Spotting stasis in cosmological perturbations

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As discussed in a number of recent papers, cosmological stasis is a phenomenon wherein the abundances of multiple cosmological energy components with different equations of state remain constant for an extended period despite the expansion of the universe. One of the most intriguing aspects of the stasis phenomenon is that it can give rise to cosmological epochs in which the effective equation-of-state parameter $\langle w \rangle$ for the universe is constant but differs from the canonical values associated with matter, radiation, vacuum energy, etc. Indeed, during such a stasis epoch, the spatial average of the energy density of the universe evolves in precisely the same manner as it would have evolved if the universe were instead dominated by a perfect fluid with an equation-of-state parameter equal to $\langle w \rangle$. However, as we shall demonstrate, this equivalence is broken at the level of the *perturbations* of the energy density. To illustrate this point, we consider a stasis epoch involving matter and radiation and demonstrate that within this stasis background the density perturbations associated with a spectator matter component with exceedingly small energy density exhibit a power-law growth that persists across the entire duration of the stasis epoch. This growth can potentially lead to significant enhancements of structure at small scales. Such enhancements are not only interesting in their own right but may also provide a way of observationally distinguishing between a stasis epoch and an epoch of perfect-fluid domination—even if the universe has the same equation of state in both cases.

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I. INTRODUCTION

It has recently been shown that many commonly studied extensions of the Standard Model give rise to cosmologies wherein the universe is driven by a dynamical interplay between particle-physics processes and cosmological expansion—an interplay which propels the universe toward a state in which the abundances Ω_i of multiple cosmological energy components with different equation-of-state parameters w_i lose their traditional time-dependent scaling behaviors and instead remain constant despite cosmological expansion. This state is known as cosmological stasis [1,2]. Such a stasis state emerges naturally in a variety of different BSM contexts [1–9]; nevertheless, all such realizations of stasis exhibit a common underlying structure [8]. Moreover, the modification of the cosmological expansion history associated with a stasis epoch can have observable impacts—for example, on the cosmic microwave background and on the stochastic gravitational-wave background [4].

One particularly intriguing aspect of the stasis phenomenon is that the effective equation-of-state parameter $\langle w \rangle \equiv \sum_i \Omega_i w_i$ for the universe as a whole can take a value which is constant and yet differs from the canonical values associated with matter ($w_i = 0$), radiation ($w_i = 1/3$), vacuum energy ($w_i = -1$), kination ($w_i = 1$), and so forth. Indeed, as far as the *global* properties of a flat Friedmann-Robertson-Walker (FRW) universe are concerned—properties such as the spatial average $\bar{\rho}$ of the total energy density or the Hubble parameter $H \equiv a^{-1}(da/dt)$—there is an equivalence between a stasis epoch involving many different cosmological energy components and an epoch dominated by a single perfect fluid with an equation-of-state parameter $w_{\rm PF}$ equal to $\langle w \rangle$.

Despite this fact, we shall demonstrate in this paper that this equivalence is broken at the level of spatial inhomogeneities. To do this, we shall consider a stasis cosmology.

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Within this cosmology, we shall then analyze the evolution of perturbations of not only the energy densities that are directly involved in the stasis dynamics but also the energy densities associated with any "spectator" components which couple only gravitationally to the stasis sector and whose energy densities are extremely small compared to the critical density during that epoch. As a concrete example, we shall consider the case in which the collective decays of a tower of massive particle species directly into massless final-state particles give rise to a matter/radiation stasis, and in which a spectator matter component is also present. We then demonstrate that once they enter the horizon, density perturbations for this spectator matter component exhibit a power-law growth that persists across the entire duration of the stasis epoch. This behavior stands in sharp contrast to that which such perturbations exhibit during an epoch of perfect-fluid domination, in which such perturbations experience only a short transient phase of growth and then plateau [10]. This power-law growth that emerges during stasis also differs from the linear growth that such perturbations exhibit [11] during an early matter-dominated era (EMDE). Nevertheless, in situations in which this spectator component plays the role of the dark matter, this enhanced growth of perturbations during stasis can potentially lead to an enhancement of structure at small scales. Such an enhancement could potentially provide observational handles which might allow one to distinguish a stasis epoch from an epoch of perfect-fluid domination in the early universe—even when the universe in each scenario has the same equation of state.

This paper is organized as follows. First, in Sec. II, we review the general formalism which describes the evolution of density perturbations in an FRW universe. Then, in Sec. III, we review how density perturbations associated with a spectator matter component evolve during an epoch of perfect-fluid domination. Finally, in Sec. IV, we investigate how such perturbations evolve during a matter/radiation stasis epoch and demonstrate that they experience power-law growth after entering the horizon. We conclude in Sec. VI with a summary of our results and a discussion of possible directions for future work.

II. PERTURBATION EVOLUTION

In general, the manner in which the spacetime metric $g_{\mu\nu}$ and the energy densities ρ_X of a set of cosmological energy components X behave as functions of the spacetime coordinates x^μ may be ascertained from the Einstein equation and the equations $\nabla_\mu (T_X)^\mu{}_\nu = (Q_X)_\nu$ which describe the transfer of stress-energy between cosmological components, where the source and sink terms $(Q_X)_\mu$ are subject to the stress-energy-conservation condition $\sum_X \nabla_\mu (T_X)^\mu{}_\nu = 0$, or equivalently $\sum_X (Q_X)_\nu = 0$. We focus on the case in which the X behave like perfect fluids, with

$$(T_X)^\mu{}_\nu = (\rho_X + p_X)(u_X)^\mu (u_X)_\nu + p_X \delta^\mu_\nu, \tag{2.1}$$

where p_X is the pressure associated with X, where $(u_X)^\mu$ is its contravariant four-velocity, and where δ^μ_ν denotes the Kronecker delta.

At the global level—i.e., neglecting inhomogeneities—we assume that the metric is that of an FRW universe with vanishing spatial curvature. Since we are primarily interested in how density perturbations evolve, we consider only scalar perturbations to $g^{\mu\nu}$. In the conformal Newtonian gauge, the most general form of the perturbed metric involving only perturbations of this sort is

$$ds^2 = -a^2(1+2\Psi)d\tau^2 + a^2(1+2\Phi)\delta_{ij}dx^i dx^j, \tag{2.2}$$

where $x^0 = \tau$ is the conformal time, where x^i with $i = \{1,2,3\}$ denote the three spatial coordinates, where a is the scale factor, where $\Psi(\tau,\vec{\mathbf{x}})$ is the Newtonian potential, and where $\Phi(\tau,\vec{\mathbf{x}})$ is the spatial curvature. In the absence of anisotropic stress, $\Psi = -\Phi$.

We decompose the energy density $\rho_X(\tau,\vec{\mathbf{x}}) = \bar{\rho}_X(\tau) + \Delta\rho_X(\tau,\vec{\mathbf{x}})$ of each cosmological component in terms of its spatial average $\bar{\rho}_X(\tau)$ and a position-dependent perturbation $\Delta\rho_X$. The pressure $p_X(\tau,\vec{\mathbf{x}}) = \bar{p}_X(\tau) + \Delta p_X(\tau,\vec{\mathbf{x}})$ and each spatial component $(u_X)^i(\tau,\vec{\mathbf{x}}) = (\bar{u}_X)^i(\tau) + (v_X)^i(\tau,\vec{\mathbf{x}})$ of $(u_X)^\mu$ may be decomposed in an analogous manner. Given that the universe is observed to be isotropic on large distance scales, we take $(\bar{u}_X)^\mu = 0$ for all X. We define the abundance Ω_X and equation-of-state parameter w_X for each X via the relations $\Omega_X \equiv \bar{\rho}_X/\rho_{\rm crit}$ and $\bar{p}_X = w_X \bar{\rho}_X$, respectively, where $\rho_{\rm crit} \equiv 3H^2/(8\pi G)$ denotes the critical density. We also define the fractional overdensity (or "density contrast") $\delta_X \equiv \Delta\rho_X/\bar{\rho}_X$ as well as the divergence $\theta_X \equiv \partial_i (v_X)^i$ of the three-velocity for each X.

In the regime wherein the perturbations are all sufficiently small, we may expand the Einstein and stress-energy-transfer equations to linear order in these perturbations in order to obtain equations of motion for δ_X, the $\tilde{\theta}_X$, and Φ—or, equivalently, for their Fourier transforms δ_{kX}, $\tilde{\theta}_{kX}$, and Φ_k, where k denotes the comoving wave number of the Fourier mode. At zeroth order in this expansion, the Einstein equation yields the Friedmann equations while the stress-energy-transfer equation for each X yields an equation of motion

$$\dot{\bar{\rho}}_X = -3\frac{\dot{a}}{a}(1+w_X)\bar{\rho}_X - (\bar{Q}_X)_0 \tag{2.3}$$

for $\bar{\rho}_X$, where a dot denotes a derivative with respect to τ, and where $(\bar{Q}_X)_\mu$ denotes the expression for $(Q_X)_\mu$ evaluated at zeroth order in the perturbations.

