

TDCOSMO XXV: Measuring H_0 to 6.5% precision with quasar strong lensing and maximally flexible mass-sheet dynamics

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We present a blind time-delay cosmography measurement of the Hubble-Lemaître constant H_0 based on the quadruply imaged quasar SDSSJ1433 + 6007. Our analysis combines deep *Hubble Space Telescope* imaging, extended time-delay monitoring from the Wendelstein and Maidanak Observatories, and spatially resolved stellar kinematics from the Keck Cosmic Web Imager and Reionization Mapper. We build a robust lens model to reconstruct the mass distribution and high-signal-to-noise kinematic maps to break the mass-sheet degeneracy, explicitly accounting for the lens galaxy's oblateness, rotation, and anisotropy. Furthermore, we constrain the external convergence (κ_{ext}) by characterizing the line-of-sight environment using wide-field photometry from the Dark Energy Spectroscopic Instrument (DESI) Legacy Survey data release 10. We incorporate these constraints into our joint lensing and dynamical model, running multiple iterations to estimate random and systematic uncertainties. Accounting for maximal flexibility of the mass-sheet

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transformation, and assuming a flat Λ cold dark matter cosmology and an $\Omega_{m,0}$ prior from DESI data release 2, we infer $H_0 = 73.2^{+4.8}_{-4.7} \text{ km s}^{-1} \text{ Mpc}^{-1}$ (a 6.5% precision) and an internal mass-sheet parameter $\lambda_{\text{int}} = 1.12^{+0.05}_{-0.06}$. Notably, λ_{int} is 2σ away from unity for this system, highlighting the importance of treating it as a free parameter. Our H_0 measurement is consistent with the result from our 2025 milestone paper, and it will be included in our next hierarchical analysis to improve the overall precision. Moving forward, the comprehensive pipeline demonstrated herein establishes a robust framework that can be readily applied to future strongly lensed systems to further refine cosmological constraints.

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I. INTRODUCTION

The standard cosmological model, Λ cold dark matter, provides a remarkably consistent description of the Universe, successfully linking the temperature fluctuations of the cosmic microwave background (CMB) to the formation of large-scale structure. However, the precision of modern cosmology has revealed a discrepancy in this picture: the “ H_0 Tension.” This refers to the statistically significant discrepancy between the value of the Hubble-Lemaître constant (H_0) predicted by early-Universe physics and the value measured directly in the local (“late”) Universe [1].

On the one side, the Planck Collaboration infers $H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ by fitting the Λ cold dark matter (CDM) model to CMB power spectra [2]. This measurement is incredibly precise but is model dependent; it relies on the assumption that standard physics (e.g., standard recombination, fixed number of relativistic species) holds true from the epoch of recombination to the present day. Other such early-Universe methods include using baryon acoustic oscillations [3] or big bang nucleosynthesis [4], both of which yield measurements comparable to the *Planck* measurement.

On the other side, the local distance ladder, anchored by Cepheid variables and Type Ia supernovae (SNIa), provides a direct, geometric measurement of the expansion rate. The SH0ES Collaboration, utilizing Hubble Space Telescope (HST) photometry, reports a value of $H_0 = 73.04 \pm 1.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [5]. This value is approximately 5σ higher than the Planck prediction. Other local probes, such as those using the Tip of the Red Giant Branch (TRGB) to calibrate SNIa, have yielded values that sit between these two extremes [6,7]. Yet another set of measurements based on megamasers and surface brightness fluctuations are in agreement with the SH0ES value [8,9]. An up-to-date review of recent measurements is given by Valentino *et al.* [1].

The persistence of this tension, despite exhaustive searches for systematic errors in both the CMB and distance ladder analyses, suggests the exciting possibility of new physics [10]. Theoretical solutions range from early dark energy and additional relativistic species to modifications of General Relativity on cosmological scales [11,12]. To

distinguish between residual systematics and genuine physical anomalies, independent probes of H_0 —completely disjoint from the systematics of the CMB and the calibration of the Cepheid distance ladder—are critically needed.

Strong gravitational lensing of time-variable sources serves as exactly such an independent arbiter. As first proposed by Refsdal [13], the arrival time difference between multiple images of a lensed quasar depends on the “time-delay distance,” $D_{\Delta t}$, which is inversely proportional to H_0 . This method is geometric, independent of the sound horizon scale used in CMB and baryon acoustic oscillations measurements, and does not require the rung-by-rung calibration of the distance ladder. Consequently, time-delay cosmography has matured into a competitive cosmological probe, providing constraints on H_0 with precision comparable to the traditional methods (e.g., Refs. [14–20]). This is most commonly achieved by identifying and studying the time delays of strongly lensed quasar systems (e.g., Refs. [21–23]) or, more rarely, strongly lensed supernovae (e.g., Refs. [24–28]).

In this paper, we showcase a comprehensive “soup-to-nuts” analysis to determine H_0 using a quadruply lensed quasar system SDSSJ1433+6007 (hereafter J1433+6007), developing a pipeline for future lensed quasar studies built upon previous work by members of our team [29,30]. This encompasses a remeasurement of the excess time delays between quasar images (Sec. IV), inspection of the nearby field imaging and line-of-sight measurement (Sec. V), reconstruction of the HST imaging with lens modeling (Sec. VI), reduction and analysis of the spatially resolved kinematic information (Sec. VII) that is used to break the mass sheet degeneracy [31–33], and finally the construction of the full dynamical model of the lens galaxy to constrain cosmology (Sec. VIII). Before the analysis, we briefly review the lensing formalism used throughout the manuscript (Sec. II), then discuss the previous literature regarding this system (Sec. III). In Sec. IX, we display and discuss our final cosmological results. Finally, in Sec. X, we summarize our results and offer concluding remarks.

In keeping with our collaboration protocol and to avoid potential experimenter biases, we blind our cosmological parameters throughout our analysis; only after we completed our analysis did we unblind these factors and reveal our results in Sec. IX, without modification. To enforce

blindness, we abstained from printing and showing unblinded values prior to the set unblinding date,¹ only displaying blinded values (by multiplying the sample by a random number) to probe relative uncertainties. The parameters we blinded were H_0 , $D_{\Delta t}$, and D_d .

II. LENSING FORMALISM

We adopt the standard formalism of strong gravitational lensing. The mapping from the source plane position β to the image plane position θ is governed by the lens equation,

$$\beta = \theta - \alpha(\theta), \quad (1)$$

where $\alpha(\theta) = \nabla\psi(\theta)$ is the deflection angle and $\psi(\theta)$ is the deflection potential. The deflection potential $\psi(\theta)$ is fundamentally related to the dimensionless surface mass density, or convergence, by

$$\kappa(\theta) \equiv \frac{1}{2} \nabla^2 \psi(\theta). \quad (2)$$

The excess time delay of an image at position θ relative to the case of no lensing is given by

$$\Delta t(\theta) = \frac{D_{\Delta t}}{c} \left[\frac{1}{2} (\theta - \beta)^2 - \psi(\theta) \right], \quad (3)$$

which consists of geometric and gravitational time-delay components within the square brackets. The time-delay distance $D_{\Delta t}$ combines the angular diameter distances to the lens (D_d), to the source (D_s), and between them (D_{ds}):

$$D_{\Delta t} \equiv (1 + z_l) \frac{D_d D_s}{D_{ds}}. \quad (4)$$

In observations of a source with variable luminosity, we can only measure the excess time delay between images (positioned at θ_A and θ_B):

$$\Delta t_{AB} = \Delta t(\theta_B) - \Delta t(\theta_A) \propto D_{\Delta t}. \quad (5)$$

Because each angular diameter distance in $D_{\Delta t}$ is inversely proportional to H_0 , it follows that $D_{\Delta t} \propto H_0^{-1}$; therefore, measuring excess time delays alongside a robust model of $\kappa(\theta)$ yields H_0 . Imaging information is used to model the convergence profile of the lensing galaxy by reconstructing source structures within the lensed images.

¹Unblinding took place during a collaboration-wide teleconference held on April 7, 2026.

A. Mass-sheet degeneracy

While time-delay cosmography is conceptually robust, it is subject to specific systematic uncertainties. By far the most important of these is the mass-sheet degeneracy (MSD) [33,34]. The MSD arises from a transformation of the lens mass profile wherein the surface mass density $\kappa(\theta)$ can be scaled by a factor λ and supplemented by a constant mass sheet $(1 - \lambda)$, such that

$$\kappa_\lambda(\theta) = \lambda\kappa(\theta) + (1 - \lambda). \quad (6)$$

This mass-sheet transform (MST), in conjunction with a linear scaling of the source plane position by $\beta_\lambda = \lambda\beta$, leaves all imaging observables identical. However, it rescales the predicted time delays and time-delay distance by $1/\lambda$, and thus the inferred H_0 is scaled by a factor λ . In other words, lens modeling alone is completely degenerate with this theoretical and often unphysical mass-sheet parameter λ . However, stellar kinematic information, such as the velocity dispersion and the root-mean-squared velocity, is not subject to the MSD, making it an invaluable tool in breaking the degeneracy.

As shown by Shajib *et al.* [30], the true galaxy convergence profile κ_{gal} , while accounting for the MSD, can be described as

$$\kappa_{\text{gal}}(\theta) \approx (1 - \kappa_{\text{ext}}) [\lambda_{\text{int}} \kappa_{\text{model}}(\theta) + (1 - \lambda_{\text{int}}) \kappa_S(\theta)]. \quad (7)$$

Here, κ_{ext} represents the external convergence contribution from line-of-sight mass structures, λ_{int} is the internal mass-sheet transform parameter that rescales the convergence profile κ_{model} recovered from lens modeling, and κ_S describes a predetermined “variable” mass-sheet profile. This profile behaves like a true mass sheet within the regime constrained by strong lens modeling but rapidly approaches zero at larger radii. An appropriate functional form we adopt in this paper is given by

$$\kappa_S(\theta) = \frac{\theta_S^2}{\theta^2 + \theta_S^2}, \quad (8)$$

where the scale radius θ_S should be large enough such that it is indistinguishable from an exact mass sheet with the images used to constrain the lens model [30,31,35].

A useful parametrization of the mass-sheet variables is

$$\lambda_{\text{total}} \equiv \lambda_{\text{int}} (1 - \kappa_{\text{ext}}), \quad (9)$$

which represents the linear transformation of the lens model convergence due to the total mass-sheet contribution [see Eq. (7)]. The external convergence can be constrained using a ground-based photometric survey of the galaxies in the lens galaxy's environment or along the line of sight [36,37], while the internal mass-sheet component can be constrained by kinematic information, as previously stated. Only by restricting these two contributions to the overall manifestation of a mass sheet can one achieve an accurate measurement of H_0 . In this paper, we will account for the MSD in our dynamical modeling of J1433 + 6007, using spatially resolved kinematic measurements.

III. SDSSJ1433 + 6007

The quadruply imaged quasar system J1433 + 6007 was initially discovered by Agnello *et al.* [38] through an outlier-selection algorithm searching the Sloan Digital Sky Survey (SDSS) [39] DR12 catalog. Since its discovery, observations have been made with the HST Wide Field Camera 3 through programs 15320 (Principal Investigator: Tommaso Treu; in the F160W, F814W, and F475X filters) and 15177 (Principal Investigator: Anna Nierenberg; in the F105W and F140W filters). See Fig. 1 for a color HST image of J1433 + 6007 and our labeling scheme. The system displays a classic quadruply imaged cusp lensing orientation, despite the presence of a satellite galaxy S near

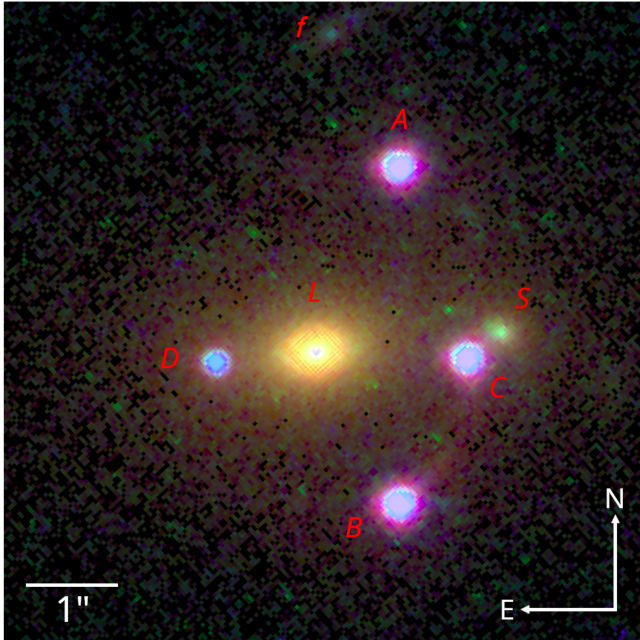


FIG. 1. HST imaging of J1433 + 6007, with the lens galaxy (L), satellite galaxy (S), and the lensed quasar images (A , B , C , and D) labeled. The faint background galaxy (f) is discussed in Sec. V A. The colored image is generated using the HST F160W, F814W, and F475X bands for the red, green, and blue channels, respectively.

image C . The source galaxy hosting the lensed quasar can be seen in a faint arc between images B and C , which is visible in all HST F160W, F814W, and F475X bands. We find that the source galaxy light profile is very prominent in the F160W band after accounting for and subtracting out the lens galaxy light (see Fig. 6). J1433 + 6007 has a lens redshift of $z_l = 0.407$ and a source redshift of $z_s = 2.737$, with an aperture-integrated velocity dispersion of $261 \pm 6(\text{statistical}) \pm 7(\text{systematic}) \text{ km s}^{-1}$ measured with the Echellette Spectrograph and Imager at the W. M. Keck Observatory [40]. From the kinematic information in Sec. VII, we find that the satellite galaxy shares the same redshift as the main perturber. Queirolo *et al.* [41] (hereafter Q25) present over three years' worth of monitoring observations from the Wendelstein Observatory, resulting in a marginalized time-delay measurement uncertainty of 3.9%.

A. Improvements from literature

Using their time-delay measurements and the HST imaging, Q25 performed a time-delay cosmography analysis on J1433 + 6007, recovering a measurement of $H_0 = 71.7^{+3.9}_{-3.6} \text{ km s}^{-1} \text{ Mpc}^{-1}$. In this paper, we independently revisit this measurement, equipped with more data, the detailed analysis required to leverage it, and a maximally flexible model to account for MSD systematics. Q25 did not utilize kinematic information nor line-of-sight information in their measurement of H_0 , relying solely on their lensing model and time-delay information. As previously discussed in Sec. II A, this may result in a value degenerate with the MSD, from both internal and external sources. In contrast, we use high-resolution spatially resolved kinematics from the Keck Cosmic Web Imager (KCWI) [42] instrument to build a dynamical model which jointly fits the kinematic and lensing/imaging data. Additionally, we use wide-field imaging of the environment around J1433 + 6007 (up to 120 arcseconds) to probe line-of-sight substructures and to identify possible perturbers that could contribute a significant flexion to the lens model. We also offer an improved lens model by accounting for a more complex source light structure in the IR range. Furthermore, we introduce an improved measurement of the time delays using additional observations from the Maidanak telescope (see Ref. [43]). Finally, by combining these advancements with an MSD-robust dynamical model, we can more accurately constrain H_0 .

