Obstructions to minimal regular black hole cosmologies

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We derive an obstruction to Friedmann–Lemaître–Robertson–Walker (FLRW) daughter cosmologies from static, asymptotically flat regular black holes. The trapped region of such a parent is Kantowski–Sachs rather than FLRW, so the daughter must be introduced as a separate matched region. For closed daughters, the angular Darmois condition is controlled by the Misner–Sharp mass: asymptotic flatness and finite Arnowitt–Deser–Misner mass force the induced density to decay as A^{-3}, while the $k = +1$ curvature term scales as A^{-2}. The minimal closed branch is therefore bounded rather than indefinitely expanding. Flat and open daughters avoid this boundedness mechanism. For their maximal homogeneous FLRW continuations, the regular-end affine-averaged null energy condition (ANEC) theorem excludes simultaneous nonstaticity, curvature regularity, null geodesic completeness, and ANEC consistency. For Bardeen, the parent source does not naturally supply the late-time support needed for an unbounded closed daughter. Within these criteria, a viable FLRW daughter therefore requires additional structure, such as modified asymptotics, nonminimal matching, non-FLRW evolution, or an additional stress-energy component.

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I. INTRODUCTION

A recurring idea in gravitational physics is that black-hole interiors may admit a cosmological interpretation. In singular examples such as Schwarzschild, the trapped region can be rewritten as a Kantowski–Sachs spacetime, while in regular black holes the singularity is replaced by a nonsingular core, often locally de Sitter–like near the center. This raises a simple question: can a regular black-hole interior furnish a genuine daughter cosmology, or does such an interpretation require additional global structure or a new stress-energy component?

To address this question, we consider static, spherically symmetric, asymptotically flat regular black holes with trapped interiors, using the Bardeen black hole as a representative example. By a *minimal daughter construction* we mean a no-shell Friedmann–Lemaître–Robertson–Walker (FLRW) matching whose evolution is fixed by the parent metric, with no added late-time bulk sector, modified asymptotics, or independent shell stress tensor. The FLRW daughter is therefore a separate homogeneous and isotropic matched continuation rather than a coordinate rewriting of the Kantowski–Sachs trapped region. Here "no-shell" means no Israel surface stress tensor; we do not assume that the parent and daughter arise from one globally defined microscopic matter field. Completeness statements about flat/open daughters refer to the maximal homogeneous FLRW continuation determined by the matched scale-factor law, not to an otherwise unspecified global parent-plus-daughter composite. We impose the averaged null energy condition (ANEC) as a minimal nonexoticity criterion: it allows controlled local violations of the pointwise null energy condition while constraining the averaged null energy along complete null geodesics [1,2]. The completeness theorem used below is formulated under the same assumption and for the regular endpoint classes specified there [3,4].

Throughout, an indefinitely expanding daughter means an unbounded branch with scale factor $A(\tau) \to \infty$, rather than a long-lived finite expansion between turning points. The main flat/open/closed dichotomy is formulated for the one-function static spherical class $g_{tt}g_{rr} = -1$; the closed-branch boundedness is extended to redshifted static parents in Sec. IV B.[1]

Historically, the broad idea that black-hole interiors may be related to cosmological regions appears in black-hole-universe proposals [5,6], limiting-curvature and baby-universe constructions [7–13], and regular de Sitter–core scenarios [14–18]. A complementary line of work concerns the geodesic completeness of nonsingular cosmologies [19–24]. More recently, Oppenheimer–Snyder-type and generalized constructions have been developed for regular black holes and

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[1]We take $c = G = 1$, except when G is restored for clarity in the effective FLRW density and pressure.

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nonsingular cosmologies with Hayward, Bardeen, and Minkowski-type cores [25–29].

We show that a regular core is not by itself enough to produce an FLRW daughter having the full list of desired properties: indefinite expansion, curvature regularity, geodesic completeness, and ANEC consistency. The result separates into two logically distinct obstructions. For closed daughters, boundedness is controlled by asymptotic flatness and finite Arnowitt–Deser–Misner (ADM) mass through the no-shell matching equation. For flat and open daughters, the obstruction is more general: for maximal FLRW continuations in the regular endpoint classes of the cited theorem, it follows from the affine-ANEC completeness result independently of the black-hole matching construction.

The rest of the paper is organized as follows. In Sec. II we review the trapped-region geometry of the parent spacetime. In Sec. III we show that the trapped interior is not itself an FLRW cosmology. In Sec. IV we derive the no-shell FLRW daughter equations and discuss a redshift-function extension. In Sec. V we prove the closed-branch boundedness theorem, explain how the flat/open branches fall under the general FLRW completeness obstruction, and analyze the expansion duration and tuning of the closed Bardeen branch. In Sec. VI we discuss core type and source limitations.

II. TRAPPED INTERIORS OF REGULAR BLACK HOLES

We begin by considering static, spherically symmetric metrics of the form

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2^2, \tag{1}$$

with an asymptotically flat exterior and a regular core [16–18,30–32]. On any trapped interval $f(r) < 0$, defining

$$T \equiv r, \qquad \chi \equiv t, \qquad d\tau = \frac{dT}{\sqrt{|f(T)|}} \tag{2}$$

recasts the metric as

$$\begin{aligned} ds^2 &= -d\tau^2 + a(\tau)^2 d\chi^2 + b(\tau)^2 d\Omega_2^2, \\ a(\tau) &= \sqrt{|f(T(\tau))|}, \qquad b(\tau) = T(\tau). \end{aligned} \tag{3}$$

For this one-function class, one has $\dot{b} = a$ with the orientation chosen so that T increases with τ; equivalently, $|\dot{b}| = a$ independent of orientation.

