Is a covariant virtual tachyon viable?

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Sidney Coleman has noted that superluminal particles or observers would be able to go back in time and have no definite trajectory according to subluminal observers, while not violating Lorentz invariance [*Sidney Coleman's Lectures on Relativity* (Cambridge University Press, Cambridge, England, 2022)]. Recently, Dragan and Ekert have significantly developed similar ideas even further, which led to formulation of a "quantum principle of relativity" that intimately links the two theories [New J. Phys. **22**, 033038 (2020); **25**, 088002(E) (2023)]. However, field theory descriptions of an on-shell tachyon, described by scalar field ϕ with negative mass squared parameter, lead to violation of basic principles of relativity or quantum mechanics. In this work, we investigate whether purely virtual tachyons can be consistent within the fakeon framework—the only known viable formulation of purely virtual particles. We identify two fatal obstructions. First, Lorentz boosts mix creation and annihilation operators, rendering the canonical commutation relations noninvariant despite formal invariance of the vacuum. Second, the real part of the tachyon Feynman propagator and Wheeler propagator have disjoint support, preventing application of both the fakeon prescription and Wheeler-Feynman absorber mechanism. Interactions with stable Standard Model fields further violate Lorentz invariance and the equivalence principle, and we provide a quantitative limit on the coupling strength of such a scenario. Our analysis excludes the possibility of formulating a covariant quantum field theory of interacting virtual tachyons.

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I. INTRODUCTION

While many authors considered superluminal observers or reference frame transformations, such objects are plagued by mathematical inconsistencies, physical paradoxes [1,2], or they break one or more basic laws of physics; for a brief overview, see Refs. [3,4]. On the other hand, their peculiar behavior has fascinated not only science-fiction writers, but also has prompted serious academic works, starting with Bilaniuk *et al.* [5].

While the question if superluminal objects are physically consistent in the framework of classical special relativity is difficult to settle, as evidenced by comments by, e.g., DeWitt [6] and Thouless [7]—who noted that while real (on-shell) tachyons present serious challenges for theory consistency, virtual (off-shell) tachyons significantly relax these constraints—the analysis is even more murky when one accounts for fundamental indeterminism of the quantum world [8]. Indeed, as Coleman has emphasized [9], superluminal particles would appear to subluminal observers as having no definite trajectory without violating Lorentz invariance—feature strikingly reminiscent of quantum-mechanical behavior.

In fact, Ref. [8] prompted recent resurgence of interest in tachyons, since it has made several observations directly linking noncausal aspects of physics of superluminal observers and fundamentals of quantum mechanics (QM). If correct, that would mean the latter theory can be understood as largely originating from a more basic and intuitive principle—the (extended) principle of relativity, precisely formulated in [8] as the "quantum principle of relativity." While the idea of explaining the origin of quantum superpositions in such a way was critically examined within special relativity and nonrelativistic QM in [10]—see also comments examining [8] along different directions [11–13] together with corresponding responses [14–16]—the ultimate test of such program is construction of a viable quantum field theory (QFT) of tachyons, defined as scalars with negative mass squared parameter.

Motivation for considering the QFT of tachyons is also provided by the fact that they are predicted by an asymptotic

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freedom scenario in higher-derivative quantum gravity [17]. Tachyons are also invoked in constructions of interesting Carrollian (which corresponds to Poincare group contraction in the limit $c \to 0$ [18,19]) models by starting with tachyonic matter in a Lorentzian theory [20–22]. Finally, we negatively settle the question of whether tachyons could be purely virtual particles, called fakeons [23–25].

We would also like to point out that purely virtual tachyons, by construction, evade the most serious problems of such theories—namely that, since the little group is not compact, the phase space of tachyons is unbounded leading to purely kinematic divergences [26]. Moreover, formulation of scattering theory for an on-shell tachyon is problematic, there are divergences in elastic scattering of Standard Model (SM) particles mediated by virtual tachyon, and optical theorem does not hold (unitarity is violated) with the tachyonic Feynman propagator (FP) [27]. Therefore, a purely virtual tachyon field deserves further investigation—its origin from covariant quantization as well as formulation of interactions with SM fields. For the first time, we show that such interactions lead to violation of Lorentz invariance (LI) and other problematic features. This negatively settles their main motivation originating from the relativity principle.

Before proceeding further, let us make several remarks. First, as recently discussed in detail in Ref. [28], the Higgs field can be viewed as self-interacting tachyon,[1] which can develop either vanishing (proper tachyon) or nonzero vacuum expectation value (the option realized within SM) through tadpole diagrams. After Dyson series resummation of the Higgs propagator, the physical mass squared is positive—such a purely diagrammatic approach does not invoke concepts like spontaneous symmetry breaking, point emphasized in [28]. Therefore, our work can be seen as exploring the vanishing tadpole solution, which was promptly dismissed in [28] due to tachyon problems with causality; we also note that the Higgs mechanism was brought up as one of main motivations for investigating tachyons in [4,32]. Since causuality problems due to tachyons are difficult to settle within the special relativity of classical field theory [8], we believe dismissing the tachyonic solution on such grounds is premature, and our work provides a comprehensive (negative) resolution of this issue.