IV. TIME-DELAY MEASUREMENTS

Q25 presented high-cadence light curves for the lensed quasar images in J1433 + 6007, derived from three years of monitoring at the Wendelstein Observatory (February 2020 to June 2023, with a mean cadence of roughly four nights). We augment that dataset with observations from the Maidanak Observatory [44] spanning May 2019 to June 2025. While the Maidanak light curve is more

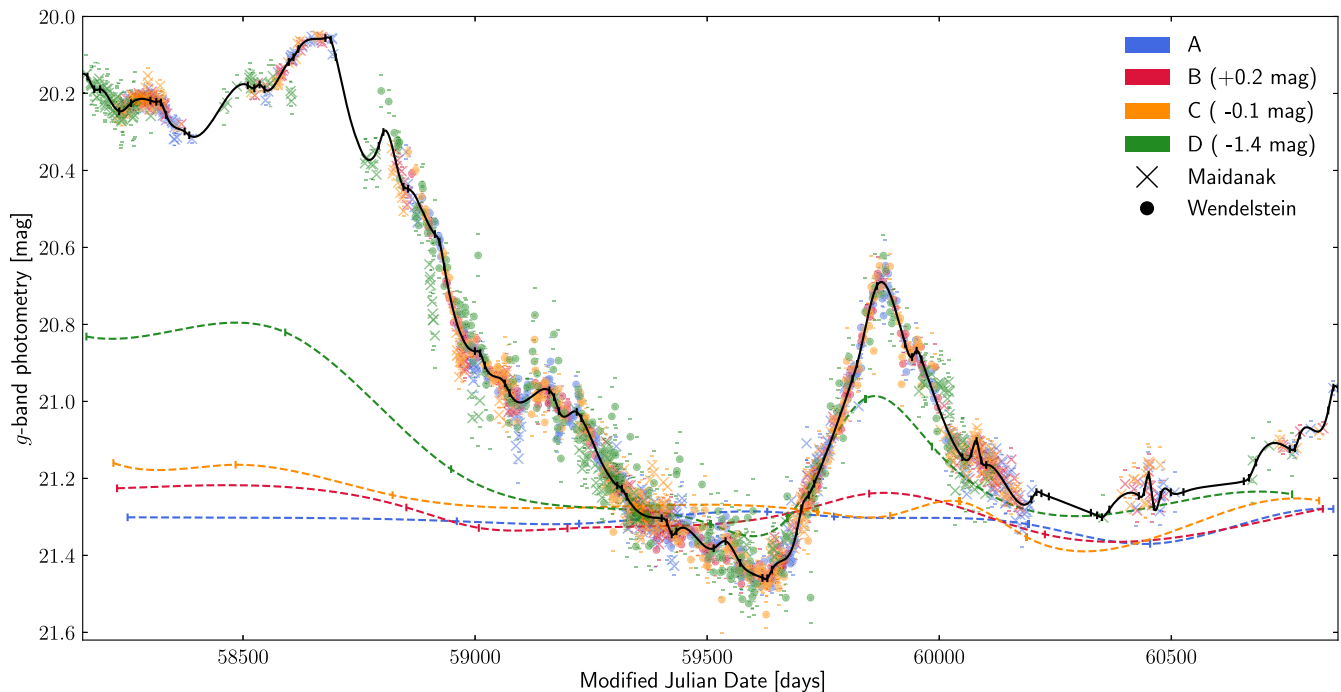


FIG. 2. The full light curve for the quasar images of J1433 + 6007. The lensed images' photometries are stacked after accounting for time delay and magnification differences (given in the top-right legend). We label measurements from the Maidanak Observatory with a cross ($\times$) and measurements from the Wendelstein Observatory with a dot ($\cdot$). As stated in the main text and shown in Fig. 3, we do not directly combine the different observatories' light curves in our analysis. For this figure, we add a flux (additive) and magnitude (multiplicative) shift to match their photometries. The light curve (in black) is then fit to the combined data, with respectively colored microlensing trends shown as dashed line for each lensed image. The purpose of this figure is to illustrate the overall coverage across both sets of observatory observations, highlighting the additional span provided by the new observations from the Maidanak Observatory.

sparse—with observations approximately every seven nights—and offers only a marginal improvement to the statistical constraint, it provides crucial confirmation that microlensing was not significant during the shorter Wendelstein baseline. This allows us to make further informed constraints on the microlensing effects of the spline light curve modeling in Q25. See Fig. 2 for the light curve coverage provided by the Maidanak and Wendelstein Observatories.

We adopt the methodology presented by Dux *et al.* [45], which is based on high-cadence r -band monitoring campaigns conducted with the 2.6 m European Southern Observatory Very Large Telescope Survey Telescope and the Max-Planck-Gesellschaft 2.2 m telescope. Photometric calibration and data preprocessing are performed using the LIGHTCURVER² pipeline [46]. We then use STARRED [47,48] to forward-model the quasar point sources and deblend the extended lensing galaxy emission. This method processes all epochs simultaneously by fitting a high-resolution pixelated model of the background and point sources convolved with epoch-specific point source function (PSF) kernels derived from field stars. Time delays are

estimated using the PyCS3³ toolbox by simultaneously aligning the light curves with free-knot splines to represent intrinsic quasar variability, while polynomials or splines are employed to model extrinsic microlensing variations [49–51]. Final uncertainties and covariance matrices are determined by applying this procedure to mock light curves containing injected red noise, thereby accounting for the degeneracies between intrinsic and extrinsic variability. Because of differences in the filters used at the two observatories, we do not directly combine the light curves; instead, we treat them as independent datasets and combine their time-delay constraints at the likelihood level.

In Fig. 3, we present the Maidanak, Wendelstein, and combined observatories' posteriors for the time delays relative to quasar image A. The total time-delay precision of each image relative to image A is 11.1%, 3.9%, and 3.6%, respectively. Assuming purely independent Gaussian errors, this yields a nominal combined precision of 2.6% (compared to 3.8% in Q25), which improves a key component of the H_0 uncertainty budget [Eq. (3)]. The true joint precision depends on the covariance between the relative delays, which we utilize in our lens model (see Sec. VI).

²<https://duxfrederic.github.io/lightcurver/>.

³<https://cosmograil.gitlab.io/PyCS3/>.

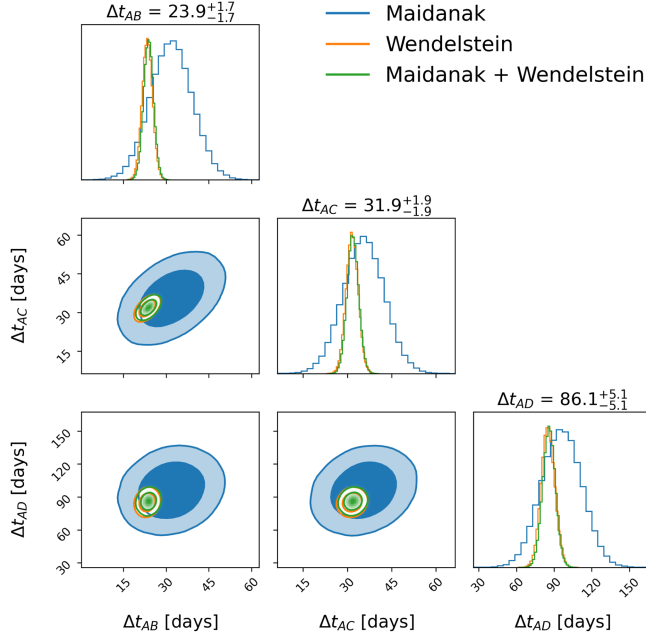


FIG. 3. The time-delay measurement (relative to quasar image A) posteriors from the Maidanak Observatory (this work; blue), Wendelstein Observatory (Q25 with more informed microlensing assumptions; orange), and the combined likelihood distribution (green). Both independent probes are consistent with each other, and the resulting combined posterior yields an uncertainty improved by an average factor of 1.5 compared to previous results.

V. LINE-OF-SIGHT MEASUREMENTS

The external convergence κ_{ext} , when not accounted for, acts as an external mass sheet to bias our cosmological values. Therefore, we constrain κ_{ext} by investigating nearby galaxies along the line-of-sight (LOS) to approximate their contribution to the MST. We use the Dark Energy Spectroscopic Instrument (DESI) Legacy Imaging Surveys data release 10 to probe structures up to 120 arcseconds away from J1433 + 6007 in the g , r , and z bands [52]. The g and r bands were observed with the 90Prime instrument (on the Bok telescope, as part of the Baryon Oscillation Spectroscopic Survey) [53] over four exposures each filter, and the z band was observed with Mosaic-3 instrument (on the Mayall telescope, as part of the Mayall z -band Legacy Survey) [54] over three exposures. From the stacked imaging, the g , r , and z bands' extended sources limiting magnitudes (5σ) are 24.23, 23.62, and 22.89, respectively [52]. These authors' source extraction algorithm Tractor photometrically catalogs these objects of interest, identifying 177 objects within the 120 arcseconds radius (and outside a 3 arcseconds radius) of J1433 + 6007 (excluding stars identified by Gaia) [55]. All the objects have photometric redshift estimates from Zhou *et al.* [56].

A. Identifying perturbers

From the stellar mass (M_*) versus velocity dispersion (σ_v) relation in the work by Zahid *et al.* [57], K -correcting the photometry according to their respective photometric redshifts, and assuming a constant M_*/L across the field (calibrated using the lens galaxy of J1433 + 6007), we estimate the velocity dispersion and uncertainty for each galaxy. If we additionally assume a spherical isothermal sphere (SIS) lens profile, the Einstein radius can be approximated through

$$\theta_E \approx 4\pi \frac{\sigma_v^2 D_{\text{ps}}}{c^2 D_s}, \quad (10)$$

where σ_v is the velocity dispersion, D_s is the angular diameter distance from the observer to the source, and D_{ps} is the angular diameter distance between the perturber and the source. We estimate these distance values using the photometric redshift distribution of the object and a fiducial Λ cold dark matter (CDM) cosmology of $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_{\text{m},0} = 0.3$. Our choice of a reasonable cosmology is negligible compared to the uncertainties of the photometric redshifts, when calculating their effective Einstein radius. The second-order lensing distortion (i.e., the flexion shift $\Delta_3 x$) can then be calculated through

$$\Delta_3 x = f(\xi) \frac{\theta_{\text{E,main}}^2 \theta_{\text{E,pert}}^2}{\delta^3}, \quad (11)$$

where $\theta_{\text{E,main}}$ is the Einstein radius of the primary lens, $\theta_{\text{E,pert}}$ is the estimated Einstein radius of the perturber, and δ is the 2D angular separation between the two objects [58]. The distance ratio ξ is defined as

$$\xi \equiv \frac{D_{\text{dp}} D_s}{D_{\text{p}} D_{\text{ds}}}, \quad (12)$$

where D_{dp} and D_{p} are the angular diameter distances between the lens and perturber and the observer and perturber, respectively. Finally, $f(\xi)$ in Eq. (11) is a piecewise function defined as

$$f(\xi) \equiv \begin{cases} 1 & \text{if } D_{\text{p}} < D_{\text{d}}, \\ (1 - \xi)^2 & \text{otherwise.} \end{cases} \quad (13)$$

From this analysis, we find that four objects exceed a conservative flexion shift threshold of $\Delta_3 x \geq 10^{-4}$ [58] and are therefore included in the lens model (see Fig. 4). Their broad Einstein radius uncertainties from this procedure are also incorporated into the lens model likelihood function. Although several assumptions are made in these measurements, this procedure allows for the identification of any possible perturbers that may

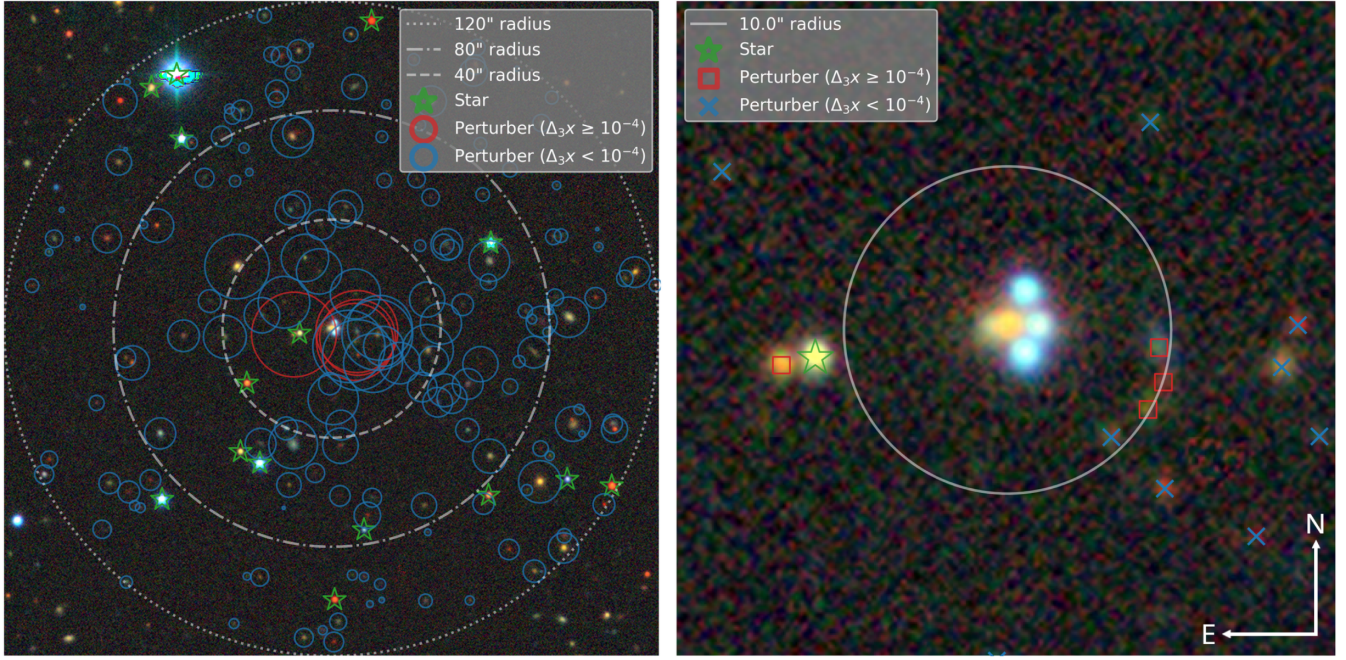


FIG. 4. Ground-based imaging of the field surrounding J1433 + 6007, from the DESI Legacy Surveys DR10. The color image is generated using the stacked Mosaic-3z, 90Prime r , and 90Prime g band imaging for the red, green, and blue channels, respectively. Left: the 120'' field around J1433 + 6007, with all objects detected by Tractor labeled. Stars are labeled with green stars, and galaxies are labeled with circles, with the area of the circle being proportional to the log $\Delta_3 x$. Galaxies with a flexion shift $\geq 10^{-4}$ are colored red. Right: the 40'' field, following the same color scheme as the left image. However, the galaxies with a flexion shift $\geq 10^{-4}$ are labeled with a square, whereas the galaxies with a flexion shift $< 10^{-4}$ are labeled with a cross symbol ($\times$). The four perturbers with flexion shifts $\geq 10^{-4}$ (captured in both fields) are accounted for in our lens model.

cause slight Fermat potential shifts in the data, which the lens model then more accurately constrains. Because three of them are in close spatial proximity to one another (contained within a 2 arcseconds radius aperture, 10 arcseconds from the main perturber) and possess similar photometric redshifts ($z_s = 0.79, 0.83$, and 0.71), their combined effect on the lens model would be highly degenerate. Therefore, we choose to aggregate these three objects before explicitly adding them into the lens model. Their combined center is calculated as an average weighted by their respective flexion shifts, and their combined Einstein radius prior is estimated by aggregating the total flexion shift contribution of the three perturbers.