We choose to work with the Bardeen geometry [16,17],

$$f(r) = 1 - \frac{2Mr^2}{(r^2 + g^2)^{3/2}}. \tag{4}$$

For $M/g > 3\sqrt{3}/4$, this solution has two horizons. Its causal structure then consists of a static exterior, a finite trapped nonstatic band $r_- < r < r_+$, and a regular static core $0 \le r < r_-$. Near the center,

$$f(r) = 1 - \frac{2M}{g^3} r^2 + O(r^4), \qquad \Lambda_{\mathrm{eff}} = \frac{6M}{g^3}, \tag{5}$$

so the core is locally de Sitter–like. The effective source may be written as

$$T^{\mu}{}_{\nu} = \mathrm{diag}(-\rho(r), p_r(r), p_T(r), p_T(r)), \tag{6}$$

with

$$\begin{aligned} \rho(r) &= \frac{3Mg^2}{4\pi(r^2 + g^2)^{5/2}}, \\ p_r(r) &= -\rho(r), \\ p_T(r) &= \frac{3Mg^2(3r^2 - 2g^2)}{8\pi(r^2 + g^2)^{7/2}}. \end{aligned} \tag{7}$$

Thus the source is anisotropic away from the center and isotropizes only in the $r \to 0$ limit.[2]

III. THE TRAPPED INTERIOR IS NOT FLRW

The following familiar observation sets our stage: the trapped region of a static spherical black hole is Kantowski–Sachs rather than FLRW. Thus the FLRW daughter considered below is a separate matched region, and not simply a coordinate reinterpretation of the trapped band.

Proposition 1 (Static trapped-region obstruction). Consider a one-function static, spherically symmetric parent spacetime of the form (1) with a trapped interval $f(r) < 0$. In the induced Kantowski–Sachs description (3), the trapped interior is not an exact FLRW cosmology in its natural slicing.

Moreover, defining

$$H_\parallel \coloneqq \frac{\dot{a}}{a}, \qquad H_\perp \coloneqq \frac{\dot{b}}{b}, \tag{8}$$

isotropic expansion on a connected open interval, $H_\parallel = H_\perp$, is equivalent to

$$a(\tau) = Cb(\tau), \tag{9}$$

for some constant $C > 0$, and hence to

$$|f(T)| = C^2 T^2. \tag{10}$$

[2]The same trapped-region geometry can also be used in time-dependent classical double-copy constructions [33,34]. Here we use only the geometric and source-side ingredients needed for the cosmological obstruction.

Thus isotropic expansion can occur only in this highly special case and is nongeneric within the one-function static class. This is a necessary condition for isotropic expansion, not a sufficient condition for an exact FLRW geometry.

Proof. At fixed τ, the induced spatial metric is

$$h_{ij}dx^i dx^j = a(\tau)^2 d\chi^2 + b(\tau)^2 d\Omega_2^2. \tag{11}$$

Thus each spatial slice has the direct-product geometry $\mathbb{R}_\chi \times S^2_{b(\tau)}$, lacking constant sectional curvature: two-planes containing the χ-direction have zero sectional curvature, while two-planes tangent to the sphere have sectional curvature $1/b(\tau)^2$. Therefore the trapped interior is not an exact FLRW cosmology in its natural slicing.

In the one-function case, one has $\dot{b} = a$. A necessary condition for exact FLRW behavior on an open interval is isotropic expansion, $H_\parallel = H_\perp$, or equivalently

$$\frac{\dot{a}}{a} = \frac{\dot{b}}{b}. \tag{12}$$

On a connected open interval this integrates to $a(\tau) = Cb(\tau)$ for some constant $C > 0$, and the converse is immediate. Since $b = T$ and $a = \sqrt{|f(T)|}$, the condition becomes

$$|f(T)| = C^2 T^2. \tag{13}$$

This is a highly special condition, and it still does not make the geometry FLRW. Even when it holds, it only equalizes the directional Hubble rates; it does not alter the intrinsic product geometry of the $\tau = \text{const}$ slices. The natural spatial slices remain $\mathbb{R} \times S^2$ rather than constant-curvature three-spaces. ∎

For the Bardeen metric (4), one has on the trapped interval

$$|f(T)| = \frac{2MT^2}{(T^2 + g^2)^{3/2}} - 1, \tag{14}$$

which is not of the form C^2T^2 on any open interval. Hence the Bardeen trapped region does not admit isotropic expansion over any finite portion of its interior evolution. More importantly, the trapped region is only the finite band $r_- < r < r_+$: it terminates at the inner horizon and is followed by a regular static core. Thus even though the core becomes locally de Sitter–like as $r \to 0$, the trapped Kantowski–Sachs phase itself does not evolve into a daughter FLRW branch.

Hence, Proposition 1 rules out the simplest possibility: the trapped region of the static parent is not already the daughter universe. For the Bardeen trapped band this conclusion is corroborated by the invariants. FLRW spacetimes are conformally flat, whereas, for a one-function spherical metric written as $F(r) = 1 - 2m(r)/r$, the quadratic Weyl invariant is

$$\begin{aligned} C_{abcd}C^{abcd} &= \frac{[r^2F'' - 2rF' + 2F - 2]^2}{3r^4} \\ &= \frac{48}{r^6}\left(m - \frac{2rm'}{3} + \frac{r^2m''}{6}\right)^2. \end{aligned} \tag{15}$$

For Bardeen, $m(r) = Mr^3/(r^2 + g^2)^{3/2}$, so

$$C_{abcd}C^{abcd} = \frac{12M^2r^4(2r^2 - 3g^2)^2}{(r^2 + g^2)^7}. \tag{16}$$

This invariant vanishes at $r = 0$ and at the isolated sphere $r = \sqrt{3/2}g$, but it is not identically zero on any open radial interval. Hence no open portion of the Bardeen trapped band is conformally flat, and the trapped band is not an FLRW spacetime.

