_{\text{pl}}$, Eq. (16) can be further analyzed in three representative limiting regimes, according to the relative magnitude of the coupling strength and the detuning.

Resonance: When the axion mass term exactly cancels the plasma effect, i.e., $\Delta_a = \Delta_{\text{pl}}$, the system is at resonance. The mixing angle reaches its maximum value $\theta = \pi/4$, and the oscillation frequency reduces to $\Delta_{\text{osc}} = 2\Delta_{a\gamma}$. The circular polarization degree simplifies to

$$P_C(z) = \frac{2 \cos(\Delta_{a\gamma} z)}{1 + \cos^2(\Delta_{a\gamma} z)}. \quad (17)$$

The corresponding evolution is shown in the left panel of Fig. 1. At resonance, energy is exchanged between photons and axions with maximum efficiency. The circular polarization degree undergoes complete periodic oscillations with an amplitude reaching unity, implying that the photon polarization state undergoes a full reversal between pure right-circularly and pure left-circularly polarization. This behavior is consistent with the standard two-level Rabi oscillation picture [31].

Strong coupling: When the coupling term dominates but the detuning is not entirely negligible, i.e., $2\Delta_{a\gamma} \gg |\Delta|$, the system is in the strong-coupling regime. The mixing angle remains close to $\pi/4$, and the oscillation frequency is approximated by

$$\Delta_{\text{osc}} \approx 2\Delta_{a\gamma} + \frac{\Delta^2}{4\Delta_{a\gamma}}. \quad (18)$$

Substituting this approximate expression into the analytic solution, the phase differences become

$$\begin{aligned}\phi_{12} &\approx \left(\frac{1}{2}\Delta - \Delta_{a\gamma} - \frac{\Delta^2}{8\Delta_{a\gamma}} \right) z, \\ \phi_{13} &\approx \left(\frac{1}{2}\Delta + \Delta_{a\gamma} + \frac{\Delta^2}{8\Delta_{a\gamma}} \right) z, \\ \phi_{23} &\approx \left(2\Delta_{a\gamma} + \frac{\Delta^2}{4\Delta_{a\gamma}} \right) z.\end{aligned} \quad (19)$$

The corresponding evolution is shown in the middle panel of Fig. 1. In this regime, the polarization oscillations retain a large amplitude, while their frequency and phase are modulated by the detuning, leading to a slight distortion of the oscillation envelope.

Weak coupling: When the detuning greatly exceeds the coupling term, i.e., $2\Delta_{a\gamma} \ll |\Delta|$, the system enters the weak-mixing regime. The mixing angle is small, $\theta \approx \Delta_{a\gamma}/\Delta \ll 1$, and the oscillation frequency is approximated by

$$\Delta_{\text{osc}} \approx |\Delta| + \frac{2\Delta_{a\gamma}^2}{|\Delta|}. \quad (20)$$

Under these conditions, the circular polarization degree reads

$$P_C(z) \approx \frac{(1 - \theta^2) \cos(\frac{\Delta_{a\gamma}^2}{\Delta} z) + \theta^2 \cos[(\Delta + \frac{\Delta_{a\gamma}^2}{\Delta})z]}{1 - \theta^2 + \theta^2 \cos[(\Delta + \frac{2\Delta_{a\gamma}^2}{\Delta})z]}. \quad (21)$$

The corresponding evolution is shown in the right panel of Fig. 1. In this regime, the photon-axion conversion

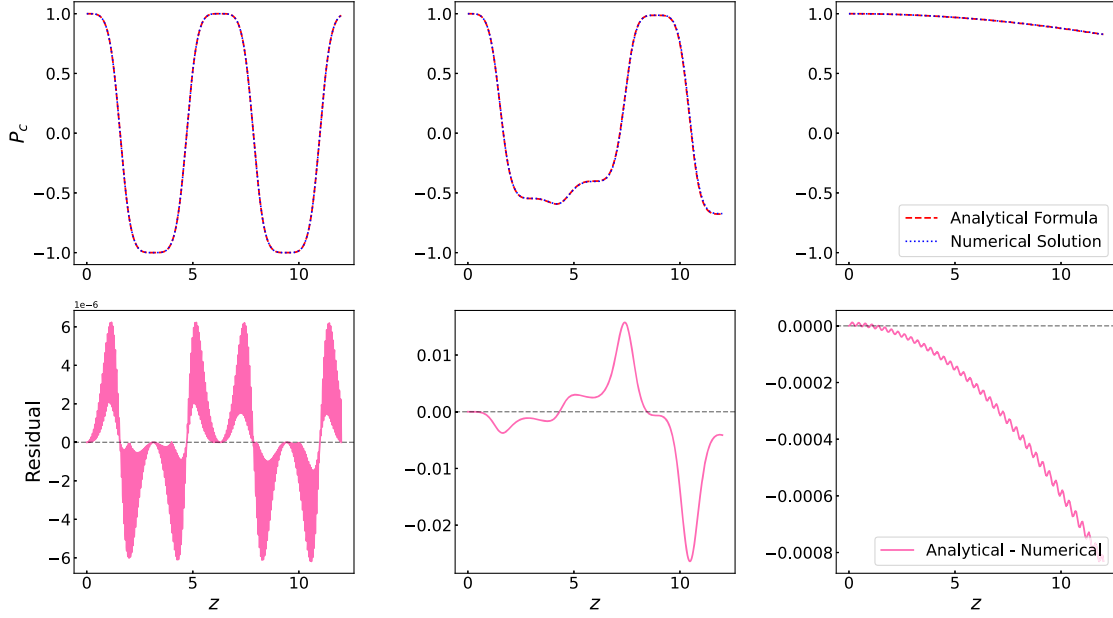


FIG. 1. Comparison of the analytic and numerical solutions for the circular polarization degree $P_C(z)$ in the three limiting cases (resonance, strong-coupling, and weak-coupling, from left to right). The upper panels show $P_C(z)$ as a function of propagation distance, while the lower panels display the corresponding residuals. Red dashed curves denote the analytic results and blue dotted curves denote the fourth-order Runge-Kutta numerical solutions. All quantities are expressed in normalized units with $\Delta_{a\gamma} = 1$, and the propagation distance z is in units of $\Delta_{a\gamma}^{-1}$. The initial state is purely right-handed circular polarization ($C_R = 1$, $C_L = 0$). The detuning $\Delta \equiv \Delta_a - \Delta_{p1}$ is set to $\Delta = 0$ (resonance, left panel, $\Delta_{a\gamma}/|\Delta| \rightarrow \infty$), $\Delta = 0.667$ (strong coupling, middle panel, $\Delta_{a\gamma}/|\Delta| = 1.5$), and $\Delta = 20$ (weak coupling, right panel, $\Delta_{a\gamma}/|\Delta| = 0.05$).

efficiency is strongly suppressed, and the circular polarization signal manifests as small-amplitude, high-frequency oscillations superimposed on a slowly varying envelope. This result is consistent with that obtained from second-order perturbation theory in the weak-mixing limit (see Appendix B for more details).