Second, already DeWitt [6] and Thouless [7] have noted, working exclusively within special relativistic setup, that off-shell, meaning nonpropagating, tachyons *might* be compatible with known laws of physics, although no clear conclusion was drawn by these authors. Within QFT, several works [33,34] have considered a tachyon field representing a virtual state, basing it on the claim that tachyon commutator identically vanishes. We believe that such an approach is not promising, since it does not correspond to the standard properties of the commutator, which is given by Eq. (4). Moreover, if the commutator vanishes identically, then so do the retarded and advanced Green's functions, which should not be the case if tachyon were to actually mediate interactions. In our approach, the commutator behaves similarly to the subluminal (bradyon) case—it vanishes only outside the light cone—and the same holds for the advanced and retarded Green's functions. On the other hand, the tachyonic commutator (and therefore also advanced and retarded Green's functions) grows exponentially in timelike direction, illustrating the unstable nature of the tachyon.

Third, consistent formulation of purely virtual particles for positive mass squared—which is motivated, e.g., by quantization of quantum gravity theories, e.g., renormalizable quadratic gravity [35], which possess ghosts—has been achieved only recently within the fakeon framework [23–25,36]. It guarantees unitarity and Lorentz invariance of the free and interacting virtual field. The cost is abandoning analyticity of the probability amplitudes, since the theory requires working in Euclidean space and continuing into Minkowski space by average continuation around the branch point $p^2=0$, $\mathcal{M}(p^2)=(\mathcal{M}(p^2-i\epsilon)+\mathcal{M}(p^2+i\epsilon))/2$ [37]. While the currently known proofs of mathematical consistency of the fakeon projection require $m^2 > 0$, it is *not* known if some generalized prescription exists or not [38]; also see Ref. [39] for discussion of the tachyonic fakeon in a different context.

Finally, tachyons are also studied in the recent bloom in exploration Carrollian physics. Indeed, since in such a setting the speed of light vanishes, no movement for ordinary matter is allowed, and any nonzero velocity is necessarily of superluminal origin, which leads to nontrivial constructions [20–22].

We work in the natural units, $\hbar = c = 1$, and Minkowski space with mostly minus metric.

II. TACHYONIC GREEN'S FUNCTIONS

We will consider the following Lagrangian, extending the SM by kinetic, mass term, and the Yukawa interaction of a tachyon with a SM massive fermion ψ,

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 + g\phi\bar{\psi}\psi, \qquad (1)$$

where $m_\phi^2 < 0$, and g is a dimensionless coupling constant. For concreteness, we will assume further that ψ represents an electron. Since the tachyon corresponds to small perturbations around the unstable, $\phi \sim 0$, saddle point of the potential, by construction we neglect the impact of ϕ self-interactions, in particular the (possibly radiatively generated) quartic term.

Let us discuss the free theory first. The equations of motion resulting from Eq. (1) are

$$(\partial_\mu\partial^\mu + m_\phi^2)\phi(x) = 0. \qquad (2)$$

[1]The bare mass squared is not directly physical quantity [29–31]. In particular, it is not an observable, which differs from tachyon that, by definition, has physical negative squared mass.

We begin by discussing the LI solutions to Eq. (2) as well as LI Green's functions in position space. While most of these expressions are already given in the literature [40,41], to the best of our knowledge, we provide the first complete expression for the tachyonic Feynman propagator (also known as the Dhar-Sudarshan propagator [42]) and the Wheeler propagator in position space.

We also provide connection of the LI solution and Green's functions to the quantum field $\chi(x)$ describing the tachyon—the details of its properties are discussed in Sec. III. Here, we only use the fact that it is proportional to the sum of a field $\phi(x)$, composed only from annihilation operators, and its Hermitian conjugate.

We will use the contours shown in Fig. 1. These are the same as in the standard case, except for the deformation in the panels (b) and (c) that are used for computation of the retarded Green's function. We have denoted the real and imaginary poles in green and red, respectively. The tachyon energy is $E_k = \pm\omega_k$ for $|\vec{k}| > m_\phi$, while $E_k = \pm i\omega_k$ for $|\vec{k}| < m_\phi$.

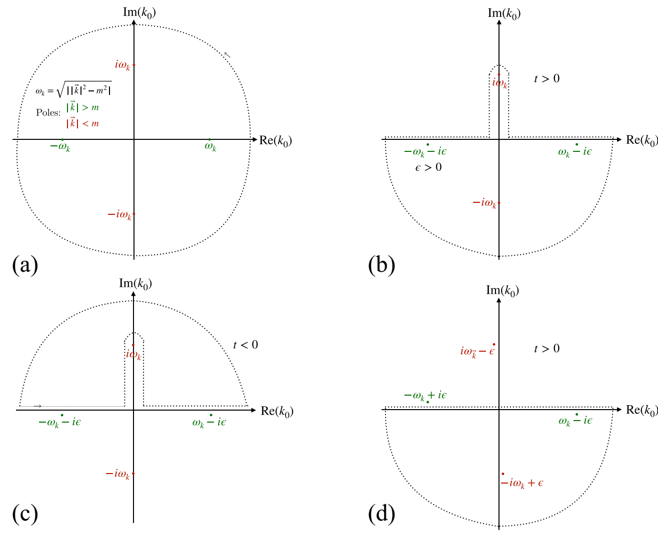

FIG. 1. Integration contours in the complex k^0 plane for: (a) the commutator function; (b) and (c) the retarded Green's functions for $t > 0$ and $t < 0$, respectively; and (d) the Feynman propagator for $t > 0$.