IV. NO-SHELL FLRW DAUGHTERS FROM STATIC PARENTS

Having shown the Kantowski–Sachs trapped interior of the parent is not itself the daughter cosmology, we now turn to the simplest global continuation one might try: a no-shell matching to an FLRW daughter region. In this matching, the FLRW daughter is a separate spacetime region attached across a spherical hypersurface and constrained by the Darmois–Israel no-shell conditions [35,36]. This is the standard spherical FLRW/static matching structure we use as a minimal control problem rather than as a full collapse model.

The use of an FLRW daughter is not arbitrary, since for regular black-holes such as Bardeen, the source isotropizes and the geometry approaches a de Sitter–like core near $r = 0$. Since de Sitter space admits homogeneous and isotropic slicings, an FLRW daughter is the natural optimistic continuation to test. The question now is whether the most symmetric no-shell daughter ansatz suggested by the regular core can support an indefinitely expanding cosmology without additional global structure or some new matter content.

A. General junction equation

Here the $+$ label denotes the static parent region, while the $-$ label denotes the FLRW daughter region. We consider a static parent geometry, written in curvature coordinates as

$$ds_+^2 = -F(R)dT^2 + \frac{dR^2}{F(R)} + R^2 d\Omega_2^2, \tag{17}$$

and an FLRW daughter region

$$ds_-^2 = -d\tau^2 + A(\tau)^2\left[\frac{d\chi^2}{1-k\chi^2} + \chi^2 d\Omega_2^2\right], \qquad k = 0, \pm 1. \tag{18}$$

We define

$$H \coloneqq \frac{\dot{A}}{A}. \tag{19}$$

For the flat or open cases, the matching hypersurface is at fixed comoving radius

$$\chi = \chi_b = \text{const}, \qquad R_b(\tau) = A(\tau)\chi_b. \tag{20}$$

On the FLRW side, the induced metric on the hypersurface is

$$ds_\Sigma^2 = -d\tau^2 + R_b(\tau)^2 d\Omega_2^2. \tag{21}$$

On the exterior side, parametrizing the hypersurface by $x_+^\mu(\tau, \theta, \varphi) = (T(\tau), R_b(\tau), \theta, \varphi)$, one finds

$$ds_\Sigma^2 = -\left(F(R_b)\dot{T}^2 - \frac{\dot{R}_b^2}{F(R_b)}\right)d\tau^2 + R_b^2 d\Omega_2^2, \tag{22}$$

so matching the first fundamental form gives

$$F(R_b)\dot{T}^2 - \frac{\dot{R}_b^2}{F(R_b)} = 1. \tag{23}$$

With the standard orientation choice for the outward normal, the angular extrinsic-curvature condition gives

$$\dot{R}_b^2 + F(R_b) = 1 - k\chi_b^2. \tag{24}$$

The remaining $\tau\tau$ component is consistent with the same comoving no-shell construction; for completeness we derive both components in the Appendix. Using $R_b = A\chi_b$ and dividing by $A^2\chi_b^2$, one obtains

$$H^2 + \frac{k}{A^2} = \frac{1 - F(A\chi_b)}{A^2\chi_b^2}. \tag{25}$$

Equation (25) is the key point of the minimal construction: with no shell stress tensor, the daughter Friedmann equation is inherited directly from the parent metric function $F(R)$. Equivalently, defining the Misner–Sharp mass function of the static parent geometry [37] by

$$m(R) \coloneqq \frac{R}{2}[1 - F(R)]. \tag{26}$$

Equation (25) becomes the invariant relation

$$H^2 + \frac{k}{A^2} = \frac{2m(A\chi_b)}{A^3\chi_b^3} \quad (k = 0, -1), \tag{27}$$

and, for the closed parametrization of Sec. IV C, in which ψ_b is the fixed comoving angular position of the closed FLRW boundary and $R_b = A\sin\psi_b$,

$$H^2 + \frac{1}{A^2} = \frac{2m(A\sin\psi_b)}{A^3\sin^3\psi_b}. \tag{28}$$

This form makes clear that the matching equation is controlled by the parent mass profile rather than by a coordinate artifact. Although the preceding derivation was written in static coordinates away from coordinate horizons, the resulting no-shell evolution equation extends by continuity across simple zeros of F.

B. Redshift functions do not rescue the closed branch

A natural question is whether the closed-branch obstruction is an artifact of the special choice $g_{TT}g_{RR} = -1$. To test this, we allow a nontrivial static redshift function. Under the same comoving no-shell assumptions, the angular Darmois condition remains controlled by the Misner–Sharp mass profile, while the redshift function enters only through the $K_{\tau\tau}$ condition.

Consider the more general static spherical metric

$$ds_+^2 = -e^{2\Phi(R)}F(R)dT^2 + \frac{dR^2}{F(R)} + R^2 d\Omega_2^2, \tag{29}$$

where $\Phi(R)$ is the redshift function. For a comoving FLRW boundary, the angular Darmois condition is unchanged:

$$\dot{R}_b^2 + F(R_b) = \beta^2, \tag{30}$$

where

$$\beta^2 = \begin{cases} 1 - k\chi_b^2, & k = 0, -1, \\ \cos^2\psi_b, & k = +1. \end{cases} \tag{31}$$

Thus, for closed daughters, the Friedmann equation and its large-A asymptotics are controlled by $F(R)$, equivalently by the Misner–Sharp mass, and not by the redshift function Φ. If $F(R) = 1 - 2M/R + o(R^{-1})$, the same asymptotic obstruction to unbounded closed expansion follows.