To verify the reliability of the analytic results, we also solve the evolution equations numerically and compare the solutions with the analytic expressions in the three limiting cases discussed above. Figure 1 shows the evolution of the circular polarization degree $P_C(z)$ as a function of propagation distance z under the resonance, strong-coupling, and weak-coupling conditions. The analytic results (red dashed lines) are in excellent agreement with the fourth-order Runge-Kutta numerical solutions (blue dotted lines) in all cases, with maximum residuals of 6.23×10^{-6} at resonance, 2.64×10^{-2} in the strong-coupling regime, and 8×10^{-4} in the weak-coupling regime.

Prior to this work, Yao *et al.* derived an analytic expression for the circular polarization degree in the weak-coupling approximation based on a perturbative expansion [30]. Masaki *et al.* obtained an analytic solution for an initially linearly polarized beam by diagonalizing the mixing matrix [29]. However, both of these works employed the conventional linear polarization basis and therefore did not directly expose the relative phase evolution of the left- and right-circular polarization modes

induced by axion-photon mixing. In the present work, we introduce the chiral basis and carry out a direct diagonalization of the mixing matrix. This approach makes the chiral phase structure explicit and provides a compact description of the circular-polarization evolution in the resonant, strong-mixing, and weak-mixing limits within the single-domain setup considered here.

III. ENERGY DEPENDENCE CHARACTERISTICS OF CIRCULAR POLARIZATION DEGREE IN ASTROPHYSICAL ENVIRONMENTS

Based on the analytic model developed in the preceding section, we investigate the characteristics of axion-induced circular polarization signals as photons propagate through various astrophysical environments. We first evaluate the energy dependence of photon-axion mixing efficiency across four representative magnetic environments—the active galactic nuclei (AGN) jet, the intracluster magnetic field (ICMF), the intergalactic magnetic field (IGMF), and the Galactic magnetic field (GMF)—to identify the energy bands in which axion-induced circular polarization may produce potentially observable features. Subsequently, we compute the energy dependence of the photon circular polarization degree in the single-domain magnetic field model for each of these environments.

The efficiency of photon-axion mixing is governed by the physical conditions of the propagation medium,

TABLE I. Fiducial parameters adopted for the four representative astrophysical environments considered in this work. The representative parameter values are compiled from [30,34].

Environment	B (μG)	n_e (cm^{-3})	L (kpc)
AGN	10^6	10^2	10^{-3}
ICMF	1	10^{-3}	10
IGMF	10^{-3}	10^{-7}	5×10^4
GMF	1	10^{-1}	10^{-2}

characterized primarily by the magnetic field strength B , the electron number density n_e , and the magnetic field coherence length L . To compare the mixing effects across distinct astrophysical contexts, we select four representative environments whose parameters are summarized in Table I.

The strength of photon-axion mixing can be characterized by the mixing angle

$$\theta = \frac{1}{2} \arctan \left(\frac{2\Delta_{a\gamma}}{\Delta_a - \Delta_{\text{pl}}} \right) = \frac{1}{2} \arctan \left(\frac{2g_{a\gamma\gamma} B_T E}{m_a^2 - \omega_{\text{pl}}^2} \right) \quad (22)$$

Fig. 2 shows the mixing angle as a function of photon energy in each of the four astrophysical environments considered above, computed for four representative sets of axion parameters. At radio and optical energies, all curves approach $\theta = 0$, indicating that the system resides in the weak-coupling regime where photon-axion mixing is highly inefficient. As the photon energy enters the x-ray band, the mixing angle begins to deviate appreciably from zero and increases with energy. At high energies (GeV–TeV band), θ asymptotically approaches $\pi/4$, signaling the onset of the resonant strong-coupling regime. These results imply that the axion-induced circular polarization degree varies with photon energy, and most pronounced energy dependence, for the representative parameters shown here, occurs from the x-ray to MeV band. To indicate the validity of the theoretical framework, we also show the range of applicability of the slowly varying envelope approximation (SVEA) in the figure. This approximation requires that the scale over which the matrix elements of the evolution Hamiltonian vary be much larger than the photon wavelength, a condition that can be quantified by the dimensionless parameter