Aside from these perturbers, there is also a faint galaxy 3.''6 North of the main galaxy (labeled as f in Fig. 1). While visible in the HST imaging ($m = 25.0, 24.1, 23.7$ in the F475X, F814W, and F160W bands, respectively), it is undetected in the DESI Legacy imaging. While this galaxy is relatively close to the lens, we do not expect this galaxy to significantly affect our Fermat potential measurements. The color, magnitude, and morphology are consistent with that of a background galaxy, which would contribute little to no flexion to our model. However, even if it were at the lens redshift, we expect it to have a much lower M_*/L

compared to the lens and other nearby galaxies simply due to its size and dimness. Therefore, we do not model any potential contributions from this object and mask out its light profile in the lens model.

B. Measuring external convergence

Using the catalog of galaxies identified by the Tractor algorithm within the 120 arcseconds radius field, we follow the pipeline by Ref. [37] to probe the LOS substructures. Succinctly, the algorithm performs a count of galaxies within the catalog (weighted by criteria such as the redshifts, inverse distance, and potential) and utilizes the Millennium Simulation [59] to generate fields similar to the weighted counts observed. In the pipeline and catalog, we impose a 23-magnitude cutoff to improve completeness, as it is well below the detection magnitudes for the 90Prime filters. From these simulations, we can infer the total convergence contribution of these LOS structures as κ_{ext} , where for J1433 + 6007 we calculate a posterior of $\kappa_{\text{ext}} = -0.041^{+0.047}_{-0.030}$ (see Fig. 5 for the full posterior). We find that J1433 + 6007 lies in a field that is slightly underdense compared to the average cosmic density ($\kappa_{\text{ext}} = 0$), though our measurement is still consistent within 1σ of being underdense, average, and overdense.

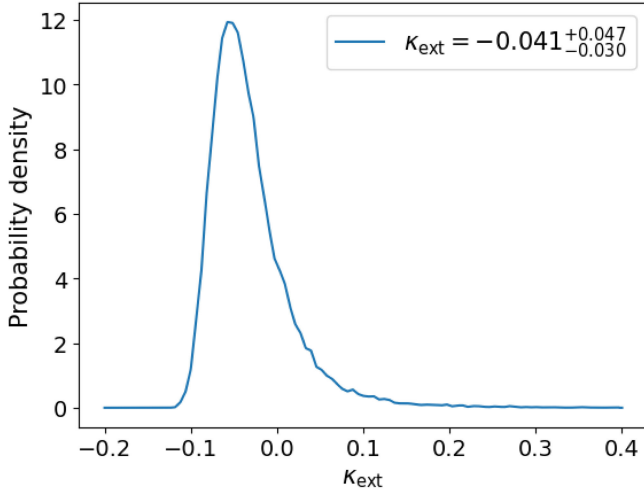


FIG. 5. The probability density function of the κ_{ext} posterior. This was obtained by using the 120 arcseconds LOS field imaging to measure a weighted count of galaxies of the nearby region, and generating different realizations from the cosmological N -body simulation Millennium simulations to probe the external convergence posterior.

VI. LENS MODEL

Using HST imaging in the F160W, F814W, and F475X filters (Program ID: 15320, Principal Investigator: Tommaso Treu), we construct a precise model of the lens galaxy 2D potential via flux based reconstruction. While there are also F105W and F140W band exposures publicly available for this system (Program ID: 15177, Principal Investigator: Anna Nierenberg), we choose to focus only on the three aforementioned bands because they have significantly higher exposure times (1500 to 2000 sec, compared to < 500 sec), and any additional features revealed in these IR bands are already emphasized in the F160W band. The source galaxy light profile, which is important in constraining the lens model, is prominent within the F160W band (after accounting for the lens light profile; see Fig. 6), whereas it is faintly visible in the F814W and F475X bands. The F160W band is then Nyquist subsampled to a pixel size of $0.''065$ (from a native pixel size of $0.''13$), as the four well-dithered exposures allowed for this higher resolution. The F814W and F475X bands were left at a pixel size of $0.''04$, as these bands do not contribute nearly as much lensing information compared to the F160W band. We use the software *Lenstronomy*⁴ [60,61] to construct our lens model. It has been shown to be robust in the Time-Delay Lens Modeling Challenge (TDLMC) [62], where two independent teams used *Lenstronomy* to recover lens model parameters within statistical consistency (in Rung 2). *Lenstronomy* has since been instrumental for its reliable lens modeling capabilities within the TDCOSMO

Collaboration and has been extensively tested both internally and externally (e.g., Refs. [63–66]).

A. Model specifications

We construct a model that jointly fits for all three HST bands, using *Lenstronomy* to constrain the same deflection model while varying certain light profile parameters between bands. All flux normalization amplitudes are linearly fit after the nonlinear parameters (e.g., deflection profile parameters and light profile parameters related to position and shape) are sampled. For the deflection profile, we use an elliptical power-law (EPL) [67] profile for each of the main deflectors and the satellites, an external shear component, and an SIS profile for the perturbors with flexion shifts $\geq 10^{-4}$. We also include broad priors on the satellite and perturbors' Einstein radii estimated from their luminosities (K -corrected from the HST F160W and the 90Prime photometry, respectively; see Sec. VA). As we are allowing for a flexible mass sheet in our dynamical analysis, we only account for a power-law profile in our lens model. The systematics introduced by a composite lens profile are already accounted for by our implementation of a maximally conservative mass sheet (e.g., Refs. [17,29]). The lens galaxy and satellite are positioned at the same redshift (as confirmed by our integral field spectroscopy data in Sec. VII) of $z_l = 0.407$.

As discussed in Sec. VA, we use two SIS profiles to account for the nearby perturbors that have exceeded the flexion shift threshold of $\geq 10^{-4}$. The aggregated perturber is set at the average photometric redshift (weighted by their inverse variance) of $z_p = 0.76$, and the isolated perturber is set at its measured photometric redshift of $z_p = 0.48$. We incorporate the uncertainty of their photo- z measurements into the Einstein radius prior uncertainty, which dominates the error budget compared to the photometry uncertainties.

To model the lens-plane light of the main deflector, we employ two elliptical Sérsic profiles [68] in each band: an exponential profile (Sérsic index of 1) and a de Vaucouleurs [69] profile (Sérsic index of 4). Because the deflector's emission in the F814W and F475X bands is significantly fainter and less complex than in the F160W band and since the F160W center is masked due to correlated noise (discussed below), we jointly constrain the lens light centroids across all bands and fix them to the center of the main deflector's EPL profile. Furthermore, we tie the axis ratios of the two Sérsic components together for the F814W and F475X bands, as their simpler light distributions do not warrant an additional degree of freedom. Although the F160W band provides the strongest lensing constraints, we adopt the F475X light model for our dynamical analysis; its wavelength range (~ 4000 to 6000 Å) closely matches our spectral coverage (see Sec. VII A). For the satellite galaxy, we fit a single Sérsic profile per band, fixing its center and ellipticity to its corresponding EPL profile. Finally, the lensed quasar

⁴<https://lenstronomy.readthedocs.io/en/latest>.

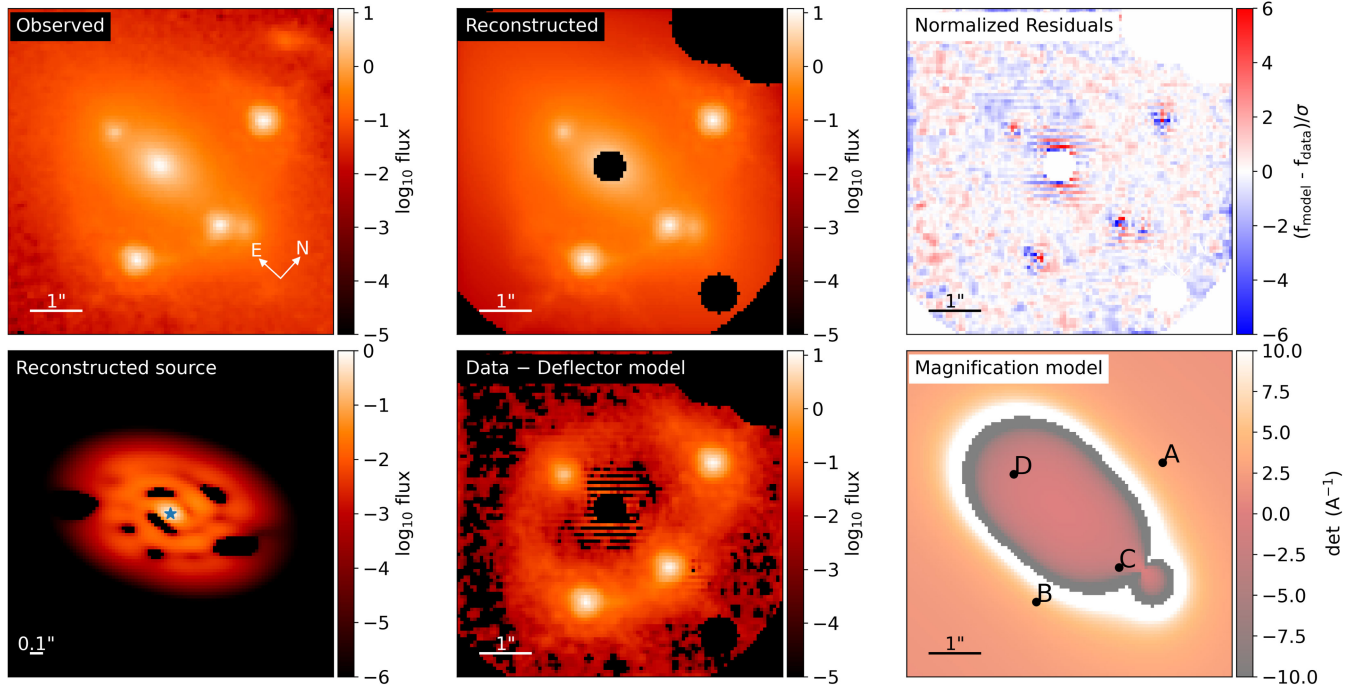


FIG. 6. Representative plots illustrating the lens model of J1433 + 6007 and its reconstruction of the HST F160W filter data. The top left panel shows the HST F160W observed image. The top middle panel shows the reconstructed image from our best-fit model. The top right panel shows the normalized residuals that are minimized in fitting for the model. The bottom left panel shows the reconstructed source of the best-fit model, with a star denoting the location of the point source. The bottom middle panel shows the HST observed image subtracted by the best-fit light profiles of the lens and satellite galaxies, illustrating the lensed arcs. The bottom right panel shows the magnification map of the lensing profile of the best-fit model, with the lensed point source locations labeled. We measure a $\chi^2_{\nu=8813}$ of 0.76 for the best-fit model to the F160W data.

images are modeled using a PSF. For each band, this PSF is initially constructed by stacking nearby stars in the HST field and then iteratively reconstructed at the pixel level. Our choice to Nyquist subsample the F160W band allows us to achieve higher resolution within the arcs and reduce large-scale aliasing artifacts. However, this in turn introduces correlated noise within the brightest regions of the image. Therefore, we opt to add masks near the center of the lens galaxy in the F160W model. We note that the F160W band is most informative to the lens model because of its ability to resolve the source arcs (as the arcs are inherently brighter in the observed IR band); a robust model is achieved by combining this with the astrometry-constraining power of the optical F814W and F475X bands; this is achieved by iteratively aligning the astrometry between all three filters.

We use an exponential profile to describe the source light for each band. Because the F160W band holds the most information regarding the source galaxy light, we also include a component modeled using a shapelet basis set for its source galaxy light profile [70,71]. This allows us to reconstruct and incorporate subfeatures within the source galaxy profile, providing a more accurate and constraining fit. We use shapelet orders of $n_{\max} = 9, 10$, and 11 (see Sec. VIC), which we find are sufficiently complex to

capture the details within the source. The center of the shapelets basis is fixed to the center of the exponential profile, which is shared across all three of the source exponential profiles (across filters).

We fit for the lensed quasar images individually on the image plane and include Gaussian priors (on the source plane) to ensure that the lensed image positions, when traced back to the source plane, are close ($\sigma = 0.''0003$) to the center of the exponential light profile. We also allow a deviation from the predicted quasar image position on the image plane (with a prior of $\sigma = 0.''001$) for each image to account for slight instrumental or systematic differences.

B. Incorporating time delays

To better inform our lens model, we include the time-delay measurements into the modeling process as an additional multivariate Gaussian prior into the likelihood function, derived from the time-delay covariance matrix (see Fig. 3). However, as the time delays are expected to be directly proportional to $D_{\Delta t}$ in addition to the Fermat potential, we must also sample and constrain $D_{\Delta t}$ in our pipeline. The resulting posterior on $D_{\Delta t}$ from our lens model is then marginalized over in the dynamical model (Sec. VIII).

TABLE I. Description and posteriors of the parameters sampled by the lens model, which are passed into the dynamical model in Sec. VIII. The lens model is constrained by the HST F160W, F814W, and F475X imaging data ($\mathcal{D}_I$), as well as with the time-delay information ($\mathcal{D}_T$). The $D_{\Delta t}$ parameter in our lens model was blinded during our analysis.

Parameter	Description	Lens model posterior $p(\omega \mathcal{D}_I, \mathcal{D}_T)$
θ_E ["]	Einstein radius	$1.538^{+0.005}_{-0.007}$
γ	Logarithmic slope of the power-law convergence profile	$1.954^{+0.015}_{-0.015}$
q_m	Minor-to-major axis ratio of the power-law convergence profile	$0.754^{+0.006}_{-0.006}$
R_{eff} ["]	Effective radius of the HST F475X light profile	$3.03^{+0.17}_{-0.13}$
q_l	Minor-to-major axis ratio of the HST F475X light profile	$0.70^{+0.01}_{-0.01}$
$D_{\Delta t}$ [Mpc]	Time-delay distance	2317^{+111}_{-111}

Our procedure for lens modeling consists of using a particle swarm optimization operation [72] to locate a maximum of the lens likelihood function that is likely close to the global maximum, followed by a Markov chain Monte Carlo (MCMC) pipeline to obtain the posterior of the model parameters [73]. The chains are then run until convergence is reached, before extracting the final posteriors.

C. Systematic tests

We run the pipeline modeling with the three different shapelets orders of the F160W source light profile ($n_{\text{max}} = 9, 10, \text{ and } 11$). We also test two marginally different masks for the F160W image to probe additional systematics, varying the full aperture radius of the imaging data (from the lens galaxy center) from 60 to 61 pixels ($\Delta r \approx 0.''065$) while slightly increasing the masked region North of image A to ensure that the total number of unmasked pixels is conserved. These tests result in six different model configurations. We find that the models are consistent with each other within the uncertainties. To account for these different choices in the error budget, the distribution used in our dynamical fit is a simple combination of the resulting chains of each model. We generate our final posterior distributions by sampling each set of these systematic chains, using their Bayesian information criteria (BIC) to calculate their respective weights.

From our lens modeling, we pass the six parameter posteriors ($\omega \in \{\theta_E, \gamma, q_m, R_{\text{eff}}, q_l, \tilde{D}_{\Delta t}\}$) into our dynamical modeling to describe the mass model, as well as the best-fit variables describing the F475X lens light profile shape. In Table I, we present the lens model posterior results for these parameters. The full lens model posteriors are provided in Appendix A (Tables V–VII). The lens model and residuals for the best-fit model (with shapelets order of $n_{\text{max}} = 10$ in the F160W filter) are shown in Fig. 6, resulting in a reduced chi-square of $\chi^2_{\nu=52695} = 1.06$. We also show the lens modeling results in the F814W and F475X filters in Appendix A (Figs. 12 and 13).