The $\tau\tau$ condition is more restrictive: because the FLRW boundary is comoving, $K_{\tau\tau}^- = 0$, and hence the parent-side trajectory must be geodesic. For Eq. (29), the radial geodesic equation is

$$\ddot{R}_b = -\frac{1}{2}F'(R_b) - \Phi'(R_b)(\dot{R}_b^2 + F(R_b)). \tag{32}$$

On any nonstatic trajectory, differentiating Eq. (30) away from isolated turning points gives

$$\ddot{R}_b = -\frac{1}{2}F'(R_b), \tag{33}$$

and the equality extends through such turning points by continuity. Combining Eqs. (32) and (33) gives

$$\Phi'(R_b)(\dot{R}_b^2 + F(R_b)) = 0. \tag{34}$$

Using Eq. (30), this becomes

$$\Phi'(R_b)\beta^2 = 0. \tag{35}$$

For flat/open boundaries and for nonequatorial closed boundaries, $\beta^2 > 0$, so a nonstatic full Darmois matching requires $\Phi'(R_b) = 0$ throughout the swept trajectory. In the equatorial closed case $\beta^2 = 0$, Eq. (35) imposes no condition on Φ, but the angular condition itself gives $\dot{R}_b^2 + F(R_b) = 0$, which forbids unbounded expansion when $F \to 1$.

An identically static boundary is not covered by differentiating the first integral. If $R_b = R_0$ is timelike and static, the angular condition gives $F(R_0) = \beta^2$, while the remaining Darmois condition must be imposed separately:

$$\frac{1}{2}F'(R_0) + \Phi'(R_0)F(R_0) = 0.$$

For $\Phi = 0$ this reduces to $F'(R_0) = 0$. Thus a redshift function does not alter the large-radius closed-branch obstruction, which remains controlled by the invariant mass profile.

C. Closed daughter universes

For the closed case $k = +1$, it is convenient to write the daughter metric as

$$ds_-^2 = -d\tau^2 + A(\tau)^2(d\psi^2 + \sin^2\psi\, d\Omega_2^2), \tag{36}$$

with the boundary at fixed

$$\psi = \psi_b = \text{const}, \qquad R_b(\tau) = A(\tau)\sin\psi_b. \tag{37}$$

Since the areal radius depends on $\sin\psi_b$, we take $0 < \psi_b \le \pi/2$ without loss of generality for the nondegenerate branch considered below. The matching equation becomes

$$H^2 + \frac{1}{A^2} = \frac{1 - F(A\sin\psi_b)}{A^2\sin^2\psi_b}. \tag{38}$$

For the Bardeen exterior, this gives

$$H^2 = \frac{2M}{(A^2\sin^2\psi_b + g^2)^{3/2}} - \frac{1}{A^2}, \tag{39}$$

which clearly displays the competition between the finite-mass contribution and the $k = +1$ positive-curvature term. At large A, the positive contribution falls as A^{-3}, whereas the curvature term falls as A^{-2} and therefore dominates. In the next section we show that this obstruction is not special to Bardeen, but follows from asymptotic flatness and finite ADM mass. As shown in Sec. IV B, adding a static redshift function does not change the large-radius conclusion.

We may express the same result as an effective closed-FLRW source. Below, the density $\rho_{\rm eff}$ is the homogeneous density inferred from the daughter Friedmann equation and not the unchanged anisotropic Bardeen nonlinear-electrodynamics stress tensor. Writing

$$H^2 + \frac{1}{A^2} = \frac{8\pi G}{3}\rho_{\rm eff}(A), \tag{40}$$

one obtains

$$\rho_{\rm eff}(A) = \frac{3M}{4\pi G(A^2\sin^2\psi_b + g^2)^{3/2}}. \tag{41}$$

As a function of the boundary areal radius $R_b = A\sin\psi_b$, this effective source approaches vacuum-like behavior only in the deep-core limit $R_b \ll g$, and it falls as A^{-3} at large A.

The corresponding effective pressure is fixed by the conservation equation,

$$\frac{d\rho_{\rm eff}}{d\tau} + 3H(\rho_{\rm eff} + p_{\rm eff}) = 0,$$

or, equivalently,

$$p_{\rm eff}(A) = -\rho_{\rm eff}(A) - \frac{A}{3}\frac{d\rho_{\rm eff}}{dA}.$$

For Bardeen this gives (with $s = \sin\psi_b$ for the closed branch)

$$p_{\rm eff}(A) = -\frac{3Mg^2}{4\pi G(A^2s^2 + g^2)^{5/2}},$$

and hence

$$w_{\rm eff}(A) = \frac{p_{\rm eff}}{\rho_{\rm eff}} = -\frac{g^2}{A^2s^2 + g^2}.$$

Thus, as a function of $R_b = As$, the matched source approaches vacuum-like behavior $w_{\rm eff} \to -1$ for $R_b/g \to 0$ and dust-like behavior $w_{\rm eff} \to 0$ at large R_b/g. The physical inner turning point occurs at finite R_b, so the equation of state at the bounce need not be parametrically close to -1. The source supplies acceleration only for $A^2s^2 < 2g^2$, and therefore cannot provide the persistent $w \le -1/3$ support needed for an unbounded closed daughter.