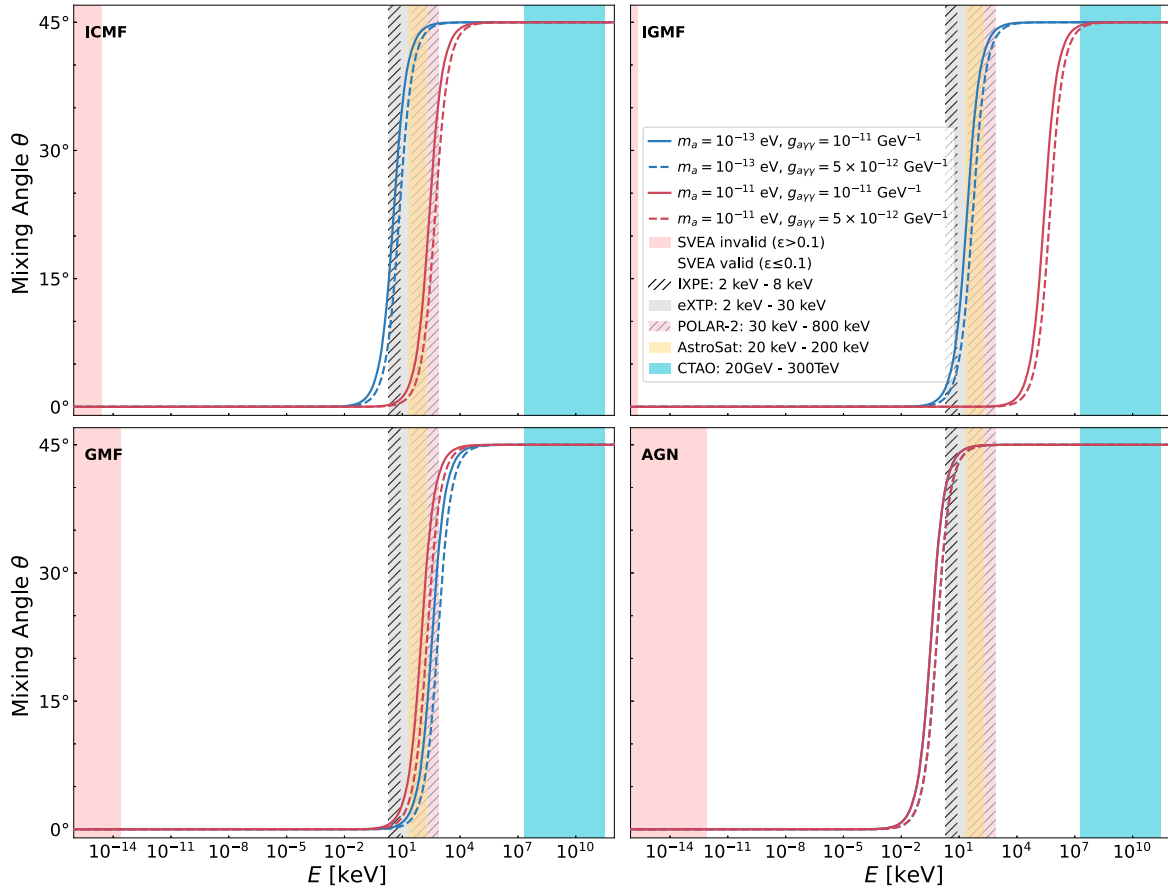


FIG. 2. Mixing angle as a function of photon energy in four representative astrophysical environments. Each panel corresponds to one environment (ICMF, IGMF, GMF, and AGN), and each curve corresponds to a different set of axion parameters. The red shaded region on the left marks the energy range where the SVEA may fail. The colored shaded regions on the right indicate the approximate energy coverage of different telescopes (IXPE [35,36], eXTP [37], POLAR-2 [38], AstroSat [39], and CTAO [40,41]).

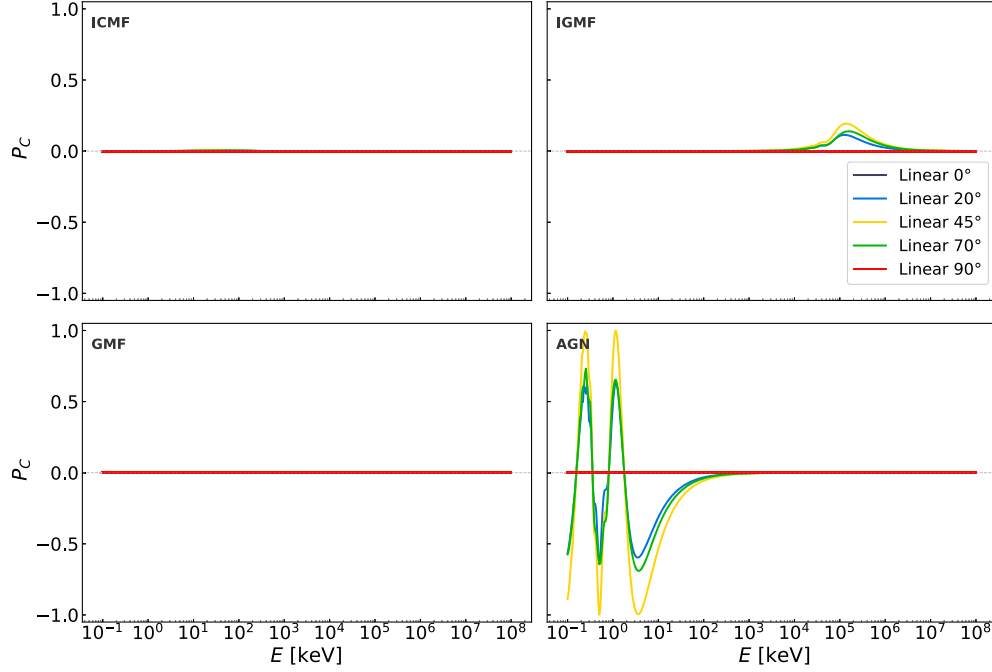


FIG. 3. Circular polarization degree as a function of photon energy for different initial linear polarization angles in the single-domain magnetic-field model. Each panel corresponds to one astrophysical environment (ICMF, IGMF, GMF, and AGN), and each curve corresponds to a different initial polarization angle β .

$$\epsilon = \max \left(\frac{g_{a\gamma\gamma} B_T}{2k}, \frac{|m_a^2 - \omega_{\text{pl}}^2|}{2\omega k} \right) \ll 1, \quad (23)$$

where $k \simeq \omega$ is the photon wave number. The low-energy region in Fig. 2 for which $\epsilon \gtrsim 0.1$, namely the region where the SVEA may break down, is excluded in the subsequent analysis.

The photon-axion coupling strength exhibits a strong energy dependence. Motivated by this property, we compute the circular polarization degree as a function of photon energy for propagation through a single homogeneous magnetic domain in each of the four environments. In the single-domain model, the magnetic field strength and orientation are held constant, and the propagation distance is taken to be the magnetic coherence length L . We adopt a representative set of axion parameters, $m_a = 1 \times 10^{-11}$ eV and $g_{a\gamma\gamma} = 1 \times 10^{-11}$ GeV $^{-1}$, and consider two initial polarization states, namely a purely linearly polarized state (Fig. 3) and a purely right-handed circularly polarized state (Fig. 4).

Figure 3 shows the results for an initially linearly polarized beam. The circular polarization degree P_C depends strongly on the initial linear polarization angle β . In the ICMF and GMF, P_C remains negligibly small for all values of β . This is because both environments possess relatively weak magnetic fields ($B \sim 1$ μ G) and coherence lengths L much shorter than the oscillation length $L_{\text{osc}} \equiv 2\pi/\Delta_{\text{osc}}$, so that the accumulated

circular-polarization signal remains small. In the IGMF, P_C is maximized at $\beta = \pi/4$ and vanishes at $\beta = 0$ and $\pi/2$, in agreement with the analytic expressions derived in Sec. II. Furthermore, the long path length of the IGMF ($L \sim 50$ Mpc) provides sufficient propagation distance for the polarization signal to accumulate, resulting in a broad, slowly varying peak in P_C at the MeV energy band. In the AGN jet, the strong magnetic field ($B \sim 10^6$ μ G) enhances photon-axion coupling. In the x-ray band, the system lies in the off-resonance regime ($\theta \ll \pi/4$), where the interference phase $\phi_{23}(E)$ is highly sensitive to energy, causing P_C to oscillate rapidly with large amplitude. By contrast, although the system enters the strong-coupling regime at TeV energies, the phase evolution becomes less sensitive to energy, and P_C consequently tends to stabilize.

Figure 4 presents the results for an initially right-handed circularly polarized beam. In the ICMF and GMF, the variation of P_C remains likewise negligible. In the IGMF, P_C exhibits a broad, shallow dip structure. In the AGN jet, P_C undergoes large-amplitude oscillations with an amplitude spanning the full range $[-1, 1]$, indicating that the polarization state can undergo nearly complete conversion between left- and right-handed circular polarization as a function of photon energy. This distinctive energy-dependent signature could provide a useful diagnostic for identifying axion-induced circular-polarization features.