At first order in the perturbations, the $\mu = i$, $\nu = j$, and $\mu = \nu = 0$ components of the Einstein equation respectively yield the relations

$$4\pi G a^2 \sum_X \delta_X \bar{\rho}_X c_{sX}^2 = -\ddot{\Phi} - 3\frac{\dot{a}}{a}\dot{\Phi} - \left(2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right)\Phi$$
$$4\pi G a^2 \sum_X \delta_X \bar{\rho}_X = 3\frac{\dot{a}}{a}\dot{\Phi} + 3\frac{\dot{a}^2}{a^2}\Phi - \nabla^2\Phi, \tag{2.4}$$

where $c_{sX} \equiv \sqrt{\Delta p_X/\Delta\rho_X}$ is the effective sound speed of X. In general, this c_{sX} differs from the adiabatic sound speed. However, we note that $c_{sX} \approx 0$ for a highly nonrelativistic matter component, whereas for radiation we have $c_{s\gamma} \approx 1/\sqrt{3}$. Indeed, the effective sound speed for a population of particles is related to their underlying phase-space distribution $f(\vec{\mathbf{x}}, \vec{\mathbf{p}})$ by

$$c_{sX}^2 = \frac{\Delta P_X}{\Delta\rho_X} = \frac{\int d^3p \Delta f_X(\vec{\mathbf{x}}, \vec{\mathbf{p}}) \frac{|\vec{\mathbf{p}}|^2}{3E_p}}{\int d^3p \Delta f_X(\vec{\mathbf{x}}, \vec{\mathbf{p}}) E_p} \tag{2.5}$$

where E_p is the energy of a particle of species X with momentum $\vec{\mathbf{p}}$ and where $\Delta f_X(\vec{\mathbf{x}}, \vec{\mathbf{p}}) \equiv f_X(\vec{\mathbf{x}}, \vec{\mathbf{p}}) - \bar{f}_X(\vec{\mathbf{p}})$ represents the position-dependent perturbation in $f_X(\vec{\mathbf{x}}, \vec{\mathbf{p}})$ away from its spatial average $\bar{f}_X(\vec{\mathbf{p}})$. Within the regime wherein $f_X(\vec{\mathbf{x}}, \vec{\mathbf{p}})$ and therefore $\Delta f_X(\vec{\mathbf{x}}, \vec{\mathbf{p}})$ is highly nonrelativistic, the above expression reduces to $c_s^2 \approx \langle v^2\rangle_{\Delta f_X}/3 \approx 0$, where $\langle v^2\rangle_{\Delta f_X}$ denotes the expectation value of the speed v of the particles which constitute the perturbation $\Delta f_X(\vec{\mathbf{x}}, \vec{\mathbf{p}})$ at a given location $\vec{\mathbf{x}}$. Likewise, if $f_X(\vec{\mathbf{x}}, \vec{\mathbf{p}})$ is highly relativistic, then $E_p \approx |\vec{\mathbf{p}}|$ across the population of particles and Eq. (2.5) reduces to $c_s^2 \approx 1/3$.

The stress-energy-transfer equation for each X yields equations of motion for δ_X and θ_X of the form

$$\dot{\delta}_X = -3\dot{\Phi}(1+w_X) - (1+w_X)\theta_X + \frac{1}{\bar{\rho}_X}[(\bar{Q}_X)_0\delta_X - (\Delta Q_X)_0]$$
$$\dot{\theta}_X = -\frac{\dot{a}}{a}(1-3w_X)\theta_X - \frac{w_X\nabla^2\delta_X}{1+w_X} + \nabla^2\Phi + \frac{1}{\bar{\rho}_X}\left[\frac{\delta^{ij}\partial_j(\Delta Q_X)_i}{1+w_X} + (\bar{Q}_X)_0\theta_X\right], \tag{2.6}$$

where $(\Delta Q_X)_\mu$ represents the contribution to $(Q_X)_\mu$ which arises at first order in the perturbations.

We also note that the two equations in Eq. (2.4) can be combined to yield a single second-order differential equation for Φ, the Fourier transform of which is

$$\Phi_k'' + \frac{7+3\langle w\rangle}{2a}\Phi_k' + \frac{\langle w\rangle k^2}{a^4H^2}\Phi_k = \frac{3}{2a^2}\sum_X \Omega_X\delta_{kX}(\langle w\rangle - c_{sX}^2), \tag{2.7}$$

where a prime denotes a derivative with respect to a and where $\langle w\rangle \equiv \sum_X \Omega_X w_X$. The right side of this equation can be interpreted as a source for the gravitational potential. As we shall see, this source term plays a crucial role in the evolution of density perturbations during stasis.

III. THE PERFECT-FLUID CASE

As a warm-up, let us first consider the simple case of an epoch during which the energy density of the universe is dominated by a single perfect fluid with a constant equation-of-state parameter $w_{\rm PF}$ within the range $0 < w_{\rm PF} < 1/3$. For convenience in what follows, we shall work primarily in terms of dimensionless variables obtained by scaling the corresponding dimensionful quantities by an appropriate power of the value $H^{(0)}$ at some initial time $t^{(0)}$. In this way, we define a dimensionless Hubble parameter $E \equiv H/H^{(0)}$, a dimensionless comoving wave number $\tilde{k} \equiv k/H^{(0)}$, and a dimensionless velocity gradient $\tilde{\theta}_X \equiv \theta_X/H^{(0)}$. We also adopt the convention that $a_0 \equiv a(t^{(0)}) = 1$ at this initial time.

During an epoch of perfect-fluid domination, there is effectively only one term in the sum over X in Eq. (2.7). Likewise, we have $\langle w\rangle = w_{\rm PF}$. Thus, since $c_{s{\rm PF}}^2 = w_{\rm PF}$ for such a fluid, the right side of this equation vanishes, yielding a homogeneous equation for Φ_k which is independent of the other perturbations. Accounting for boundary conditions at early times, we then find that the solution to Eq. (2.7) is given by [10]

$$\Phi_k = \Phi_{k0}\Gamma(\eta+1)\left(\frac{a^{\frac{3w_{\rm PF}+1}{2}}\tilde{k}w_{\rm PF}^{1/2}}{1+3w_{\rm PF}}\right)^{-\eta} \times J_\eta\left(\frac{2a^{\frac{3w_{\rm PF}+1}{2}}\tilde{k}w_{\rm PF}^{1/2}}{1+3w_{\rm PF}}\right) \tag{3.1}$$

where $\Gamma(x)$ denotes the Euler gamma function of x, where $J_\alpha(x)$ denotes the Bessel function of the first kind, and where $\eta \equiv (5+3w_{\rm PF})/(2+6w_{\rm PF})$.

We can also investigate how a density perturbation for a spectator matter component χ with $w_\chi = 0$ evolves in the presence of such a perfect fluid. The corresponding equations of motion for $\delta_{k\chi}$ and $\tilde{\theta}_{k\chi}$ are given by Eq. (2.6) and in this case can be written in the form

$$a^2E\delta_{k\chi}' = -3a^2E\Phi_k' - \tilde{\theta}_{k\chi}$$
$$a^2E\tilde{\theta}_{k\chi}' = -aE\tilde{\theta}_{k\chi} - \tilde{k}^2\Phi_k. \tag{3.2}$$

These equations can be combined into a single second-order equation for $\delta_{k\chi}$. Since $E = a^{-3(1+\langle w\rangle)/2}$ during any epoch wherein $\langle w\rangle$ is constant, this equation may be expressed as

$$\delta_{k\chi}'' + \frac{3}{2a}(1-\langle w\rangle)\delta_{k\chi}' = S_{k\chi} \tag{3.3}$$

where we have defined

$$S_{k\chi} \equiv -3\Phi_k'' - \frac{9}{2a}(1-\langle w\rangle)\Phi_k' + \tilde{k}^2 a^{3\langle w\rangle-1}\Phi_k. \quad (3.4)$$

The full solution to this inhomogeneous linear equation may be obtained via the method of Green's functions. In particular, applying appropriate boundary conditions for $\delta_{k\chi}$ at early times and making the additional assumption that the primordial perturbations are purely adiabatic, we find

$$\delta_{k\chi} = \frac{2\Phi_{k0}}{1+\langle w\rangle} + \int_0^a db\, G_{k\chi}(a,b) S_{k\chi}(b), \quad (3.5)$$

where Φ_{k0} denotes the value of Φ_k at very early times, well before horizon entry, and where the Green's function $G_{k\chi}(a,b)$ is given for all $0 < \langle w\rangle < 1/3$ by

$$G_{ks}(a,b) = \frac{2b}{1-3\langle w\rangle}\left[1 - \left(\frac{b}{a}\right)^{(1-3\langle w\rangle)/2}\right]. \quad (3.6)$$

In Fig. 1, we show how $\delta_{k\chi}$ evolves as a function of a for a mode with $\tilde{k} = 5\times 10^{-2}$ during an epoch of perfect-fluid domination for a variety of different choices of $w_{\rm PF}$. For all $0 < w_{\rm PF} < 1/3$, the spectator density contrast undergoes a transient period of growth shortly after horizon entry and then plateaus, as discussed in Ref. [11]. However, we note that the transient period of growth which $\delta_{k\chi}$ experiences becomes longer and longer as $w_{\rm PF}$ decreases. In the $w_{\rm PF} \to 0$ limit, this period of growth becomes infinite, and we recover the behavior that we would expect for the density contrast of a matter component during a matter-dominated epoch—namely, that $\delta_{k\chi}$ increases linearly with a for all a.