D. Flat daughter universes

For the flat case $k = 0$, Eq. (25) reduces to

$$H^2 = \frac{1 - F(A\chi_b)}{A^2\chi_b^2}. \tag{42}$$

For Bardeen,

$$H^2 = \frac{2M}{(A^2\chi_b^2 + g^2)^{3/2}}. \tag{43}$$

Unlike the closed case, there is no positive-curvature term forcing a turning point, so the flat daughter branch is monotonic. Near $A \to 0$,

$$H^2 \to \frac{2M}{g^3}, \tag{44}$$

and the early-time behavior is asymptotically de Sitter–like,

$$A(\tau) \sim e^{H_0\tau}, \qquad H_0^2 = \frac{2M}{g^3}. \tag{45}$$

At large A,

$$H^2 \sim \frac{2M}{\chi_b^3 A^3}, \tag{46}$$

so

$$A(\tau) \propto \tau^{2/3}. \tag{47}$$

Thus the flat daughter branch is nonsingular in cosmic time and evolves from an asymptotically de Sitter–like past into a dust-like late phase. As we will emphasize below, however, this does not furnish a geodesically complete nonsingular cosmology.

E. Open daughter universes

For the open case $k = -1$, Eq. (25) gives

$$H^2 - \frac{1}{A^2} = \frac{1 - F(A\chi_b)}{A^2\chi_b^2}. \tag{48}$$

For an asymptotically flat finite-mass parent,

$$H^2 = \frac{1}{A^2} + \frac{2M}{A^3\chi_b^3} + o(A^{-3}). \tag{49}$$

Equation (49) is the large-A asymptotic limit of Eq. (48). Thus the open branch, like the flat branch, avoids the closed-universe recollapse argument. Its obstruction is instead the shared flat/open completeness theorem used in Sec. V.

If the parent has a de Sitter–like core, $F(R) = 1 - H_0^2R^2 + O(R^4)$, then the open matching equation gives

$$H^2 - \frac{1}{A^2} \to H_0^2, \qquad \dot{A}^2 = 1 + H_0^2A^2 + O(A^4).$$

Locally, the leading-order solution is the open de Sitter slicing,

$$A(\tau) = H_0^{-1}\sinh[H_0(\tau - \tau_0)] + O(A^3).$$

The endpoint $A = 0$ is not a curvature singularity, but the open slicing covers only a geodesically incomplete patch of de Sitter space. Indeed, $A(\tau) \sim \tau - \tau_0$ there, so a radial null geodesic reaches the endpoint in finite affine parameter:

$$\Delta\lambda \propto \int_{\tau_0}^{\tau_1} A(\tau)d\tau < \infty.$$

This provides the local model for the flat/open incompleteness obstruction discussed in Sec. V.

V. OBSTRUCTION MECHANISMS

The preceding matching equations give the closed-branch obstruction, while the flat/open branches are controlled by the general FLRW completeness theorem.

Theorem 1 (Minimal asymptotically flat FLRW daughters). Consider a static, spherically symmetric, asymptotically flat parent spacetime in the one-function class $g_{tt}g_{rr} = -1$, with finite ADM mass $M > 0$. Attach an FLRW daughter across a nondegenerate comoving spherical Darmois boundary. Assume that the matching is no-shell and that the daughter evolution is fixed by the parent metric profile, with no additional late-time bulk component, modified asymptotics, or independent shell stress tensor. Then the closed daughter branch is bounded at finite scale factor and therefore does not yield an indefinitely expanding FLRW cosmology.

For $k = 0$ and $k = -1$, consider the maximal homogeneous FLRW continuation determined by the matched scale-factor law. If that continuation is nonstatic, curvature-regular, and has regular affine ends in the sense of Refs. [3,4], it cannot be both null geodesically complete and ANEC consistent. Consequently, within these explicit hypotheses, the minimal construction does not furnish an FLRW continuation satisfying all four desired properties: indefinite expansion, curvature regularity, geodesic completeness, and ANEC consistency.

The proof separates into two logically distinct ingredients. The closed branch is bounded by the finite-ADM matching equation. The cited flat/open theorem is a conditional statement about the regular endpoint classes specified in those references; since it obstructs null completeness, it also obstructs full geodesic completeness within that class. For the explicit Bardeen flat and open branches, null incompleteness is established directly by the affine-length estimates in Eq. (60) and Sec. IV E, respectively. These statements concern maximal FLRW continuations and do not, by themselves, decide the geodesic completeness of a separately specified global parent-plus-daughter composite. As shown in Sec. IV B, the closed-branch boundedness also extends to redshifted static parents in the same comoving no-shell setup.

A. Closed daughters from asymptotically flat parents are bounded

We now assume only that the parent geometry is asymptotically flat with finite ADM mass,

$$F(R) = 1 - \frac{2M}{R} + o(R^{-1}), \qquad M > 0. \tag{50}$$

The resulting boundedness statement is given in Proposition 2, and the mechanism is illustrated in Fig. 1 for a representative asymptotically flat Bardeen daughter. The important point is that the obstruction is not the absence of a bounce or the formation of a singular big crunch. The branch is bounded between finite turning points. The obstruction is instead that the minimal closed branch does not yield an indefinitely expanding daughter universe.

Proposition 2 (Closed daughter boundedness). For any no-shell closed FLRW daughter universe with nondegenerate matching surface $0 < \psi_b \le \pi/2$, matched to a static asymptotically flat parent geometry with finite ADM mass and curvature-coordinate function $F(R) = 1 - 2M/R + o(R^{-1})$, the daughter branch cannot extend to arbitrarily large A. If the allowed interval has a simple finite outer zero, the expanding branch reaches a finite turning point and recollapses. Degenerate endpoints are tuned limiting cases corresponding to static or asymptotically static behavior.