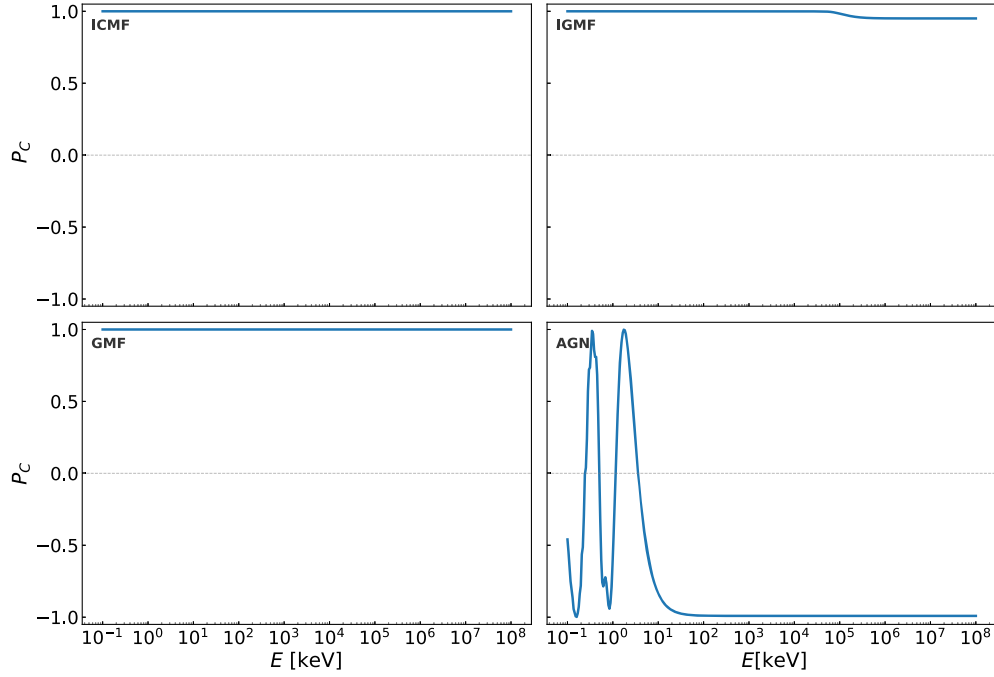


FIG. 4. Circular polarization degree as a function of photon energy for an initially purely right-handed circularly polarized state in the single-domain model. Each panel corresponds to one astrophysical environment (ICMF, IGMF, GMF, and AGN).

IV. EVOLUTION OF CIRCULAR POLARIZATION IN MULTIDOMAIN PROPAGATION AND CONSTRAINTS ON AXION PARAMETERS

In realistic astrophysical environments, magnetic field structures typically exhibit complex inhomogeneous properties, which differ significantly from the single-domain uniform magnetic field adopted in Sec. III. To describe the propagation of photons through real astrophysical medium, we study the evolution of axion-induced circular polarization under multidomain propagation. We construct a three-zone line-of-sight propagation model that includes the AGN jet, IGMF, and GMF. Taking the blazar S4 0954 + 65 as a representative source, we numerically simulate the propagation from the source to the observer, and analyze the evolution of the circular polarization degree as a function of redshift and photon energy. We then combine the result with the existing upper limit on the optical circular polarization of this source to constrain the axion parameter space.

A. Multidomain magnetic field configuration and numerical simulation method

We consider high-energy photons from the blazar S4 0954 + 65 propagating along the line of sight through three magnetized regions in sequence, namely the AGN, the IGMF, and the GMF. The source parameters are right ascension $09^h58^m47.24^s$, declination $+65^\circ33'54.81''$, and redshift $z = 0.367$ [42].

For the AGN jet, we adopt an axisymmetric helical magnetic-field model. Following the Clausen-Brown model, we decompose the magnetic field in cylindrical

coordinates (r, ϕ, z) into an axial component B_z and a toroidal component B_ϕ , whose radial profiles are given by Bessel functions [43]

$$\begin{aligned} B_z(r, z) &= B_T^{\text{jet}}(z) \frac{J_0(k\rho)}{\sqrt{J_0^2 + J_1^2}}, \\ B_\phi(r, z) &= B_T^{\text{jet}}(z) \frac{J_1(k\rho)}{\sqrt{J_0^2 + J_1^2}}, \end{aligned} \quad (24)$$

where $\rho = r/R_{\text{jet}}$, and $k = 2.405$ is the first zero of J_0 , ensuring $B_z = 0$ at the jet boundary. Here $B_T^{\text{jet}}(z)$ describes the overall evolution of the magnetic field strength with distance and serves as a unified normalization factor for all components. The magnetic field phase is

$$\phi_{\text{rot}}(z) = \phi_0 + 10 \ln\left(\frac{z}{z_E}\right), \quad (25)$$

where $\phi_0 \in [0, 2\pi]$ and $z_E = 0.01$ pc. Taking into account the viewing angle of approximately 5.5° between the jet axis and the line of sight [44,45], we rotate the magnetic field from the jet frame to the observer frame and then project it to obtain the transverse component. The overall magnetic field strength and electron number density follow power-law profiles [46,47], as

$$\begin{aligned} B_T^{\text{jet}}(z) &= B_{T,0} \left(\frac{z}{z_E}\right)^{-1}, \\ n_e^{\text{jet}}(z) &= n_{e,0} \left(\frac{z}{z_E}\right)^{-2}, \end{aligned} \quad (26)$$

where $B_{T,0} \sim 1$ G and $n_{e,0} \sim 10^2 \text{ cm}^{-3}$. The jet region is discretized into approximately 150–200 domains. In each domain, the magnetic-field strength and geometry are determined by the model above, while the field orientation varies continuously across domain boundaries.

For the IGMF, we adopt a random magnetic domain model to characterize the turbulent magnetic field [48]. The propagation path is divided into a series of cells, each with a coherence length of $L_{\text{coh}} \sim 1$ Mpc. Within each domain, the magnetic field strength $B_{\text{IGMF}} \sim 1$ nG is held constant, while the field orientation is randomly and uniformly distributed over the interval $[0, 2\pi]$. The electron number density is assigned a typical value of $n_e^{\text{IGM}} \sim 10^{-7} \text{ cm}^{-3}$. The total number of IGMF magnetic domains is determined by the source redshift. For a source at $z = 0.367$, the comoving distance is approximately 1500 Mpc, corresponding to roughly 1500 domains.

For the GMF, we adopt the Jansson-Farrar model [49], which is based on a joint fit to approximately 4×10^4 Faraday rotation measurements of extragalactic radio sources and pulsars, together with the WMAP synchrotron polarization data. This model provides a parametric description of the Galactic disk field, the halo field, and the out-of-plane field components. The distribution of the electron number density n_e is described by the Cordes-Lazio model [50].