VII. INTEGRAL FIELD SPECTROSCOPY

The KCWI instrument [42] on the W. M. Keck Observatory is a powerful integral field spectrograph located on the summit of Maunakea, Hawaii. The main KCWI instrument can be concurrently processed by a blue channel (KCWI-blue) and red channel [the Keck Cosmic Reionization Mapper (KCRM)] [74]. Our configuration uses a 1×1 binning, a dual amp readout, and the smallest integral field unit (IFU) slicer size to maximize its spatial resolution. The KCWI-blue channel has a wavelength range from 3600 to 5600 Å, while the KCRM channel has a range of 5600 to 8850 Å. The red channel has the added disadvantage of being more susceptible to cosmic ray contamination; therefore, KCRM observations are taken three at a time and median-coadded to compensate. KCRM also suffers from a slightly lower signal-to-noise ratio (S/N) compared to the KCWI-blue channel. However, despite this, the KCRM data constrain well prominent absorption lines outside of the KCWI-blue coverage (see Fig. 7). We choose to mask a region between the two channels (from 5569 to 5834 Å observer frame) to avoid any distortions in the vicinity of the dichroic. Over two nights on May 7 2024 and June 8 2024, we observed J1433 + 6007 for 8 h on KCWI-blue and 7.25 h on KCRM excluding overhead times (Program ID: U077, Principal Investigator: Tommaso Treu).

While the KCWI slicer (with 1×1 binning) has a native, rectangular pixel size of $0.''35 \times 0.''147$, we drizzle and subsample to a square pixel size of $0.''1334$ using our well-dithered observing pattern. The KCWI-blue side has a spectral resolution of $R = 3600$, while the KCRM side has a spectral resolution of $R > 2000$. We also truncate the full spectral range to 5000 to 6750 Å in the observed frame (3554 to 4797 Å rest frame) to avoid telluric lines and dichroic uncertainties at the edges of the instruments' wavelength ranges. This 1750 Å observed range captures plenty of prominent absorption lines used for constraining the lens galaxy kinematics. In Fig. 7, we present the 1 arcseconds diameter aperture spectrum of the lens galaxy,

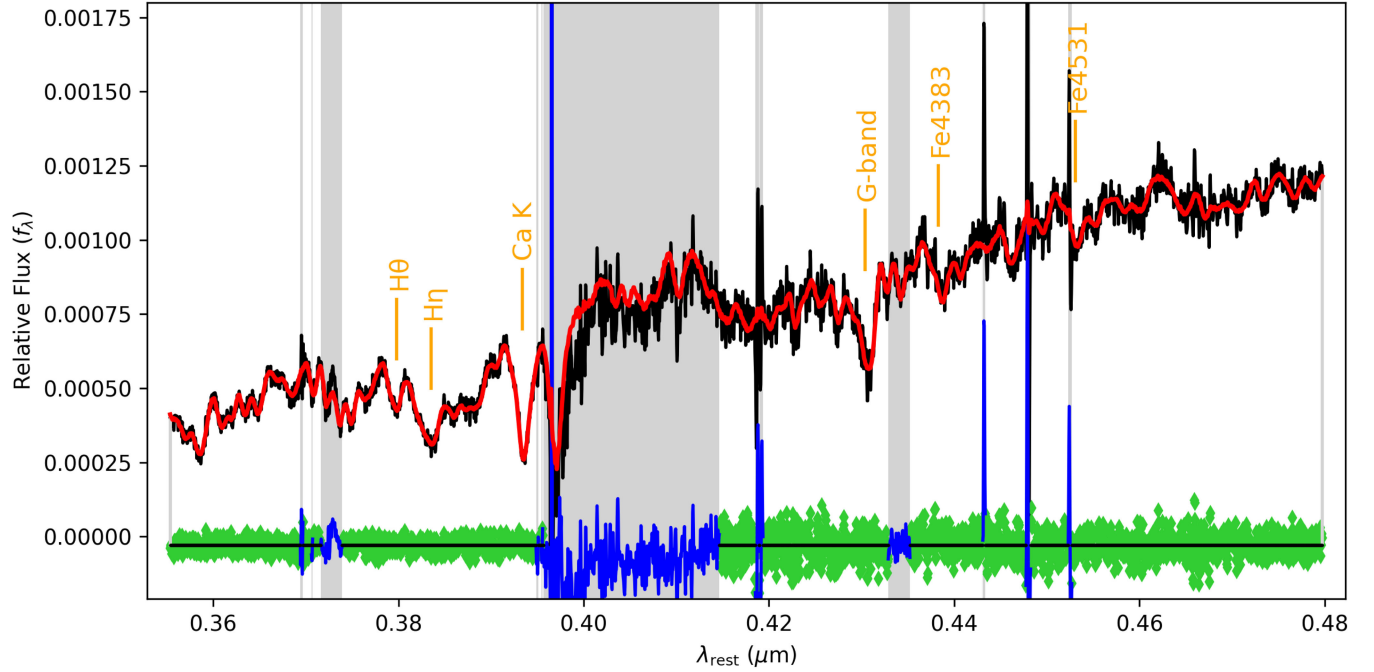


FIG. 7. The 1 arcseconds aperture spectrum of the lens galaxy of J1433 + 6007. The data are in black, the pPXF fit (using the MILES stellar library) is in red, and the residuals are in green. The shaded region represents the wavelength ranges excluded from the fit, and the blue shows the data points that are being masked out. Prominent absorption lines in the spectra have been annotated in orange. The pPXF model fits the data remarkably well ($\chi^2_{\nu=2755} = 0.98$), and there are strong absorption lines to tightly constrain the velocity dispersion at $\sigma_v = 260 \pm 5 \text{ km s}^{-1}$ (statistical).

along with the fitted model, masked region, and strong absorption lines present.

A. Spectral reduction and fitting

In Fig. 7, we show the integrated spectra and fit of J1433 + 6007 within an aperture of 1 arcseconds diameter. We use the pPXF⁵ package [75,76] for all kinematic fitting of the spectra in this analysis; an example is illustrated in Fig. 7. Additionally, because some spaxels are in close proximity to the lensed quasar images, we use the brightest spaxels of quasar images *A*, *B*, and *C* as additional stellar templates to fit for the quasar contribution to the spectra, following Shajib *et al.* [30]. We mask out $>3\sigma$ outliers in the spectral fit, as well as the H γ , H δ , and H κ lines, to avoid contamination from nebular gas emission and thermal broadening. We allow H θ and H η to remain in the fit, as they are useful stellar absorption lines and their nebular emission should be negligible. From probing a 0."5 diameter aperture of the satellite galaxy using this procedure, we have also confirmed that it lies at the same redshift as the main perturber.

To analyze the data while properly accounting for both statistical and systematic uncertainties of the velocity dispersion σ_v , we follow the procedures laid out by Knabel *et al.* [77]. As the line-of-sight velocity v_{LOS}

measurement is expected to be fairly simple and robust (i.e., the centers of absorption features), we do not probe the systematic uncertainties for v_{LOS} . Knabel *et al.* [77] has shown that the main systematic uncertainty contribution comes from differences in stellar libraries. Therefore, we experiment with different stellar libraries (Indo-US [78]; X-Shooter [79]; and MILES [80]), in addition to different additive polynomial orders (from four to eight) and multiplicative polynomial orders (from zero to four) in our pPXF fits. We note that each of these stellar libraries has been “cleaned” of irregular spectra which would contribute to nonphysical features in the fit. First, we vary different combinations of additive and multiplicative polynomial orders, and measure the BIC-weighted variance of σ_v across the three stellar libraries on the 1 arcseconds aperture spectra. We find that an additive polynomial order of 7 and a multiplicative polynomial order of 1 minimize the scatter across libraries (see Fig. 8). The overall effect on the BIC-weighted mean value of σ_v (with respect to an additive polynomial order of 7 and a multiplicative polynomial order of 1) ranges from -0.25% to $+1.88\%$, with the higher end attributed to an overfitted regime at an additive polynomial order of 8. If we exclude the plausible overfitted regime of an additive polynomial order of 8, this range shrinks to -0.25% to $+1.08\%$, a percent-level accuracy.

The analysis of the data cube involves binning the spaxels into Voronoi bins such that their total S/N are

⁵<https://pypi.org/project/ppxf>.

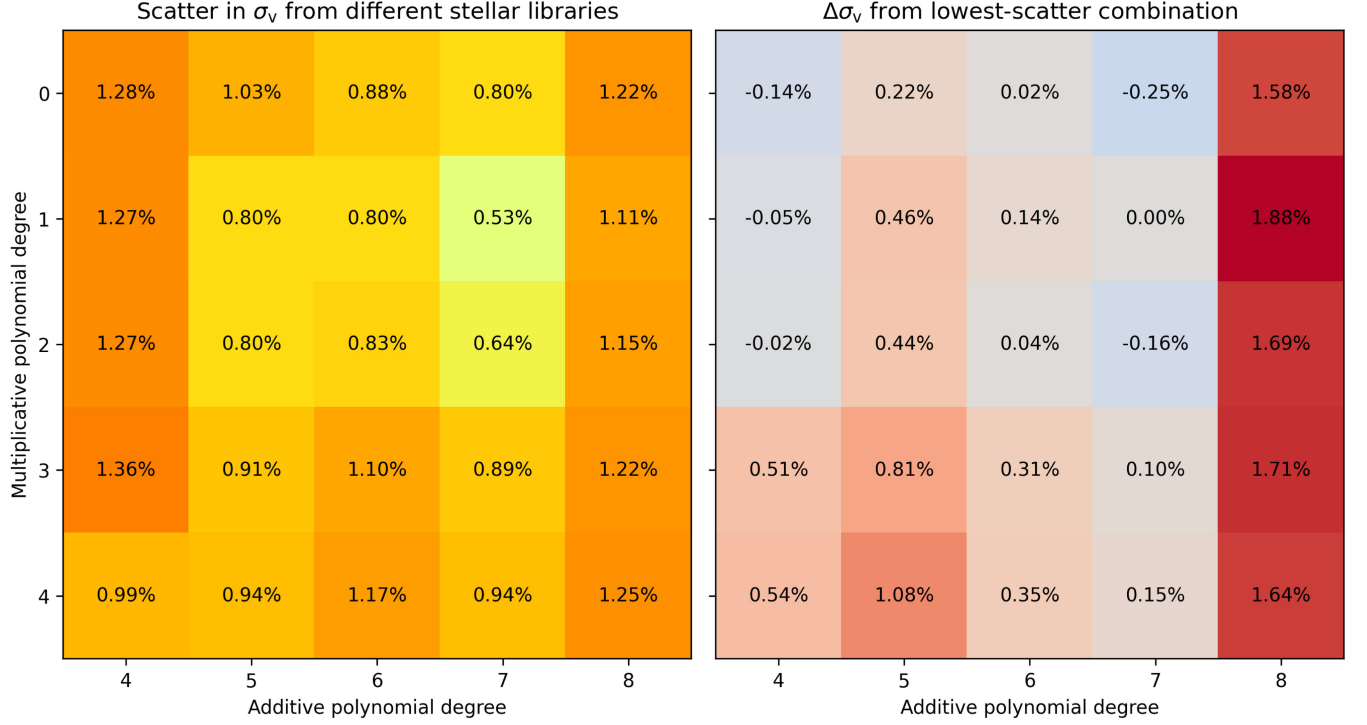


FIG. 8. Comparison plots to gauge the effects of different additive and multiplicative polynomial orders on the measurement of σ_v across three stellar libraries. Left: the effect of different polynomial orders on the BIC-weighted scatter of σ_v across stellar libraries. We find that additive and multiplicative degrees of 7 and 1, respectively, best reduce the scatter between stellar libraries. Hence, we use this configuration moving forward. Right: the effect of different polynomial orders on the BIC-weighted mean of σ_v across stellar libraries, relative to the measurement at the smallest-scatter configuration. We observe a clear overfitting regime at an additive degree of 8. If we exclude this regime, $\Delta\sigma_v$ between different polynomial degrees ranges from -0.25% to $+1.08\%$.

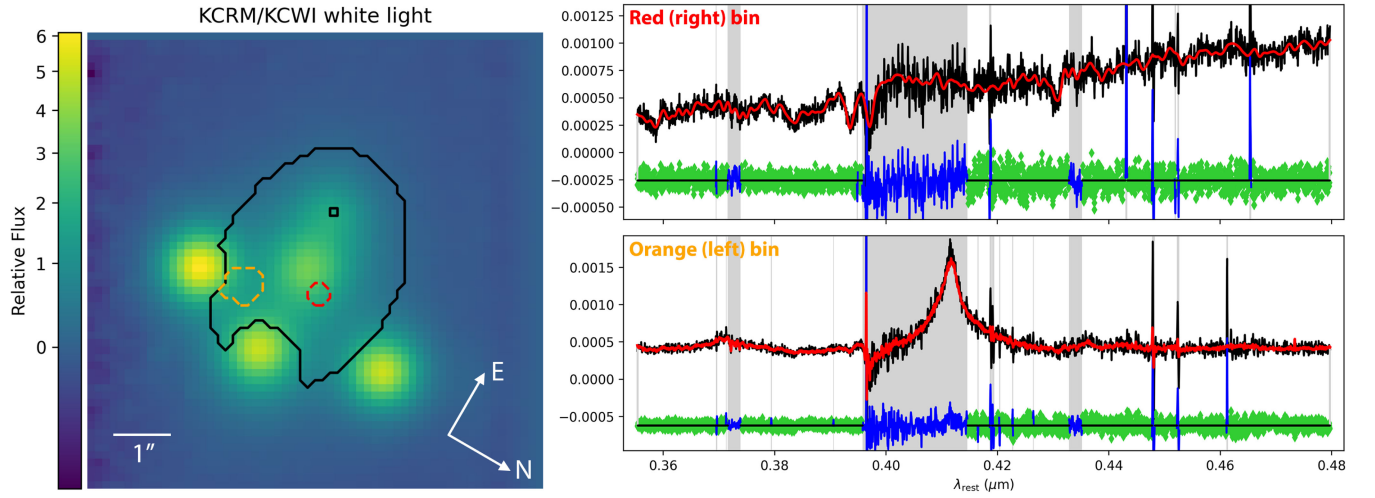


FIG. 9. Plots describing the KCRM/KCWI integral-field data. Left: the white-light image integrated from 5000 to 6750 Å (observer frame). The solid black outline describes the full extent of our kinematic map, and the dashed red and orange outlines denote specific bins from our final kinematic map for which we show their spectra to the right. Right: the 1D spectra integrated over the spaxels in their respective region on the left. Both spectra plots follow the same layout as Fig. 7, using ppxf and the MILES stellar library. The red bin (top right) shows the spectrum near the lens galaxy center, relatively isolated from the quasar light, while the orange bin (bottom right) shows the spectrum near quasar image *B*. Despite the quasar light contamination in the orange bin, a decent fit to the lens galaxy kinematics is achieved.