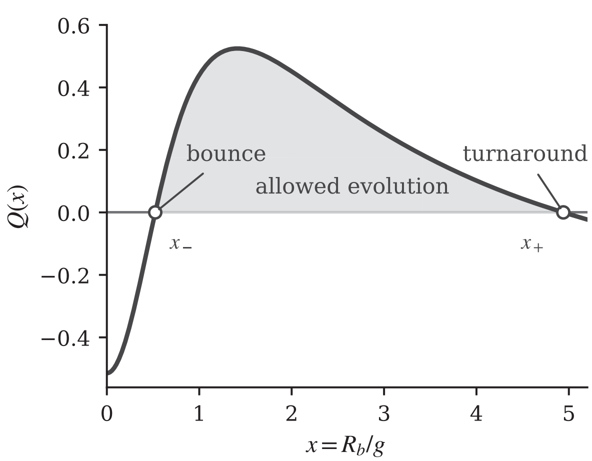

FIG. 1. Closed Bardeen daughter evolution in the asymptotically flat case. The plotted function is $Q(x) := 2\mu x^2/(1+x^2)^{3/2} - \sin^2\psi_b$, with $x = R_b/g = A\sin\psi_b/g$, $\eta = \tau/g$, $\mu = M/g$, and representative parameters $(\mu, \psi_b) = (1.35, 0.80)$. On the dynamically allowed interval, $Q(x) = (dx/d\eta)^2 \ge 0$, which is the shaded region. The two zeros x_- and x_+ are finite turning points: a nondegenerate expanding branch can emerge from the inner turning point and reach an outer turnaround before recollapsing. The obstruction is the boundedness of the closed branch in the asymptotically flat minimal construction, with evolution between finite turning points rather than a singular big crunch.

Proof. The closed mechanical form gives

$$\dot{R}_b^2 = E - F(R_b), \qquad E = \cos^2\psi_b.$$

For $0 < \psi_b \le \pi/2$, one has $E < 1$, while asymptotic flatness gives $F(R) \to 1$. Hence

$$E - F(R) \longrightarrow -\sin^2\psi_b < 0,$$

so the physical region $\dot{R}_b^2 \ge 0$ cannot extend to arbitrarily large R_b. Since $R_b = A\sin\psi_b$ and $\sin\psi_b > 0$, the daughter branch cannot extend to arbitrarily large A.

Equivalently, the large-A Friedmann form is

$$H^2 = -\frac{1}{A^2} + \frac{2M}{A^3\sin^3\psi_b} + o(A^{-3}) \quad (A \to \infty).$$

Thus the A^{-2} curvature term eventually dominates the induced A^{-3} matter term, and $H^2 < 0$ at sufficiently large scale factor. Along a physical daughter solution one must have $H^2 \ge 0$. Therefore an expanding branch cannot be unbounded.

If the upper endpoint of an allowed connected component is a simple zero of $E - F(R_b)$, the expanding solution reaches a finite outer turning point $A_{\max}$ with $H(A_{\max}) = 0$ and then recollapses. Degenerate zeros are tuned limiting cases and give static or asymptotically static behavior rather than an indefinitely expanding daughter universe. ∎

This is a class statement depending only on the angular junction equation and the asymptotic Schwarzschild falloff of the parent geometry, not on whether the regular core is of de Sitter type, Minkowski type, or some other nonsingular form. In mass-function language, the obstruction is simply the finite-ADM limit $m(R) \to M$.

B. Expansion duration and tuning of the closed branch

The closed Bardeen branch illustrates that boundedness means a finite duration for each expansion phase, not a finite spacetime lifetime or a singular collapse. Define

$$x := \frac{R_b}{g} = \frac{A\sin\psi_b}{g}, \qquad \eta := \frac{\tau}{g}, \qquad \mu := \frac{M}{g}. \tag{51}$$

Equation (39) becomes

$$\left(\frac{dx}{d\eta}\right)^2 = \frac{2\mu x^2}{(1+x^2)^{3/2}} - \sin^2\psi_b. \tag{52}$$

The first term on the right-hand side has maximum at $x = \sqrt{2}$, with value $4\mu/(3\sqrt{3})$. Thus an allowed non-degenerate closed Bardeen branch exists when

$$\sin^2\psi_b < \frac{4\mu}{3\sqrt{3}}, \tag{53}$$

while equality gives a tuned static limiting case. For a nonextremal Bardeen black hole, $\mu > 3\sqrt{3}/4$, this condition is automatically satisfied for $0 < \psi_b \le \pi/2$. The allowed region is therefore the finite interval between the two positive roots x_- and x_+ of the right-hand side. The simple roots are regular turning points of the second-order equation

$$\frac{d^2x}{d\eta^2} = \frac{1}{2}\frac{d}{dx}\left[\frac{2\mu x^2}{(1+x^2)^{3/2}} - \sin^2\psi_b\right].$$

Consequently, the maximal FLRW scale-factor solution can be continued smoothly through the outer turnaround and the inner bounce, giving a nonsingular periodic evolution. It fails the criterion adopted here because it is bounded, not because the scale-factor solution must end after one cycle.

The expansion time from the inner to the outer turning point is

$$\Delta\tau_{\rm exp} = g\int_{x_-}^{x_+} \frac{dx}{\sqrt{\frac{2\mu x^2}{(1+x^2)^{3/2}} - \sin^2\psi_b}}. \tag{54}$$

For order-one matching data this time is of order the black-hole scale. It can be made parametrically large by taking $\psi_b \ll 1$. In that limit $x_+ \sim 2\mu/\sin^2\psi_b$, and the large-$x$ part of the integral gives

$$\Delta\tau_{\rm exp} \sim \frac{\pi M}{\sin^3\psi_b} \quad (\psi_b \ll 1). \tag{55}$$

This long expansion-time limit is a tuning of the matched branch rather than a way to circumvent the result. The parameter ψ_b is global junction data fixing the comoving angular location of the FLRW boundary. Thus an arbitrarily long expansion phase can be engineered by tuning the matching construction, but it is not a prediction of the asymptotically flat parent geometry itself.