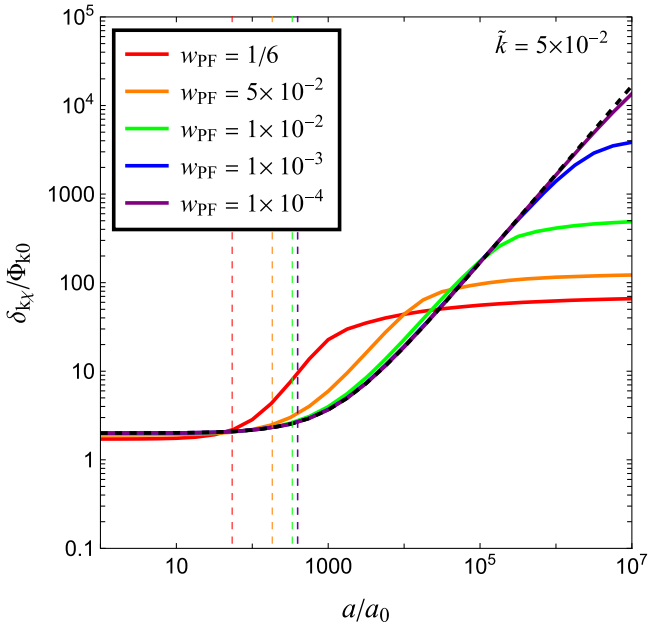

FIG. 1. Density contrast $\delta_{k\chi}$ for a spectator matter field ($w_\chi = 0$), normalized to the value of Φ_{k0} and plotted as a function of the normalized scale factor a/a_0 within a cosmology in which a perfect fluid with a constant equation-of-state parameter $w_{\rm PF}$ dominates the energy density of the universe. The solid curves of different colors correspond to different values of $w_{\rm PF}$, while the black dashed curve corresponds to the case in which $w_{\rm PF} = 0$. For each nonzero $w_{\rm PF}$, the vertical dashed line of the corresponding color indicates the value of a/a_0 at horizon entry.

IV. GROWTH THROUGH STASIS

We now proceed to our primary goal, that of calculating how perturbations in the energy density of a spectator matter component χ evolve during a stasis epoch. The primary new feature in this analysis, relative to the analysis presented in Sec. III, turns out to be the appearance of a nonzero, stasis-induced source term in the equation of motion for Φ_k in Eq. (2.7). As we shall see, this source term leads to a number of surprising and important consequences.

For concreteness, we shall consider a matter/radiation stasis involving a tower of N massive particle species ϕ_ℓ, where the index $\ell = 0, 1, \ldots, N-1$ labels the species in order of increasing mass. As in Refs. [1,2], we consider a mass spectrum for the ϕ_ℓ of the form $m_\ell = m_0 + \ell^\delta \Delta m$, where m_0, Δm, and δ are free parameters. Each ϕ_ℓ is unstable, with a nonzero decay width Γ_ℓ, and is presumed to decay directly into a final state comprising exceedingly light or massless particles γ which behave like radiation, with an equation-of-state parameter $w_\gamma = 1/3$. We take the decay widths Γ_ℓ and the initial abundances $\Omega_\ell(t_I)$ at some early time t_I significantly smaller than the proper lifetime of the most unstable of the ϕ_ℓ to scale across the tower according to power laws of the form

$$\Gamma_\ell = \Gamma_0\left(\frac{m_\ell}{m_0}\right)^\gamma, \qquad \Omega_\ell(t_I) = \Omega_0\left(\frac{m_\ell}{m_0}\right)^\alpha, \quad (4.1)$$

where $\Omega_0(t_I)$ and Γ_0 respectively represent the initial abundance and decay width of the lightest tower mode and where α and γ are scaling exponents. Indeed, as discussed in Refs. [1,2], scaling relations of this sort emerge naturally in many commonly studied models of BSM physics. Moreover, we shall assume that the cosmological population of each ϕ_ℓ is already highly nonrelativistic, with an equation-of-state parameter $w_\ell \approx 0$, by the time the particles of any of these species begin decaying appreciably. Finally, in addition to the matter fields ϕ_ℓ and the radiation component, we shall assume that our cosmology includes an additional spectator matter field χ with $w_\chi = 0$ and an abundance Ω_χ which is negligible throughout the stasis epoch. This field will be assumed to couple to the stasis sector—i.e., to the ϕ_ℓ fields and the relativistic particles into which they decay—only through gravity.

Given this cosmological setup, the sink term $(Q_\ell)_\mu$ for each ϕ_ℓ within Eq. (2.3) takes the canonical form $(Q_\ell)_\mu = \Gamma_\ell (T_\ell)_{\nu\mu} (u_\ell)^\nu$ associated with particle decay, while $(Q_\chi)_\mu = 0$. It therefore follows from stress-energy conservation that $(Q_\gamma)_\mu = -\sum_{\ell=0}^{N-1} (Q_\ell)_\mu$. At zeroth order in the perturbations, Eq. (2.3) then yields

$$\begin{aligned}
\bar{\rho}'_\ell &= -\frac{3}{a}\bar{\rho}_\ell - \frac{\tilde{\Gamma}_\ell}{aE}\bar{\rho}_\ell \\
\bar{\rho}'_\gamma &= -\frac{4}{a}\bar{\rho}_\gamma + \frac{1}{aE}\sum_\ell \tilde{\Gamma}_\ell \bar{\rho}_\ell \\
\bar{\rho}'_\chi &= -\frac{3}{a}\bar{\rho}_\chi,
\end{aligned} \tag{4.2}$$

where we have defined $\tilde{\Gamma}_\ell \equiv \Gamma_\ell / H^{(0)}$. As discussed in Refs. [1,2], an FRW universe with energy densities governed by such equations is dynamically driven toward a stasis in which the total matter abundance $\Omega_M \equiv \sum_{\ell=0}^{N-1} \Omega_\ell$ and the radiation abundance Ω_γ take fixed values $\bar{\Omega}_M$ and $\bar{\Omega}_\gamma \approx 1 - \bar{\Omega}_M$ which depend on the values of the model parameters α, γ, and δ. During such a stasis, we have $\langle w \rangle = \bar{\Omega}_\gamma / 3$.

The equations of motion for the spectator perturbations $\delta_{k\chi}$ and $\tilde{\theta}_{k\chi}$ in this stasis scenario are given by Eq. (3.2), just as they are in the perfect-fluid-dominated scenario we discussed in Sec. III. Given the results in Eq. (2.6), the corresponding equations of motion for the perturbations associated with each ϕ_ℓ and with the radiation component γ are given by

$$\begin{aligned}
a^2 E \delta'_{k\ell} &= -3a^2 E \Phi'_k - \tilde{\theta}_{k\ell} + a\tilde{\Gamma}_\ell \Phi_k \\
a^2 E \tilde{\theta}'_{k\ell} &= -aE\tilde{\theta}_{k\ell} - \tilde{k}^2 \Phi_k \\
a^2 E \delta'_{k\gamma} &= -4a^2 E \Phi'_k - \frac{4}{3}\tilde{\theta}_{k\gamma} + a\sum_{\ell=0}^{N-1} \tilde{\Gamma}_\ell \frac{\tilde{\bar{\rho}}_\ell}{\tilde{\bar{\rho}}_\gamma}(\delta_{k\ell} - \delta_{k\gamma} - \Phi_k) \\
a^2 E \tilde{\theta}'_{k\gamma} &= \frac{1}{4}\tilde{k}^2 \delta_{k\gamma} - \tilde{k}^2 \Phi_k + a\sum_{\ell=0}^{N-1} \tilde{\Gamma}_\ell \frac{\tilde{\bar{\rho}}_\ell}{\tilde{\bar{\rho}}_\gamma}\left(\frac{3}{4}\tilde{\theta}_{k\ell} - \tilde{\theta}_{k\gamma}\right).
\end{aligned} \tag{4.3}$$

A first-order equation of motion for Φ_k may also be obtained from the second equation in Eq. (2.4):

$$\begin{aligned}
a^3 E^2 \Phi'_k = &-\frac{1}{3}\tilde{k}^2 \Phi_k - a^2 E^2 \Phi_k \\
&+ \frac{1}{2} a^2 E^2 \left(\Omega_\gamma \delta_{k\gamma} + \sum_{\ell=0}^{N-1} \Omega_\ell \delta_{k\ell}\right).
\end{aligned} \tag{4.4}$$

In order to obtain an expression for $\delta_{k\chi}$ as a function of a, we numerically integrate the coupled system of equations for Φ_k, $\delta_{k\gamma}$, $\tilde{\theta}_{k\gamma}$, the $\delta_{k\ell}$, and the $\tilde{\theta}_{k\ell}$ in Eq. (4.3). Initial conditions for our perturbations at the beginning of the stasis epoch are established by starting our simulation during a matter-dominated epoch at a time $t_I \ll \Gamma_{N-1}^{-1}$ during which the ϕ_ℓ collectively dominate the energy density for the universe and have not decayed appreciably. Under the assumption that the primordial perturbations are adiabatic, these initial conditions are the same as those discussed in Ref. [11] for the corresponding cosmological components in an EMDE involving a single decaying matter field, with all of the $\delta_{k\ell}$ given by identical expressions equal to the expression for the initial density contrast of that decaying field. We then time-evolve our system toward stasis. In order to establish a meaningful comparison between the results we obtain in this way and the results for perfect-fluid domination, we choose a time at which the universe has already entered stasis as the reference time $t^{(0)}$ at which we take $a = a_0$. In what follows, we shall only consider modes with $\tilde{k} \leq 1$, which enter the horizon after the stasis epoch has begun.