A. Commutator function

The vacuum expectation value of the commutator of the field χ given by Eq. (19) is

$$\begin{aligned}\langle 0|[\chi(x), \chi(y)]|0\rangle &= \langle 0|[\phi(x), \phi^\dagger(y)]|0\rangle \\ &= \langle 0|\phi(x)\phi^\dagger(y)|0\rangle = i\Delta(x-y), \end{aligned} \tag{3}$$

where $i\Delta(x)$ is the (Pauli-Jordan) commutator function, and we used the fact that $\phi(x)|0\rangle = 0$ for all x.

Using the contour shown in panel (a) in Fig. 1, one calculates $i\Delta(x)$ by straightforward computation analogous (discussed in detail in Sec. 2.3 of Ref. [43]) to the bradyon case,

$$\begin{aligned} i\Delta(x-y) &= \int_{C_{(a)}} \frac{d^4k}{(2\pi)^4} \frac{-ie^{-ik(x-y)}}{k^2 - m_\phi^2} \\ &= -i\int \frac{d^3\vec{k}}{(2\pi)^3 E_k} e^{-i\vec{k}\cdot(\vec{x}-\vec{y})} \sin(E_k(x^0-y^0)) \\ &= -i\int_{|\vec{k}|>|m_\phi|} \frac{d^3\vec{k}}{(2\pi)^3\omega_k} e^{-i\vec{k}\cdot(\vec{x}-\vec{y})} \sin(\omega_k(x^0-y^0)) - i\int_{|\vec{k}|<|m_\phi|} \frac{d^3\vec{k}}{(2\pi)^3 i\omega_k} e^{-i\vec{k}\cdot(\vec{x}-\vec{y})} \sin(i\omega_k(x^0-y^0)) \\ &= \frac{-i\mathrm{sgn}(x^0-y^0)}{2\pi}\left[\delta((x-y)^2) - \theta((x-y)^2)\frac{|m_\phi|}{2\sqrt{(x-y)^2}} I_1\left(|m_\phi|\sqrt{(x-y)^2}\right)\right], \end{aligned} \tag{4}$$

where $I_1(x)$ is the modified Bessel function of the first kind and $\theta(x)$ is the Heaviside step function. This matches the expressions stated in [44,45]. It is worth pointing out that $\Delta(x-y)$ can be obtained from $\Delta_{\mathrm{bradyon}}(x-y)$ by analytic continuation, $m \to \pm im$.

The desirable properties of the commutator are: $\Delta(x-y)$ is the local, LI solution of Eq. (2), and vanishes outside the lightcone, so the field $\chi(x)$ satisfies microcausality. Moreover, $\Delta(x-y)$ satisfies the initial conditions for Eq. (2),

$$\Delta(0, \vec{x}) = 0, \qquad \partial_t\Delta(0, \vec{x}) = -\delta^3(\vec{x}), \tag{5}$$

which translates into satisfying the equal time commutation relations,

$$[\chi(0, \vec{x}), \partial_t\chi(0, \vec{y})] = i\delta^3(\vec{x}-\vec{y}). \tag{6}$$

However, the asymptotic behavior of the commutator in the timelike regime is problematic, since it grows exponentially [41]. Indeed, for large proper time $\tau = \sqrt{x^2} \gg 1/|m_\phi|$,

$$i\Delta(x) \sim \frac{i\mathrm{sgn}(x^0)}{4\pi}\sqrt{\frac{|m_\phi|}{2\pi(x^2)^{3/2}}}e^{|m_\phi|\sqrt{x^2}}. \tag{7}$$

The instability has an obvious physical interpretation—the field wants to roll to the true minimum, resulting in a phase transition. This well-known fact provides the first check that our covariant formalism is on the right track—since we want to understand tachyons, we must work around the $\chi = 0$ configuration, and the resulting commutator of such a field indeed indicates the presence of instability.

B. Advanced/retarded Green's functions

For bradyons, the advanced/retarded Green's functions play a key role in solving the initial value problem, and are given by

$$\Delta_A(x) = -\theta(-x^0)\Delta(x), \qquad \Delta_R(x) = \theta(x^0)\Delta(x). \tag{8}$$

The same relation holds for the tachyonic analogues. The difference is that for the retarded Green's functions, we need to deform the contour, as shown in panels (b) and (c), since only such a contour (and homologically equivalent ones) guarantees satisfying causality, $\Delta_R(x^0 < 0) = 0$.

The resulting formulas are as follows:

$$\begin{aligned}\Delta_R(x) &= \Delta_A(-x) = \theta(x^0)\Delta(x)\\ &= \frac{-\theta(x^0)}{2\pi}\left(\delta(x^2) - \theta(x^2)\frac{|m_\phi|}{2\sqrt{x^2}}I_1\left(|m_\phi|\sqrt{x^2}\right)\right).\end{aligned} \tag{9}$$

Therefore, $\Delta_R(x)$ grows exponentially in the future timelike direction, while $\Delta_A(x)$ in the past. As as result, they cannot be used to obtain a stable solution to the initial value problem.