The coalescing-root endpoint does not provide a second long-lived limit. Write

$$\sin^2\psi_b = \frac{4\mu}{3\sqrt{3}} - \delta, \qquad \delta \downarrow 0.$$

Expanding about $x_0 = \sqrt{2}$ gives

$$\frac{2\mu x^2}{(1+x^2)^{3/2}} - \sin^2\psi_b = \delta - \frac{4\sqrt{3}\mu}{27}(x-\sqrt{2})^2 + O((x-\sqrt{2})^3).$$

It follows that

$$\Delta\tau_{\rm exp} \longrightarrow \frac{33^{1/4}\pi g}{2\sqrt{\mu}},$$

which is finite. Moreover, for a nonextremal Bardeen black hole, $\mu > 3\sqrt{3}/4$ makes $4\mu/(3\sqrt{3}) > 1$, so this equality is not reachable with real matching data $\sin^2\psi_b \le 1$. The only parametrically long expansion regime exhibited here is therefore $\psi_b \to 0$.

C. Flat and open daughters avoid boundedness but fail completeness

For the flat daughter branch, the large-A asymptotic from Eqs. (42) and (50) is

$$H^2 \sim \frac{2M}{A^3\chi_b^3}, \qquad A(\tau) \propto \tau^{2/3}. \tag{56}$$

Thus the specific boundedness mechanism of the closed branch is absent. In applying the flat/open completeness condition, the object being tested is the maximal FLRW daughter spacetime determined by the matched scale-factor law $A(\tau)$, not merely the finite comoving ball bounded by the junction surface. This maximal-FLRW test is not a completeness theorem for an otherwise unspecified global parent-plus-daughter composite.

If the parent also has a de Sitter–like regular core,

$$F(R) = 1 - H_0^2R^2 + O(R^4) \quad (R \to 0), \tag{57}$$

then Eq. (42) implies

$$H^2 \to H_0^2 \quad (A \to 0), \tag{58}$$

so the early-time branch is asymptotically de Sitter–like,

$$A(\tau) \sim e^{H_0\tau} \quad (\tau \to -\infty). \tag{59}$$

This makes the incompleteness mechanism explicit. For a radial null geodesic in the flat FLRW metric, the conserved comoving momentum gives $d\lambda \propto A(\tau)d\tau$, where λ is an affine parameter. Thus the past affine length of the asymptotic de Sitter branch is finite:

$$\Delta\lambda \propto \int_{-\infty}^{\tau_0} A(\tau)d\tau \sim \int_{-\infty}^{\tau_0} e^{H_0\tau}d\tau < \infty. \tag{60}$$

Thus the flat daughter model is nonsingular in cosmic time and monotonic, but it is past null incomplete via the general flat/open FLRW completeness obstruction. More generally, nonstatic curvature-regular flat/open FLRW spacetimes with regular affine ends cannot be both null geodesically complete and ANEC-consistent [3,4]. Hence the flat daughter branch avoids the closed-universe boundedness mechanism, but it does not satisfy our full desired set of properties.

The open branch is in the same completeness class. As discussed in Sec. IV E, it avoids the closed-branch boundedness mechanism, but the local open de Sitter behavior is only a curvature-regular FLRW patch and is not geodesically complete as an FLRW spacetime. Combining this flat/open completeness obstruction with Proposition 2 proves Theorem 1: the closed branch is bounded by finite ADM mass, while flat and open branches fail the regular-complete-ANEC conditions by the general FLRW completeness theorem.

D. Escape routes

The assumptions enter the closed and flat/open models differently. In the closed model, the large-A failure follows from the finite-ADM limit $m(R) \to M$, which makes the induced no-shell density scale as A^{-3}. An unbounded closed daughter therefore requires an ingredient that changes this late-time balance: modified asymptotics for which the effective mass function does not approach a constant, an independent shell stress tensor, a non-FLRW or non-comoving daughter geometry, or an additional smooth bulk component whose density redshifts no faster than A^{-2}. A positive vacuum-energy component is the simplest example, but adding it lies outside the asymptotically flat minimal construction considered here.

Within the regular-end class of the cited theorem, a flat/open evasion must give up at least one of the desired conditions: curvature regularity, null completeness, ANEC consistency, the FLRW ansatz, or the flat/open curvature class. A construction with irregular affine-end behavior instead lies outside the theorem's stated hypotheses.

VI. CORE TYPE AND SOURCE LIMITATIONS

The preceding theorem is a structural statement about asymptotically flat constructions, but does not apply to all black-hole-universe models. Two possible escape attempts are especially natural: changing the core type and using the parent matter source as the daughter source.

A. De Sitter cores versus Minkowski cores

The Bardeen geometry has a locally de Sitter–like core, which is often one of the motivations for viewing regular black-hole interiors as cosmological seeds. This local behavior is compatible with recent convergence-condition analyses: de Sitter-core regular black holes can preserve the null condition while evading singularity theorems through strong-energy-condition violation [38]. In our case, however, the core type does not control the large-A fate of the daughter.

Minkowski-core parents provide a useful comparison. Their early-time daughter patch can approach a Minkowski-like nonsingular regime rather than an asymptotically de Sitter-like one, as illustrated in Ref. [28]. This changes the early-time patch but not the closed-branch obstruction: for $k = +1$, boundedness follows from asymptotic flatness and finite ADM mass. For $k = 0$ and $k = -1$, changing the core type likewise does not by itself evade the general flat/open completeness theorem.

B. Can the parent matter source rescue the daughter universe?

One might also ask whether the matter source associated with the parent regular black hole can stabilize the daughter cosmology. We do not prove a general no-go theorem, but the minimal Bardeen/nonlinear-electrodynamics route is disfavored for two simple reasons.