For the three regions described above, we generate the corresponding sequence of magnetic domains. The evolution of the photon-axion system within each domain is governed by Eq. (2). The state vector $\psi = (|R\rangle, |L\rangle, |a\rangle)^T$ evolves within each domain as $\psi(z) = \mathcal{U}(z)\psi(0)$, where $\mathcal{U}(z) = \exp(-i\mathcal{H}_{\text{chiral}}z)$, and $\mathcal{H}_{\text{chiral}}$ is the mixing matrix given in Sec. II. For a propagation path consisting of N magnetic domains, the overall evolution operator is the ordered product of the individual domain operators,

$$\psi_{\text{final}} = \mathcal{U}_N(z_N) \cdots \mathcal{U}_2(z_2)\mathcal{U}_1(z_1)\psi_{\text{initial}}, \quad (27)$$

where $\mathcal{U}_i(z_i)$ denotes the evolution operator of the i th domain. In the numerical calculation, the state vector is updated iteratively domain by domain as $\psi_{i+1} = \mathcal{U}_i(z_i)\psi_i$ until the final state is obtained. Given that the initial polarization state of an astrophysical source may be a mixture of linear polarization and unpolarized components, we employ a Monte Carlo ensemble method to compute the final ensemble-averaged circular polarization degree. Optical observations of the blazar S4 0954 + 65 show a linear polarization degree of $\Pi_l = 30.9\%$ with an electric-vector position angle (EVPA) of 83° [42], and we construct a photon ensemble of 2000 initial pure states to simulate its polarization properties. Among these, 30.9% of the samples are assigned the same linearly polarized state (with polarization angle fixed at the observed EVPA), while the remaining 69.1% are generated by randomly sampling pure states with uniformly random phases and polarization

angles, which represents the unpolarized component. The multidomain iterative evolution is performed independently for each initial state in the ensemble, and the circular polarization degree $\langle P_C \rangle$ is obtained by statistically averaging over all final states.

B. Redshift evolution and energy-dependent features

Using the multidomain magnetic field model described above, we compute the ensemble-averaged circular polarization degree as a function of source redshift z . Figure 5 shows the ensemble-averaged circular polarization degree $\langle P_C \rangle$ as a function of source redshift z for the optical, x-ray, MeV gamma-ray, and TeV gamma-ray bands, computed for axion parameters $m_a = 10^{-13} \text{ eV}$ and $g_{a\gamma\gamma} = 1 \times 10^{-11} \text{ GeV}^{-1}$. The calculation assumes that photons traverse the entire propagation medium at their emission-frame energy. In the figure, the solid curves represent the ensemble mean obtained from multidomain evolution, and the shaded regions indicate the 1σ uncertainty. The results show that in the x-ray and MeV bands, the circular polarization degree exhibits stochastic fluctuations with increasing redshift, arising from phase accumulation as photons propagate through multiple magnetic domains. As the redshift z increases, photons traverse a larger number of IGMF domains. Since azimuthal angle of the transverse magnetic field within each domain is randomly distributed, random phase differences are continuously introduced during the evolution. This random superposition of interference phases across different domains causes the net polarization degree $\langle P_C \rangle$ to fluctuate irregularly about zero with redshift, while the 1σ uncertainty broadens with propagation distance following the statistics of a random walk. This redshift-dependent feature carries clear observational

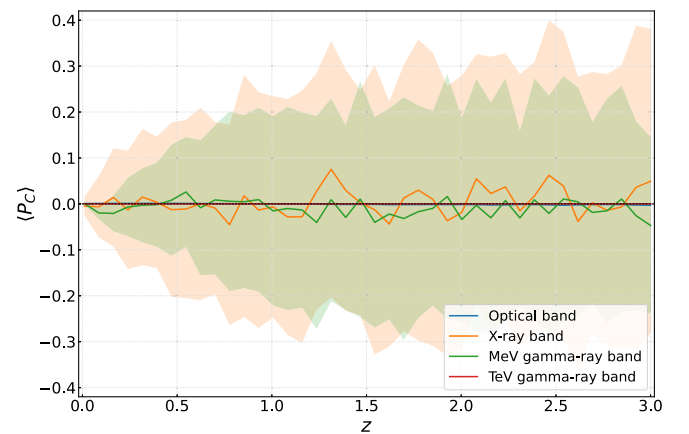


FIG. 5. Average circular polarization degree $\langle P_C \rangle$ as a function of source redshift z under multidomain propagation for an ALP with $m_a = 10^{-13} \text{ eV}$ and $g_{a\gamma\gamma} = 1 \times 10^{-11} \text{ GeV}^{-1}$. Blue, orange, green, and red lines denote the optical, x-ray, MeV gamma-ray, and TeV gamma-ray bands, respectively.

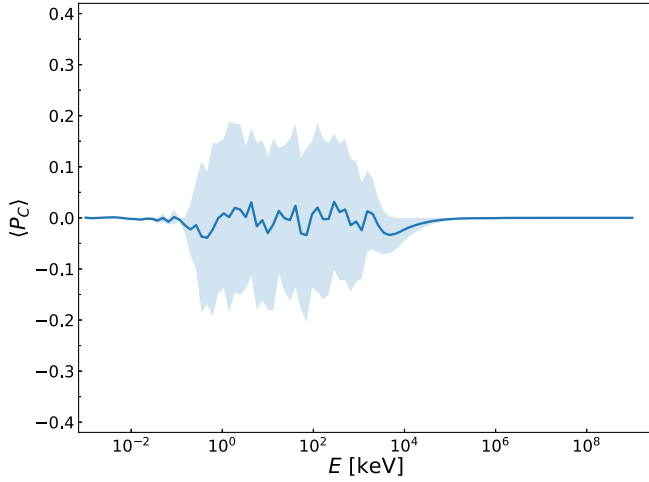


FIG. 6. Average circular polarization degree $\langle P_C \rangle$ as a function of photon energy E under multidomain propagation for an ALP with $m_a = 10^{-13}$ eV and $g_{a\gamma\gamma} = 1 \times 10^{-11}$ GeV $^{-1}$.

implications. From a theoretical standpoint, the intrinsic polarization properties of astrophysical sources are unlikely to naturally produce such pronounced fluctuations spanning a wide range of redshifts. Thus, comparing circular polarization measurements for AGN at different redshifts could help separate the propagation-induced photon-axion mixing signal from the source-intrinsic contribution, thereby providing an independent probe of the axion parameter space.

Furthermore, we investigate the energy dependence of the circular polarization degree within the multidomain model at a fixed source redshift of $z = 0.367$, as shown in Fig. 6. The results show a clear energy dependence. In the x-ray to MeV band ($E \sim 1\text{--}10^4$ keV), the ensemble-averaged circular polarization degree $\langle P_C \rangle$ exhibits large stochastic fluctuations, whereas the amplitude of the fluctuations is suppressed at lower energies (radio to optical) and at very high energies. This nontrivial energy-dependent feature provides a key observational test for axion searches. Observationally, upper limits on the circular polarization degree from a single waveband (e.g., optical or x-ray) already allow independent constraints on the axion parameter space. Looking ahead, broadband multiwavelength circular polarization observations spanning a wide energy range, particularly in the x-ray to MeV band, will more effectively break the physical degeneracy between the photon-axion mixing propagation effect and intrinsic emission mechanisms.