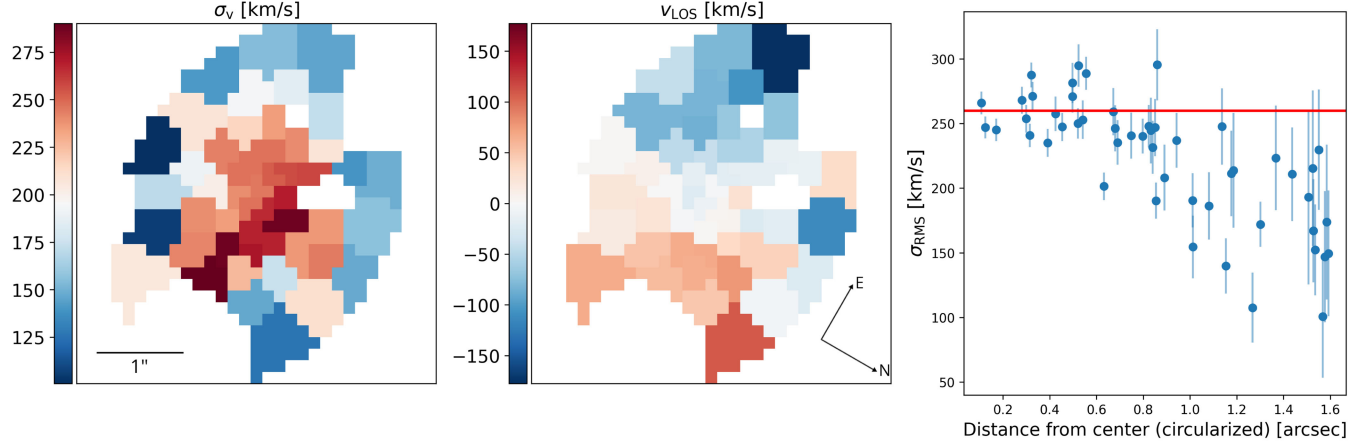


FIG. 10. Kinematic map for J1433 + 6007 using KCRM/KCWI. Left and middle: the velocity dispersion and the line-of-sight velocity maps, respectively. There is clear rotation displayed in the v_{LOS} map which aligns well with the light profile major axis ($\Delta\text{PA} = 15.7 \pm 8.5$ degrees), indicating an oblate symmetry. Right: the root-mean-squared velocity versus the distance between the flux-weighted bin center and the center of the lens, circularized to account for its light profile ellipticity. The red horizontal line denotes the 1 arcseconds aperture velocity dispersion measurement of 260 km s^{-1} . The kinematic map for σ_{RMS} is utilized in our dynamical model described in Sec. VIII.

comparable between bins. This is done using the Python package `Vorbin`⁶ [81]. However, to assess the systematic variance and covariance between bins, we first only use six bins each with a $\text{S/N} \approx 30 \text{ \AA}^{-1}$. This way, the statistical contribution to the uncertainties becomes negligible, and the systematic contribution can be accurately probed. Using the additive and multiplicative polynomial orders that minimize σ scatter (7 and 1, respectively), we measure the ΔBIC between the different stellar libraries by bootstrapping these six bins and calculate the libraries' associated weights w , as described by Knabel *et al.* [77]. For the Indo-US, X-Shooter, and MILES stellar libraries, we measure ΔBIC values of 3.4 ± 16.9 , 18.9 ± 25.5 , and 0.0 ± 16.6 and weights of 0.39, 0.16, and 0.45, respectively. Using w to weight each library's measurements of σ , we calculate the covariance matrix between the six Voronoi bins and find an average off-diagonal square root covariance of 0.88% and an average diagonal standard deviation of 1.20%.

Using the white-light image of the datacube (see Fig. 9), we fit for the KCWI/KCRM PSF, which needs to be accounted for in our dynamical model (e.g., see Ref. [82], for the potential uncertainties associated with errors on the PSF). From the white-light image, we forward model a simulated image using an exponential and a de Vaucouleurs profile to model the lens and four point sources for the quasar images. We do not subtract the source galaxy flux because, based on the HST F475X band (which shares a very similar wavelength coverage to the KCWI/KCRM region we are probing), the source flux is negligible (see Fig. 13). We also mask out any flux contribution from the satellite galaxy, and fit for the PSF with two concentric 2D

Gaussian distributions. This configuration should replicate well a Moffat PSF profile [83], capturing the core and wings of the light as distinct Gaussian profiles.

B. Kinematic maps

In our final kinematic maps, we bin to an $\text{S/N} \approx 10 \text{ \AA}^{-1}$, up to approximately the elliptical Einstein radius, resulting in 55 Voronoi bins. We also exclude spaxels near quasar images *B*, *C*, and *D* to avoid cases where the quasar light and associated Poisson noise overwhelm the lens galaxy signal. Additionally, we also exclude spaxels near the satellite galaxy, which may bias kinematic measurements. See Fig. 9 for the full binned region, as well as a set of representative binned-spectra fits. We also exclude four outlier bins with $\sigma_{\text{RMS}} > 300 \text{ km s}^{-1}$ measurements, which we ascribe to a combination of quasar light contamination, a low signal-to-noise ratio in the deflector, and spurious noise spikes coinciding with critical absorption lines. Figure 10 illustrates our final kinematic maps for σ_v , v_{LOS} , and σ_{RMS} , where $\sigma_{\text{RMS}} \equiv \sqrt{\sigma_v^2 + v_{\text{LOS}}^2}$.

We add in quadrature the systematic uncertainties and covariance we measured previously to the statistical uncertainties, measured from `pPXF`, for each bin. Most remarkable is the v_{LOS} map, which shows that the J1433 + 6007 lens galaxy is a rotating elliptical galaxy. By measuring the specific angular momentum of $\lambda_{\text{R}} = 0.149$ (defined in Ref. [84]), we find that this system is very near the nominal border distinguishing fast from slow rotating galaxies in Cappellari [85]. However, given that J1433 + 6007 is a massive elliptical galaxy lens (suggesting a density-supported rather than rotationally supported structure), strong evidence indicates that this is likely a slow-rotating system with a large inclination angle [86]. From measuring the

⁶<https://pypi.org/project/vorbin>.

kinematic position angle using the Python package PaFit,⁷ we find that it aligns well with the light position angle ($\Delta\text{PA} = 15.7 \pm 8.5$ degrees) and hence has an oblate symmetry, with a 99.97% certainty [87]. The σ_{RMS} map, which tracks the second moment of the line-of-sight velocity, is the constraint used in our dynamical model.

VIII. DYNAMICAL MODEL

Using the Python package Jampy,⁸ we construct a dynamical model for J1433 + 6007, constrained by both the lensing model and kinematic maps from the previous sections [88,89]. Given a 2D convergence profile, a 2D light profile, inclination angle, anisotropy parametrization, and angular diameter distance to the lens, our model generates kinematic measurement predictions at a given position relative to the lens galaxy. We assume a constant anisotropy for our analysis and hence sample the radius-independent anisotropy parameter; see Table II for definitions. The likelihood function we maximize is

$$p(\pi|\mathcal{D}) = \frac{p(\pi)}{p(\mathcal{D})} p(\mathcal{D}|\pi), \quad (14)$$

where $\mathcal{D}$ represents the superset of data used for our analysis (kinematic data $\mathcal{D}_{\text{K}}$, HST imaging $\mathcal{D}_{\text{I}}$, time-delay measurements $\mathcal{D}_{\text{T}}$, and LOS data $\mathcal{D}_{\text{E}}$) and π represents the parameters that we fit for in our dynamical model. These parameters are listed and described in Table II. By expanding $\mathcal{D}$ to components, we arrive at

$$p(\pi|\mathcal{D}) \propto \frac{p(\pi)}{p(\mathcal{D})} p(\mathcal{D}_{\text{K}}|\pi) p(\mathcal{D}_{\text{E}}|\pi) p(\mathcal{D}_{\text{I}}, \mathcal{D}_{\text{T}}|\pi). \quad (15)$$

Note that, while the datasets should be independently measured, they are still marginally dependent due to observing the same system. Therefore, the r.h.s. is proportional, not equal, to $p(\pi|\mathcal{D})$. This does not affect our log-likelihood maximization, only resulting in an additional additive constant. Further simplifying the r.h.s. via Bayes' theorem results in

$$p(\pi|\mathcal{D}) \propto p(\pi)^j p(\pi|\mathcal{D}_{\text{K}}) p(\pi|\mathcal{D}_{\text{E}}) p(\pi|\mathcal{D}_{\text{I}}, \mathcal{D}_{\text{T}}), \quad (16)$$

where $p(\pi)$ represents the prior on π shown in Table II and $p(\pi|\mathcal{D}_{\text{X}})$ is the dynamical model posterior based on its fit to the respective dataset(s). The exponent j is to ensure that the prior is not applied multiple times. As we only use these priors in the dynamical modeling likelihood (i.e., uniform priors are assumed for relevant parameters in π in the posteriors from the lens model and LOS analysis), $j = 1$.

$\mathcal{D}_{\text{E}}$ constrains only κ_{ext} in our dynamical model, while $\mathcal{D}_{\text{I}}$ and $\mathcal{D}_{\text{T}}$ (i.e., the lens model) constrain the parameter set ω .

All parameters in ω , with the exception of $D_{\Delta t}$, are in π ($\pi \cap \omega = \omega - \{D_{\Delta t}\}$), and thus these posteriors can be probed directly in our likelihood function. The constraint on $D_{\Delta t}$ can be translated to a constraint on the fitted cosmology ($\Omega_{\text{m},0}$ and H_0). Therefore, we can further transform the r.h.s. as

$$p(\pi|\mathcal{D}) \propto p(\pi) p(\pi|\mathcal{D}_{\text{K}}) p(\kappa_{\text{ext}}|\mathcal{D}_{\text{E}}) \times p(\pi \cap \omega, \Omega_{\text{m},0}, H_0|\mathcal{D}_{\text{I}}, \mathcal{D}_{\text{T}}), \quad (17)$$

and finally

$$p(\pi|\mathcal{D}) \propto p(\pi) p(\pi|\mathcal{D}_{\text{K}}) p(\kappa_{\text{ext}}|\mathcal{D}_{\text{E}}) p(\pi \cap \omega|\mathcal{D}_{\text{I}}, \mathcal{D}_{\text{T}}) \times p(D_{\Delta t}|\mathcal{D}_{\text{I}}, \mathcal{D}_{\text{T}}) p(\Omega_{\text{m},0}, H_0|D_{\Delta t}). \quad (18)$$

$p(\pi|\mathcal{D}_{\text{K}})$ represents the likelihood constraints provided by fitting our dynamical model to the kinematic data, $p(\kappa_{\text{ext}}|\mathcal{D}_{\text{E}})$ represents the posterior on κ_{ext} from the LOS measurements (see Sec. VB), $p(\pi \cap \omega|\mathcal{D}_{\text{I}}, \mathcal{D}_{\text{T}})$ represents the posterior from our lens model using both the imaging and time-delay information (see Sec. VI), and finally $p(\Omega_{\text{m},0}, H_0|D_{\Delta t})$ is the likelihood of a cosmology parameterization for a given $D_{\Delta t}$ value.

When building our dynamical model within the Jampy framework, we must provide a 2D surface profile of the tracer stellar population and a 2D surface potential profile of the mass profile (alongside other secondary parameters such as i , the kinematic PSF, the anisotropy configuration, black hole mass, etc). The 2D surface profile distribution is taken from our F475X light profile in the lens model, where we fix the best-fitting profile and vary its expansion/contraction by the total effective radius R_{eff} posterior (see Sec. VI for more details on the light model configuration). The 2D surface potential profile is parametrized as Eq. (7), accounting for a power-law convergence, an external convergence, and a variable mass-sheet contribution. To convert the unitless convergence profile into physical mass units $[\Sigma(\theta)]$,

$$\Sigma_{\text{gal}}(\theta) = \frac{c^2}{4\pi G} \frac{D_{\Delta t}}{D_{\text{d}}^2(1+z_l)} \kappa_{\text{gal}}(\theta), \quad (19)$$

where $D_{\Delta t}$ and D_{d} are derived from the sampled cosmology. Internally, Jampy converts the provided 2D light and density profiles into 3D profiles, which cannot be analytically done with an arbitrary parametrization. Therefore, we first decompose these 2D profiles through multi-Gaussian expansion (using MgeFit⁹ [90]), since each individual 2D Gaussian component can be analytically converted into a 3D component. For both mass and light, we fit 20 Gaussian components to the 2D surface profile, logarithmically sampled up to $50R_{\text{eff}}$.

⁷<https://pypi.org/project/pafit>.

⁸<https://pypi.org/project/jampy>.

⁹<https://pypi.org/project/mgefit>.

TABLE II. Description and priors of the parameters sampled by our dynamical model.

Parameter		Description	Dynamical model prior $p(\pi)$
θ_E	["]	Einstein radius of the convergence profile	$\sim \mathcal{U}(0, 10)$
γ		Logarithmic slope of the convergence profile	$\sim \mathcal{U}(0, 5)$
q_m		Minor-to-major axis ratio of the convergence profile	$\sim \mathcal{U}(0.05, 1)$
R_{eff}	["]	Effective radius of the HST F475X light profile	$\sim \mathcal{U}(0, 20)$
q_l		Minor-to-major axis ratio of the HST F475X light profile	$q_{l,\text{int}}(q_l, i) \sim \mathcal{N}(0.74, 0.08)$
i	[°]	Inclination angle	
κ_{ext}		External convergence	$\sim \mathcal{U}(-0.2, 0.4)$
θ_S	["]	Scale radius of the variable mass-sheet	$\sim \mathcal{U}(7.72, 15.43)$
$\beta_{\text{ani,cyl}}$		Cylindrically aligned, constant anisotropy parameter	$\sigma_z/\sigma_R \sim \mathcal{U}(0, 1)$
$\beta_{\text{ani,sph}}$		Spherically aligned, constant anisotropy parameter	$\sigma_r/\sigma_\theta \sim \mathcal{N}(1, 0.07)$
$\log(M_{\text{BH}})$	[log $M_\odot$]	Logarithmic central black hole mass	$\sim \mathcal{N}(8.78, 0.35)$
λ_{int}		Internal mass-sheet parameter	$\sim \mathcal{U}(0.88, 1.21)$
$\Omega_{m,0}$		Matter density parameter	DES-DR2 or Pantheon+
H_0	[km s ⁻¹ Mpc ⁻¹]	Hubble-Lemaître constant	$\sim \mathcal{U}(0, 150)$

Throughout our fitting process, we implement an iterative process to prune Voronoi bins that result in overwhelming χ^2 contributions $> 3\sigma$ (or a 0.28% statistical outlier) to the overall likelihood function. This is to ensure that our dynamical model is not being overfitted by these outlying bins, which could be caused by isolated velocity features in the lens galaxy. MCMC chains are well converged before extracting the model posteriors.

We take many precautions to prevent any possible source of bias in our dynamical model. Using toy models, Forés-Toribio *et al.* [82] demonstrated where these systematics may arise in the process. They showed that it is necessary to properly model the PSF kinematic data, which we accurately account for in Sec. VII A. Based on real data and simulations, constant anisotropy is a better description than Osipkov-Merritt [82,91], which is what we adopt in this analysis. Recent Schwarzschild modeling [92] indicates that orbital distributions are nearly isotropic [93]. This evidence effectively addresses the concerns raised by Forés-Toribio *et al.* [82], which were predicated on more extreme orbital scenarios. We agree with Forés-Toribio *et al.* [82] that inconsistencies may occur from a radial color gradient within the tracer population profile, which is why we build the tracer profile using observations from a similar wavelength range as the kinematic measurements. Modern analysis techniques, high-fidelity data, and refined stellar libraries (see Sec. VII and the work by Knabel *et al.* [77]) keep our kinematic systematics low, including spatial bin covariance. This methodology has been thoroughly tested (see the work by Knabel *et al.* [77] and Birrer *et al.* [17]), superseding the overly simplified scenarios presented in Forés-Toribio *et al.* [82].

A. Intrinsic axis ratio prior

The inclination angle (i), or the orientation of the lens galaxy relative to our LOS, can be defined in the following equation

$$q_{l,\text{int}}(q_l, i) \equiv \sqrt{\frac{q_l^2 - \cos^2 i}{\sin^2 i}}, \quad (20)$$

where q_l is the observed minor-to-major axis ratio of the 2D light profile and $q_{l,\text{int}}$ is the intrinsic axis ratio of the 3D galaxy. We use the posterior of $q_{l,\text{int}}(q_l, i) \sim \mathcal{N}(0.74, 0.08)$ from Li *et al.* (See the first row of Table 1 in Ref. [94]) as a joint prior on our sampled values q_l and i , which was derived using a sample of slow rotator early-type galaxies in SDSS-IV DR14 MaNGA. While formally $i \in [0^\circ, 90^\circ]$, we allow for $i \in [0^\circ, 180^\circ]$ as this is analytically identical and allows for the Markov chains to better capture the expected posterior.