First, the unchanged static source remains anisotropic away from the center and decays too rapidly at large areal radius. For Bardeen, the static density falls as r^{-5}, while the no-shell Friedmann density induced by the exterior mass profile falls as A^{-3}. Neither behavior supplies the persistent A^{-2}-or-slower support required to evade the closed-branch obstruction.

Second, promoting the same broad matter class to a homogeneous or coarse-grained cosmological source is not automatic. A homogeneous magnetic sector must be isotropized or averaged, and in an ordinary Maxwellian weak-field completion the late-time behavior is radiation-like rather than dark-energy-like. The original Ayón-Beato–García realization of Bardeen is not itself a standard Maxwell weak-field model, so this should not be read as a no-go theorem for all nonlinear-electrodynamics cosmologies. Our point is that neither the unchanged anisotropic parent source nor ordinary weak-field homogeneous continuations naturally provide the persistent $w \leq -1/3$ support needed for an unbounded closed daughter.

VII. CONCLUSIONS

We have examined whether asymptotically flat regular black holes can admit minimal FLRW daughter cosmologies. We identify a structural obstruction. The trapped interior of the parent spacetime is not itself an FLRW cosmology, so an FLRW daughter must be introduced as a separate matched region. In the minimal no-shell construction, closed daughters are bounded rather than indefinitely expanding. Flat and open daughters can avoid this boundedness mechanism, but, for their maximal homogeneous FLRW continuations within the regular endpoint classes of the affine-ANEC theorem, nonstatic curvature-regular models cannot be both null geodesically complete and ANEC-consistent.

These conclusions are not tied to one special metric. The closed-branch result follows from asymptotic flatness and finite ADM mass, while the flat/open obstruction follows from the stated completeness constraints on non-static FLRW cosmologies. Changing the detailed core structure from de Sitter–like to Minkowski-like modifies the early-time daughter patch but does not evade this dichotomy within the same hypotheses. Likewise, the parent matter source does not naturally provide the late-time support required to evade the closed-branch obstruction. We have not proved that every separately specified global spacetime formed from a finite FLRW cap and a parent region is geodesically incomplete; that is a distinct global-extension problem.

Our result does not exclude more elaborate black-hole-universe constructions with shells, modified asymptotics, non-FLRW daughters, or additional smooth bulk components. Rather, it identifies which ingredients must be changed within the minimal criteria adopted here. Closed FLRW daughters require additional late-time support absent from finite-ADM no-shell matching, while flat/open FLRW continuations must leave the hypotheses of the flat/open completeness theorem. The main lesson is that a regular core, even a de Sitter–like one, is not by itself sufficient to produce a viable, indefinitely expanding FLRW daughter cosmology in the sense considered here.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX: DARMOIS CONDITIONS FOR THE COMOVING NO-SHELL BOUNDARY

Here we present the full no-shell matching conditions used in Sec. IV A. For the FLRW daughter metric (18), a boundary at fixed $\chi = \chi_b$ has areal radius $R_b = A\chi_b$. The outward unit normal on the FLRW side is

$$n_-^\mu \partial_\mu = \frac{\sqrt{1-k\chi_b^2}}{A}\partial_\chi, \tag{A1}$$

and therefore

$$K_{\theta\theta}^- = R_b n_-^\mu \partial_\mu R = R_b\sqrt{1-k\chi_b^2}. \tag{A2}$$

On the parent side, with trajectory $x_+^\mu = (T(\tau), R_b(\tau), \theta, \varphi)$, the induced-metric condition is

$$F(R_b)\dot{T}^2 - \frac{\dot{R}_b^2}{F(R_b)} = 1. \tag{A3}$$

Choosing the outward normal consistently with the orientation in the main text gives

$$K_{\theta\theta}^+ = R_b F(R_b)\dot{T} = R_b\sqrt{F(R_b) + \dot{R}_b^2}. \tag{A4}$$

Equating Eqs. (A2) and (A4) yields

$$\dot{R}_b^2 + F(R_b) = 1 - k\chi_b^2, \tag{A5}$$

which is Eq. (24). For the closed parametrization $R_b = A\sin\psi_b$, the same calculation gives

$$K_{\theta\theta}^- = R_b\cos\psi_b, \qquad \dot{R}_b^2 + F(R_b) = \cos^2\psi_b. \tag{A6}$$

For a smooth nonstatic trajectory, the remaining $\tau\tau$ component is consistent with the same comoving no-shell construction; the identically static case is treated separately in this appendix. On the FLRW side the boundary is comoving and geodesic, hence $K_{\tau\tau}^- = 0$. On the parent side, differentiating the first integral away from an isolated turning point gives

$$\ddot{R}_b = -\frac{1}{2}F'(R_b), \tag{A7}$$

which is precisely the radial geodesic equation for the trajectory in the one-function parent metric. Thus $K_{\tau\tau}^+ = 0$. At an isolated turning point this relation is understood by continuity, or equivalently by using the local second-order geodesic equation rather than dividing by $\dot{R}_b$. Hence the angular condition together with induced-metric matching gives the full Darmois no-shell matching for a smooth nonstatic comoving construction.

An identically static boundary is exceptional: differentiation of the first integral gives no information. For a timelike static boundary at $R_b = R_0$, full matching additionally requires $F'(R_0) = 0$ in the one-function metric, besides the angular condition $F(R_0) = 1 - k\chi_b^2$ (or $F(R_0) = \cos^2\psi_b$ in the closed parametrization). For the redshifted metric (29), the angular condition is unchanged, while the $\tau\tau$ condition gives the additional constraint discussed in Sec. IV B.

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