C. Feynman propagator

To compute the FP, one uses the contour shown in panel (d) in Fig. 1 for $t > 0$, and its complement for $t < 0$. The result is

$$\begin{aligned}\Delta_F(x) &= \int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik(x-y)}}{k^2 - m_\phi^2 + i\epsilon}\\ &= \frac{\delta(x^2)}{4\pi} - i\frac{\theta(x^2)|m_\phi|}{4\pi^2\sqrt{x^2}}K_1\left(|m_\phi|\sqrt{x^2}\right) - \frac{\theta(-x^2)|m_\phi|}{8\pi\sqrt{-x^2}}\\ &\quad\times\left[J_1\left(|m_\phi|\sqrt{-x^2}\right) - iY_1\left(|m_\phi|\sqrt{-x^2}\right)\right].\end{aligned} \tag{10}$$

Similarly to the commutator, tachyonic FP can be obtained from the bradyon case by analytic continuation in mass, $m \to -im$.

On the other hand, the causal behavior of bradyon and tachyon FP are different. For $x^2 > 0$, the tachyon FP is purely imaginary and decays exponentially,

$$\Delta_F(x) \sim -i\frac{\sqrt{|m_\phi|}}{4\sqrt{2}\pi^{3/2}(x^2)^{3/4}}e^{-|m_\phi|\sqrt{x^2}}, \tag{11}$$

while for $x^2 < 0$, the FP is oscillating,

$$\Delta_F(x) \sim \frac{\sqrt{|m_\phi|}(1-i)}{8\pi^{3/2}(-x^2)^{3/4}}e^{-i|m_\phi|\sqrt{-x^2}}. \tag{12}$$

In particular, the real part of FP has support on and *outside* the light cone, and is proportional to $\cos(\pi/4 + |m_\phi|\sqrt{-x^2})$. In Sec. IV, we discuss in detail physical consequences of such behavior.

D. Wheeler propagator

The arithmetic average of advanced and retarded Green's functions is called the Wheeler propagator [46,47],

$$\Delta_W(x) = \frac{1}{2}(\Delta_R(x) + \Delta_A(x)) = \frac{1}{2}\mathrm{sgn}(x^0)\Delta(x), \tag{13}$$

because it was notably used in Wheeler-Feynman (time-symmetric) electrodynamics [48,49], which can also be viewed as relativistic action at a distance between a source and an absorber; it was also used within general relativity as a computational device [50].

For tachyons, the Wheeler propagator in position space is

$$\Delta_W(x) = \frac{-1}{4\pi}\left(\delta(x^2) - \theta(x^2)\frac{|m_\phi|}{2\sqrt{x^2}}I_1\left(|m_\phi|\sqrt{x^2}\right)\right). \tag{14}$$

Therefore, it has support inside the lightcone, and it blows up exponentially for both past and future in the timelike regime.

For fields with $m^2 \geq 0$, the Wheeler propagator overlaps with the principal value propagator, $\bar{\Delta}(x)$, which is defined as the arithmetic average of Feynman and Dyson (also called anti-Feynman) propagators,

$$\bar{\Delta}(x) = \frac{1}{2}(\Delta_F(x) + \Delta_D(x)). \tag{15}$$

This equation follows from the Sochocki-Plemelj formula,

$$\lim_{\epsilon\to 0^+}\frac{1}{x \pm i\epsilon} = \mp i\pi\delta(x) + \mathrm{PV}\left(\frac{1}{x}\right), \tag{16}$$

applied to $x = k^2 - m^2$,

$$\begin{aligned}&\lim_{\epsilon\to 0^+}\frac{1}{2}\left[\frac{1}{k^2 - m^2 + i\epsilon} + \frac{1}{k^2 - m^2 - i\epsilon}\right]\\ &\quad = \lim_{\epsilon\to 0^+}\frac{k^2 - m^2}{(k^2 - m^2)^2 + \epsilon^2} = \mathrm{PV}\left(\frac{1}{k^2 - m^2}\right).\end{aligned} \tag{17}$$

Clearly, the principal value propagator is time symmetric and never on shell. Moreover, it equals the real part of $\Delta_F(x)$ and, since for bradyons there are no imaginary poles,

$$\bar{\Delta}(x) = \Delta_W(x). \tag{18}$$

The last relation does *not* hold for tachyons, resulting in important physical consequences for viability of the covariant virtual tachyon theory.

In fact, the two functions entering Eq. (18) have very different behavior: $\Delta_W(x)$ has support within and on the light cone, while $\bar{\Delta}(x)$ has support outside it. Therefore, for virtual tachyon interacting with SM fields, the equations of motion for the latter obtained after integrating out the tachyon (called classicization [39]) will be governed by data *outside* the light cone. This means that (1) virtual tachyons can induce arbitrary large causality violation, contrary to fakeons, where such violation takes place only at timescales $\tau \sim 1/m$ and (2) one cannot employ the Wheeler-Feynman absorber-emiter idea—which is supposed to restore causality with a purely virtual photon in the time-symmetric electrodynamics—to tachyons since they behave like superluminal (and not advanced-retarded but confined to the lightcone as in Wheeler-Feynman electrodynamics) action at a distance.