In Fig. 2, we plot $\delta_{k\chi}$ as a function of a/a_0 for different values of $\tilde{k}$ (solid curves) during a stasis epoch in which $\langle w \rangle = 1/6$. We observe that once each perturbation enters the horizon, it begins to grow with a. However, unlike during an EMDE, wherein $\delta_{k\chi}$ grows *linearly* with a after horizon entry [11], we find that during stasis the spectator density contrast is instead described by a power law of the form $\delta_{k\chi} \sim a^{q(\langle w \rangle)}$, where $0 < q(\langle w \rangle) < 1$. Indeed, for the particular choice of $\langle w \rangle$ shown in Fig. 2, we have $\delta_{k\chi} \sim a^{3/4}$.

Moreover, the results shown in Fig. 2 illustrate not only how $\delta_{k\chi}$ evolves during stasis, but also how it evolves at the end of the stasis epoch, as the universe transitions from this epoch to the radiation-dominated epoch that follows. Indeed, a matter/radiation stasis of this sort ends at a time $t \sim \Gamma_0^{-1}$, when the longest-lived of the ϕ_ℓ decays into radiation and Ω_M becomes negligible. Moreover, as discussed in Ref. [1], discretization effects which arise when only a small number of ϕ_ℓ with lifetimes $\Gamma_\ell^{-1} \gtrsim t$ remain can cause the value of $\langle w \rangle$ to fluctuate around $\bar{w}$ during the last few e-folds of the stasis epoch. However, the results in Fig. 2 illustrate that $\delta_{k\chi}$ evolves smoothly from scaling like $\delta_{k\chi} \sim a^{q(\langle w \rangle)}$ to scaling like $\delta \sim \log a$ as the universe transitions from the stasis epoch to the subsequent radiation-dominated epoch without exhibiting similar fluctuations.

The growth of $\delta_{k\chi}$ during stasis is ultimately a consequence of the source term on the right side Eq. (2.7). During an epoch of perfect-fluid domination, only a single cosmological component has a non-negligible abundance, whereupon we have seen that this source term vanishes. By contrast, during stasis, *multiple* cosmological components with different equation-of-state parameters have non-negligible Ω_X. Moreover, the δ_{kX} for these components evolve differently with time. As a result, the source term in Eq. (2.7) generically does *not* vanish during stasis.

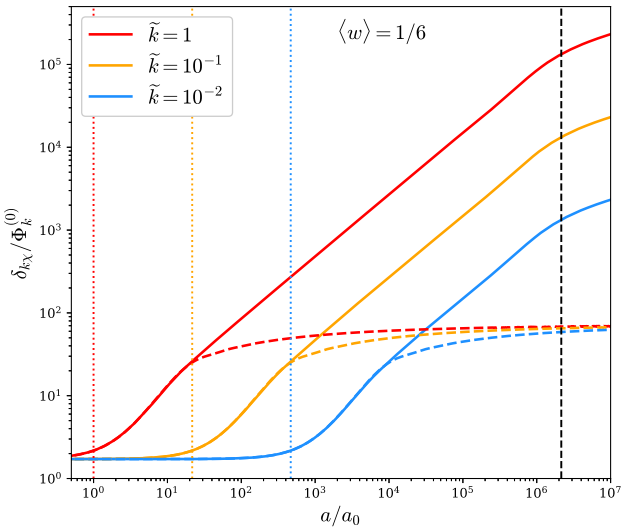

FIG. 2. Density contrast $\delta_{k\chi}$, plotted as a function of a/a_0 for a spectator matter field during a matter/radiation stasis epoch with $\langle w\rangle = 1/6$ (solid curves) as well as during an epoch dominated by a perfect fluid with $w_{\rm PF} = 1/6$ (dashed curves of the corresponding color). Curves are shown for three different dimensionless wave numbers $\tilde{k}$ (different colors). In each case, the dotted vertical line indicates the time at which the corresponding mode enters the horizon, while the dashed black vertical line indicates the value of a/a_0 at which the stasis epoch ends and the subsequent epoch of radiation domination begins in the stasis cosmology. We observe that after a given mode enters the horizon, its density perturbation in all cases initially experiences a transient period of growth. However, in the case of a stasis epoch, $\delta_{k\chi}$ subsequently experiences sustained power-law growth as long as the stasis persists. By contrast, for the case of an epoch of perfect-fluid domination, $\delta_{k\chi}$ subsequently plateaus to an asymptotic value which is independent of $\tilde{k}$. These results provide a dramatic illustration of the differences between stasis and perfect-fluid domination which arise at the perturbation level.

We may analytically determine the manner in which the power-law index $q(\langle w\rangle)$ depends on $\langle w\rangle$ from the fundamental equations of motion in Eqs. (2.7), (4.3), and (4.4). We begin by noting that the ϕ_ℓ which contribute significantly to Ω_M at each moment are those with dimensionless decay widths $\tilde{\Gamma}_\ell \lesssim E$, whose energy densities have not yet been impacted significantly by particle decay. For all such ϕ_ℓ, the term proportional to $\tilde{\Gamma}_\ell$ in the first equation in Eq. (4.3) is small in comparison to the other terms and can to a reasonable approximation be neglected. Since this is the only term in this equation which depends meaningfully on ℓ, it follows that the equations of motion for $\delta_{k\ell}$ and $\tilde{\theta}_{k\ell}$ for all such ϕ_ℓ are identical. Moreover, they are also identical in form to the corresponding equations of motion for the spectator perturbations $\delta_{k\chi}$ and $\tilde{\theta}_{k\chi}$. This implies that each $\delta_{k\ell}$ obeys a second-order differential equation whose form is identical to that of Eq. (3.3). Thus, for these ϕ_ℓ, we have

$$\delta''_{k\ell} + \frac{3}{2a}(1-\langle w\rangle)\delta'_{k\ell} \approx -3\Phi''_k - \frac{9}{2a}(1-\langle w\rangle)\Phi'_k + \tilde{k}^2 a^{3\langle w\rangle-1}\Phi_k. \tag{4.5}$$

We also note that the contributions to Ω_M from ϕ_ℓ with decay widths in the opposite regime—the regime in which $\tilde{\Gamma}_\ell \gtrsim E$—are highly suppressed. Thus, at any given moment, the ϕ_ℓ which contribute meaningfully to the sums over ℓ in both Eq. (2.7) and the last equation in Eq. (4.3) are those for which $\tilde{\Gamma}_\ell \lesssim E$, whose $\delta_{k\ell}$ are essentially independent of ℓ. This implies that during stasis, we may pull $\delta_{k\ell}$ outside of each of these sums and approximate

$$\sum_{\ell=0}^{N-1} \delta_{k\ell}\Omega_\ell \approx \delta_{k\ell}\bar{\Omega}_M \approx (1-3\langle w\rangle)\delta_{k\ell}. \tag{4.6}$$

With this simplification, Eqs. (2.7), (4.4), and (4.5) can be combined into a single fourth-order differential equation for Φ_k. Within the regime in which $\tilde{k} \gg aE$ and in which the perturbation mode is well within the horizon, this equation reduces to

$$0 \approx \left(\frac{\tilde{k}}{aE}\right)^{-2}[a^4\Phi''''_k + (10-3\langle w\rangle)a^3\Phi'''_k] + \frac{1}{3}a^2\Phi''_k + \frac{9\langle w\rangle+7}{6}a\Phi'_k + \frac{3\langle w\rangle^2+7\langle w\rangle}{2}\Phi_k. \tag{4.7}$$