C. Constraints on axion parameters from S4 0954 + 65

The blazar S4 0954 + 65 is one of the few low-synchrotron-peaked blazars detected at TeV gamma-ray energies, exhibiting a high degree of optical linear polarization, with an observational upper limit on the optical circular polarization of $P_c < 0.184\%(3\sigma)$ [42]. This upper limit was obtained in the z-SDSS broad-band

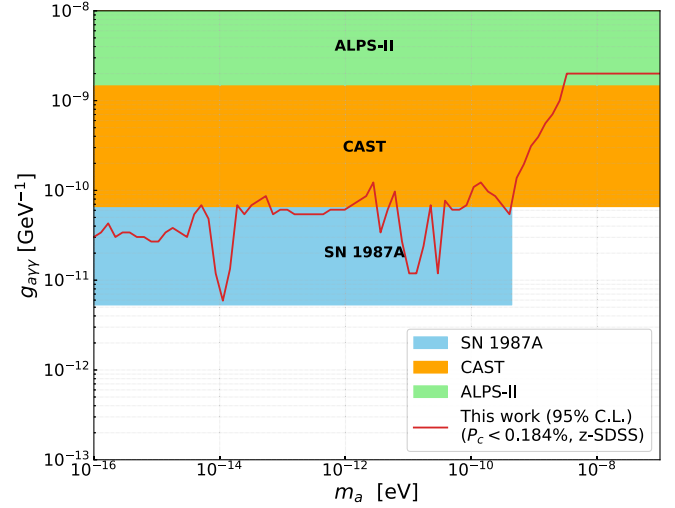


FIG. 7. Constraints on the axion parameter space derived from the upper limit on circular polarization of the blazar S4 0954 + 65. The red solid curve represents the exclusion boundary obtained in this work based on $P_c < 0.184\%$ in the z-SDSS band ($E \simeq 1.36$ eV), at the 95% confidence level (i.e. $|P_C| > 0.184\%$ in at least 95% of the random magnetic-field realizations) (the region above the curve is excluded). For comparison, the blue, orange, and green shaded regions represent previously obtained bounds from SN 1987A, CAST, and ALPS-II, respectively.

filter ($\Delta\lambda \simeq 830\text{--}1000$ nm), corresponding to a central frequency $\nu_{\text{obs}} \simeq 3.28 \times 10^{14}$ Hz, i.e. a photon energy $E \simeq 1.36$ eV. Based on this upper limit, we derive constraints on the axion parameter space. Because the predicted circular polarization degree in the multidomain model depends on the specific stochastic realization of the random magnetic fields (cf., Fig. 6), for a fixed point in the $(m_a, g_{a\gamma\gamma})$ plane the computed P_C fluctuates from one field realization to another and can lie either above or below the observational upper limit. We therefore formulate the exclusion statistically. For each point in the $(m_a, g_{a\gamma\gamma})$ plane, we generate 300 independent random magnetic-field realizations. For each realization, we compute the ensemble-averaged circular polarization degree at the observed energy $E \simeq 1.36$ eV using the multidomain propagation model described above. A parameter point is excluded at the 95% confidence level if $|P_C| > 0.184\%$ in at least 95% of the magnetic-field realizations. The resulting exclusion region is shown in Fig. 7. For $m_a \sim 10^{-16} \text{--} 10^{-10}$ eV, we constrain $g_{a\gamma\gamma} \lesssim 3 \times 10^{-11}$ GeV $^{-1}$ (95% CL), with the most stringent bound occurring near $m_a \sim 10^{-14}$ eV, corresponding to $g_{a\gamma\gamma} \sim 6 \times 10^{-12}$ GeV $^{-1}$. Over $m_a \sim 10^{-16} \text{--} 10^{-10}$ eV, this exclusion is broadly comparable to the CAST bound and, at the most sensitive masses, reaches the level of the SN 1987A limit, while lying well below the projected ALPS-II sensitivity. Being based on circular polarization, it provides a complementary constraint.

Constraining the axion parameter space via astrophysical energy spectra is another widely adopted approach. Recently, Zhu *et al.* [12] derived a constraint of $g_{a\gamma\gamma} \sim 10^{-11} \text{ GeV}^{-1}$ using the very-high-energy gamma-ray spectrum of the Galactic center source HESS J1745-290. Malyshev *et al.* [51] improved this bound to $g_{a\gamma\gamma} \sim 2 \times 10^{-12} \text{ GeV}^{-1}$ by stacking the gamma-ray spectra of active galactic nuclei behind galaxy clusters. These results are broadly comparable in magnitude to the constraint obtained in the present work. However, spectral methods are sensitive to assumptions about the intrinsic spectral shape of the source and require modeling of the magnetic field and radiation background along the line of sight, leading to systematic uncertainties. Moreover, intrinsic source variability can produce spurious signals that may mimic photon-axion oscillations. In contrast, polarization-based methods are independent of the spectral shape and thus less affected by these spectral uncertainties. Zhang *et al.* [52] placed stringent constraints of $g_{a\gamma\gamma} \lesssim 10^{-13} \text{ GeV}^{-1}$ by using x-ray linear polarization observations of gamma-ray bursts. Compared with the linear polarization results of Zhang *et al.*, the constraint on the axion coupling constant derived in this work is weaker, which can be primarily attributed to systematic differences in the observational waveband and sample redshift. This work employs the optical band, whereas Zhang *et al.* utilize the x-ray band, in which the photon-axion mixing efficiency is substantially higher. In addition, their gamma-ray burst sample spans higher redshifts ($z \sim 1$), providing longer mixing path lengths. It is worth noting that linear polarization is susceptible to contamination from intrinsic polarization of the astrophysical environment. By contrast, circular polarization offers a distinctive advantage of intrinsically low background. This is because standard astrophysical emission mechanisms produce negligible circular polarization, and any significant detection would therefore constitute a strong indicator of new physics. Future observations combining x-ray circular polarimetry with broadband, multiredshift joint analyses are expected to substantially enhance the constraining power on axion parameters.

V. CONCLUSION

This study investigates the impact of photon-axion mixing on the evolution of photon circular polarization in astrophysical environments. Within the chiral basis framework, we solve the evolution equations of the photon-axion mixing system and derive an analytic expression for the circular polarization degree $P_C(z)$, which provides a compact description of circular polarization evolution in the resonant, strong coupling, and weak coupling regimes. Specifically, at resonance, the photon polarization state undergoes nearly complete periodic reversals. In the strong coupling regime, polarization

oscillations maintain substantial amplitudes, although their frequencies and phases are modulated by the detuning, resulting in slight distortions of the oscillation envelope. In the weak coupling regime, the circular polarization signal exhibits small-amplitude, high frequency oscillations superimposed on a slowly varying envelope. This analytic treatment does not rely on a weak-mixing expansion and makes explicit how the opposite chiral phases of the left- and right-handed circular polarization modes generate relative phase shifts and amplitude differences, allowing initially linearly polarized light to acquire a nonzero circular polarization component.