To briefly summarize the main result of this section—we have provided complete expressions for the tachyonic LI solution (commutator) and Green's functions in position space. Crucially, we have shown that the Wheeler and real part of the FP do *not* overlap for tachyon. As a consequence, the interacting virtual tachyon field leads to a nonlocal initial value problem for matter it interacts with, contrary to fakeons.

III. CANONICAL QUANTIZATION OF A FREE TACHYON FAKEON

Purely virtual fields have been recently consistently constructed with the fakeon prescription [23–25], which posits that the proper propagator is the real part of FP. In particular, the relativistic covariance and perturbative unitarity of the S matrix after projecting out the virtual field was proved [51], which was applied to ghosts, plagued by inconsistencies within standard formalism.

In the canonical QFT formalism, describing fakeons requires working with a Hermitian field composed of a *pair* of particle ghosts—see Sec. 3.4 in [52]. Since our goal is showing that constructing a covariant virtual field for a tachyon is *impossible*, we adapt an ansatz inspired by the fakeon decomposition; *nota bene*, it is particularly suitable to handle the unstable, $|\vec{k}| < |m_\phi|$ modes.

Within such a framework, we show that the tachyon field $\chi(x)$ satisfies microcausality and equations of motion, as well as that its commutator and Green's functions are LI. However, full relativistic covariance is not achieved because the boosts that change the sign of the energy do not preserve the commutation relations of the annihilation/creation operators [3,4], which prevents construction of LI Fock space. Despite this fundamental problem, we believe such construction to be worth elucidating because the field $\chi(x)$ is purely virtual, i.e., it has no particle excitations and is not part of asymptotic states (this is a fundamental property of fakeons, for which such construction—contrary to tachyons—can be proved to result in unitary and relativistically invariant theory). Therefore, it is not entirely clear if LI Fock space construction is actually necessary.

Let us consider the following tachyon field:

$$\sqrt{2}\chi(x) = \phi(x) + \phi^\dagger(x), \tag{19}$$

where the non-Hermitian field $\phi(x)$ contains only annihilation operators, as follows:

$$\begin{aligned}\phi(x) &= \phi_>(x) + \phi_<(x)\\ &= \int_{|\vec{k}|>|m_\phi|} \frac{d^3\vec{k}}{2\omega_k(2\pi)^3}\left[a_{+\vec{k}}e^{-i(\omega_k x^0 - \vec{k}\cdot\vec{x})} + a_{-\vec{k}}e^{i(\omega_k x^0 - \vec{k}\cdot\vec{x})}\right]\\ &\quad + \int_{|\vec{k}|<|m_\phi|} \frac{d^3\vec{k}}{2\Omega_k(2\pi)^3}\left[b_{+\vec{k}}e^{-\Omega_k x^0 + i\vec{k}\cdot\vec{x}} + b_{-\vec{k}}e^{\Omega_k x^0 + i\vec{k}\cdot\vec{x}}\right],\end{aligned} \tag{20}$$

and $\omega_k = \sqrt{|\vec{k}|^2 - |m_\phi|^2}$ and $\Omega_k = \sqrt{|m_\phi|^2 - |\vec{k}|^2}$.

The key difference between this approach and cases discussed in the literature is that an indefinite metric is imposed for both stable ($|\vec{k}| > |m_\phi|$) and unstable modes ($|\vec{k}| < |m_\phi|$). The following canonical commutation relations (CCRs) hold:

$$\begin{aligned}[][a_{+\vec{k}}, a^\dagger_{+\vec{l}}] &= +(2\pi)^3(2\omega_k)\delta^3(\vec{k}-\vec{l}),\\ [a_{-\vec{k}}, a^\dagger_{-\vec{l}}] &= -(2\pi)^3(2\omega_k)\delta^3(\vec{k}-\vec{l}),\\ [b_{-\vec{k}}, b^\dagger_{+\vec{l}}] &= -i(2\pi)^3(2\Omega_k)\delta^3(\vec{k}-\vec{l}),\\ [b_{+\vec{k}}, b^\dagger_{-\vec{l}}] &= i(2\pi)^3(2\Omega_k)\delta^3(\vec{k}-\vec{l}),\end{aligned} \tag{21}$$

and other commutators vanish, in particular, $[b_{+k}, b^\dagger_{+l}] = 0$. We define a vacuum as the state normalized to unity that is annihilated by all operators entering $\phi(x)$, i.e., $a_{\pm\vec{k}}|0\rangle = b_{\pm\vec{k}}|0\rangle = 0$ for all real[2] three-momenta $\vec{k}$. Then, $\phi(x)|0\rangle = 0$ for all x.

[2]We note that there can be imaginary momenta in some reference frames. Therefore, one would need to demand that the $a_\pm$, $b_\pm$ operators annihilate $|0\rangle$ *also* for complex momenta. This requires working with the Dirac delta in a complex plane, which introduces technical difficulties that are beyond our scope. On the other hand, since in Sec. IV we show that interacting ϕ leads to Lorentz violation, it may be present, but somehow hidden, already for free theory with complex momenta—we leave this point (which was glossed over in all local tachyon theories [41,44,45]) for the future.