As a increases, the higher-order terms become increasingly suppressed by the overall prefactor $(\tilde{k}/aE)^{-2} = \tilde{k}^{-2}a^{-(3\langle w\rangle+1)}$. This fourth-order differential equation for Φ_k then effectively reduces to a second-order equation. For $0 < \langle w\rangle < 1/3$, the two linearly independent solutions $\Phi_k^{(+)}$ and $\Phi_k^{(-)}$ to this second-order equation are given by $\Phi_k^{(\pm)} = a^{\xi_\pm(\langle w\rangle)}$, where we have defined

$$\xi_\pm(w) \equiv \frac{1}{4}[-9w-5 \pm (9w^2-78w+25)^{1/2}]. \tag{4.8}$$

At late times, when a is large, the $\Phi_k^{(+)}$ solution dominates over $\Phi_k^{(-)}$. Thus, up to an overall normalization constant, we find $\Phi_k \sim \Phi_k^{(+)}$. Indeed, inserting this expression for Φ_k into $S_{k\chi}$ and evaluating the integral in Eq. (3.5), we find that the spectator density contrast at late times scales with a according to a power law of the form $\delta_{k\chi} \sim a^{q(\langle w\rangle)}$, where

$$q(w) \equiv \frac{1}{4}[3w-1 \pm (9w^2-78w+25)^{1/2}]. \tag{4.9}$$

In Fig. 3, we plot $\delta_{k\chi}$ as a function of a/a_0 for a mode with $\tilde{k} = 0.1$ during stasis epochs with different values of $\langle w\rangle$. We observe that for large a, the spectator density

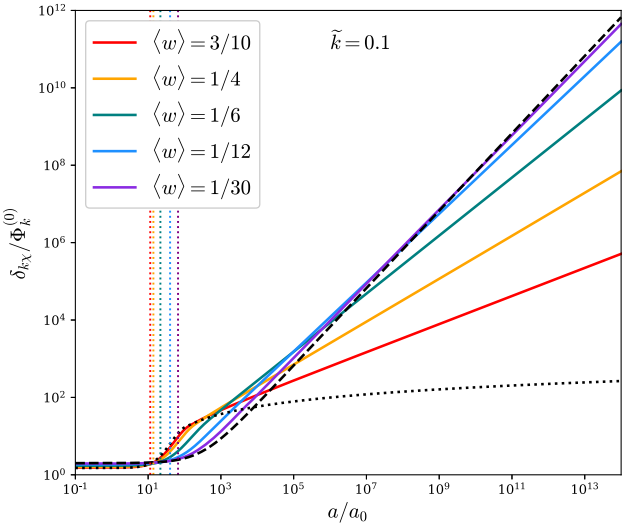

FIG. 3. Density contrast $\delta_{k\chi}$ for a spectator matter field during stasis epochs with several different values of $\langle w\rangle$ (solid colored curves), plotted as a function of a/a_0 for a perturbation with $\tilde{k} = 0.1$. The vertical line of the corresponding color indicates when the mode enters the horizon for that choice of $\langle w\rangle$. The dashed and dotted black curves indicate the results for matter domination ($\langle w\rangle = 0$) and radiation domination ($\langle w\rangle = 1/3$), respectively. We observe that perturbations which enter the horizon during stasis exhibit power-law growth at large a, with a power-law exponent which depends on $\langle w\rangle$ in the manner indicated in Eq. (4.9). We observe that for $\langle w\rangle$ within the range $0 < \langle w\rangle < 1/3$, the behavior of $\delta_{k\chi}$ interpolates between the linear and logarithmic growth which perturbations in matter components exhibit during matter and radiation domination, respectively.

contrast indeed grows with a according to a power law with an exponent given by Eq. (4.9). As such, for $\langle w\rangle$ within the range $0 < \langle w\rangle < 1/3$, the behavior of $\delta_{k\chi}$ interpolates between two extremes: the linear growth expected for matter perturbations during a matter-dominated epoch ($\langle w\rangle = 0$), and the logarithmic growth which such perturbations exhibit during a radiation-dominated epoch ($\langle w\rangle = 1/3$). Of course, since our initial conditions imply that $\delta_{k\chi} \approx 2\Phi_k^{(0)}/(1 + \langle w\rangle)$ for modes well outside the horizon at the beginning of the stasis epoch, these curves do not coincide at $a/a_0 = 1$.

It has already been noted in previous works that the matter/radiation stasis phenomenon allows the effective equation-of-state parameter $\langle w\rangle$ of the universe to assume nontraditional values within the range $0 < \langle w\rangle < 1/3$, with the energy density of the universe evolving accordingly. However, we now find that a similar result also holds for the manner in which the *perturbations* to the energy density evolve. Thus, matter/radiation stasis represents a middle ground between matter and radiation domination, not only for the evolution of the universe as a whole but also for the evolution of its density perturbations. This novel scaling behavior of $\delta_{k\chi}$ with a can potentially have an impact on small-scale structure, with features that do not arise in either of these standard epochs. However, as we have also seen, the growth of these density perturbations is wholly unlike that which would emerge for a universe whose energy content is dominated by the energy associated with a single perfect fluid, even if we could somehow arrange $w_{\rm PF} = \langle w\rangle$. Indeed, during a stasis epoch, the power-law growth persists throughout the duration of the stasis. By contrast, during an epoch dominated by a perfect fluid, the initial growth is only transient, with the density perturbations $\delta_{k\chi}$ quickly reaching a plateau which is independent of the wave number $\tilde{k}$.

V. EFFECT ON THE MATTER POWER SPECTRUM

Given that $\delta_{k\chi}$ evolves differently during stasis than it does in the standard cosmology, it is interesting to consider the effect that this has on the linear matter power spectrum $P(k,t)$ in the case in which our spectator component χ plays the role of the dark matter. In general, $P(k,t)$ is defined as the Fourier transform of the two-point correlation function $\xi(\vec{\mathbf{y}},t) = \langle\delta_M(\vec{\mathbf{x}},t)\delta_M(\vec{\mathbf{x}}+\vec{\mathbf{y}},t)\rangle$, where $\delta_M(\vec{\mathbf{x}},t) \equiv [\rho_M(\vec{\mathbf{x}},t) - \bar{\rho}_M(t)]/\bar{\rho}_M(t)$ denotes the total perturbation in the density of matter, including contributions from all nonrelativistic species. This implies that up to an overall large-volume factor,

$$P(k,t) = |\delta_{Mk}(t)|^2, \tag{5.1}$$

where $\delta_{Mk}(t)$ is the Fourier transform of $\delta_M(\vec{\mathbf{x}},t)$. We may also define a corresponding dimensionless power spectrum according to the usual convention

$$\Delta^2(k,t) \equiv \frac{k^3}{2\pi^2}P(k,t). \tag{5.2}$$

Under the assumption that the spectator ϕ_s plays the role of the dark matter, we may establish a relationship between Φ_k and $\delta_{k\chi}$ at late times through the second equation in Eq. (2.4), the Fourier transform of which may be expressed as

$$\left[k^2 + 3(aH)^2\left(1 + \frac{a\Phi_k'}{\Phi_k}\right)\right]\Phi_k = 4\pi G a^2\sum_X \bar{\rho}_X\delta_{Xk}. \tag{5.3}$$

We first consider how this equation simplifies at times $t_{\rm MRE} < t < t_{\rm DE}$, where $t_{\rm MRE}$ denotes the time at matter-radiation equality and $t_{\rm DE}$ is the time at which the universe transitions from matter domination to dark-energy domination. Since the universe is matter dominated while t lies within this range, we can neglect the abundances of all species excepting those which behave like matter. Moreover, since we are also assuming that our perturbations are initially adiabatic, and therefore that the δ_{Xk} for

baryonic and dark matter, which both have $w_X \approx 0$, are approximately equal, we have

$$\sum_X \bar{\rho}_X \delta_{Xk} \approx \bar{\rho}_M \delta_{Mk}, \tag{5.4}$$

where $\bar{\rho}_M$ is the spatial average of the total matter energy density.

We are primarily interested in perturbation modes δ_{Mk} which have already entered the horizon well before the range of t we are considering—i.e., modes for which $k \gg aH$ throughout this range of t. For such modes, the second term in square brackets in Eq. (5.3) is negligible, and we obtain

$$k^2 \Phi_k = 4\pi G a^2 \bar{\rho}_M \delta_{Mk}, \tag{5.5}$$

which is simply the Fourier transform of the Poisson equation. Since the energy density of matter at such times can be written as

$$\bar{\rho}_M = \frac{3H_{\rm now}^2}{8\pi G} \Omega_{M,\rm now} \left(\frac{a}{a_{\rm now}}\right)^{-3}, \tag{5.6}$$

Eq. (5.5) implies that

$$\delta_{Mk} = \frac{2ak^2 \Phi_k}{3(aH_{\rm now})^2 \Omega_{M,\rm now}}. \tag{5.7}$$

Thus, in order to determine the matter power spectrum $P(k,t)$ at times within this range, we simply need to know the corresponding gravitational potential Φ_k.

The manner in which the value of Φ_k depends on k and t—or, equivalently, on k and a—for times in this range is typically parametrized as

$$\Phi_k = \frac{9}{10} \Phi_k^{(p)} T(k) \frac{\mathcal{D}(a)}{a}, \tag{5.8}$$

where $\Phi_k^{(p)}$ is the "primordial" value that Φ_k would have had in the standard cosmology immediately after the reheating epoch following inflation had concluded and the universe had become radiation dominated. The transfer function $T(k)$ parametrizes whatever additional k-dependence Φ_k on k develops beyond whatever dependence was already present in $\Phi_k^{(p)}$, while the growth $\mathcal{D}(a)$ parametrizes the dependence of Φ_k on a.