Building upon the analytic model, we investigate photon-axion mixing and its energy dependence in four representative astrophysical environments. The results show that mixing efficiency increases with photon energy, with the x-ray to MeV band being the key window for searching for axion-induced circular polarization signals. Single-domain magnetic field calculations show that the circular polarization degree is sensitive to energy, initial polarization angle, and environmental parameters, with its energy-dependent structure providing identifiable observational signatures for axion searches.

To more realistically describe photon propagation through astrophysical environments, we construct a multidomain cascade model incorporating helical magnetic fields in AGN jets, intergalactic random magnetic fields, and large-scale regular Galactic magnetic fields. Using the blazar S4 0954 + 65 as a case study, we perform numerical simulations. The results show that phase accumulation in random magnetic domains causes the circular polarization degree to exhibit irregular fluctuations with redshift and produces pronounced energy-dependent structures in the x-ray to MeV band. These features provide potential diagnostics for distinguishing axion-induced propagation effects from intrinsic source polarization mechanisms. Based on the optical circular polarization observational upper limit $P_C < 0.184\%$ (z-SDSS band, $E \simeq 1.36 \text{ eV}$) from this source, we statistically constrain the axion parameter space by excluding parameter points for which $|P_C| > 0.184\%$ in at least 95% of the random magnetic-field realizations, obtaining an upper limit at the level of $g_{a\gamma\gamma} \lesssim 3 \times 10^{-11} \text{ GeV}^{-1}$ (95% CL) for $m_a \sim 10^{-16} - 10^{-10} \text{ eV}$, with the strongest constraint reaching $g_{a\gamma\gamma} \sim 6 \times 10^{-12} \text{ GeV}^{-1}$ near $m_a \sim 10^{-14} \text{ eV}$. Compared to spectral methods and linear polarization methods, the circular polarization approach is less susceptible to astrophysical background contamination and does not depend on assumptions about the intrinsic spectrum of the source, thereby providing unique advantages for axion detection. Future circular polarization measurements in the x-ray to MeV band could further test this scenario and improve sensitivity to ultralight axions.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: ANALYTIC DERIVATION OF THE CIRCULAR-POLARIZATION DEGREE BY DIAGONALIZATION

We solve the evolution of the photon-axion system by diagonalizing the Hamiltonian in the chiral basis Eq. (10). Solving the secular equation $\det(\mathcal{H}_{\text{chiral}} - E\mathbb{I}) = 0$, we obtain three eigenvalues:

$$\begin{aligned} E_1 &= \Delta_{\text{pl}}, \\ E_2 &= \frac{\Delta_{\text{pl}} + \Delta_a - \Delta_{\text{osc}}}{2}, \\ E_3 &= \frac{\Delta_{\text{pl}} + \Delta_a + \Delta_{\text{osc}}}{2}, \end{aligned} \quad (\text{A1})$$

where $\Delta_{\text{osc}} = \sqrt{(\Delta_a - \Delta_{\text{pl}})^2 + 4\Delta_{a\gamma}^2}$ is the oscillation frequency. The corresponding normalized eigenstates are

$$\begin{aligned} |\psi_1\rangle &= \frac{1}{\sqrt{2}}(|R\rangle + |L\rangle), \\ |\psi_2\rangle &= N_2 \left(-\frac{i\sqrt{2}\Delta_{a\gamma}}{B} |R\rangle + \frac{i\sqrt{2}\Delta_{a\gamma}}{B} |L\rangle + |a\rangle \right), \\ |\psi_3\rangle &= N_3 \left(\frac{i\sqrt{2}\Delta_{a\gamma}}{B'} |R\rangle - \frac{i\sqrt{2}\Delta_{a\gamma}}{B'} |L\rangle + |a\rangle \right), \end{aligned} \quad (\text{A2})$$

where

$$\begin{aligned} B &= \frac{\Delta_a - \Delta_{\text{pl}} - \Delta_{\text{osc}}}{2}, & B' &= \frac{\Delta_a - \Delta_{\text{pl}} + \Delta_{\text{osc}}}{2}, \\ N_2 &= \left(1 + \frac{4\Delta_{a\gamma}^2}{B^2} \right)^{-1/2}, & N_3 &= \left(1 + \frac{4\Delta_{a\gamma}^2}{B'^2} \right)^{-1/2}. \end{aligned}$$

For a general initial state $|\psi(0)\rangle = C_R|R\rangle + C_L|L\rangle + C_a|a\rangle$, projecting the initial state onto each eigenstate $|\psi_j\rangle$, the projection coefficients $C_n = \langle\psi_n|\psi(0)\rangle$ are given by

$$\begin{aligned} C_1 &= \frac{C_R + C_L}{\sqrt{2}}, \\ C_2 &= N_2 \left[\frac{i\sqrt{2}\Delta_{a\gamma}}{B} (C_R - C_L) + C_a \right], \\ C_3 &= N_3 \left[-\frac{i\sqrt{2}\Delta_{a\gamma}}{B'} (C_R - C_L) + C_a \right]. \end{aligned} \quad (\text{A3})$$

The state of the system after propagating a distance z can be expanded as $|\psi(z)\rangle = \sum_{n=1}^3 C_n e^{-iE_n z} |\psi_n\rangle$. We focus on the case with no initial axion component, i.e., $C_a = 0$. Substituting the projection coefficients and eigenstates into the expansion and rearranging, the amplitudes of the right- and left-handed circular polarization components are obtained as

$$\begin{aligned} R(z) &= \frac{C_R + C_L}{2} e^{-iE_1 z} \\ &\quad + A e^{-iE_2 z} (C_R - C_L) + A' e^{-iE_3 z} (C_R - C_L), \\ L(z) &= \frac{C_R + C_L}{2} e^{-iE_1 z} \\ &\quad - A e^{-iE_2 z} (C_R - C_L) - A' e^{-iE_3 z} (C_R - C_L), \end{aligned} \quad (\text{A4})$$

where

$$A = \frac{2\Delta_{a\gamma}^2}{B^2 + 4\Delta_{a\gamma}^2}, \quad A' = \frac{2\Delta_{a\gamma}^2}{B'^2 + 4\Delta_{a\gamma}^2}.$$

The circular-polarization degree can be written as

$$P_C(z) = \frac{|R(z)|^2 - |L(z)|^2}{|R(z)|^2 + |L(z)|^2} = \frac{2\text{Re}(\alpha\eta^*) + 2\text{Re}(\alpha\delta^*)}{|\alpha|^2 + |\eta|^2 + |\delta|^2 + 2\text{Re}(\eta\delta^*)}, \quad (\text{A5})$$

where

$$\begin{aligned} \alpha &= \frac{C_R + C_L}{2} e^{-iE_1 z}, \\ \eta &= A e^{-iE_2 z} (C_R - C_L), \\ \delta &= A' e^{-iE_3 z} (C_R - C_L). \end{aligned}$$

For an initially linearly polarized photon with polarization angle β ,

$$C_R = \frac{e^{i\beta}}{\sqrt{2}}, \quad C_L = \frac{e^{-i\beta}}{\sqrt{2}}, \quad (\text{A6})$$

Equation (A5) reduces to the analytic expression given in

Eq. (13). For an initially purely circularly polarized photon state, it reduces to Eq. (14).