B. Scale radius prior

We sample scale radii θ_S [which defines our variable mass sheet in Eq. (8)] such that the variable mass sheet is large enough to be agnostic of the lensing information. Through simulations, Birrer *et al.* [31] find that at $\theta_S \geq 5\theta_E$, the variable mass sheet follows very closely to a true mass sheet, which we use as lower bound for θ_S . We also set an upper bound of $\theta_S \leq 10\theta_E$, as unreasonably high values θ_S result in an artificially tighter, physically motivated constraints on λ_{int} (see Sec. VIII E). Since the error of the Einstein radius from the lens model is negligible compared to the range spanned by these bounds, we essentially sample $\theta_S \sim \mathcal{U}(7.''72, 15.''43)$.

C. Anisotropy parameter prior

We use constant anisotropy, which has been found to be a good description of both observed [93] and simulated galaxies [91], and is more accurate than the Osipkov-Merritt model [95,96] used a few years ago by our collaboration. Because J1433 + 6007 is a rotating elliptical galaxy (see Fig. 10), a cylindrically aligned velocity ellipsoid is more appropriate as a solution for the Jeans

equation in our dynamical model [88]. However, as an additional systematics check, we fit for two different models: with a cylindrically aligned or spherically aligned velocity ellipsoid solution. These different orientations also necessitate a slightly different definition of the constant anisotropy parameter β_{ani} . For the cylindrically aligned case β_{ani} is defined as

$$\beta_{\text{ani,cyl}} \equiv 1 - (\sigma_z/\sigma_R)^2, \quad (21)$$

and for the spherically aligned case,

$$\beta_{\text{ani,sph}} \equiv 1 - (\sigma_\theta/\sigma_r)^2. \quad (22)$$

σ_z and σ_R are the LOS and radial velocity dispersions in a cylindrical coordinate system, and σ_θ and σ_r are the tangential and radial velocity dispersions in a spherical coordinate system. We include a prior of $\sigma_z/\sigma_R \sim \mathcal{U}(0, 1)$ for the cylindrically-aligned model, based on physically and empirically motivated bounds [97].

For the spherically aligned model, we establish a Gaussian prior on the velocity dispersion ratio of $\sigma_r/\sigma_\theta \sim \mathcal{N}(1, 0.07)$. This prior was derived empirically by analyzing a sample of 13 representative early-type galaxies from Cappellari (see Ref. [93], Fig. 10). For each galaxy, the ratio $\sigma_r/\sigma_{\text{tang}}$ (which is equivalent to σ_r/σ_θ in our spherical model) was averaged across the radial range $R_e/30 < r < R_e$. This specific radial annulus was chosen because it exhibits relatively low scatter, meaning the residual radial variation is primarily driven by data or numerical noise rather than intrinsic physical features. Crucially, this selection intentionally excludes the very center of the galaxy, where the dynamics can be significantly influenced by the supermassive black hole and would necessitate a logistic Jeans anisotropic model rather than the constant anisotropy assumed here.

Averaging the values across these 13 galaxies yields a mean of 0.98 and a standard deviation of 0.07. Given the remarkable proximity of the mean to isotropy, we adopt a rounded mean of 1.0. We do not derive separate priors for fast versus slow rotators, as the resulting subsamples would be too small for reliable statistics and the anisotropy values are comparable between the two populations.

D. Central black hole mass prior

In our dynamical model, we also include a prior on the central black hole mass M_{BH} , as it has been found that it may significantly affect the kinematics near the center of the elliptical galaxy (see, e.g., Refs. [17,98]). As such, we include a prior on $\log(M_{\text{BH}})$, calculated using the well-studied linear scaling relation between the $\log(M_{\text{BH}})$ and aperture velocity dispersion for our lens galaxy ($\sigma_v = 260 \text{ km s}^{-1}$; see Fig. 7):

$$\log\left(\frac{M_{\text{BH}}}{M_\odot}\right) = \alpha_{\text{BH}} + \beta_{\text{BH}} \log\left(\frac{\sigma_v}{200 \text{ km s}^{-1}}\right). \quad (23)$$

We use the zero point ($\alpha_{\text{BH}} \sim 8.23 \pm 0.05$), slope ($\beta_{\text{BH}} \sim 4.86 \pm 0.50$), and intrinsic root-mean squared scatter ($\Delta_{\log(M_{\text{BH}})} = 0.34$) distributions of this linear relation from Graham and Scott [99], derived from their sample of 28 massive elliptical galaxies. And so, we include a prior of $\log(M_{\text{BH}}/M_\odot) \sim \mathcal{N}(8.78, 0.35)$.

E. Internal mass-sheet parameter prior

As shown in Sec. II A, accounting for the internal mass-sheet parameter is paramount in recovering an unbiased measurement of cosmology. Therefore, we set a physical upper bound for λ_{int} to ensure that the sampled $\kappa_{\text{gal}}(\theta)$ realization must be monotonic. We find that when $\lambda_{\text{int}} \geq 1.21$, there exists no reasonable sampling of the θ_E , γ , and θ_S such that the true galaxy convergence maintains monotonicity. We also ensure that the total mass of the variable mass-sheet within a three-dimensional radius does not exceed the mass of the NFW profile in the same sphere [31]. For our posteriors and range of scale radii, this effectively sets a lower limit of $\lambda_{\text{int}} \geq 0.88$. Previous analyses show that λ_{int} is close to unity and well within this range [17,30,100,101].

To facilitate comparison with analyses that break the MSD by enforcing a power-law model without the parameter λ_{int} [20,41,102] in Appendix B, we also run a set of dynamical modeling configurations where we fix $\lambda_{\text{int}} = 1$. This check highlights the importance of allowing it to be free for this system and provides a lower bound on the precision attainable with this system alone using present data.

F. Matter density parameter prior

We include a prior on the matter density parameter of $\Omega_{\text{m},0} = 0.3169 \pm 0.0065$ in the Λ cold dark matter (CDM) cosmology, from the DESI data release 2 (DESI-DR2), derived using baryon acoustic oscillation measurement [3]. As an additional test for systematics, we also impose an alternative prior of $\Omega_{\text{m},0} = 0.334 \pm 0.018$, from the Pantheon+ analysis of 1550 Type Ia supernovae from redshifts $0.001 \leq z \leq 2.26$ and also assuming a Λ cold dark matter (CDM) cosmology [103]. Both priors are measurements from only using relative distances and hence are independent of potential distance ladder biases. Although the time-delay distance $D_{\Delta t}$ for a relatively low-redshift lens system such as J1433 + 6007 ($z_1 \approx 0.407$) is only weakly sensitive to $\Omega_{\text{m},0}$ (with the effect suppressed by higher-order terms in the Taylor expansion of redshift), we explicitly marginalize over $\Omega_{\text{m},0}$ rather than assuming a fixed value. This ensures that any minor residual dependency is fully accounted for and its uncertainty is correctly propagated to our final H_0 error budget. Additionally, comparing results across different independent priors

for $\Omega_{m,0}$ (i.e., DESI-DR2 vs Pantheon+) serves as a crucial sanity check to demonstrate the robustness and insensitivity of our H_0 measurement to the assumed matter density prior.

We first run a given dynamical model with a uniform prior of $\Omega_{m,0} \sim \mathcal{U}(0.05, 0.5)$, then impose either the DESI-DR2 or Pantheon + prior on $\Omega_{m,0}$ by importance sampling the converged chains. This is effectively equivalent to rerunning the model with the normal prior established beforehand (assuming a sufficient number of samples).

IX. COSMOLOGICAL RESULTS

For our primary results, we run four different dynamical model configurations, with different assumptions on cosmology and anisotropy. See Table III for our labeling scheme moving forward. We aggregate our models to generate a final set of posteriors by using the ΔBIC (relative to the model with the lowest BIC) of each model to weight each individual distribution according to $w_{\text{BIC}} \propto \exp(-\frac{1}{2}\Delta\text{BIC})$. For each model, the BIC will be calculated using the median value for a given sampled parameter. Hereafter, whenever we combine dynamical model results, it is assumed we weight each result by w_{BIC} , which is given in Table VIII.

We present a table of our cosmological results in Table IV. We also show the full directly sampled posteriors for our dynamical models in Table VIII.

A. Discussion

In DyM1+2 (combining our anisotropy-alignment model and assuming a DESI-DR2 prior on $\Omega_{m,0}$, which we treat as a baseline), we measure $H_0 = 73.2^{+4.8}_{-4.7}$, a 6.5%-level precision. Our result is most in line with late-Universe measurements, such as those from the SH0ES team (from the Cepheid-calibrated distance ladder [5]). However, given our uncertainty, we remain in approximate 1σ agreement with measurements from the CCHP team (derived from the TRGB-calibrated distance ladder [104]), and even with early-Universe measurements (derived from the CMB power spectrum [2]).

Our results highlight aspects we expect to see in a dynamical model such as this. For example, we see that H_0 is positively correlated with λ_{int} and/or λ_{total} , corresponding to the mass-sheet degeneracy. Additionally, we find that our constraints on H_0 are robust against $\Omega_{m,0}$ (e.g., comparing

TABLE III. Our labeling scheme different dynamical model configurations.

Dynamical model ID	Anisotropy model	$\Omega_{m,0}$ prior
DyM1	Cylindrical	DESI-DR2
DyM2	Spherical	DESI-DR2
DyM3	Cylindrical	Pantheon+
DyM4	Spherical	Pantheon+

DyM1 and 2 with DyM3 and 4), as expected, since time-delay cosmography for a single system should only be weakly dependent on cosmology outside of H_0 .

We note some particularities that are more specific to J1433 + 6007 compared to other analyses of time-delay lenses. First, we find that cylindrically aligned anisotropic models are heavily preferred over spherically aligned ones for J1433 + 6007 (comparing the w_{BIC} values of DyM1 and 3 versus DyM2 and 4 in Table VIII). We attribute this to J1433 + 6007 being better described as a cylinder in the dynamical model, as evidenced by its significant rotation. We do not believe the spherically aligned model priors on anisotropy are overly constraining and therefore causing this preference, because the β_{ani} constraints in Table VIII are very similar between the alignment models.

Another feature to note is that we find that J1433 + 6007 prefers an internal mass-sheet parameter greater than one ($\lambda_{\text{int}} = 1.14^{+0.05}_{-0.06}$) at the 2σ level, implying the presence of a negative mass sheet. This is not unrealistic, as some systems have been found to have $\lambda_{\text{int}} > 1$ using kinematics [105]. Birrer *et al.* [17] measure a population mean of $\lambda_{\text{int}} = 1.06^{+0.06}_{-0.06}$ (population $\sigma \approx 0.11$) from eight lensed quasars, the Sloan Lens ACS (SLACS), and the Strong Lensing Legacy Survey (SL2S) lens datasets, which is in agreement with our measurement for J1433 + 6007.

B. Comparison with previous work

The most comparable analysis to the one presented here is that carried out by Shajib *et al.* [30] using KCWI data of the lens RXJ1131-1231 (henceforth RXJ1131). While many elements of the analyses are similar, we highlight the differences here to gain insight into why our analysis achieves higher precision. The main differences are the following:

- (1) We exploit the superior stellar velocity dispersion techniques introduced by Knabel *et al.* [77],

TABLE IV. Our final cosmological results for our dynamical models, aggregated by the anisotropy configurations (cylindrically and spherically aligned). Left of the dashed line are cosmological parameters directly sampled from the dynamical modeling pipeline; right of the dashed line are informative parameters derived directly from the left parameters.

Dynamical model ID	H_0 [km s ⁻¹ Mpc ⁻¹]	$\Omega_{m,0}$	λ_{int}	κ_{ext}	λ_{total}	$D_{\Delta t}$ [Mpc]	D_d [Mpc]
DyM1+2 (DESI-DR2)	$73.2^{+4.8}_{-4.7}$	$0.30^{+0.01}_{-0.01}$	$1.12^{+0.05}_{-0.06}$	$-0.02^{+0.04}_{-0.04}$	$1.14^{+0.05}_{-0.06}$	2032^{+141}_{-125}	1072^{+74}_{-68}
DyM3+4 (Pantheon+)	$72.6^{+5.0}_{-4.7}$	$0.33^{+0.02}_{-0.02}$	$1.11^{+0.06}_{-0.06}$	$-0.02^{+0.04}_{-0.03}$	$1.13^{+0.05}_{-0.05}$	2046^{+142}_{-133}	1069^{+75}_{-67}

developed after the Shajib *et al.* [30] analysis. As a result, the covariance between stellar kinematics measurements is reduced from a few percent to subpercent levels, improving overall accuracy. This is the main factor improving our ability to break the MSD.

- (2) While the overall precision of the time delays is comparable between J1433 + 6007 and RXJ1131, the latter is a cusp configuration with only one time delay measured to percent-level precision. In the lens analyzed here, the long delay is the most precise, but the other two are still measured with 6%–7% precision, providing some information to break degeneracies in the models.
- (3) The clear detection of a kinematic axis aligned with the galaxy major axis allows us to constrain the intrinsic axis ratio of light, the inclination, and oblateness (versus prolateness) probability of J1433 + 6007 much more tightly than RXJ1131. In the case of RXJ1131, Shajib *et al.* [30] find a probability of oblateness of 65%, compared to a probability of oblateness of 99.97% in J1433 + 6007 (see Sec. VII B).
- (4) Following Cappellari [93], we have imposed an informative prior on the anisotropy. Because of the clear rotation in J1433 + 6007, we are able to test the systematics between two different anisotropy alignments (cylindrical and spherical), whereas a