The numerical prefactor in Eq. (5.8) accounts for the fact that the value of Φ_k for modes with $k \lesssim aH$ experiences a shift in its otherwise constant value during transitions between cosmological epochs with different values of $\langle w \rangle$. This shift can be determined from the relationship between Φ_k and the corresponding comoving curvature perturbation $\mathcal{R}_k$, which is conserved while $k \lesssim aH$. In particular, prior to horizon entry, these quantities are related by

$$\mathcal{R}_k = \frac{5 + 3\langle w \rangle}{3 + 3\langle w \rangle} \Phi_k. \tag{5.9}$$

Since the primordial $\Phi_k^{(p)}$ is defined during a radiation-dominated epoch, it then follows that modes which enter the horizon during a matter-dominated epoch have $\Phi_k = (9/10)\Phi_k^{(p)}$. Thus, our choice of numerical prefactor in Eq. (5.8) ensures that modes which enter the horizon at times $t_{\rm MRE} < t < t_{\rm DE}$ have $T(k) = 1$.

Substituting the result in Eq. (5.8) into Eq. (5.7), we have

$$\delta_{Mk} = \frac{3k^2 \Phi_k^{(p)}}{5(aH_{\rm now})^2 \Omega_{M,\rm now}} T(k) \mathcal{D}(a). \tag{5.10}$$

This is the usual parametrization for δ_{Mk}, and it is the one used in Ref. [11]. Cosmic-microwave-background (CMB) measurements [12] indicate that $P(k, t_{\rm now}) \sim k$ for modes which enter the horizon at times $t > t_{\rm MRE}$. Thus, for an approximately scale-invariant dimensionless power spectrum of primordial curvature perturbations after inflation which obeys a scaling relation of the form

$$\Delta^2_{\mathcal{R}}(k) \equiv \frac{k^3}{2\pi^2} \langle |\mathcal{R}_k|^2 \rangle = A_* \left(\frac{k}{k_*}\right)^{n_s - 1} \tag{5.11}$$

with a spectral index n_s very near unity, where k_* is a suitably chosen pivot scale and where A_* is a dimensionless constant, Eq. (5.9) implies that $\Phi_k^{(p)} \sim k^{-3/2}$. For the choice $k_* = 0.05\ {\rm Mpc}^{-1}$, CMB measurements yield $n_s \approx 0.965$ and $A_* \approx 2.099 \times 10^{-9}$ [13]. Thus, Eqs. (5.1) and (5.10) together imply that

$$P(k, t_{\rm now}) \sim k^4 |\Phi_k^{(p)}|^2 T^2(k) \sim k T^2(k) \tag{5.12}$$

for modes which enter the horizon at times $t > t_{\rm MRE}$. Consistency with CMB data therefore implies that $T(k)$ is independent of k for such modes, as desired.

We emphasize, however, that $\Phi_k^{(p)}$ is not the same as the initial value $\Phi_k^{(0)}$ for the corresponding perturbation mode in our stasis cosmology. Rather, if we choose $t^{(0)}$ to be a time at which the universe is already in stasis, Eq. (5.9) implies that

$$\Phi_k^{(0)} = \frac{9(1 + \bar{w})}{2(5 + 3\bar{w})} \Phi_k^{(p)}. \tag{5.13}$$

By contrast, if we choose $t^{(0)}$ to be a time at which the stasis epoch has not yet begun and the universe is dominated by the ϕ_ℓ, which behave like massive matter, then $\Phi_k^{(0)} = (9/10)\Phi_k^{(p)}$.

Within the context of the standard cosmology, perturbations in the energy density of cold dark matter—here identified with $\delta_{k\chi}$—are, to a good approximation, described by a function of the form [14]

$$\delta_{k\chi}^{(\mathrm{SC})} = A_{\mathrm{SC}}\Phi_k^{(p)} \log\left(B_{\mathrm{SC}}\frac{a}{a_k}\right) \tag{5.14}$$

at times $t \ll t_{\mathrm{MRE}}$, where $A_{\mathrm{SC}} \approx 9.11$ and $B_{\mathrm{SC}} \approx 0.594$ are constants and where a_k denotes the value of a at which the condition $k = aH$ is satisfied and the mode enters the horizon. At times $t_{\mathrm{MRE}} < t \ll t_{\mathrm{BD}}$ after the time of matter-radiation equality but before the time t_{BD} of baryon decoupling, these perturbations are described by a function which can be expressed as

$$\delta_{k\chi}^{(\mathrm{SC})} = \frac{3}{2}A_{\mathrm{SC}}\Phi_k^{(p)} f_1 \log\left(B_{\mathrm{SC}}\left(\frac{4}{e^3}\right)^{f_2/f_1}\frac{a_{\mathrm{MRE}}}{a_k}\right)\mathcal{D}(a), \tag{5.15}$$

where e denotes the natural log base and where the constants

$$\begin{aligned} f_1 &\equiv 1 - 0.568 f_b + 0.094 f_b^2 \\ f_2 &\equiv 1 - 1.156 f_b + 0.149 f_b^2 - 0.074 f_b^3 \end{aligned} \tag{5.16}$$

are defined in terms of the late-time baryon fraction $f_b \equiv \Omega_b/(\Omega_b + \Omega_\chi)$. For $t_{\mathrm{MRE}} \ll t < t_{\mathrm{BD}}$, the growth function $\mathcal{D}(a)$ is given by

$$\begin{aligned} \mathcal{D}(a) = &\left(\frac{a_{\mathrm{MRE}}}{a + a_{\mathrm{MRE}}}\right)^{\alpha} \\ &\times {}_2F_1\left(\alpha, \alpha + \frac{1}{2}; 2\alpha + \frac{1}{2}; \frac{a_{\mathrm{MRE}}}{a + a_{\mathrm{MRE}}}\right), \end{aligned} \tag{5.17}$$

where ${}_2F_1(a, b; c; z)$ denotes Gauss's hypergeometric function and where

$$\alpha \equiv -\frac{1}{4}\left[\sqrt{1 + 24(1 - f_b)} - 1\right]. \tag{5.18}$$

Comparing Eqs. (5.15) and (5.10), one finds that in the standard cosmology, the transfer function takes the form

$$\begin{aligned} T_{\mathrm{SC}}(k) = &\frac{5A_{\mathrm{SC}}f_1}{2k^2}(aH)_{\mathrm{now}}^2\Omega_{M,\mathrm{now}} \\ &\times \log\left(B_{\mathrm{SC}}\frac{a_{\mathrm{MRE}}}{a_k}\left(\frac{4}{e^3}\right)^{f_2/f_1}\right). \end{aligned} \tag{5.19}$$

Within the context of the standard cosmology, wherein the universe is radiation dominated between the time t_{RH} at the end of reheating and the time of matter-radiation equality, the Friedmann equation implies that

$$\frac{a_{\mathrm{MRE}}}{a_k} = \frac{\sqrt{2}k}{k_{\mathrm{MRE}}} \tag{5.20}$$

for modes which enter the horizon at times $t \ll t_{\mathrm{MRE}}$, where $k_{\mathrm{MRE}} \equiv a_{\mathrm{MRE}}H_{\mathrm{MRE}}$ denotes the wave number of the mode which enters the horizon at $t = t_{\mathrm{MRE}}$. Thus, our final expression for $T(k)$ for short-wavelength modes in the standard cosmology is

$$\begin{aligned} T_{\mathrm{SC}}(k) = &\frac{5A_{\mathrm{SC}}f_1}{2k^2}(aH_{\mathrm{now}})^2\Omega_{M,\mathrm{now}} \\ &\times \log\left(\frac{\sqrt{2}B_{\mathrm{SC}}k}{k_{\mathrm{MRE}}}\left(\frac{4}{e^3}\right)^{f_2/f_1}\right). \end{aligned} \tag{5.21}$$

By contrast, within the context of nonstandard cosmological scenarios, $\delta_{k\chi}$ may scale differently with a at times $t < t_*$, where t_* denotes the time after which the expansion history is equivalent to that of the standard cosmology. Nevertheless, at times $t_* < t$, they scale with a in the same manner as they do in the standard cosmology. For example, they scale as $\delta_{k\chi} \propto \log(a)$ at times $t_* < t < t_{\mathrm{MRE}}$. Thus, without loss of generality, $\delta_{k\chi}$ may be parametrized in such cosmologies by a function of the same form as Eq. (5.14) prior to matter-radiation equality, but with the constants A_{SC} and B_{SC} replaced by general functions $A(k)$ and $B(k)$ of k:

$$\delta_{k\chi} = A(k)\Phi_k^{(p)} \log\left(B(k)\frac{a}{a_k}\right). \tag{5.22}$$

During the later period wherein $t_{\mathrm{MRE}} \ll t < t_{\mathrm{BD}}$, the evolution of $\delta_{k\chi}$ will accordingly be described by a function of the same general form as Eq. (5.15) but with A_{SC} and B_{SC} likewise replaced by $A(k)$ and $B(k)$. The transfer function in our stasis cosmology therefore takes the form

$$\begin{aligned} T(k) = &\frac{5A(k)f_1}{2k^2}(aH)_{\mathrm{now}}^2\Omega_{M,\mathrm{now}} \\ &\times \log\left(B(k)\frac{a_{\mathrm{MRE}}}{a_k}\left(\frac{4}{e^3}\right)^{f_2/f_1}\right). \end{aligned} \tag{5.23}$$