As a result, the single particle states that correspond to imaginary energies have zero norm (which is mandatory for self-adjoint Hamiltonian with complex eigenvalues). In fact, since the total tachyon field is given by $\chi(x)$, *all* of its single particle states have zero norm. Indeed, they are $(a^\dagger_{+k} + a^\dagger_{-k})|0\rangle$ for stable modes and $(b^\dagger_{+k} + b^\dagger_{-k})|0\rangle$ for unstable ones, and both have zero norm. Since physical states have positive norm, they are unobservable, which confirms that $\chi(x)$ is a purely virtual field that only can have zero four-energy state $|0\rangle$, which, however, is not the ground state.

By direct calculation, one verifies Eq. (3) and shows that the time-ordered two point function of $\chi(x)$ is given by the principle value propagator (the real part of tachyonic FP). This shows that expectation values of fields constructed from $\chi(x)$ lead to a manifestly LI commutator and Green's functions. Note that $|0\rangle$ is an invariant state, in the sense that under boosts, annihilation operators are always mapped (with appropriately boosted four-momentum) to themselves or to other annihilation operators, which also annihilate $|0\rangle$.

On the other hand, the CCRs, given by Eq. (21) are not LI, which precludes constructing LI Fock space. For this reason, we show that the standard formalism applied to Eq. (20) leads to Poincare generators that are rotationally invariant, but covariant *only* under boosts preserving the sign of energy.

Indeed, consider the following generators of time and space translations:

$$H = \int_{|\vec{k}|>|m_\phi|} \frac{d^3\vec{k}}{2(2\pi)^3} (a^\dagger_{+k} a_{+k} + a^\dagger_{-k} a_{-k}) - \int_{|\vec{k}|<|m_\phi|} \frac{d^3\vec{k}}{2(2\pi)^3} (b^\dagger_{-k} b_{+k} + b^\dagger_{+k} b_{-k}), \tag{22}$$

$$P^j = \int_{|\vec{k}|>|m_\phi|} \frac{d^3\vec{k}}{2\omega_k(2\pi)^3} k^j (a^\dagger_{+k} a_{+k} + a^\dagger_{-k} a_{-k}) - i \int_{|\vec{k}|<|m_\phi|} \frac{d^3\vec{k}}{2\Omega_k(2\pi)^3} k^j (b^\dagger_{-k} b_{+k} - b^\dagger_{+k} b_{-k}), \tag{23}$$

where due to the $|\vec{k}| < |m_\phi|$ modes, the Hamiltonian and momentum operators do not have uniform form like for the $m^2 > 0$ case. This ansatz satisfies

$$[P^\mu, \phi(x)] = -i\partial^\mu \phi(x), \tag{24}$$

which explicitly reads ($j = 1, 2, 3$),

$$[H, \phi(x)] = -i\partial_0 \phi(x) \quad \text{and} \quad [P^j, \phi(x)] = i\partial_j \phi(x). \tag{25}$$

Since $P^\mu|0\rangle = 0$, the vacuum is translation invariant. On the other hand, spectrum of H is not positive definite—it contains not only positive and negative real eigenstates, but also imaginary ones. Therefore, $|0\rangle$ is an eigenstate with zero four-momentum, but it is not ground state—there are infinitely many zero-energy eigenstates as well as lower energy states.

For boosts preserving the sign of energy, one can show LI of the vacuum, $M^{\mu\nu}|0\rangle = 0$, by providing explicit form of the generators of the homogeneous Lorentz group. For rotations and boosts, they read as follows for the stable modes:

$$M_{ij,>} = i \int_{|\vec{k}|>|m_\phi|} \frac{d^3\vec{k}}{2\omega_k(2\pi)^3} [a^\dagger_{+k}(k_i\partial_{k_j} - k_j\partial_{k_i}) a_{+k} - a^\dagger_{-k}(k_i\partial_{k_j} - k_j\partial_{k_i}) a_{-k}], \tag{26}$$

$$M_{0i,>} = i \int_{|\vec{k}|>|m_\phi|} \frac{d^3\vec{k}}{2\omega_k(2\pi)^3} [a^\dagger_{+k}(\omega_k\partial_{k_i}) a_{+k} - a^\dagger_{-k}(\omega_k\partial_{k_i}) a_{-k}]. \tag{27}$$

Analogous expressions hold for the $|\vec{k}| < |m_\phi|$ modes,

$$M_{ij,<} = -\int_{|\vec{k}|<|m_\phi|} \frac{d^3\vec{k}}{2\Omega_k(2\pi)^3} [b^\dagger_{-k}(k_i\partial_{k_j} - k_j\partial_{k_i}) b_{+k} - b^\dagger_{+k}(k_i\partial_{k_j} - k_j\partial_{k_i}) b_{-k}], \tag{28}$$

$$M_{0i,<} = -i \int_{|\vec{k}|<|m_\phi|} \frac{d^3\vec{k}}{2\Omega_k(2\pi)^3} [b^\dagger_{-\vec{k}}(\Omega_k\partial_{k_i}) b_{+\vec{k}} + b^\dagger_{+\vec{k}}(\Omega_k\partial_{k_i}) b_{-\vec{k}}]. \tag{29}$$

We used integration by parts and dropped the surface terms, which are used to verify that the Lorentz algebra is satisfied.