APPENDIX B: ANALYTICAL DERIVATION OF CIRCULAR POLARIZATION DEGREE IN WEAK COUPLING APPROXIMATION VIA SECOND-ORDER PERTURBATION THEORY

In the axion-photon mixing system, a weak-mixing regime is established when the detuning greatly exceeds the coupling strength ($2\Delta_{a\gamma} \ll |\Delta_a - \Delta_{\text{pl}}|$), allowing for a perturbative treatment. Starting from the perturbative Hamiltonian, we derive the analytical expression for the circular polarization degree. In the chiral basis $\psi = (|R\rangle, |L\rangle, |a\rangle)^T$, the total Hamiltonian is decomposed into the unperturbed part H_0 and the perturbation H'

$$H_0 = \begin{pmatrix} \Delta_{\text{pl}} & 0 & 0 \\ 0 & \Delta_{\text{pl}} & 0 \\ 0 & 0 & \Delta_a \end{pmatrix}, \quad H' = \begin{pmatrix} 0 & 0 & \frac{i}{\sqrt{2}}\Delta_{a\gamma} \\ 0 & 0 & -\frac{i}{\sqrt{2}}\Delta_{a\gamma} \\ -\frac{i}{\sqrt{2}}\Delta_{a\gamma} & \frac{i}{\sqrt{2}}\Delta_{a\gamma} & 0 \end{pmatrix}, \quad (\text{B1})$$

where $\Delta_{\text{pl}} = -\omega_{\text{pl}}^2/(2E)$ is the plasma contribution, $\Delta_a = -m_a^2/(2E)$ is the axion mass term, and $\Delta_{a\gamma} = g_{a\gamma\gamma}B_T/2$ is the axion-photon coupling term, following the notation used in Sec. II. The eigenstates of the unperturbed Hamiltonian H_0 are denoted as $|R\rangle$, $|L\rangle$, and $|a\rangle$, with the corresponding zeroth-order eigenvalues $E_1^{(0)} = E_2^{(0)} = \Delta_{\text{pl}}$ and $E_3^{(0)} = \Delta_a$, respectively. Since all diagonal elements of H' vanish, the first-order energy corrections for all states are zero. Thus, we need to compute the second-order energy corrections.

The axion state $|a\rangle$ is nondegenerate and the standard nondegenerate perturbation formula is

$$E_3^{(2)} = \sum_{n \neq 3} \frac{|\langle n|H'|3\rangle|^2}{E_3^{(0)} - E_n^{(0)}}. \quad (\text{B2})$$

Substituting the nonzero matrix elements $\langle 1|H'|3\rangle = i\Delta_{a\gamma}/\sqrt{2}$ and $\langle 2|H'|3\rangle = -i\Delta_{a\gamma}/\sqrt{2}$, we obtain

$$E_3^{(2)} = \frac{\Delta_{a\gamma}^2}{\Delta_a - \Delta_{\text{pl}}}. \quad (\text{B3})$$

The photon states $|R\rangle$ and $|L\rangle$ are degenerate under H_0 , so they cannot be treated with nondegenerate perturbation theory directly. We construct an effective Hamiltonian H_{eff} within this two-dimensional degenerate subspace to obtain the second-order corrections. The matrix elements of the

effective Hamiltonian are given by

$$H_{ij}^{\text{eff}} = \sum_{n \neq 1,2} \frac{\langle i|H'|n\rangle \langle n|H'|j\rangle}{E^{(0)} - E_n^{(0)}}, \quad i, j \in \{1, 2\}, \quad (\text{B4})$$

where $E^{(0)} = \Delta_{\text{pl}}$ is doubly degenerate. Taking $|a\rangle$ as the only intermediate state, the effective Hamiltonian simplifies to

$$H_{\text{eff}} = \frac{\Delta_{a\gamma}^2}{2(\Delta_{\text{pl}} - \Delta_a)} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}. \quad (\text{B5})$$

Diagonalizing H_{eff} , the second-order energy corrections within the degenerate subspace are

$$E_1^{(2)} = 0, \quad E_2^{(2)} = \frac{\Delta_{a\gamma}^2}{\Delta_{\text{pl}} - \Delta_a}. \quad (\text{B6})$$

The corresponding normalized eigenstates are

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|R\rangle + |L\rangle), \quad |\psi_2\rangle = \frac{1}{\sqrt{2}}(|R\rangle - |L\rangle). \quad (\text{B7})$$

With the eigenvalues and eigenstates of the weakly mixed limit now fully determined, we solve for the system's evolution. Consider an initial state with no axion component

$$|\psi_0\rangle = C_R|R\rangle + C_L|L\rangle. \quad (\text{B8})$$

Expanding the initial state in the eigenstate basis is

$$|\psi_0\rangle = \frac{C_R + C_L}{\sqrt{2}}|\psi_1\rangle + \frac{C_R - C_L}{\sqrt{2}}|\psi_2\rangle. \quad (\text{B9})$$

Since each eigenmode evolves independently in the eigenbasis, after propagating through distance z , the state vector becomes

$$|\psi(z)\rangle = \frac{C_R + C_L}{\sqrt{2}}e^{-iE_1z}|\psi_1\rangle + \frac{C_R - C_L}{\sqrt{2}}e^{-iE_2z}|\psi_2\rangle, \quad (\text{B10})$$

where $E_1 = \Delta_{\text{pl}}$ and $E_2 = \Delta_{\text{pl}} + \frac{\Delta_{a\gamma}^2}{\Delta_{\text{pl}} - \Delta_a}$. Substituting the eigenstates $|\psi_1\rangle$ and $|\psi_2\rangle$ and simplifying, the amplitudes of the right- and left-circularly polarization components are obtained as

$$R(z) = \frac{(C_R + C_L)e^{-iE_1z} + (C_R - C_L)e^{-iE_2z}}{2}, \quad (\text{B11})$$

$$L(z) = \frac{(C_R + C_L)e^{-iE_1z} - (C_R - C_L)e^{-iE_2z}}{2}. \quad (\text{B12})$$

Applying the definition of circular polarization degree from Eq. (12) to the case of purely right-handed circular polarized incidence, i.e. $C_R = 1$, $C_L = 0$, the circular polarization degree simplifies to

$$P_C(z) = \cos\left(\frac{\Delta_{ay}^2}{\Delta_{pl} - \Delta_a} z\right). \quad (\text{B13})$$

The above result indicates that, in the weak-mixing limit, the polarization degree of purely circularly polarized incident light undergoes simple harmonic oscillation with a characteristic frequency $\frac{\Delta_{ay}^2}{|\Delta_a - \Delta_{pl}|}$. This expression is consistent with the leading term of Eq. (21) in the main text in the limit $\theta \rightarrow 0$, providing an independent check of the general analytic solution derived in Sec. II.

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