The forms of $A(k)$ and $B(k)$ can be determined as follows. Modes which enter the horizon at times $t_s < t < t_{\mathrm{MRE}}$, where t_s denotes the time at which stasis effectively ends, evolve exactly as they evolve within the standard cosmology. Thus, for such modes, $A(k)$ and $B(k)$ are independent of k and given by the constants A and B. By contrast, we have seen that modes which enter the horizon during stasis grow with a like a power law of the form $\delta_{k\chi} \sim a^{q(\langle w\rangle)}$, with $q(\bar{w})$ given by Eq. (4.9), until stasis ends. The normalization for these $\delta_{k\chi}$ follows from the fact that $\delta_{k\chi} \approx 2\Phi_k^{(0)}/(1 + \bar{w}) = 9\Phi_k^{(p)}/(5 + 3\bar{w})$ for each such mode after the stasis epoch has begun but prior to horizon entry. Thus, we find that the growth of these modes during the radiation-dominated epoch can be modeled as

$$\delta_{k\chi} \approx \frac{9\Phi_k^{(p)}}{5 + 3\bar{w}}\left(\frac{a_s}{a_k}\right)^{q(\bar{w})}\left[1 + \log\left(\frac{a}{a_s}\right)\right], \tag{5.24}$$

where $a_s \equiv a(t_s)$. Thus, noting that

$$\frac{a_s}{a_k} = \left(\frac{k}{k_s}\right)^{2/(1+3\bar{w})}, \tag{5.25}$$

where k_s denotes the wave number of the mode which enters the horizon at $t = t_s$, we find that

$$\begin{aligned} A(k) &= \frac{9}{5+3\bar{w}}\left(\frac{k}{k_s}\right)^{2q(\bar{w})/(1+3\bar{w})} \\ B(k) &= e\left(\frac{k}{k_s}\right)^{-2/(1+3\bar{w})}. \end{aligned} \tag{5.26}$$

Moreover, we note that the ratio $a_{\rm MRE}/a_k$ which appears in Eq. (5.23) can be expressed as

$$\frac{a_{\rm MRE}}{a_k} = \frac{\sqrt{2}k_s}{k_{\rm MRE}}\left(\frac{k}{k_s}\right)^{2/(1+3\bar{w})}. \tag{5.27}$$

One additional effect which can modify $T(k)$ at large k is the free streaming of dark-matter particles out of overdense regions. This effect depends on the primordial velocity distribution for the cosmological population of χ particles that plays the role of the dark matter in this scenario. Since the production of χ particles is unrelated to the stasis epoch, the primordial velocity distribution of the χ particles can take a variety of forms. In what follows, we shall simply assume that this velocity distribution is unimodal and reasonably narrow, such that the effect that the free streaming of χ particles has on structure formation is well characterized by the average velocity $\langle v_\chi \rangle$ of these particles. In particular, for such a velocity distribution, the free-streaming length at a given value of the scale factor a is

$$\begin{aligned} \lambda_{\rm fs}(a) = {}& \frac{2a_s\langle v_{\chi s}\rangle}{(aH)_{\rm now}\Omega_{\gamma,\rm now}^{1/2}}\left(\frac{a_s}{a_{\rm now}}\right) \\ & \times \left[\operatorname{arcsinh}\left(\sqrt{\frac{a_{\rm MRE}}{a_s}}\right) - \operatorname{arcsinh}\left(\sqrt{\frac{a_{\rm MRE}}{a}}\right)\right], \end{aligned} \tag{5.28}$$

where $\langle v_{\chi s}\rangle$ denotes the value of $\langle v_\chi\rangle$ at $t = t_s$ and where $\Omega_{\gamma,\rm now}$ denotes the present-day abundance of radiation. At late times, when $a \gg a_{\rm MRE}$, this expression becomes approximately independent of a. We model the effect of free streaming on $P(k,t)$ at late times by incorporating an additional exponential suppression factor into our expression for $T(k)$ in Eq. (5.23) such that it becomes

$$\begin{aligned} T(k) = {}& \frac{5A(k)f_1}{2k^2}a_{\rm now}^2 H_{\rm now}^2 \Omega_{M,\rm now} e^{-\frac{k^2}{2k_{\rm fs}^2(a_{\rm now})}} \\ & \times \log\left(B(k)\frac{a_{\rm MRE}}{a_k}\left(\frac{4}{e^3}\right)^{f_2/f_1}\right), \end{aligned} \tag{5.29}$$

where we have defined $k_{\rm fs}(a) \equiv \lambda_{\rm fs}^{-1}(a)$.

For any primordial spectrum of perturbations $\Phi_k^{(p)}$ to the gravitational potential, the matter power spectrum in our stasis cosmology can be parametrized as

$$P(k, t_{\rm now}) = R^2(k) P_{\rm SC}(k, t_{\rm now}), \tag{5.30}$$

where $P_{\rm SC}(k, t_{\rm now})$ denotes the present-day matter power spectrum in the standard cosmology and where we have defined the ratio

$$R(k) \equiv \frac{T(k)}{T_{\rm SC}(k)} = \frac{\delta_{sk}}{\delta_{sk,\rm SC}}. \tag{5.31}$$

A similar parametrization was adopted in Ref. [11] in order to characterize the matter power spectrum which emerges in cosmologies involving an EMDE. We also note that the ratio $R(k)$ accords with the alternative definition of the transfer function adopted in, e.g., Ref. [15]. Explicitly, Eqs. (5.21) and (5.23) imply that

$$R(k) = \frac{A(k)\log\left(B(k)\frac{a_{\rm MRE}}{a_k}\left(\frac{4}{e^3}\right)^{f_2/f_1}\right)e^{-\frac{k^2}{2k_{\rm fs}^2(a_{\rm now})}}}{A_{\rm SC}\log\left(B_{\rm SC}\frac{\sqrt{2}k}{k_{\rm MRE}}\left(\frac{4}{e^3}\right)^{f_2/f_1}\right)}. \tag{5.32}$$

We emphasize that in this expression, a_k denotes the scale factor at horizon entry *in the stasis cosmology*, which, in general, differs from the corresponding result in the standard cosmology. We also note that the analytic results in Eqs. (5.26) and (5.27) together imply that

$$B(k)\frac{a_{\rm MRE}}{a_k} = \frac{\sqrt{2}k_s e}{k_{\rm MRE}}. \tag{5.33}$$

In Fig. 4, we plot the enhancement factor $R^2(k)$ (differently colored curves) for the matter power spectrum which results from a period of matter/radiation stasis as a function of k for different values of $\bar{w}$. All curves shown correspond to the parameter choices $\delta = 1$, $\gamma = 7$, and $\Delta m = m_0$, with α determined by the value of $\bar{w}$ in each case. By varying α while holding δ, γ, and Δm fixed, we ensure that the mass spectrum of the tower is the same in all cases. The solid curves represent our full numerical results for $A(k)$ and $B(k)$. The dashed curves represent the results obtained from the analytic expressions in Eq. (5.26). The solid and dashed black curves represent the corresponding enhancement factors which result from a period of early matter domination. For both stasis and EMDE cosmologies, we take the temperature of the SM radiation bath at t_s to be

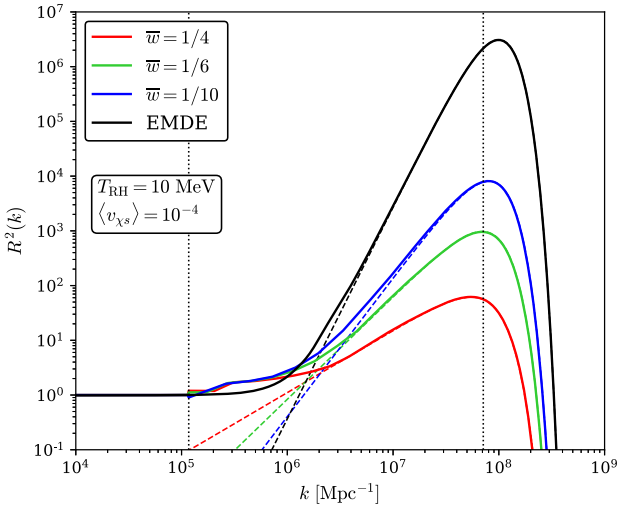

FIG. 4. Factor $R^2(k)$ by which the present-day linear matter power spectrum $P(k, t_{\rm now})$ is enhanced in our stasis scenario relative to the standard cosmology, shown as a function of the physical wave number k for several different values of $\bar{w}$. All curves shown correspond to the parameter choices $\delta = 1$, $\gamma = 7$, and $\Delta m = m_0$, with α determined by the value of $\bar{w}$. The solid curves represent our full numerical results for $A(k)$ and $B(k)$. The dashed curves represent the results obtained from the analytic expressions in Eq. (5.26). The solid and dashed black curves represent the corresponding enhancement factors which result from a period of early matter domination. For all curves shown, we have taken $\langle v_{\chi s} \rangle = 10^{-4}$ and $T(t_s) = 10$ MeV. We observe that a period of cosmological stasis can lead to a considerable enhancement in the matter power spectrum at small k and that the degree of enhancement becomes larger as $\bar{w}$ decreases toward zero.