In summary, in this section we have determined the consequences of the fakeonlike ansatz for a purely virtual tachyon. We showed that it indeed corresponds to the LI commutator and propagator that is equal to the real part of the FP. However, since the CCRs are not preserved, constructing LI Fock space is impossible.

IV. TACHYON-BRADYON INTERACTIONS: EQUIVALENCE PRINCIPLE AND LORENTZ INVARIANCE VIOLATION

In this section, we show that interactions of virtual tachyon with ordinary matter introduce violation of Lorentz symmetry and equivalence principle in the observable sector. We base our analysis on the peculiar behavior of the real part of the tachyonic FP, which is given by Eq. (10).

Let us first clarify the role of the tachyon mass squared parameter m^2_ϕ, which is a *bare* parameter that has physical meaning only in free theory. Once interactions are included, the physical mass $\mu = m - i\Gamma/2$ of any quantum field is defined as the pole of its resummed FP in the complex energy plane [53]. The real part of μ, m, is the physical mass, while Γ is the decay width entering the Breit-Wigner distribution.

For a stable bradyon, $\mu = m$, while $\Gamma \neq 0$ is obtained only if a decay channel becomes kinematically open. Since the S matrix is unitary (and the same holds for fakeons [51]), Γ can be also calculated by invoking the optical theorem. It equals Γ with the imaginary part of the field self-energy, $\Gamma = \mathrm{Im}(\mathcal{M})$. Therefore, for a tachyon to have nonzero μ^2, it would need to decay. However, since the tachyon is kinematically forbidden to decay into two bradyons, the imaginary part of its self-energy vanishes,

$$\mathrm{Im}(\mathcal{M}) = -\pi \int_0^1 dx\theta[p^2 x(1-x) - m_0^2(1-x) - m_1^2 x] = 0, \tag{30}$$

where $p^2 < 0$ and the masses squared of the particles in the loop are positive. Even tachyon self-interactions (which can modify the value of m^2 but have to keep it real) or decays into other tachyons are not viable, since such states would also need to be purely virtual—then, $\mathrm{Im}(\mathcal{M}) = 0$ also takes place. This shows *stability* of tachyons against particle decays, and thus μ_ϕ^2 cannot have a nonzero imaginary part; it can only change its value due to renomalization. However, the field itself clearly is *unstable*, since by equations of motion it wants to roll down the potential.

In particular, renormalization group evolution can turn it into a sufficiently massive bradyon, which can then decay into lighter states. Moreover, once tachyon interacts with SM fields, it 1) mixes with the Higgs and 2) receives contribution to its effective mass squared parameter after the Higgs condensates—see panels (b) and (c) in Fig. 2, respectively. Therefore, the interacting tachyon may be turned into a bradyon not only by developing nonzero vacuum expectation value (for the case of positive quartic self-interactions), or equivalently by resumming tadpole diagrams [28], but also by renormalization group evolution or dynamics associated with Higgs mixing. Since we are interested in an interacting tachyon, we assume further that such dynamics does not take place.

Let us now discuss the consequences of introducing Yukawa interactions between a tachyon (ϕ) and electrons (ψ) by examining elastic scattering of the latter, mediated by exchange of the virtual latter state—see the panel (a) in Fig. 3 for illustration (s and u channels are not shown). Using Eq. (10) in momentum space and the contour shown in the panel (b) in Fig. 3, we obtain the following nonrelativistic potential resulting from tachyon exchange:

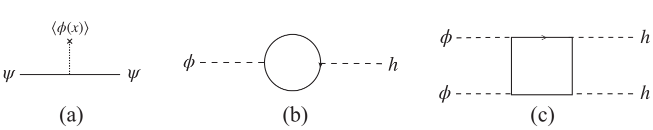

FIG. 2. Interactions of the virtual tachyon ϕ: (a) exchange between fermions inducing a long-range oscillating potential that violates Lorentz invariance; (b) mixing with the Higgs boson; (c) mixed quartic interaction $\phi^2 h^2$.

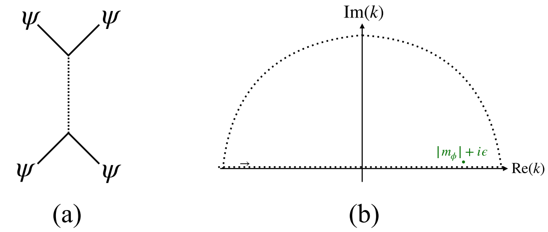

FIG. 3. (a) Elastic electron scattering mediated by virtual tachyon exchange. (b) The integration contour used to evaluate the nonrelativistic potential.

$$V_T(|\vec{r}|) = \mathrm{Re}\int \frac{d^3\vec{k}}{(2\pi)^3} \frac{e^{i\vec{k}\cdot\vec{r}}}{|\vec{k}|^2 + m_\phi^2 - i\epsilon} = \mathrm{Re}\frac{g^2 e^{i(|m_\phi|+i\epsilon)|x|}}{4\pi r} = \frac{g^2}{4\pi r}\cos(|m_\phi| r). \tag{31}$$

We used the fact that both bare and interacting tachyons correspond to $\epsilon \to 0^+$, leading to a new long-range and oscillating force. We note that in Ref. [54,55], bounds on such oscillating potentials have been discussed. In particular, in notation of [55], the following bound holds: $\alpha_0 < 10^7$ for $\lambda \simeq 35\mu$, which (assuming tachyon interactions are extended to other SM fields) translates into the bound $g < 2 \times 10^{-13}$ for $|m_\phi| \simeq 0.04$ eV.