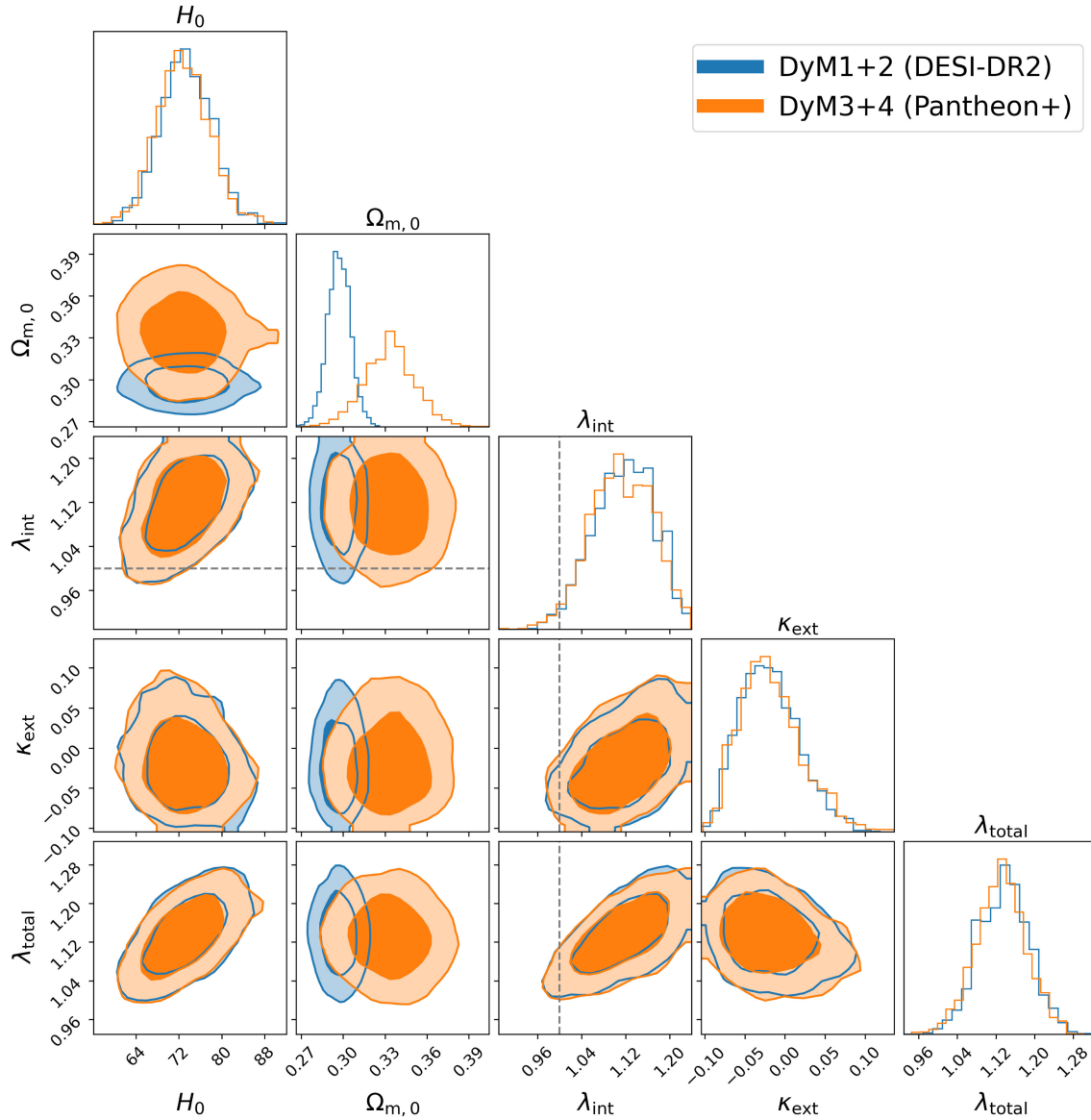


FIG. 11. Our dynamical modeling results, illustrating our cosmological constraints. See Table III for our labeling scheme, denoting different dynamical model configurations. The dotted line indicates $\lambda_{\text{int}} = 1$. We find that our measurement of H_0 is robust against different assumptions of $\Omega_{m,0}$, and we report a significant measurement of $\lambda_{\text{int}} > 1$.

spherically aligned model was most suited for RXJ1131. In the work by Shajib *et al.* [30], the authors use priors of $\sigma_\theta/\sigma_r \sim \mathcal{U}(0.78, 1.14)$ [97], whereas we use updated constraints of $\sigma_r/\sigma_\theta \sim \sigma_\theta/\sigma_r \sim \mathcal{N}(1, 0.07)$ [93].

- (5) We have allowed for the presence of a supermassive black hole in the center of the deflector, which was not considered in the analysis of RXJ1131.

X. CONCLUSIONS

We present a blind time-delay cosmography measurement of the Hubble-Lemaître constant H_0 using the quadruply imaged quasar system SDSSJ1433 + 6007. To achieve this, we develop a comprehensive soup-to-nuts pipeline that combines data from multiple independent observations to reconstruct the lens and constrain cosmology. Our analysis incorporates deep high-resolution HST imaging, extended time-delay monitoring from the Wendelstein and Maidanak Observatories, spatially resolved stellar kinematics from KCWI/KCRM, and wide-field photometry from the DESI Legacy Survey DR10.

By combining these datasets, we construct a robust lens model and incorporate high signal-to-noise kinematic maps to break the mass-sheet degeneracy. We lay out our reduction and analysis procedure for the IFU data, where we measure the lens galaxy’s specific physical characteristics, identifying it as a rotating elliptical galaxy with significant inclination and oblate symmetry. The inclusion of Maidanak Observatory data alongside the Wendelstein data yields a time-delay measurement uncertainty improved by a factor of 1.5 when compared to previously published results. Furthermore, by characterizing the local environment and line-of-sight structures using wide-field photometry, we successfully constrain the external convergence to $\kappa_{\text{ext}} = -0.041^{+0.047}_{-0.030}$.

We integrate these components into a highly flexible dynamical model to constrain cosmology while explicitly accounting for the mass-sheet transform. Assuming a flat Λ cold dark matter (CDM) cosmology and utilizing an $\Omega_{m,0}$ prior from DESI data release 2, our maximally flexible baseline models (which sample the internal mass-sheet parameter λ_{int}) measure $H_0 = 73.2^{+4.8}_{-4.7} \text{ km s}^{-1} \text{ Mpc}^{-1}$, with 6.5% precision.

Extensive systematic testing prior to unblinding reveals several key physical insights about J1433 + 6007. We find that our cosmological constraints on H_0 are remarkably robust against the choice of matter density priors. The system prefers a value greater than one for the mass-sheet parameter ($\lambda_{\text{int}} = 1.14^{+0.05}_{-0.06}$ for our baseline model) at the 2σ level. This implies the presence of a negative mass sheet, which is physically consistent with the galaxy being baryon-dominated at the Einstein radius. Additionally, the kinematic data heavily prefer a cylindrically aligned anisotropic model over a spherically aligned one, which

aligns with the clear rotational velocity profile observed in the lens.

J1433 + 600714 is expected to receive further improvements, with cycle-5 observations by the James Webb Space Telescope Near-Infrared Spectrograph to better assess the spatially resolved kinematics of the lens galaxy at higher angular resolution. Furthermore, adaptive-optics-assisted IR imaging from the Keck Observatory OSIRIS instrument may further improve the resolution of arc features and hence better constrain the lens model.

Ultimately, this work demonstrates the efficacy of our comprehensive, multiprobe pipeline in delivering a robust and independent measurement of the Hubble-Lemaître constant. Explicitly accounting for the mass-sheet transform and breaking the mass-sheet degeneracy requires a highly integrated approach; our precise H_0 constraints rely equally on the synthesis of deep HST imaging, extended time-delay monitoring, careful environmental characterization, and high-resolution spatially resolved kinematics. Looking forward, by combining this fully integrated analysis of SDSSJ1433 + 6007 with the broader TDCOSMO dataset, we aim to drive down systematic uncertainties even further and achieve tighter constraints on H_0 to help resolve the current cosmological tension. Furthermore, while the absolute distance scale measured via time-delay cosmography is primarily sensitive to H_0 and is robust to the choice of the $\Omega_{m,0}$ prior, the latter parameter becomes highly significant when combining these results with other cosmological datasets. Joint analyses of time-delay constraints with CMB anisotropy, galaxy clustering, and weak lensing measurements from forthcoming surveys (such as DESI, Euclid, Rubin, and Roman) will break the degeneracies between H_0 and $\Omega_{m,0}$, yielding tighter simultaneous constraints on both parameters and offering a powerful test of the standard Λ cold dark matter (CDM) framework and dark energy models.

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DATA AVAILABILITY

The Hubble Space Telescope data used for the modeling in this work are publicly available under the dataset title "Data for 17916 HST modeling" and can be accessed via [106]. The modeling was performed using the publicly available Lenstronomy software package [3,5].

APPENDIX A: ADDITIONAL FIGURES AND DATA REGARDING THE LENS MODEL

Here, we present the lens modeling results for the F814W and F475X filters (Figs. 12 and 13, respectively). We use the F475X fitted light profile of the main lens for our dynamical model in Sec. VIII.

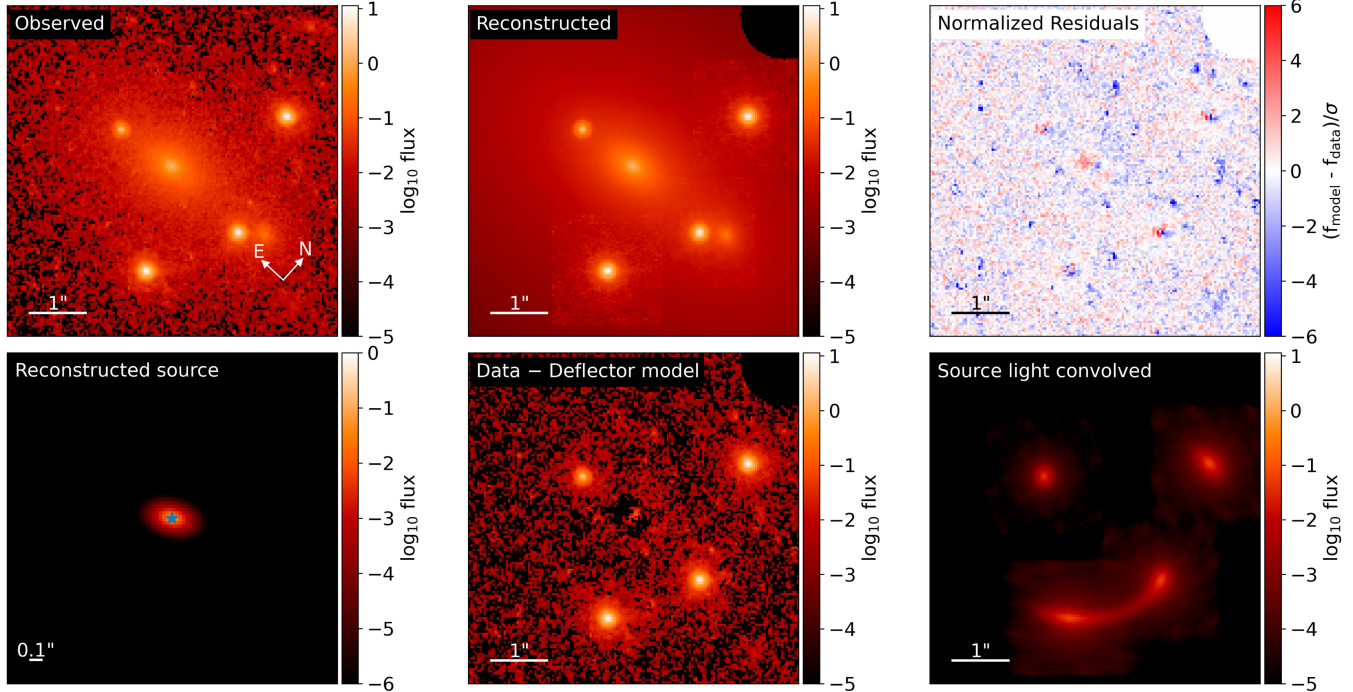


FIG. 12. Representative plots illustrating the lens model of J1433 + 6007 and its reconstruction of the HST F814W filter data. The layout of this figure (with the exception of the bottom right panel) is described in Fig. 6. The bottom right panel shows the convolved and lensed source light (excluding the point source) of our best-fit reconstruction. We measure a $\chi^2_{\nu=21941}$ of 1.16 for the F814W data fit.

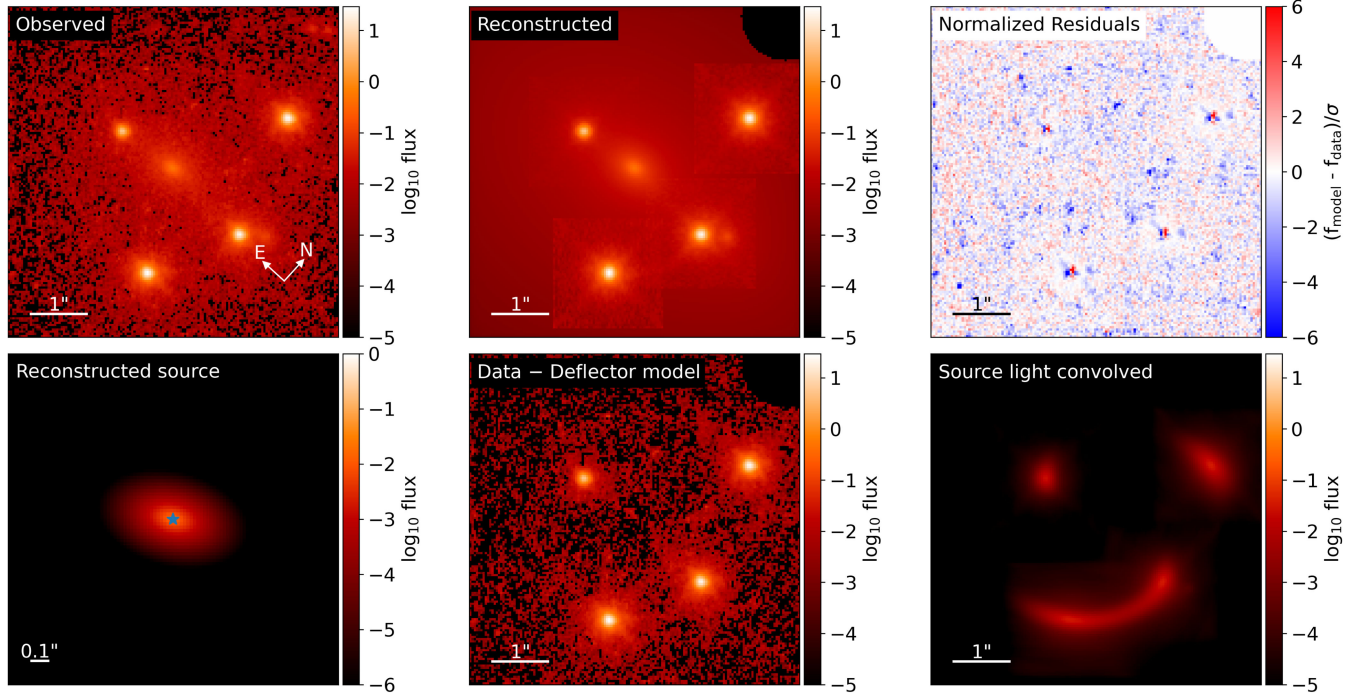


FIG. 13. Representative plots illustrating the lens model of J1433 + 6007, and its reconstruction of the HST F475X filter data. The layout of this figure (with the exception of the bottom right panel) is described in Fig. 6. The bottom right panel shows the convolved and lensed source light (excluding the point source) of our best-fit reconstruction. We measure a $\chi^2_{\nu=21941}$ of 1.10 for the F475X data fit.

TABLE V. Full lensing parameter posteriors derived from our modeling procedures (see Sec. VI). θ_E represents the Einstein radius, γ represents the logarithmic power-law profile slope, q represents the minor-to-major axis ratio, ϕ represents the east-of-north position angle of the major axis, γ_{ext} represents the total external shear strength, and ϕ_{ext} represents the position angle of the external shear.

Parameter		Lens	Satellite	Aggregated perturber	Isolated perturber	External shear
θ_E	[$''$]	$1.538^{+0.005}_{-0.007}$	$0.159^{+0.003}_{-0.002}$	$1.17^{+0.08}_{-0.07}$	$0.64^{+0.08}_{-0.09}$	...
γ		$1.954^{+0.015}_{-0.015}$	2	2	2	...
q		$0.754^{+0.006}_{-0.006}$	$0.756^{+0.016}_{-0.015}$	1	1	...
ϕ	[$^\circ$]	$-8.2^{+0.3}_{-0.3}$	$18.5^{+2.0}_{-2.1}$	...	...	...
γ_{ext}		...	...	...	...	$0.053^{+0.004}_{-0.004}$
ϕ_{ext}	[$^\circ$]	...	...	...	...	$-77.1^{+1.2}_{-1.2}$

TABLE VI. Full lens light parameter posteriors derived from our modeling procedures (see Sec. VI). For each filter, we give the distributions for the bulge (B) and disk (D) components of the lens, and the satellite. A represents the relative amplitude, R_{eff} represents the effective (half-light) radius, n represents the Sérsic index, q represents the minor-to-major axis ratio, and ϕ represents the east-of-north position angle of the major axis.