$T(t_s) = 10$ MeV and the average velocity of dark-matter particles at this same time to be $\langle v_{\chi s} \rangle = 10^{-4}$. We define t_s in both the stasis and EMDE cosmologies to be the time at which Ω_M falls below $\Omega_M = 0.001$ and the universe may be considered radiation dominated. We note that this convention for the time at which the expansion history of the universe begins to coincide with that of the standard cosmology differs from the convention adopted in Ref. [11] in the context of a modified cosmology involving an EMDE, wherein this time was taken to be the canonical reheating time $t_{\rm RH}$—i.e., the time at which $\Omega_M \simeq 0.5$. The values of k for the modes which cross the horizon at t_s and $t_{\rm RH}$ are related by $k_{\rm RH} \simeq 2.85 k_s$.

We see from Fig. 4 that $P(k, t_{\rm now})$ is significantly enhanced for modes with wave numbers which lie within a finite range of values—demarcated here by the two dotted vertical lines. Modes with k below this range enter the horizon only after the end of the stasis epoch and evolve exactly as they would in the standard cosmology. By contrast, modes with k above this range are washed out by free-streaming effects. Unlike in the case of an EMDE, the behavior of the curves at the lower end of this range is sensitive to the edge effects which result from the discrete spacing between the masses of the tower states. We observe that the extent to which the matter power spectrum is enhanced within this range of k can be quite significant and that this enhancement increases as $\bar{w}$ approaches zero.

The primary observational implication of these results is that an epoch of matter/radiation stasis prior to the BBN epoch can lead to the formation of dark-matter substructures similar to those which can arise from an epoch of early matter domination [11,16,17]. The smallest dark-matter substructures to which experiments are currently sensitive are those with masses $M \sim 10^{-11} M_\odot$ (for reviews, see, e.g., Ref. [18]). Constraints on both compact and diffuse substructures with masses on this scale have been derived from the analysis of data from stellar surveys of the Andromeda galaxy in light of gravitational-microlensing considerations [19,20]. Future pulsar-timing arrays (PTAs) could also, in principle, be sensitive to both compact and diffuse substructures on this mass scale—and potentially on mass scales down to $M \sim 10^{-13}$ due to the characteristic imprints they would leave on the timing residuals obtained for a set of pulsars at the population level [21,22]. Substructures of this mass correspond to wave numbers $k_{\rm phys} \sim 10^9$ Mpc^{-1}.

While dark-matter substructures of this sort can arise as a consequence of either an epoch of matter/radiation stasis or an EMDE, the results shown in Fig. 4 indicate that the morphology of the feature in $P(k, t_{\rm now})$ which results from such a stasis epoch can differ appreciably from that of the feature which results from an EMDE—especially in situations in which $\bar{\Omega}_\gamma$ is non-negligible. These differences in $P(k, t_{\rm now})$ can, in principle, lead to population-level differences in the properties of dark-matter substructures within galaxies and clusters. Such differences could, in principle, be used in order to determine whether a population of dark-matter substructures might have originated from a stasis epoch or from an EMDE. We leave the investigation of this possibility for future work.

VI. CONCLUSIONS

In this paper, we have analyzed the behavior of density perturbations during a stasis epoch. We have demonstrated that density perturbations for a spectator component—i.e., a component which couples to the stasis sector only through gravity—evolve differently during a stasis epoch than they would during an epoch in which the universe is dominated by a single perfect fluid, even if the effective equation-of-state parameter $\langle w \rangle$ for both universes is the same. In particular, we have shown that density perturbations associated with a spectator matter component exhibit power-law growth during stasis after horizon entry, with a power-law exponent which is determined solely by the value of $\langle w \rangle$. As a result, a stasis epoch prior to BBN can lead to an enhancement in small-scale structure reminiscent

of that which results from an EMDE, but with the magnitude of that enhancement depending on the balance between the abundances of the cosmological components which participate in the stasis. This provides a sharp observational handle that can be used to distinguish a stasis epoch from more traditional cosmological epochs. The phenomenological consequences of this observation are currently under study [23].

A few comments are in order. First, for simplicity, we have taken the effective sound speed for the radiation component to be $c_{s\gamma} \approx 1/\sqrt{3}$. In scenarios wherein additional effects are present which modify $c_{s\gamma}$, the evolution of cosmological perturbations becomes more complicated. However, the qualitative results that we have obtained are unchanged within the regime in which the impact that these effects have on $c_{s\gamma}$ is small.

Second, in this paper, we have assumed for simplicity that the perturbations for all of the fields in this scenario, including the spectator, are initially purely adiabatic. Such initial conditions arise naturally in scenarios in which the cosmological populations of χ and all of the ϕ_ℓ are generated predominately either directly via inflaton decay during reheating or subsequently via interactions among particles in the radiation bath—e.g, via freeze-in or freeze-out. By contrast, such initial conditions are less natural in scenarios in which these populations of χ and/or the ϕ_ℓ are generated predominately via processes such as vacuum-misalignment production, wherein the primordial fluctuations of these fields are uncorrelated with those of the inflaton. Nevertheless, we emphasize that even in realizations of matter/radiation stasis wherein the initial conditions for χ and the ϕ_ℓ are not adiabatic, the right side of Eq. (2.7) is still, in general, nonvanishing, and thus $\delta_{k\chi}$ still evolves differently than it does during an epoch of perfect-fluid domination.

Many extensions of this work are also possible. For example, in this paper we have studied the implications of a matter/radiation stasis, but since any realization or stasis generically gives rise to a nonvanishing source term for Φ_k in Eq. (2.7), we expect that similar results will emerge from other realizations of stasis as well—including those realizations in which the stasis involves more than two cosmological energy components. The effects we have uncovered here may also have an impact on the spectrum of primordial perturbations which emerges from models of stasis inflation [5].

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

[1] K. R. Dienes, L. Heurtier, F. Huang, D. Kim, T. M. P. Tait, and B. Thomas, Phys. Rev. D **105**, 023530 (2022).
[2] K. R. Dienes, L. Heurtier, F. Huang, T. M. P. Tait, and B. Thomas, Phys. Rev. D **109**, 083508 (2024).
[3] J. D. Barrow, E. J. Copeland, and A. R. Liddle, Mon. Not. Roy. Astron. Soc. **253**, 675 (1991).
[4] K. R. Dienes, L. Heurtier, F. Huang, D. Kim, T. M. P. Tait, and B. Thomas, Phys. Rev. D **112**, 083546 (2025).
[5] K. R. Dienes, L. Heurtier, F. Huang, T. M. P. Tait, and B. Thomas, Phys. Rev. D **110**, 123514 (2024).
[6] J. Halverson and S. Pandya, Phys. Rev. D **110**, 075041 (2024).
[7] J. Barber, K. R. Dienes, and B. Thomas, Phys. Rev. D **110**, 123515 (2024).
[8] J. Barber, K. R. Dienes, and B. Thomas, Phys. Rev. D **111**, 063519 (2025).
[9] F. Huang and V. Knapp-Perez, J. High Energy Phys. 09 (2025) 072.
[10] K. Redmond, A. Trezza, and A. L. Erickcek, Phys. Rev. D **98**, 063504 (2018).
[11] A. L. Erickcek and K. Sigurdson, Phys. Rev. D **84**, 083503 (2011).
[12] N. Aghanim *et al.* (Planck), Astron. Astrophys. **641**, A1 (2020).
[13] N. Aghanim *et al.* (Planck), Astron. Astrophys. **641**, A6 (2020); **652**, C4(E) (2021).
[14] W. Hu and N. Sugiyama, Astrophys. J. **471**, 542 (1996).

[15] K. R. Dienes, F. Huang, J. Kost, S. Su, and B. Thomas, Phys. Rev. D **101**, 123511 (2020).
[16] A. L. Erickcek, Phys. Rev. D **92**, 103505 (2015).
[17] M. S. Delos, K. Redmond, and A. L. Erickcek, Phys. Rev. D **108**, 023528 (2023).
[18] K. Bechtol *et al.*, in *Snowmass 2021* (2022); arXiv: 2203.07354.
[19] H. Niikura *et al.*, Nat. Astron. **3**, 524 (2019).
[20] D. Croon, D. McKeen, N. Raj, and Z. Wang, Phys. Rev. D **102**, 083021 (2020).
[21] H. Ramani, T. Trickle, and K. M. Zurek, J. Cosmol. Astropart. Phys. 12 (2020) 033.
[22] V. S. H. Lee, S. R. Taylor, T. Trickle, and K. M. Zurek, J. Cosmol. Astropart. Phys. 08 (2021) 025.
[23] K. R. Dienes, L. Heurtier, F. Huang, A. Paulsen, T. M. P. Tait, and B. Thomas (to be published).