Before discussing physical consequences of Eq. (31), let us make few remarks. First, the cross-section for the process $\psi\psi \to \psi\psi$ becomes divergent in some kinematic regimes if one works with momentum eigenstates of ψ. This is because such scattering corresponds to a spacelike regime, where the tachyon pole can be hit, resulting in on-shell production, $\psi\psi \to \phi$. We checked that this takes place for *both* FP and principal value propagators describing the tachyon. Therefore, it seems that using the real part of the FP does not consistently enforce the off-shell condition. However, analogous singularities in the t (or u) channel also take place for unstable bradyons. For example, in colliders the natural solution is to take into account the finite size of the beam by changing the virtual field propagator as: $1/(k^2 - m^2 + i\epsilon) \to 1/(k^2 - m^2 + i|k|/a)$, where a is the beam width [56]. Applying this prescription to tachyons, the divergence is removed, therefore, arguments invoking t-channel divergence may be inconclusive.

On the other hand, the nonrelativistic potential provides a different argument against the viability of virtual tachyons.

Indeed, since the potential is *static*, it corresponds to $\partial\phi/\partial t = 0$. Therefore, in Fourier space the energy must vanish, $\omega = 0$. Moreover, the potential corresponds to real Green's function of the Helmholtz equation. These hold for any mediator: massless, massive, or tachyonic, but only in the last case does the condition of vanishing energy corresponds to on-shell states, the shell $|\vec{k}| = |m_\phi|$. Therefore, the potential given by Eq. (31) actually includes *on-shell* tachyon states.

Moreover, due to the cosine term, a tachyon field carries *phase* information with characteristic wavelength $\lambda = 2\pi/m$, which leads to the spatiotemporal dependence of the tachyon field background resulting from all bradyons interacting by tachyon exchange. Indeed, at any point x, the total potential comes from contributions of all electrons in the Universe, and reads

$$\langle\phi(t,\vec{x})\rangle = g\sum_i \frac{\cos(|m_\phi||\vec{x}-\vec{x}_i|)}{4\pi|\vec{x}-\vec{x}_i|}. \tag{32}$$

Since the density of matter is nonuniform, the electrons moving away from a fixed observer will emit a blue-shifted (the zero-frequency field in rest frame becomes a high-frequency signal for the boosted observer) tachyon field, resulting in a whole spectrum of frequencies. Therefore, $\langle\phi(x)\rangle \neq \mathrm{const}$ becomes a frame-dependent quantity that varies in both time and space.

It also leads to breaking of both the equivalence principle and Lorentz invariance in the observable sector. This takes place since by Eq. (1), a virtual ϕ modifies the electron mass as follows:

$$m_e^{\text{effective}} = m_e + g\langle\phi(x)\rangle. \tag{33}$$

In summary, in this section, we showed that virtual tachyon field results in *unscreened*, frame-dependent background. It modifies the effective mass of the electrons that varies in both time and space, hence, breaking not only LI but also the equivalence principle. In particular, it defines a preferred frame, one in which $\langle\phi(t,\vec{x})\rangle$ is close to being isotropic. As a result, the relativity principle is broken.

V. CONCLUSIONS

Tachyonic behavior of classical or quantum fields is conventionally viewed as an instability in the long wavelength regime, as indicated by the presence of complex energies in the infrared mode decomposition. Our analysis of a covariant virtual tachyonic scalar further supports this interpretation since one cannot cure the instability by the fakeon prescription. Moreover, we have shown that interactions of the virtual tachyon with stable SM fields induce Lorentz violation effects for the latter, and we have provided quantitative bounds on such a scenario.

In conjunction with our previous work [27], we have established that formulation of a covariant QFT of superluminal objects—whether propagating or purely off shell—is *not* viable. While we have not excluded the formulation of tachyon QFT that would explicitly abandon one of the standard axioms, such as relativistic covariance, locality, and unitarity, any such attempt would not only face stringent constraints[3] but also questions about naturalness of such an approach.

Finally, our work provides further support to the lack of viability of asymptotic freedom in higher-derivative quantum gravity [17]. It also shows that constructions employing tachyons within Carrollian physics face obstacles in Minkowski space, as well as conclusively shows that application of generalized fakeon prescription to negative mass-squared fields is impossible.

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DATA AVAILABILITY

No data were created or analyzed in this study.

[3]For example, the recent work [57] has put a bound on tachyon mass within an unspecified nonlocal field theory of tachyons. Since our starting point is relativistic covariance, and keeping only the $|\vec{k}| > |m_\phi|$ modes is LI only on shell, such theories lie beyond our scope.

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