Parameter	F160W			F814W			F475X		
	Lens (B)	Lens (D)	Satellite	Lens (B)	Lens (D)	Satellite	Lens (B)	Lens (D)	Satellite
A	$112.7^{+7.5}_{-7.9}$	$2.61^{+0.16}_{-0.16}$	$46.5^{+1.2}_{-1.4}$	$34.2^{+1.2}_{-1.1}$	$1.92^{+0.08}_{-0.08}$	$5.88^{+0.10}_{-0.10}$	$9.54^{+0.71}_{-0.67}$	$1.29^{+0.07}_{-0.07}$	$2.03^{+0.03}_{-0.04}$
R_{eff}	[$''$]	$0.73^{+0.03}_{-0.03}$	$4.92^{+0.24}_{-0.22}$	$0.31^{+0.01}_{-0.01}$	$0.48^{+0.01}_{-0.01}$	$2.56^{+0.08}_{-0.08}$	$0.42^{+0.01}_{-0.01}$	$0.52^{+0.03}_{-0.03}$	$3.36^{+0.19}_{-0.15}$
n	$4.48^{+0.19}_{-0.18}$	1	4	4	1	4	4	1	4
q	$0.597^{+0.002}_{-0.002}$	$0.37^{+0.02}_{-0.02}$	$0.76^{+0.02}_{-0.02}$	$0.628^{+0.004}_{-0.004}$	$0.628^{+0.004}_{-0.004}$	$0.76^{+0.02}_{-0.02}$	$0.702^{+0.009}_{-0.009}$	$0.702^{+0.009}_{-0.009}$	$0.76^{+0.02}_{-0.02}$
ϕ	[$^\circ$]	$-9.1^{+0.1}_{-0.2}$	$1.5^{+0.8}_{-0.7}$	$18.5^{+2.0}_{-2.1}$	$-8.2^{+0.3}_{-0.3}$	$-8.2^{+0.3}_{-0.3}$	$18.5^{+2.0}_{-2.1}$	$-8.2^{+0.3}_{-0.3}$	$-8.2^{+0.3}_{-0.3}$

TABLE VII. Sérsic source light parameter posteriors derived from our modeling procedures (see Sec. VI). A represents the relative amplitude, R_{eff} represents the effective (half-light) radius, n represents the Sérsic index, q represents the minor-to-major axis ratio, and ϕ represents the east-of-north position angle of the major axis.

Parameter	F160W	F814W	F475X
A	339^{+88}_{-99}	21^{+11}_{-8}	74^{+140}_{-62}
R_{eff} ["]	$0.099^{+0.016}_{-0.017}$	$0.038^{+0.006}_{-0.004}$	$0.031^{+0.025}_{-0.008}$
n	1	1	1
q	$0.64^{+0.03}_{-0.03}$	$0.64^{+0.03}_{-0.03}$	$0.64^{+0.03}_{-0.03}$
ϕ [°]	-27^{+3}_{-3}	-27^{+3}_{-3}	-27^{+3}_{-3}

We also present the constraints for all relevant lens model parameters, including the lensing parameters (Table V), the lens light parameters (Table VI), and the source light parameters (Table VII).

APPENDIX B: ANALYSIS WITHOUT ACCOUNTING FOR THE INTERNAL MASS SHEET

To illustrate the importance of the MSD for this system, we run additional dynamical models where $\lambda_{\text{int}} = 1$ (i.e., removing the effect of an internal mass sheet). This is equivalent to enforcing a pure power-law model. In Table VIII, we present the full posteriors of our final dynamical models (for both our primary models DyM1–4, as well as for models where we fix $\lambda_{\text{int}} = 1$, denoted with a “— $\lambda_{\text{int}} = 1$ ” suffix in the dynamical model identification). All parameters sampled are described in Table II.

We see that by removing the internal mass sheet altogether we get a tighter constraint on H_0 (from 6.5% to 5.2%, while also shifting the median by -7%). This is to be expected, as H_0 is to first order linearly dependent on λ_{total} . The increase in precision is appreciable but not dramatic, since other terms in the error budget (time delay, Fermat potential, κ_{ext}) are comparable to the residual MSD and set an effective floor.

Despite the tension between our models with free and fixed λ_{int} , we observe a compensating effect with κ_{ext} (and to a lesser extent γ , which we address in Appendix C) in Fig. 14. Without the freedom provided by an internal mass sheet, the model compensates by manifesting an external mass sheet such that the total mass sheet (λ_{total}) is somewhat conserved. This counterbalancing phenomenon decreases the effect of fixing λ_{int} on cosmology and H_0 .

TABLE VIII. The results from our final dynamical models, labeled by the first column, based on the configurations defined in Table III. Excluding the columns that are directly being sampled (which are defined in Table II), we also add the column w_{BIC} which represents the associated BIC weight of a given model. We see that our results for H_0 are especially robust against different model configurations.

Dynamical model ID	w_{BIC}	θ_E ["]	γ	q_{in}	R_{eff} ["]	q_1	i [°]	κ_{ext}	θ_S ["]	β_{ani}	$\log(M_{\text{BH}})$ [log $M_{\odot}$]	λ_{int}	$\Omega_{\text{m},0}$	H_0 [km s $^{-1}$ Mpc $^{-1}$]
DyM1	42.3	$1.54^{+0.01}_{-0.01}$	$1.96^{+0.02}_{-0.01}$	$0.75^{+0.01}_{-0.01}$	$3.05^{+0.13}_{-0.14}$	$0.70^{+0.01}_{-0.01}$	89^{+22}_{-20}	$-0.02^{+0.04}_{-0.04}$	$9.5^{+2.9}_{-1.4}$	$0.11^{+0.10}_{-0.08}$	$8.88^{+0.35}_{-0.38}$	$1.12^{+0.05}_{-0.06}$	$0.30^{+0.01}_{-0.01}$	$73.1^{+4.9}_{-4.7}$
DyM2	3.6	$1.54^{+0.01}_{-0.01}$	$1.96^{+0.01}_{-0.01}$	$0.75^{+0.01}_{-0.01}$	$3.05^{+0.14}_{-0.16}$	$0.70^{+0.01}_{-0.01}$	88^{+20}_{-17}	$-0.03^{+0.04}_{-0.03}$	$9.6^{+2.7}_{-1.5}$	$0.10^{+0.10}_{-0.11}$	$8.89^{+0.38}_{-0.38}$	$1.13^{+0.05}_{-0.05}$	$0.30^{+0.01}_{-0.01}$	$74.4^{+4.8}_{-4.8}$
DyM3	50.3	$1.54^{+0.01}_{-0.01}$	$1.96^{+0.02}_{-0.01}$	$0.75^{+0.01}_{-0.01}$	$3.04^{+0.16}_{-0.14}$	$0.70^{+0.01}_{-0.01}$	90^{+21}_{-20}	$-0.02^{+0.05}_{-0.03}$	$9.6^{+2.9}_{-1.4}$	$0.11^{+0.09}_{-0.07}$	$8.86^{+0.38}_{-0.36}$	$1.11^{+0.06}_{-0.06}$	$0.33^{+0.02}_{-0.02}$	$72.5^{+5.1}_{-4.7}$
DyM4	3.8	$1.54^{+0.01}_{-0.01}$	$1.96^{+0.01}_{-0.01}$	$0.75^{+0.01}_{-0.01}$	$3.04^{+0.16}_{-0.16}$	$0.70^{+0.01}_{-0.01}$	91^{+19}_{-19}	$-0.03^{+0.04}_{-0.03}$	$9.5^{+2.7}_{-1.3}$	$0.09^{+0.10}_{-0.11}$	$8.88^{+0.40}_{-0.39}$	$1.13^{+0.05}_{-0.05}$	$0.33^{+0.02}_{-0.02}$	$74.5^{+4.7}_{-4.9}$
DyM1— $\lambda_{\text{int}} = 1$	49.0	$1.54^{+0.01}_{-0.01}$	$1.97^{+0.01}_{-0.01}$	$0.75^{+0.01}_{-0.01}$	$3.04^{+0.17}_{-0.17}$	$0.70^{+0.01}_{-0.01}$	90^{+20}_{-20}	$-0.05^{+0.03}_{-0.02}$	$11.7^{+2.4}_{-2.7}$	$0.16^{+0.09}_{-0.09}$	$9.08^{+0.50}_{-0.47}$	1	$0.30^{+0.01}_{-0.01}$	$68.1^{+3.8}_{-3.3}$
DyM2— $\lambda_{\text{int}} = 1$	2.4	$1.54^{+0.01}_{-0.01}$	$1.97^{+0.01}_{-0.01}$	$0.75^{+0.01}_{-0.01}$	$3.04^{+0.15}_{-0.15}$	$0.70^{+0.01}_{-0.01}$	89^{+19}_{-17}	$-0.06^{+0.02}_{-0.02}$	$11.6^{+2.8}_{-2.6}$	$0.16^{+0.08}_{-0.10}$	$9.11^{+0.47}_{-0.43}$	1	$0.30^{+0.01}_{-0.01}$	$68.4^{+3.5}_{-3.7}$
DyM3— $\lambda_{\text{int}} = 1$	46.5	$1.54^{+0.01}_{-0.01}$	$1.97^{+0.01}_{-0.01}$	$0.75^{+0.01}_{-0.01}$	$3.04^{+0.15}_{-0.16}$	$0.70^{+0.01}_{-0.01}$	91^{+20}_{-20}	$-0.05^{+0.03}_{-0.02}$	$11.8^{+2.4}_{-3.0}$	$0.15^{+0.09}_{-0.09}$	$9.05^{+0.45}_{-0.46}$	1	$0.33^{+0.02}_{-0.02}$	$67.7^{+3.7}_{-3.3}$
DyM4— $\lambda_{\text{int}} = 1$	2.1	$1.54^{+0.01}_{-0.01}$	$1.97^{+0.01}_{-0.01}$	$0.75^{+0.01}_{-0.01}$	$3.04^{+0.16}_{-0.14}$	$0.70^{+0.01}_{-0.01}$	90^{+18}_{-18}	$-0.06^{+0.03}_{-0.02}$	$11.6^{+2.6}_{-2.7}$	$0.15^{+0.09}_{-0.10}$	$9.07^{+0.46}_{-0.45}$	1	$0.34^{+0.02}_{-0.02}$	$68.3^{+3.7}_{-3.8}$

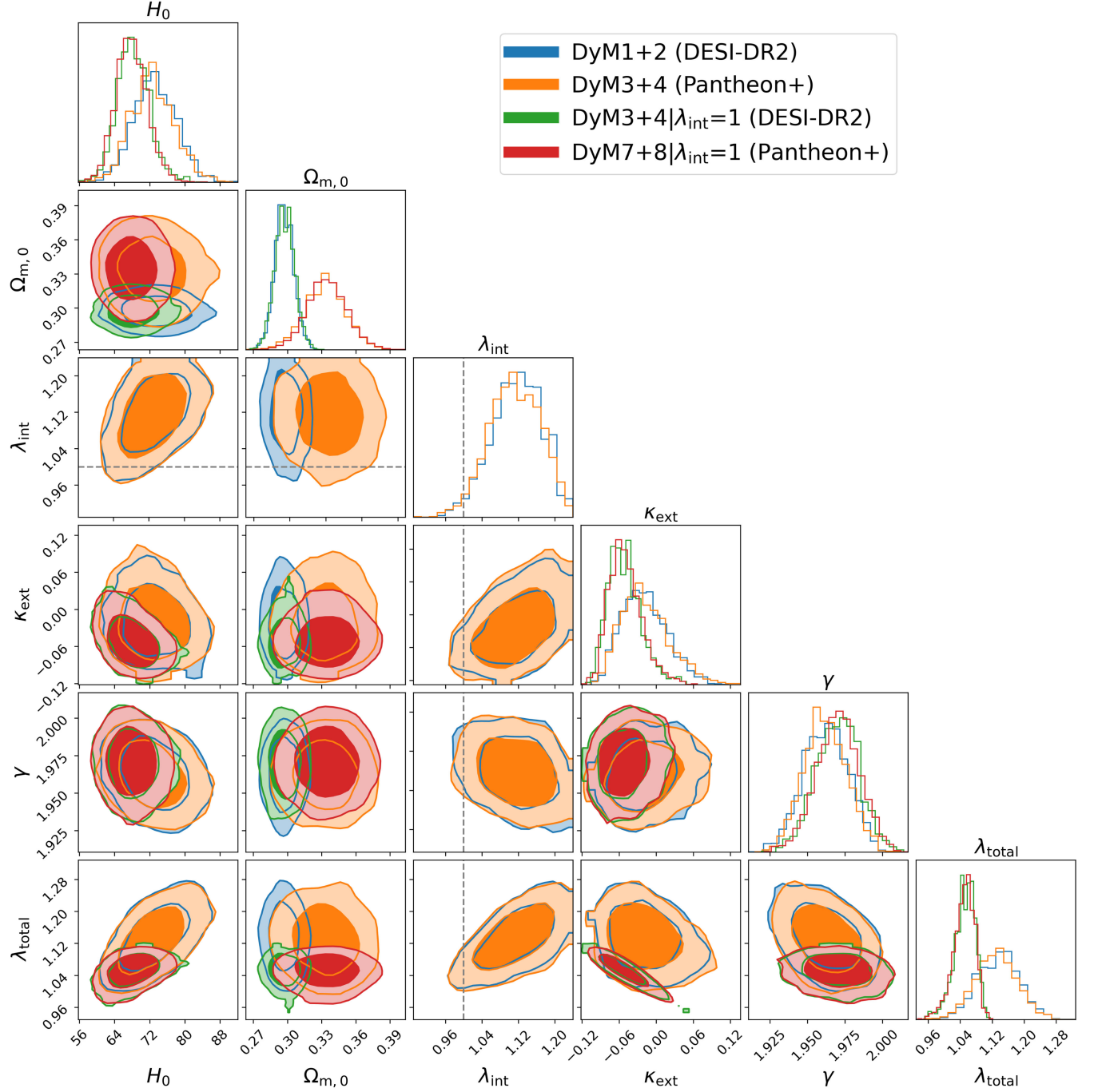


FIG. 14. Our dynamical modeling results, with our primary and $\lambda_{\text{int}} = 1$ model shown. See Table III for our labeling scheme, denoting different dynamical model configurations. The dotted line indicates $\lambda_{\text{int}} = 1$. By fixing $\lambda_{\text{int}} = 1$, we observe a compensating effect on κ_{ext} (and γ to a lesser extent), leading to a smaller discrepancy with λ_{total} and H_0 . All models have measurements of H_0 consistent with each other.

APPENDIX C: ROBUSTNESS TESTING WITH THE CONVERGENCE SLOPE

Our results show that when the dynamical model is allowed additional freedom by not fixing λ_{int} , it shifts the γ distribution slightly higher (by 0.5%; see Fig. 11 and Table VIII). This indicates that the kinematic data prefer a somewhat higher γ than what is predicted by the lens model. To test whether this has an appreciable impact on H_0 , we rerun DyM1+2 with double the uncertainty on γ

from the lens model posteriors. This is done by multiplying the row and column corresponding to γ in the lens model covariance matrix by 2.

We find that even with double the lensing uncertainty on γ , the resulting γ dynamical posterior peak only shifts 1% (from $\gamma = 1.96^{+0.02}_{-0.01}$ in DyM1+2, to $\gamma = 1.98^{+0.03}_{-0.02}$ in case 1). The measured H_0 posterior peak only shifts by -0.9% , a subpercent level, much less than our estimated errors.

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